SU(3) hybrid static potentials at small quark-antiquark separations from fine lattices


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Main goals / literature

(1) Compute hybrid static potentials, i.e. potentials of a static quark antiquark pair ($\bar{Q}Q$), where the flux tubes are excited with quantum numbers different from the ground state.
   → SU(3) lattice gauge theory

(2) Use these potentials to approximately compute the spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.
   → Born-Oppenheimer approximation, effective field theories, quantum mechanics

(3) Explore hybrid static potential flux tubes by computing the chromoelectric and chromomagnetic energy density.
   → SU(2) and SU(3) lattice gauge theory

• The talk focuses on (1) and summarizes
  [C. Schlosser, M.W., [arXiv:2111.00741]]

• Further references on (1):
   [arXiv:1811.11046 [hep-lat]]]
Hybrid static potentials: quantum numbers

- "(Hybrid) static potential states" can be characterized by the following quantum numbers:
  - Absolute total angular momentum with respect to the $\bar{Q}Q$ separation axis ($z$ axis):
    \[ \Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots \]
  - Parity combined with charge conjugation:
    \[ \eta = +, - = g, u. \]
  - Reflection along an axis perpendicular to the $\bar{Q}Q$ separation axis ($x$ axis):
    \[ \epsilon = +, - \]

- For $\Lambda \geq 1$ potentials are degenerate with respect to $\epsilon$, i.e. $V_{\Lambda \eta}^+(r) = V_{\Lambda \eta}^-(r)$
  \[ \rightarrow \text{use quantum numbers } \Lambda_{\eta}^\epsilon \text{ for } \Lambda = \Sigma \]
  \[ \rightarrow \text{use quantum numbers } \Lambda_{\eta}^\epsilon \text{ for } \Lambda = \Pi, \Delta, \ldots \]

- The ordinary static potential has quantum numbers $\Lambda_{\eta}^\epsilon = \Sigma_g^+.$

- In this talk I focus on the two lowest hybrid static potentials with quantum numbers $\Lambda_{\eta}^\epsilon = \Pi_u, \Sigma_u^-.$
Hybrid static potentials: trial states (1)

- To determine the hybrid static potential with quantum numbers $\Lambda_\eta^\xi$, compute the temporal correlation functions of suitable trial states,

$$W_{S, S'; \Lambda_\eta^\xi}(r, t) = \langle \Psi_{\text{hybrid}}(t)|_{S; \Lambda_\xi^\eta} | \Psi_{\text{hybrid}}(0)\rangle_{S'; \Lambda_\xi^\eta} \sim_{t \to \infty} \exp\left(-V_{\Lambda_\eta^\xi}(r)t\right).$$

- Trial states are

$$|\Psi_{\text{hybrid}}\rangle_{S; \Lambda_\eta^\xi} = \bar{Q}(-r/2) a_{S; \Lambda_\eta^\xi}(-r/2, +r/2) Q(+r/2) |\Omega\rangle$$

with gluonic parallel transporters (on the lattice products of gauge links)

$$a_{S; \Lambda_\eta^\xi}(-r/2, +r/2) =$$

$$= \frac{1}{4} \sum_{k=0}^{3} \exp\left(\frac{i \pi \Lambda k}{2}\right) \left\{ U(-r/2, r_1) \left( S(r_1, r_2) + \epsilon S_{\mathcal{P}_x}(r_1, r_2) \right) U(r_2, +r/2) +
$$

$$U(-r/2, -r_2) \left( \eta S_{\mathcal{P}_x C}(-r_2, -r_1) + \eta \epsilon S_{(\mathcal{P}_x C)\mathcal{P}_x}(-r_2, -r_1) \right) U(-r_1, +r/2) \right\}$$

generating quantum numbers $\Lambda_\eta^\xi$. 
Hybrid static potentials: trial states (2)

- For $a_{S;\Lambda_{ij}^\ell}(-r/2, +r/2)$, which define the trial states

$$|\Psi_{\text{hybrid}}\rangle_{S;\Lambda_{ij}^\ell} = \bar{Q}(-r/2)a_{S;\Lambda_{ij}^\ell}(-r/2, +r/2)Q(+r/2)|\Omega\rangle,$$

we have explored many different shapes and variations of their extents.

- For the final computation of each hybrid static potential $V_{\Lambda_{ij}^\ell}(r)$ we have used an optimized set of 3 to 4 creation operators and have solved generalized eigenvalue problems for the corresponding correlation matrices (arXiv:1811.11046) or the operator with the largest ground state overlap (arXiv:2111.00741).
Hybrid static potentials: results (1)


- Discrepancies to existing results for $V_{\Pi g}(r)$ and $V_{\Delta u}(r)$ at small $\bar{Q}Q$ separation $r \leq 0.25$ fm. [K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90, 161601 (2003) [hep-lat/0207004]]

- Observed degeneracies of $V_{\Sigma_g^+}(r)$, $V_{\Pi g}(r)$ and $V_{\Sigma_u^+}(r)$, $V_{\Delta u}(r)$ at small $r$ expected from pNRQCD.
Hybrid static potentials: results (2)


- Computations of the $\Pi_u$ and the $\Sigma_u^{-}$ hybrid static potentials (the two lowest hybrid static potentials) on four additional ensembles with lattice spacing $a$ as small as 0.04 fm (previously a single ensemble with $a = 0.093$ fm).
  → Lattice data points at significantly smaller separations.
  → Determination and elimination of discretization errors possible ("continuum limit").

