Dynamical lattice computation of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$

Benoit Blossier, Marc Wagner, Olivier Pène

Humboldt-Universität zu Berlin, Institut für Physik

mcwagner@physik.hu-berlin.de

http://people.physik.hu-berlin.de/∼mcwagner/

July 30, 2009

Introduction

- Consider semileptonic decays of B mesons (B, B^*) into orbitally excited P wave D mesons (D^{**}):

$$B^{(*)} \quad \rightarrow \quad D^{**} \, l \, \nu.$$

- Precise knowledge of the corresponding branching fractions important, e.g. to reduce the systematic uncertainty in the measurements of the CKM matrix element $|V_{cb}|$.

- There is a persistent conflict ("$1/2$ versus $3/2$ puzzle") between theory and experiment:
  - Experiment favors the decay into "$1/2$ P wave D^{**}'s".
  - Theory favors the decay into "$3/2$ P wave D^{**}'s".
  - Lattice calculations can help to resolve this conflict.

Outline

- Heavy-light mesons.
- The $1/2$ versus $3/2$ puzzle.
- Lattice computation of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$.

Heavy-light mesons

- Heavy-light meson: a meson made from a heavy quark (b, c) and a light quark (u, d), i.e. $B = \{\bar{b}u\,,\,\bar{b}d\}$, $D = \{\bar{c}u\,,\,\bar{c}d\}$.
- Static limit, i.e. $m_b, m_c \rightarrow \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $\mathcal{P}$ and total angular momentum of the light cloud (light quarks and gluons) j.
- m_b, m_c finite, but heavy:
  - Classify states according to parity $\mathcal{P}$ and total angular momentum J.
  - Although j is not a "true quantum number" anymore, it is still an approximate quantum number $\rightarrow$ notation D_J^j.
  - $D^{**} = \{D_0^*\,,\,D_1'\,,\,D_1\,,\,D_2^*\}$.

$j^{\mathcal{P}}$	$J^{\mathcal{P}}$
$(1/2)^- \equiv S$	$0^- \equiv B, D$ $1^- \equiv B^*, D^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv D_0^* \equiv D_0^{1/2}$ $1^+ \equiv D_1' \equiv D_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv D_1 \equiv D_1^{3/2}$ $2^+ \equiv D_2^* \equiv D_2^{3/2}$

$1/2$ versus $3/2$: experimental side

- Consider the semileptonic decay $B \rightarrow X_c \, l \, \nu$.
- Experiments, which have studied this decay: ALEPH, BaBar, BELLE, CDF, DELPHI, DØ.
- What is X_c?
  - $\approx 75\%$ D and D^*, i.e. S wave states (agreement with theory).
  - $\approx 10\%$ $D_1^{3/2}$ and $D_2^{3/2}$, i.e. $j = 3/2$ P wave states (agreement with theory).
  - For the remaining $\approx 15\%$ the situation is not clear:
    * A natural candidate would be $D_0^{1/2}$ and $D_1^{1/2}$, i.e. $j = 1/2$ P wave states.
    * This would imply $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) > \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$, which is in conflict with theory.
    * This conflict between experiment and theory is called the "$1/2$ versus $3/2$ puzzle".

$1/2$ versus $3/2$: theory side (1)

- Static limit, i.e. $m_b, m_c \to \infty$.

- Parameterization of the matrix elements relevant for decays $B \to D^{**}\,l\,\nu$ by two form factors (Isgur-Wise functions) due to heavy quark symmetry:

$$\begin{aligned} \langle D_0^{1/2}(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle &\propto \tau_{1/2}(w)(v-v')_\mu \\ \langle D_2^{3/2}(v',\epsilon)|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle &\propto \tau_{3/2}(w)\Big((w+1)\epsilon^*_{\mu\alpha}v^\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\nu\Big). \end{aligned}$$

  where $w = (v' \cdot v) \geq 1$.

  [N. Isgur and M. B. Wise, Phys. Rev. D **43**, 819 (1991)]

- Relation to decay rates:

$$\begin{aligned} \frac{d\Gamma(B \to D_J^{1/2} l\nu)}{dw} &\propto G_F^2|V_{cb}|^2 K_J^{1/2}(w)\Big|\tau_{1/2}(w)\Big|^2 \quad , \quad J = 0,1 \\ \frac{d\Gamma(B \to D_J^{3/2} l\nu)}{dw} &\propto G_F^2|V_{cb}|^2 K_J^{3/2}(w)\Big|\tau_{3/2}(w)\Big|^2 \quad , \quad J = 1,2, \end{aligned}$$

  where K_J^j are analytically known kinematical factors.

