

# Static-light meson masses from twisted mass lattice QCD



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# European Twisted Mass Collaboration

- **Cyprus:** University of Nikosia.
- **France:** University of Paris Sud, LPSC Grenoble.
- **Germany:** Humboldt University Berlin, University of Münster, DESY Hamburg, DESY Zeuthen.
- **Great Britain:** University of Glasgow, University of Liverpool.
- **Italy:** University of Rome I, University of Rome II, University of Rome III, ECT\* Trento.
- **Netherlands:** University of Groningen.
- **Spain:** University of Valencia.
- **Switzerland:** University of Bern.



# Introduction

- **Static-light meson:** a bound state of an infinitely heavy quark and a light quark (“a  $B$ -meson in leading order”).
- Static-light mesons can be classified according to total angular momentum  $F = 0, 1, 2, 3, \dots$  and parity  $P = \pm$ .
- **Goal:** compute static-light meson masses for low lying states (ground state, first excited state) for different quantum numbers  $F$  and  $P$ .
- **Related papers:**
  - C. Michael and J. Peisa [UKQCD], Phys. Rev. D **58**, 034506 (1998).
  - A. M. Green, J. Koponen, C. McNeile, C. Michael and G. Thompson [UKQCD], Phys. Rev. D **69**, 094505 (2004).
  - T. Burch and C. Hagen, Comput. Phys. Commun. **176**, 137 (2007).
  - J. Koponen, Acta Phys. Polon. B **38**, 2893 (2007).
  - J. Foley, A. O’Cais, M. Peardon and S. M. Ryan, Phys. Rev. D **75**, 094503 (2007).

# Outline

- Basic principle.
- Twisted mass lattice QCD.
- Static-light meson creation operators on the lattice.
- Simulation setup and numerical results.
- Summary and outlook.

# Basic principle (1)

- Let  $\mathcal{O}(\mathbf{x})$  be a suitable “static-light meson creation operator”, i.e. an operator such that  $\mathcal{O}(\mathbf{x})|\Omega\rangle$  is a state containing a static-light meson at position  $\mathbf{x}$  ( $|\Omega\rangle$ : vacuum).
- Determine the mass of the ground state of the corresponding static-light meson from the exponential behavior of the corresponding correlation function  $\mathcal{C}$  at large Euclidean times  $T$ :

$$\begin{aligned}\mathcal{C}(T) &= \langle \Omega | \left( \mathcal{O}(\mathbf{x}, T) \right)^\dagger \mathcal{O}(\mathbf{x}, 0) | \Omega \rangle = \\ &= \langle \Omega | e^{+HT} \left( \mathcal{O}(\mathbf{x}, 0) \right)^\dagger e^{-HT} \mathcal{O}(\mathbf{x}, 0) | \Omega \rangle = \\ &= \sum_n \left| \langle n | \mathcal{O}(\mathbf{x}, 0) | \Omega \rangle \right|^2 \exp \left( - (E_n - E_\Omega) T \right) \approx \quad (\text{for } T \gg 1) \\ &\approx \left| \langle 0 | \mathcal{O}(\mathbf{x}, 0) | \Omega \rangle \right|^2 \exp \left( - \underbrace{(E_0 - E_\Omega)}_{\text{meson mass}} T \right).\end{aligned}$$

## Basic principle (2)

- To compute the static-light spectrum, i.e. meson masses for different quantum numbers, consider extended meson creation operators with different spatial structure and different spin structure yielding well defined total angular momentum  $F$ .
- Static-light meson masses are degenerate with respect to the static spin.
- Therefore, it is more appropriate to label static-light mesons by  $J = L \pm 1/2$ , where  $L$  is the angular momentum quantum number and  $\pm$  describes the coupling of the light spin.
- Parity  $P$  is also a good quantum number.
- Since static-light mesons are made from non-identical quarks, charge conjugation is not a useful quantum number (static-light meson masses are degenerate with respect to charge conjugation).

# Basic principle (3)

- General form of a static-light meson creation operator:

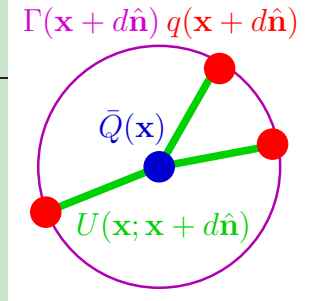
$$\mathcal{O}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- $\bar{Q}(\mathbf{x})$  creates an infinitely heavy i.e. static antiquark at position  $\mathbf{x}$ .
- $q(\mathbf{x} + d\hat{\mathbf{n}})$  creates a light quark at position  $\mathbf{x} + d\hat{\mathbf{n}}$  separated by a distance  $d$  from the static antiquark.
- The spatial parallel transporter

$$U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) = P \left\{ \exp \left( +i \int_{\mathbf{x}}^{\mathbf{x}+d\hat{\mathbf{n}}} dz_j A_j(\mathbf{z}) \right) \right\}$$

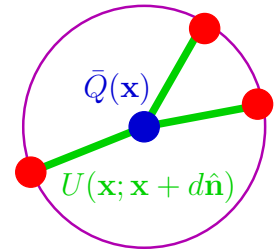
connects the antiquark and the quark in a gauge invariant way via gluons.

