Kaon and D meson masses with $N_f = 2+1+1$ twisted mass lattice QCD

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Abstract

We discuss the computation of the Kaon and D meson masses in the $N_f = 2+1+1$ twisted mass lattice QCD setup, where explicit heavy flavor and parity breaking occurs at finite lattice spacing. We present three methods suitable in this context and verify their consistency.

$N_f = 2+1+1$ ETMC simulation setup

- Iwasaki gauge action.
- $N_f = 2+1+1$ flavors of dynamical Wilson twisted mass quarks:

$$S_{\mathrm{F,light}}[\chi^{(l)}, \bar{\chi}^{(l)}, U] = a^4 \sum_x \bar{\chi}^{(l)}(x)\Big(D_{\mathrm{W}}(m_0) + i\mu\gamma_5\tau_3\Big)\chi^{(l)}(x)$$

$$S_{\mathrm{F,heavy}}[\chi^{(h)}, \bar{\chi}^{(h)}, U] = a^4 \sum_x \bar{\chi}^{(h)}(x)\Big(D_{\mathrm{W}}(m_0) + i\mu_\sigma\gamma_5\tau_1 + \tau_3\mu_\delta\Big)\chi^{(h)}(x)$$

(D_{W} is the standard Wilson Dirac operator).

- $\kappa = 1/(2m_0+8)$ is tuned to maximal twist by requiring $m^{\mathrm{PCAC}}_{\chi^{(l)}} = 0$
  $\rightarrow$ automatic $\mathcal{O}(a)$ improvement for physical quantities.
- All results shown in the following are for the ensemble with

$$\beta = 1.95\ ,\ L^3 \times T = 32^3 \times 64\ ,\ \mu = 0.0035\ ,\ \kappa = 0.161240\ ,\ \mu_\sigma = 0.135\ ,\ \mu_\delta = 0.170,$$

which amounts to $a \approx 0.078\,\mathrm{fm}$ and $m_{\mathrm{PS}} \approx 318\,\mathrm{MeV}$.

- Cf. talk by Siebren Reker "Light hadrons from $N_f = 2+1+1$ dynamical twisted mass fermions".

Quantum numbers, physical and twisted basis

- Goal: compute the mass of the Kaon and the D meson.
- In twisted mass lattice QCD at finite lattice spacing parity is not a symmetry and the heavy flavors cannot be diagonalized
  $\rightarrow$ instead of the four sectors $(s,-)$, $(s,+)$, $(c,-)$, $(c,+)$ there is only a single combined sector $(s/c,-/+)$ in twisted mass lattice QCD.
- Twist rotation in the continuum:

$$\begin{pmatrix}\psi^{(u)}\\ \psi^{(d)}\end{pmatrix} = \exp\Big(i\gamma_5\tau_3\omega_l/2\Big)\begin{pmatrix}\chi^{(u)}\\ \chi^{(d)}\end{pmatrix}\quad,\quad \begin{pmatrix}\psi^{(s)}\\ \psi^{(c)}\end{pmatrix} = \exp\Big(i\gamma_5\tau_1\omega_h/2\Big)\begin{pmatrix}\chi^{(s)}\\ \chi^{(c)}\end{pmatrix}$$

(ψ denotes quark fields in the physical basis, χ in the twisted basis).

- We use spatially extended versions of the twisted basis meson creation operators

$$\mathcal{O}_j \in \Big\{ +i\bar{\chi}^{(d)}\gamma_5\chi^{(s)}\ ,\ -i\bar{\chi}^{(d)}\gamma_5\chi^{(c)}\ ,\ +\bar{\chi}^{(d)}\chi^{(s)}\ ,\ -\bar{\chi}^{(d)}\chi^{(c)}\Big\},$$

to access the $J=0$, $(s/c,-/+)$ sector (in particular the Kaon and the D).

- Twist rotation of local meson creation operators at finite lattice spacing:

$$\begin{pmatrix} +i\bar{\psi}^{(d)}\gamma_5\psi^{(s)}\\ -i\bar{\psi}^{(d)}\gamma_5\psi^{(c)}\\ +\bar{\psi}^{(d)}\psi^{(s)}\\ -\bar{\psi}^{(d)}\psi^{(c)}\end{pmatrix}^R = \underbrace{\begin{pmatrix} +c_lc_h & -s_ls_h & -s_lc_h & -c_ls_h\\ -s_ls_h & +c_lc_h & -c_ls_h & -s_lc_h\\ +s_lc_h & +c_ls_h & +c_lc_h & -s_ls_h\\ +c_ls_h & +s_lc_h & -s_ls_h & +c_lc_h\end{pmatrix}}_{=\mathcal{M}(\omega_l,\omega_h)}\begin{pmatrix} +iZ_P\bar{\chi}^{(d)}\gamma_5\chi^{(s)}\\ -iZ_P\bar{\chi}^{(d)}\gamma_5\chi^{(c)}\\ +Z_S\bar{\chi}^{(d)}\chi^{(s)}\\ -Z_S\bar{\chi}^{(d)}\chi^{(c)}\end{pmatrix}$$

($c_x = \cos(\omega_x/2)$, $s_x = \sin(\omega_x/2)$ and Z_P and Z_S are operator dependent renormalization constants).