$1/2$ versus $3/2$: theory side (2)

- By means of OPE a couple of sum rules have been derived in the static limit:
  - Most prominent sum rule in this context: Uraltsev sum rule,

    $$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \left|\tau_{1/2}^{(n)}(1)\right|^2 \ = \ \frac{1}{4}$$

    ($\tau_{1/2} \equiv \tau_{1/2}^{(0)}$ and $\tau_{3/2} \equiv \tau_{3/2}^{(0)}$; the sum is over all $1/2$ and $3/2$ P wave meson states respectively).

    [N. Uraltsev, Phys. Lett. B **501**, 86 (2001) [arXiv:hep-ph/0011124]]
  - From experience with sum rules one expects approximate saturation from the ground states, i.e.

    $$\left|\tau_{3/2}^{(0)}(1)\right|^2 - \left|\tau_{1/2}^{(0)}(1)\right|^2 \ \approx \ \frac{1}{4},$$

    which implies $|\tau_{1/2}(1)| < |\tau_{3/2}(1)|$. This strongly suggests $\Gamma(B \rightarrow D_{0,1}^{1/2}\, l\, \nu) < \Gamma(B \rightarrow D_{1,2}^{3/2}\, l\, \nu)$, which is in conflict with experiment.

$1/2$ versus $3/2$: theory side (3)

- Phenomenological models:
  - $|\tau_{1/2}(1)| < |\tau_{3/2}(1)|$ and $\Gamma(B \to D_{0,1}^{1/2}\, l\, \nu) < \Gamma(B \to D_{1,2}^{3/2}\, l\, \nu)$, which is in "conflict" with experiment.

    [V. Morenas, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **56** 5668 (1997) [arXiv:hep-ph/9706265]]

    [D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B **434**, 365 (1998) [arXiv:hep-ph/9805423]]

    [...]
  - Same qualitative picture also beyond the static limit, i.e. for finite m_b and m_c.

    [D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **61**, 014016 (2000) [arXiv:hep-ph/9906415]]

$1/2$ versus $3/2$: possible explanations (1)

- **Experiment:**

  (A) The signal for the remaining 15% of X_c is rather vague; therefore, only a small part might be $D_{0,1}^{1/2}$.

- **OPE:**

  – Sum rules might not be saturated by the ground states.

  (B) Sum rules hold in the static limit and might change for finite quark masses.

  (C) Sum rules make statements about $\tau_{1/2}(w = 1)$ and $\tau_{3/2}(w = 1)$; to obtain decay rates, however, one has to integrate over w.

- **Phenomenological models:**

  – Models might give a wrong answer.

- **Most probable scenario:** a combination of (A), (B) and (C).

  [N. Uraltsev, arXiv:hep-ph/0409125]

$1/2$ versus $3/2$: possible explanations (2)

- A lattice calculation of $\tau_{1/2}$ and $\tau_{3/2}$ could shed some light on this puzzle.
- Exploratory quenched lattice study confirmed the theory side:
  $\tau_{1/2}(1) = 0.38(4)$, $\tau_{3/2}(1) = 0.53(8)$.
  [D. Becirevic *et al.*, Phys. Lett. B **609**, 298 (2005) [arXiv:hep-lat/0406031]]
- In the following I will report about the first unquenched lattice calculation of $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$.
  [B. Blossier, M. Wagner and O. Pene [ETM Collaboration], JHEP **0906**, 022 (2009)
  [arXiv:0903.2298 [hep-lat]]]

Lattice calculation of $\tau_{1/2}$ and $\tau_{3/2}$ (1)

- The "Isgur-Wise relations"

$$\begin{aligned}
\langle D_0^{1/2}(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle &\propto \tau_{1/2}(w)(v-v')_\mu \\
\langle D_2^{3/2}(v',\epsilon)|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle &\propto \tau_{3/2}(w)\Big((w+1)\epsilon^*_{\mu\alpha}v^\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\nu\Big).
\end{aligned}$$

  are note directly useful to compute $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$.