- The integration over the unit sphere  $\int d\hat{\mathbf{n}}$  combined with a suitable weight factor  $\Gamma(\hat{\mathbf{n}})$  yields well defined total angular momentum  $J$  and parity  $P$  ( $\Gamma(\hat{\mathbf{n}})$  is a combination of spherical harmonics [ $\rightarrow$  angular momentum] and  $\gamma$ -matrices [ $\rightarrow$  spin]; Wigner-Eckart theorem).



# Basic principle (4)

$$\Gamma(\mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}})$$



- **General form of a static-light meson creation operator:**

$$\mathcal{O}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **List of operators** ( $L$ : angular momentum;  $S$ : total spin;  $F$ : total angular momentum;  $J$ : angular momentum and light spin;  $P$ : parity):

| common notation | $\Gamma(\mathbf{x})$  | $L^P$          | $S^P$          | $F^P$ | $J^P$     |
|-----------------|---|----------------|----------------|-------|-----------|
| $S$             | $\gamma_5$<br>$\gamma_5 \gamma_j x_j$                               | $0^+$<br>$1^-$ | $0^-$<br>$1^+$ | $0^-$ | $(1/2)^-$ |
| $P_-$           | $1$<br>$\gamma_j x_j$   | $0^+$<br>$1^-$ | $0^+$<br>$1^-$ | $0^+$ | $(1/2)^+$ |
| $P_+$           | $\gamma_1 x_1 - \gamma_2 x_2$                                       | $1^-$          | $1^-$          | $2^+$ | $(3/2)^+$ |
| $D_-$           | $\gamma_5 (\gamma_1 x_1 - \gamma_2 x_2)$                            | $1^-$          | $1^+$          | $2^-$ | $(3/2)^-$ |
| $D_+$           | $\gamma_1 x_2 x_3 + \gamma_2 x_3 x_1 + \gamma_3 x_1 x_2$            | $2^+$          | $1^-$          | $3^-$ | $(5/2)^-$ |
| $F_-$           | $\gamma_5 (\gamma_1 x_2 x_3 + \gamma_2 x_3 x_1 + \gamma_3 x_1 x_2)$ | $2^+$          | $1^+$          | $3^+$ | $(5/2)^+$ |



# Twisted mass lattice QCD

- Twisted mass action (two degenerate flavors, “continuum version”):

$$S_{\text{fermionic}} = \int d^4x \bar{\chi} \left( \gamma_\mu D_\mu + m + \underbrace{i\mu\gamma_5\tau_3}_{\text{twisted mass term}} - \underbrace{\frac{a}{2}\square}_{\text{Wilson term}} \right) \chi$$
$$\psi = e^{i\omega\gamma_5\tau_3/2} \chi$$

( $\psi$ : physical basis quark fields;  $\chi$ : twisted basis quark fields;  $\mu$ : twisted mass;  $\tau_3$ : third Pauli matrix acting in flavor space;  $a$ : lattice spacing).

- Wilson term: removes fermionic doublers.
- Twisted mass term: automatic  $\mathcal{O}(a)$  improvement, when tuned to maximal twist ( $\omega = \pi/2$ ).

+ Automatic  $\mathcal{O}(a)$  improvement.

+ Numerically cheap, i.e. large lattices and small lattice spacings possible.

– Explicit breaking of parity and flavor symmetry.