- Starting point for all our analysis methods: 4×4 correlation matrices

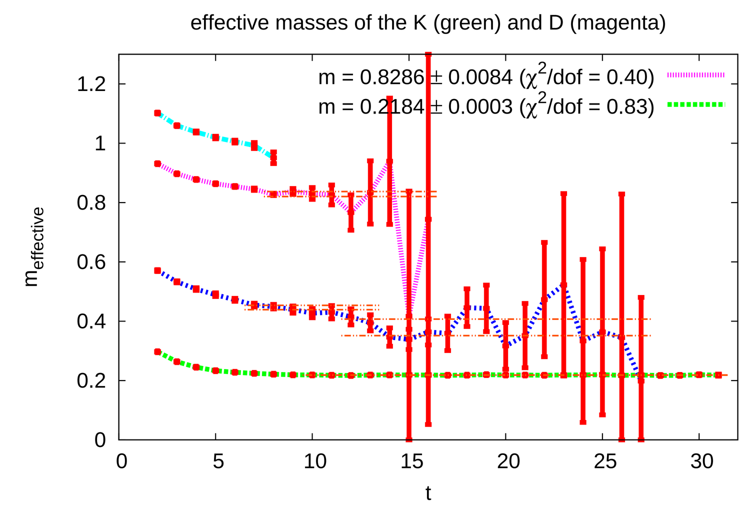
$$C_{jk}(t) = \langle\Omega|\mathcal{O}_j(t)\Big(\mathcal{O}_k(0)\Big)^\dagger|\Omega\rangle.$$

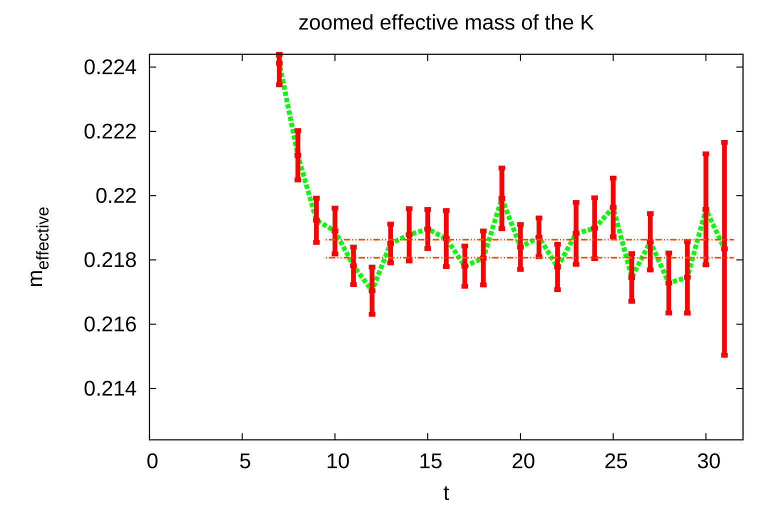
Method 1: generalized eigenvalue problem

- Generalized eigenvalue problem (GEP), effective meson masses:

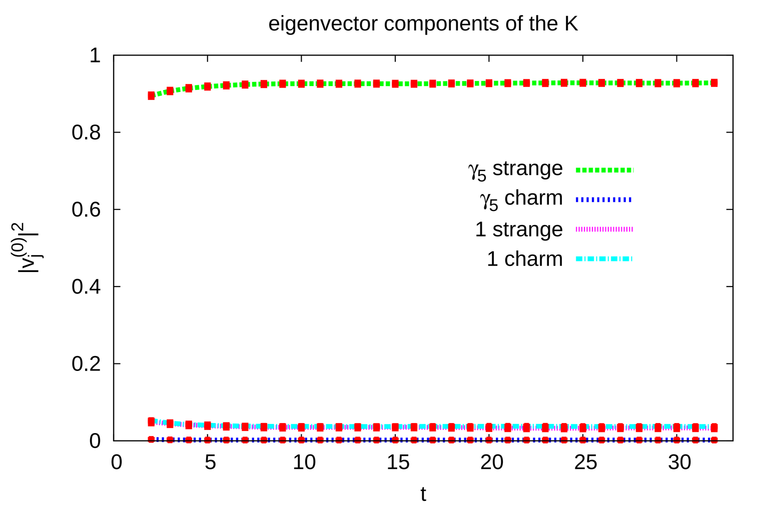
$$C_{jk}(t)v_j^{(n)}(t,t_0) = C_{jk}(t_0)v_j^{(n)}(t,t_0)\lambda^{(n)}(t,t_0)\quad,\quad m^{(n)}_{\mathrm{effective}}(t,t_0) = \ln\left(\frac{\lambda^{(n)}(t,t_0)}{\lambda^{(n)}(t+a,t_0)}\right).$$