- They can be rewritten in the following form, which is directly accessible to a lattice calculation:

$$\begin{aligned}
\langle D_0^{1/2}(v)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle &= -ig_{jk}\Big(m(D_0^{1/2}) - m(B)\Big)\tau_{1/2}(1) \\
\langle D_2^{3/2}(v,\epsilon)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle &= +i\sqrt{3}\epsilon_{jk}\Big(m(D_2^{3/2}) - m(B)\Big)\tau_{3/2}(1).
\end{aligned}$$

  [A. K. Leibovich, Z. Ligeti, I. W. Stewart and M. B. Wise, Phys. Rev. D **57**, 308 (1998) [arXiv:hep-ph/9705467]]

Lattice calculation of $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- We compute

$$\langle D_0^{1/2}(v)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle \ = \ -i g_{jk}\Big(m(D_0^{1/2}) - m(B)\Big)\tau_{1/2}(1)$$

via

$$\tau_{1/2}(1) \ = \ \lim_{t_0-t_1\rightarrow\infty\,,\,t_1-t_2\rightarrow\infty} \tau_{1/2,\mathrm{effective}}(t_0-t_1,t_1-t_2)$$

$$\tau_{1/2,\mathrm{effective}}(t_0-t_1,t_1-t_2) \ =$$

$$= \ \frac{1}{Z_\mathcal{D}}\left|\frac{N(P_-) \ \ N(S) \ \ \Big\langle\Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger \ (\bar{Q}\gamma_5\gamma_3 D_3 Q)(t_1) \ \mathcal{O}^{(S)}(t_2)\Big\rangle}{\Big(m(P_-) - m(S)\Big) \ \ \Big\langle\Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger \mathcal{O}^{(P_-)}(t_1)\Big\rangle \ \ \Big\langle\Big(\mathcal{O}^{(S)}(t_1)\Big)^\dagger \mathcal{O}^{(S)}(t_2)\Big\rangle}\right|.$$

- We need:
  - Static-light meson creation operators $\mathcal{O}^{(S)}$, $\mathcal{O}^{(P_-)}$, $\mathcal{O}^{(P_+)}$.
  - Static-light meson masses $m(S)$, $m(P_-)$ and $m(P_+)$.
  - 2-point and 3-point functions (and norms $N(S)$, $N(P_-)$, $N(P_+)$).

Simulation setup

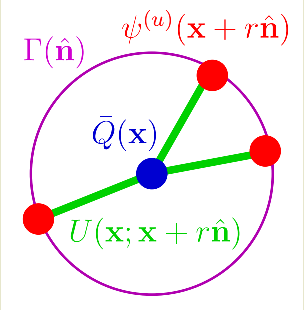
- $N_f = 2$ ETMC gauge configurations.
- Gauge action: tree-level Symanzik improved.
- Fermionic action: Wilson twisted mass at maximal twist
  $\rightarrow$ "automatic $\mathcal{O}(a)$ improvement of physical quantities".
- Lattice volume: $L^3 \times T = 24^3 \times 48$.
- Lattice spacing: $a = 0.0855\,\mathrm{fm}$.
- "Pion masses":

m_{PS} in MeV	number of gauge configurations
$314(2)$	1400
$391(1)$	1450
$448(1)$	1350

Static-light meson creation operators

- In the continuum, physical basis:

$$\mathcal{O}^{(\Gamma)}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + r\hat{\mathbf{n}}) \psi^{(u)}(\mathbf{x} + r\hat{\mathbf{n}}).$$



- List of operators (J: total angular momentum; j: total angular momentum of the light cloud; $\mathcal{P}$: parity):

$\Gamma(\hat{\mathbf{n}})$	$J^{\mathcal{P}}$	$j^{\mathcal{P}}$	$\mathrm{O_h}$	lattice $j^{\mathcal{P}}$	notation
γ_5	0^-	$(1/2)^-$	A_1	$(1/2)^-$, $(7/2)^-$, ...	S
1	0^+	$(1/2)^+$		$(1/2)^+$, $(7/2)^+$, ...	P_-
$\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2$ (cyclic)	2^+	$(3/2)^+$	E	$(3/2)^+$, $(5/2)^+$, ...	P_+
$\gamma_5(\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2)$ (cyclic)	2^-	$(3/2)^-$		$(3/2)^-$, $(5/2)^-$, ...	$D_\pm$