# Meson operators on the lattice (1)

- **Static-light meson creation operators in the continuum:**

$$\mathcal{O}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **Static-light meson creation operators on the lattice:**

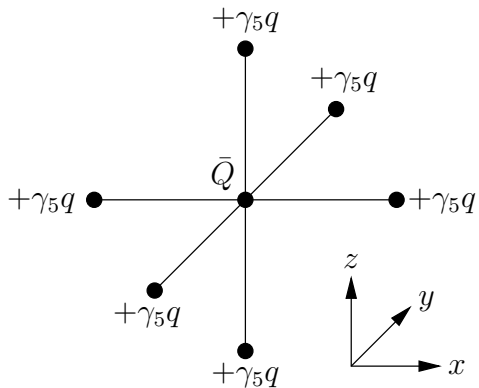
$$\mathcal{O}^{6\text{-path}}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}}=\pm\mathbf{e}_1, \pm\mathbf{e}_2, \pm\mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+$$

$$\mathcal{O}^{8\text{-path}}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}}=\pm\mathbf{e}_1 \pm \mathbf{e}_2 \pm \mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+.$$

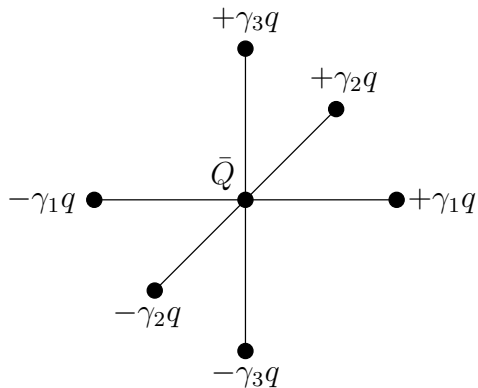
- **Main difference:**

- The integrations over spheres  $\int d\hat{\mathbf{n}}$  are replaced by finite sums  $\sum_{\hat{\mathbf{n}}}$ .
- Spherical harmonics contained in  $\Gamma$  are approximated by six or eight points respectively.

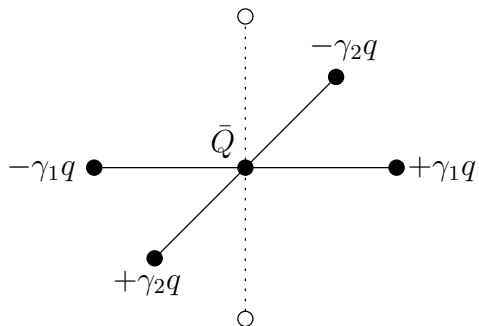
$S_+$  operator ( $J^P = (1/2)^-$ )



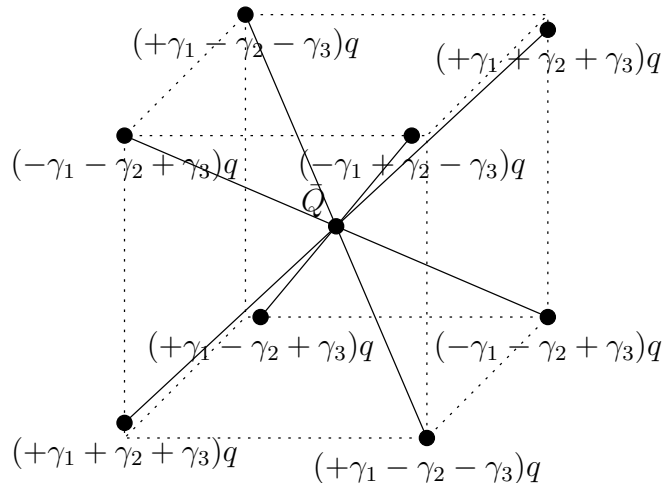
$P_-$  operator ( $J^P = (1/2)^+$ )



$P_+$  operator ( $J^P = (3/2)^+$ )



$D_+$  operator ( $J^P = (5/2)^-$ )



# Meson operators on the lattice (2)

- To determine the total angular momentum quantum numbers of lattice meson creation operators, expand them in terms of spherical harmonics:
  - Expansions are infinite sums.
  - Lattice operators have no well defined total angular momentum; they always create an infinite superposition of total angular momentum eigenstates.
  - In contrast to the continuum, where there is an infinite number of fixed angular momentum representations (continuous rotation group  $SO(3)$ ), on the lattice there are only five different representations (discrete rotation group  $O_h$ ):

$$A_1 \rightarrow L = 0, 4, 6, 8, \dots$$

$$A_2 \rightarrow L = 3, 6, 7, 9, \dots$$

$$E \rightarrow L = 2, 4, 5, 6, \dots$$

$$T_1 \rightarrow L = 1, 3, 4, 5(2\times), \dots$$

$$T_2 \rightarrow L = 2, 3, 4, 5, \dots$$

# Further lattice techniques

- **Stochastic propagators:**

- Statistical noise is significantly reduced.
- Spatial smearing is easy.

- **Smearing techniques:**

- HYP2 smearing of links in time direction to reduce the self energy of the static quark (→ statistical noise is reduced).
- Jacobi smearing of light quark operators and APE smearing of spatial links to increase ground state overlaps (→ allows to extract static-light meson masses at smaller temporal separations, where the signal quality is better).