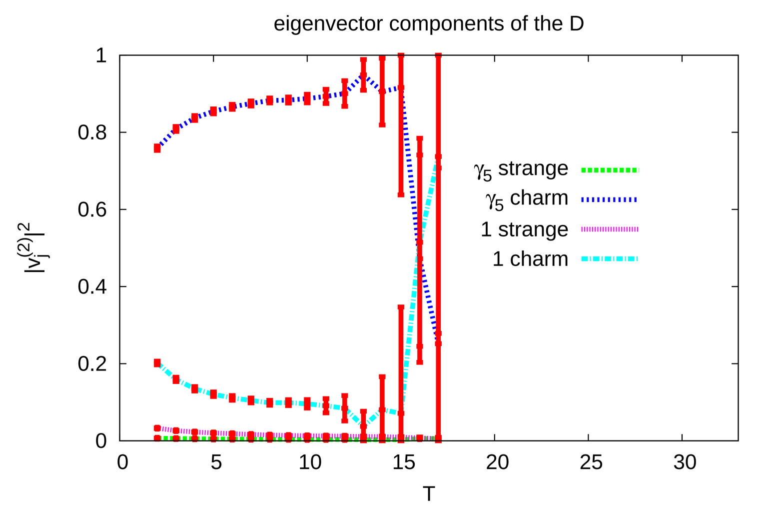
- Fitting constants to effective mass plateaus at $t \gg a$ yields meson masses.





- After rotating the eigenvectors $\mathbf{v}^{(n)}$ to the pseudo physical basis (physical basis with $Z_P = Z_S = 1$) one can read off the quantum numbers heavy flavor and parity, i.e. $(s,-)$, $(s,+)$, $(c,-)$ or $(c,+)$.





Problem:

- For $t \gg a$ GEP yields the lowest four states in the combined $(s/c,-/+)$ sector; the D is not among them:
  - $m(K) \approx 496\,\mathrm{MeV}$, $\quad m(K(1460)) = 1400\,\mathrm{MeV} - 1460\,\mathrm{MeV}$, $\quad \ldots\ (J^{\mathcal{P}} = 0^-)$.
  - $m(K_0^*(800)) = 672(40)\,\mathrm{MeV}$, $\quad m(K_0^*(1430)) = 1425(50)\,\mathrm{MeV}$, $\quad \ldots\ (J^{\mathcal{P}} = 0^+)$.
  - $m(K+\pi)$, $\quad m(K+2\times\pi)$, $\quad\ldots$
  - $m(D) \approx 1868\,\mathrm{MeV}$ $(J^{\mathcal{P}} = 0^-)$.

Why can we still expect to get an estimate for $m(D)$ from GEP?

- In the continuum an exact diagonalization of C_{jk} is possible yielding one correlator for each of the four sectors $(s,-)$, $(s,+)$, $(c,-)$, $(c,+)$
  $\rightarrow$ GEP would not yield the four lowest masses but m_K, $m_{(s,+)}$, m_D and $m_{(c,+)}$.
- At finite lattice spacing corrections are $\mathcal{O}(a)$; at not too large temporal separations one of the four effective masses should be dominated by the D.

Method 2: fitting exponentials

- Perform a χ^2 minimizing fit of

$$\tilde{C}_{jk}(t) = \sum_{n=1}^{N}\Big(a_j^{(n)}\Big)^\dagger a_k^{(n)}\exp\Big(-m_nt\Big),$$

i.e. of N exponentials to the computed correlation matrix $C_{jk}(t)$, $t_{\min} \le t \le t_{\max}$.