Static-light meson masses

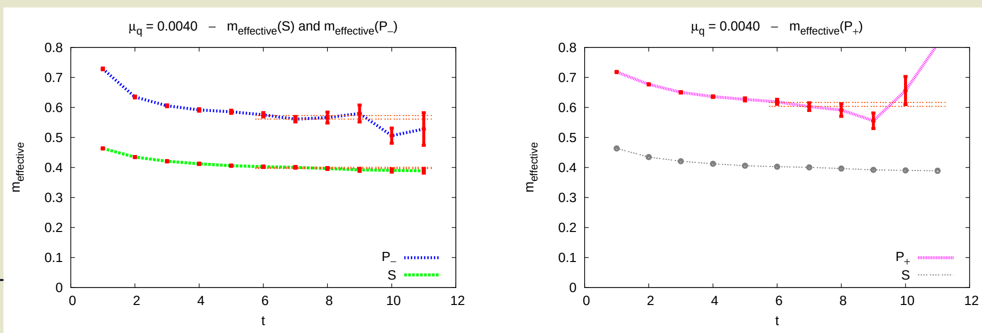
- Compute 2-point functions

$$\mathcal{C}^{(\Gamma)}(t) \quad = \quad \left\langle \left(\mathcal{O}^{(\Gamma)}(t) \right)^\dagger \mathcal{O}^{(\Gamma)}(0) \right\rangle \quad , \quad \Gamma \quad \in \quad \{\gamma_5 \, , \, 1 \, , \, \gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2\}.$$

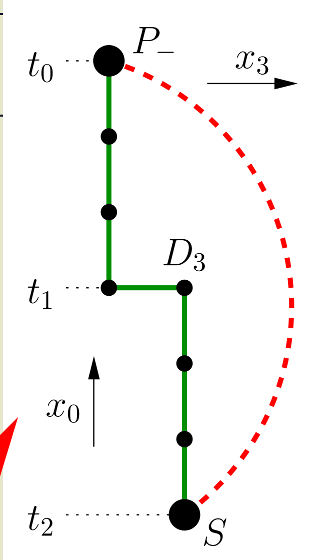
- Determine static-light meson masses $m(S)$, $m(P^-)$ and $m(P^+)$ from effective mass plateaus:

$$m_{\mathrm{effective}}^{(\Gamma)}(t) \quad = \quad \ln \left(\frac{\mathcal{C}^{(\Gamma)}(t)}{\mathcal{C}^{(\Gamma)}(t+1)} \right).$$

- Obtain ground state norms $N(S)$, $N(P_-)$ and $N(P_+)$ by fitting exponentials to the 2-point functions at large temporal separations.



3-point functions, $\tau_{1/2}$ and $\tau_{3/2}$



- Compute the Isgur-Wise function

$$\tau_{1/2}(1) \quad = \quad \left|\frac{\langle P_-|\bar{Q}\gamma_5\gamma_3 D_3 Q|S\rangle}{m(P_-) - m(S)}\right|$$

via "effective form factors":

$$\tau_{1/2}(1) \quad = \quad \lim_{t_0-t_1\rightarrow\infty\,,\,t_1-t_2\rightarrow\infty} \tau_{1/2,\text{effective}}(t_0-t_1,t_1-t_2)$$

$$\tau_{1/2,\text{effective}}(t_0-t_1,t_1-t_2) \quad =$$

$$= \quad \frac{1}{Z_{\mathcal{D}}}\left|\frac{N(P_-)\quad N(S)\quad \Big\langle\Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger\;(\bar{Q}\gamma_5\gamma_3 D_3 Q)(t_1)\;\mathcal{O}^{(S)}(t_2)\Big\rangle}{\Big(m(P_-)-m(S)\Big)\;\Big\langle\Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger\mathcal{O}^{(P_-)}(t_1)\Big\rangle\;\Big\langle\Big(\mathcal{O}^{(S)}(t_1)\Big)^\dagger\mathcal{O}^{(S)}(t_2)\Big\rangle}\right|.$$

- $\tau_{3/2}(1)$ analogously: replace

$$P_- \quad\rightarrow\quad P_+ \quad , \quad \gamma_3 D_3 \quad\rightarrow\quad \frac{\gamma_5(\gamma_1 D_1 - \gamma_2 D_2)}{\sqrt{6}}.$$