- **Correlation matrices:**

- Increase ground state overlaps.
- Extract excited states.

# Simulation setup

- $24^3 \times 48$  lattices.
- Twisted mass Dirac operator with two degenerate flavors,

$$Q^{(\chi)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\square \quad , \quad m + 4 = \frac{1}{2\kappa}$$

with  $\kappa = 0.160856$ .

- Tree-level Symanzik improved gauge action with  $\beta = 3.9$ .
- Lattice spacing  $a \approx 0.0855(5)$  fm, spatial lattice extension  $24 \times a \approx 2.05$  fm.

| $\mu$  | $m_\pi$ in MeV | number of gauges              |
|--------|----------------|-------------------------------|
| 0.0040 | 314(2)         | 1400                          |
| 0.0064 | 391(1)         | 1450                          |
| 0.0085 | 448(1)         | 1350                          |
| 0.0100 | 485(1)         | 350 ( $\approx 1000$ planned) |
| 0.0150 | 597(2)         | 500 ( $\approx 1000$ planned) |

# Results (1)

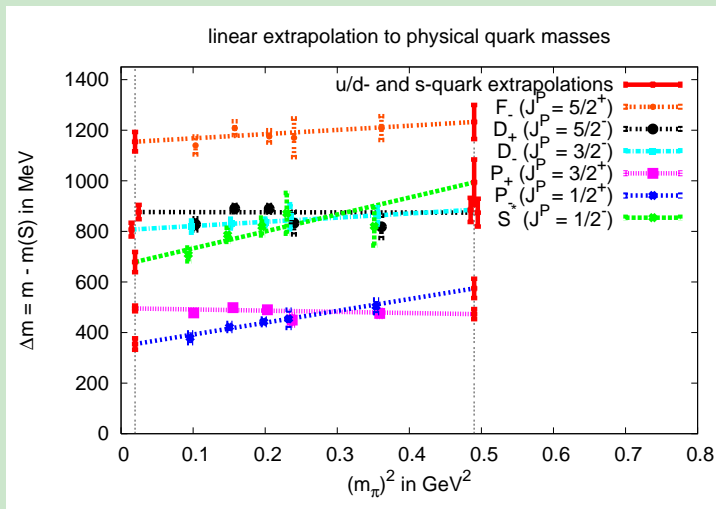
- To compute ground states and excited states, consider  $6 \times 6$  correlation matrices

$$\mathcal{C}_{jk}(T) = \langle \Omega | \left( \mathcal{O}_j(\mathbf{x}, T) \right)^\dagger \mathcal{O}_k(\mathbf{x}, 0) | \Omega \rangle.$$

- Different smearing levels, i.e. different meson extensions.
  - Operators with parity  $P = +$  and  $P = -$  in the same correlation matrix, because of parity mixing induced by the twisted mass Dirac operator.
  - Fixed total angular momentum  $J$  for each correlation matrix.
- Two approaches:
    - Effective masses by solving a generalized eigenvalue problem (visualization of static-light meson masses and their statistical accuracy).
    - $\chi^2$  fitting of an ansatz of exponentials to the correlation matrices (numerical values and statistical errors for static-light meson masses).
    - Both approaches yield consistent results.

# Results (2)

- Linear extrapolation in  $(m_\pi)^2$  to physical light quark masses:
  - $B$  mesons:  $u/d$  quark extrapolation ( $m_\pi = 139.6$  MeV).
  - $B_s$  mesons:  $s$  quark extrapolation ( $"m_\pi = 700.0$  MeV").
- \* However: sea of two degenerate  $s$  quarks.