Method 3: heavy flavor/parity restoration

- Express the correlation matrix $C_{jk}(t)$ in the physical basis in terms of ω_l and ω_h and Z_P/Z_S:

$$C^{\mathrm{physical},R}(t;\omega_l,\omega_h,Z_P/Z_S) = \mathcal{M}(\omega_l,\omega_h)\mathrm{diag}(Z_P,Z_P,Z_S,Z_S)C(t)\mathrm{diag}(Z_P,Z_P,Z_S,Z_S)\mathcal{M}^\dagger(\omega_l,\omega_h).$$

- Determine ω_l, ω_h and Z_P/Z_S by requiring

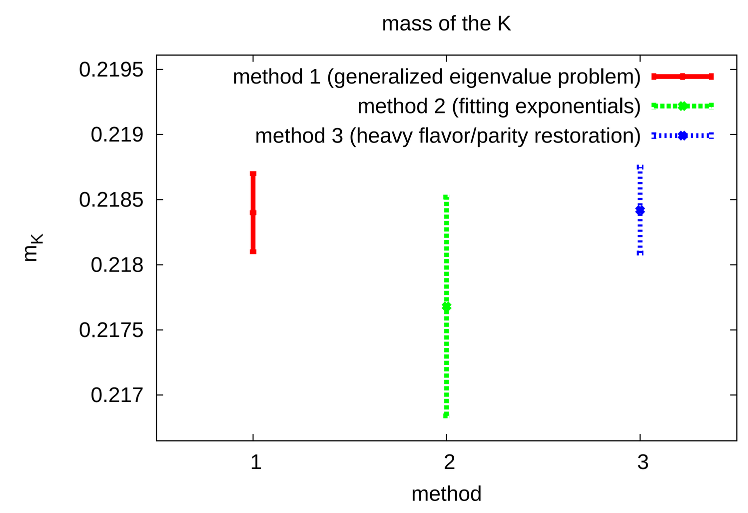
$$C^{\mathrm{physical},R}_{jk}(t;\omega_l,\omega_h,Z_P/Z_S)\Big|_{j\neq k} = 0\ :$$

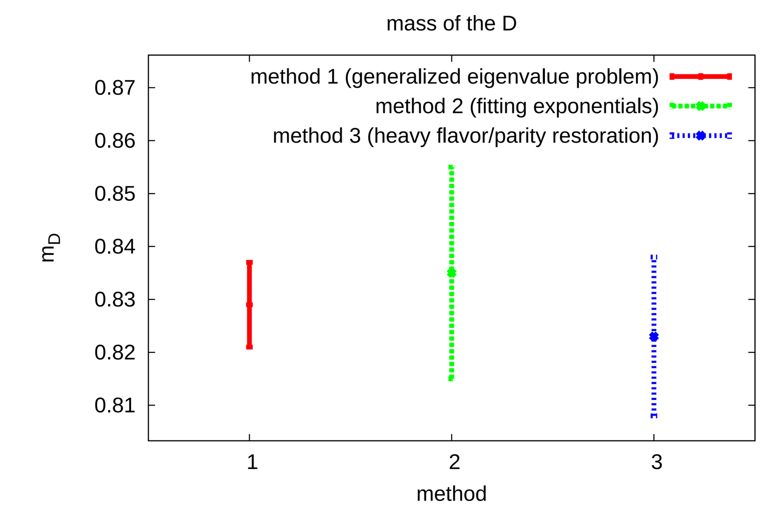
  - At finite lattice spacing and small t this cannot be achieved exactly (excited states, $\mathcal{O}(a)$ effects).
  - It can be realized at large t (when only the K survives).
  - Amounts to removing any K contribution from the diagonal correlators $C^{\mathrm{physical},R}_{jj}$, $j \neq (s,-)$.
- Analyze the diagonal correlators $C^{\mathrm{physical},R}_{jj}$ separately; there is one correlator for each of the four sectors $(s,-)$, $(s,+)$, $(c,-)$, $(c,+)$.

Conclusions

- Results obtained with our three methods agree within statistical and systematic errors.

	method 1	method 2	method 3
m_K	0.2184(3)	0.21768(84)	0.21842(33)
m_D	0.829(8)	0.835(20)	0.823(15)





- Precise results for m_K; statistical error $\lesssim 0.4\%$.
- All three methods require assumptions for m_D, i.e. there is a systematical error involved; combined statistical and systematical error $\lesssim 2.5\%$.
- For precision charm physics we intend to use a mixed action Osterwalder-Seiler setup (cf. talk by Carsten Urbach, "Pseudoscalar decay constants from $N_f = 2+1+1$ twisted mass lattice QCD"); m_K and m_D are needed/helpful for matching bare quark masses.

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