3-point functions, $\tau_{1/2}$ an

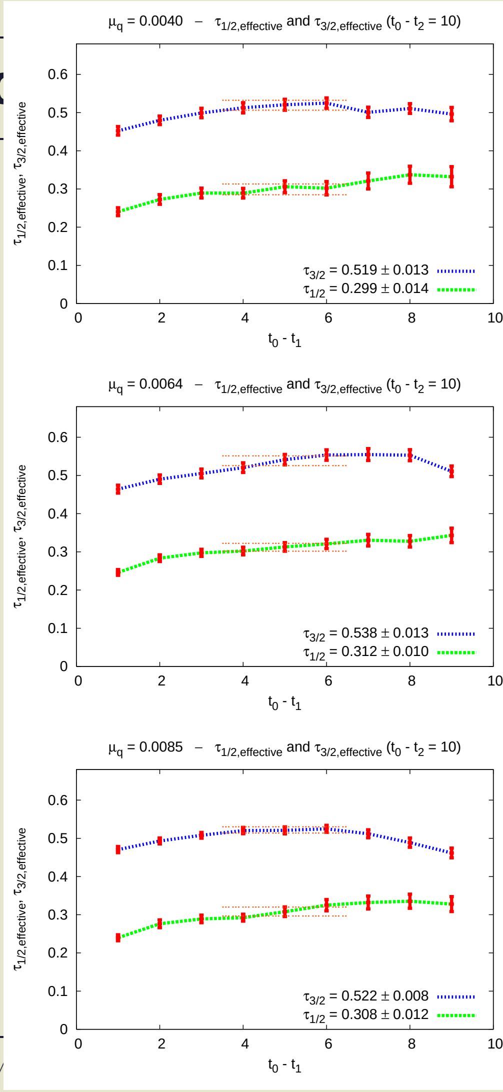
- Results for various light quark masses:

	$t_0 - t_2 = 10$		
μ_q	$\tau_{1/2}(1)$	$\tau_{3/2}(1)$	$(\tau_{3/2})^2 - (\tau_{1/2})^2$
0.0040	$0.299(14)$	$0.519(13)$	$0.180(16)$
0.0064	$0.312(10)$	$0.538(13)$	$0.193(13)$
0.0085	$0.308(12)$	$0.522(8)$	$0.177(9)$

- The Uraltsev sum rule,

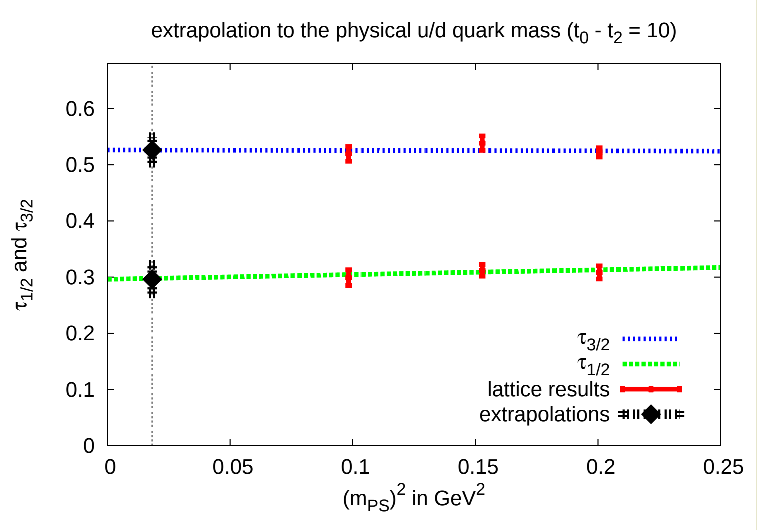
$$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \left|\tau_{1/2}^{(n)}(1)\right|^2 \quad = \quad \frac{1}{4},$$

is almost fulfilled by the ground state contributions $\tau_{1/2}^{(0)}(1) \equiv \tau_{1/2}(1)$ and $\tau_{3/2}^{(0)}(1) \equiv \tau_{3/2}(1)$.



Extrapolation to the u/d quark mass

- Linear extrapolation in $(m_{\mathrm{PS}})^2$ to the u/d quark mass $m_{\mathrm{PS}} = 135\,\mathrm{MeV}$:
  - $\tau_{1/2} = 0.296(26)$.
  - $\tau_{3/2} = 0.526(23)$.



Conclusions

- First dynamical lattice computation of the Isgur-Wise functions $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$:
  - $\tau_{1/2}(1) = 0.296(26)$, $\tau_{3/2}(1) = 0.526(23)$.
  - This indicates $\Gamma(B \rightarrow D_{0,1}^{1/2}\, l\, \nu) < \Gamma(B \rightarrow D_{1,2}^{3/2}\, l\, \nu)$ in the static limit.
  - Expectation from sum rules confirmed:
    - Uraltsev sum rule is approximately fulfilled by the ground states.
    - $\tau_{1/2}(1) \ll \tau_{3/2}(1)$.
    - Numerical values in agreement with sum rule expectation.
  - Phenomenological models qualitatively and quantitatively confirmed.
  - Experiment:
    - Fair agreement with the experimentally measured $\tau_{3/2}(1) \approx 0.75$.
    - No agreement with the experimentally measured $\tau_{1/2}(1) \approx 1.28$.