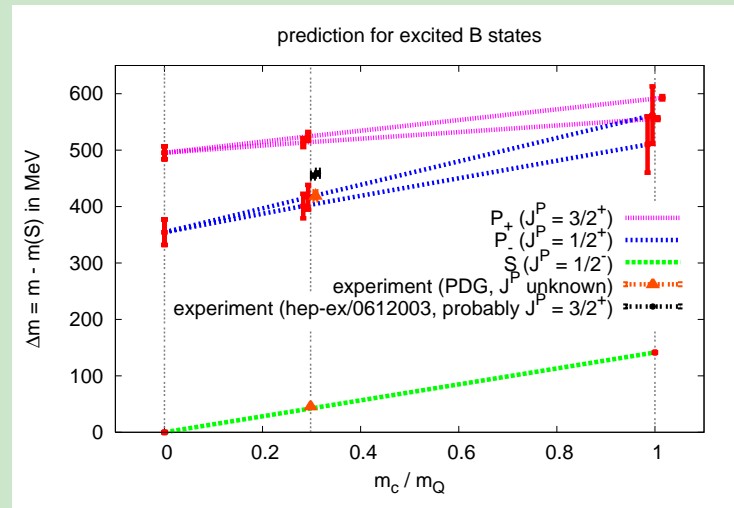




# Results (3)

- Prediction for excited  $B$  states  $B_0^*$ ,  $B_1^*$ ,  $B_1$  and  $B_2^*$  ( $P$  wave states):
  - Linear interpolation in  $m_c/m_Q$  to physical  $b$  quark mass (input:  $u/d$  extrapolated lattice data for  $m_Q = \infty$ , experimental data for  $m_Q = m_c$ ).
- Status of experimental results:
  - PDG: one excited state,  $J^P$  unknown.
  - CDF and CØ collaborations (hep-ex/0612003): two excited states,  $B_1$  and  $B_2^*$ .

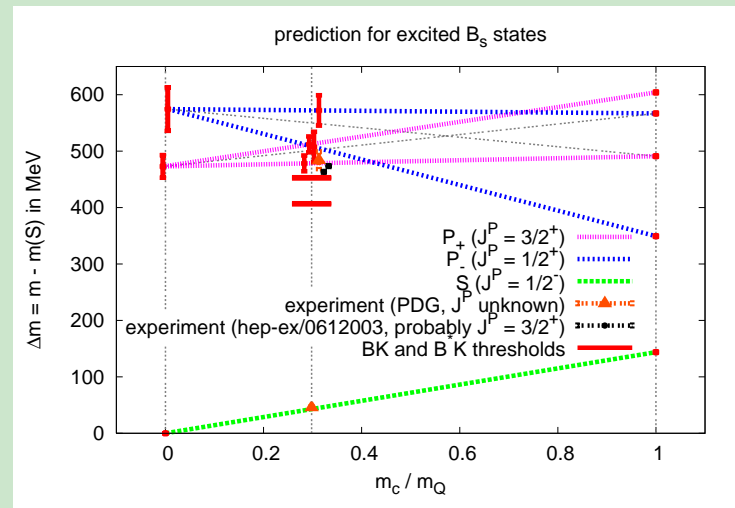
|         | $m - m(S)$ in MeV |        |            |
|---------|-------------------|--------|------------|
| state   | lattice           | PDG    | hep-ex/... |
| $B_0^*$ | 401(22)           |        | -          |
| $B_1^*$ | 416(22)           | ↑      | -          |
| $B_1$   | 513(8)            | 419(8) | 455(5)     |
| $B_2^*$ | 524(8)            | ↓      | 459(6)     |



# Results (4)

- Prediction for excited  $B_s$  states  $B_{s0}^*$ ,  $B_{s1}^*$ ,  $B_{s1}$  and  $B_{s2}^*$  ( $P$  wave states):
  - Linear interpolation in  $m_c/m_Q$  to physical  $b$  quark mass (input:  $s$  extrapolated lattice data for  $m_Q = \infty$ , experimental data for  $m_Q = m_c$ ).
- Status of experimental results:
  - PDG: one excited state,  $J^P$  unknown.
  - CDF and CØ collaborations (hep-ex/0612003): two excited states,  $B_1$  and  $B_2^*$ .

|            | $m - m(S)$ in MeV |         |            |
|------------|-------------------|---------|------------|
| state      | lattice           | PDG     | hep-ex/... |
| $B_{s0}^*$ | 507(27)           |         | -          |
| $B_{s1}^*$ | 572(27)           | ↑       | -          |
| $B_{s1}$   | 478(14)           | 484(16) | 463(1)     |
| $B_{s2}^*$ | 512(14)           | ↓       | 474(2)     |



# Summary

- Static-light meson masses have been computed via twisted mass lattice QCD at a small value of the lattice spacing ( $a = 0.0855$  fm) and at small values of the pion mass ( $m_\pi = 314$  MeV,  $\dots$ ,  $597$  MeV):
  - Total angular momentum  $J = 1/2, 3/2, 5/2$ .
  - Parity  $P = +, -$ .
  - Ground states and first excited states.
- Interpolation/extrapolation to physical quark masses allow predictions for the spectrum of  $B$  mesons and  $B_s$  mesons. Results are in agreement with currently available experimental results within statistical errors.

# Outlook

- Extrapolate to the continuum by considering other values for the lattice spacing.
- Include a sea of  $u/d$  quarks for  $B_s$  computations by using 2+1+1 flavor twisted mass lattice QCD.
- Compute static-light decay constants  $f_B$  and  $f_{B_s}$ .