      [D. Liventsev *et al.* [Belle Collaboration], Phys. Rev. D **77**, 091503 (2008) [arXiv:0711.3252 [hep-ex]]]

Outlook

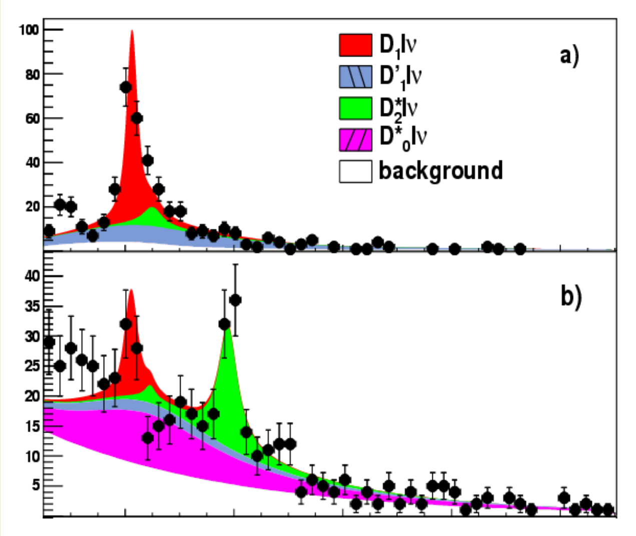
- “To do list” and “wish list”:
  - Perform the continuum limit.
  - Use $N_f = 2+1+1$ flavors of dynamical quarks.
  - Non-perturbative renormalization.
  - Compute $1/m_Q$ corrections or do a computation with finite heavy quark masses.
  - Compute the slopes of the form factors $d\tau_{1/2}/dw\Big|_{w=1}$, $d\tau_{3/2}/dw\Big|_{w=1}$.
  - Study other decays of the type $B \rightarrow X_c \, l \, \nu$, e.g. decays into radially excited states.

$1/2$ versus $3/2$: experimental side (A)

- Example plot from BaBar/SLAC:
  - Horizontal axis: $m(D^{(*)}\pi) - m(D^{(*)})$ in GeV/c^2.
  - Vertical axis: $\mathrm{events}/(20\,\mathrm{MeV}/c^2)$.
  - Simultaneous fit of four probability distribution functions (D_0^*, D_1', D_1, D_2^*) to $m(D^{(*)}\pi) - m(D^{(*)})$ data:

    a) $B^- \rightarrow D^{*+}\pi^- l^- \bar{\nu}_l$.

    b) $B^- \rightarrow D^{+}\pi^- l^- \bar{\nu}_l$.



  - Two states (D_1 and D_2^*, i.e. the $j = 3/2$ P wave states) have small widths and can "clearly" be identified.
  - Two states (D_0^* and D_1', i.e. the $j = 1/2$ P wave states) have very large widths.

[B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **101**, 261802 (2008) [arXiv:0808.0528 [hep-ex]]]

Simulation setup (A)

- Fermionic action: Wilson twisted mass, $N_f = 2$ degenerate flavors,

$$S_\mathrm{F}[\chi, \bar{\chi}, U] = a^4 \sum_x \bar{\chi}(x) \Big(D_\mathrm{W} + i\mu_\mathrm{q} \gamma_5 \tau_3 \Big) \chi(x)$$

$$D_\mathrm{W} = \frac{1}{2} \Big(\gamma_\mu (\nabla_\mu + \nabla^*_\mu) - a \nabla^*_\mu \nabla_\mu \Big) + m_0$$

  (m_0: untwisted mass; μ_q: twisted mass; τ_3: third Pauli matrix acting in flavor space).

- Relation between the physical basis ψ and the twisted basis χ (in the continuum):

$$\psi = \frac{1}{\sqrt{2}} \Big(\cos(\omega/2) + i \sin(\omega/2) \gamma_5 \tau_3 \Big) \chi$$

$$\bar{\psi} = \frac{1}{\sqrt{2}} \bar{\chi} \Big(\cos(\omega/2) + i \sin(\omega/2) \gamma_5 \tau_3 \Big)$$

  (ω: twist angle; $\omega = \pi/2$: maximal twist).