<table>
<thead>
<tr>
<th>ensemble</th>
<th>$\beta$</th>
<th>$a$ in fm</th>
<th>$(L/a)^3 \times T/a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>6.000</td>
<td>0.093</td>
<td>$12^3 \times 26$</td>
</tr>
<tr>
<td>$B$</td>
<td>6.284</td>
<td>0.060</td>
<td>$20^3 \times 40$</td>
</tr>
<tr>
<td>$C'$</td>
<td>6.451</td>
<td>0.048</td>
<td>$26^3 \times 50$</td>
</tr>
<tr>
<td>$D$</td>
<td>6.594</td>
<td>0.040</td>
<td>$30^3 \times 60$</td>
</tr>
</tbody>
</table>

- Self energy is $a$-dependent, needs to be subtracted, before all lattice data point can be shown together in a meaningful plot.
  → Fits needed (see next slides).

- Unsmearred temporal links (previously: HYP2 static action).

- Multilevel algorithm (previously: without multilevel algorithm).
Hybrid static potentials: results (3)

- 8-Parameter-Fit for the ordinary ($\Sigma_g^+$) static potential:

\[
V_{\Sigma_g^+}^{\text{fit},e}(r) = V_{\Sigma_g^+}(r) + C^e + \Delta V_{\Sigma_g^+}^{\text{lat},e}(r)
\]

\[
V_{\Sigma_g^+}(r) = -\frac{\alpha}{r} + \sigma r \quad \text{(Cornell ansatz)}
\]

\[
\Delta V_{\Sigma_g^+}^{\text{lat},e}(r) = \alpha' \left( \frac{1}{r} - \frac{G^e(r/a)}{a} \right) \quad \text{(difference of tree-level continuum and lattice result)}.
\]

- $C^e$: $a$-dependent self energies.
- $\Delta V_{\Sigma_g^+}^{\text{lat},e}(r)$: lattice discretization errors at tree-level.
- $V_{\Sigma_g^+}(r)$: parameterization of the ordinary static potential; useful to set the energy scale for $b$ and for $c$ quarks (via a Born-Oppenheimer computation of the quarkonium ground state $\eta_b(1S) \equiv \Upsilon(1S)$ or $\eta_c(1S) \equiv J/\Psi(1S)$ and identification with the corresponding experimental result).

- Fit parameters allow to define data points, with the self-energy subtracted and discretization errors at tree-level removed:

\[
\tilde{V}_{\Sigma_g^+}^{e}(r) = V_{\Sigma_g^+}^{e}(r) - C^e - \Delta V_{\Sigma_g^+}^{\text{lat},e}(r).
\]
Marc Wagner, "Hybrid static potentials in SU(3) lattice gauge theory at small quark-antiquark separations", November 09, 2021.
Hybrid static potentials: results (5)

• 10-Parameter-Fit for the $\Pi_u$ and $\Sigma^-_u$ hybrid static potentials:

$$V^\text{fit, e}_{\Lambda^{e}}(r) = V^e_{\Lambda^{e}}(r) + C^e + \Delta V^\text{lat, e}_{\text{hybrid}}(r) + A^e_{2,\Lambda^{e}} a^2,$$

$$V_{\Pi_u}(r) = \frac{A_1}{r} + A_2 + A_3 r^2,$$

$$V_{\Sigma^-_u}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 + \frac{B_1 r^2}{1 + B_2 r + B_3 r^2}.$$

$$\Delta V^\text{lat, e}_{\text{hybrid}}(r) = -\frac{1}{8} \Delta V^\text{lat, e}_{\Sigma^-_u}(r).$$

- $A^e_{2,\Lambda^{e}} a^2$: leading order ($\propto a^2$) lattice discretization error in the difference to the ordinary static potential.

- $V_{\Pi_u}(r), V_{\Sigma^-_u}(r)$: parameterizations of the $\Pi_u$ and $\Sigma^-_u$ hybrid static potentials; consistent with and motivated by the pNRQCD prediction at small $r$.

• Fit parameters allow to define data points, with the self-energy subtracted and discretization errors to a large part removed:

$$\tilde{V}^e_{\Lambda^{e}}(r) = V^e_{\Lambda^{e}}(r) - C^e - \Delta V^\text{lat, e}_{\text{hybrid}}(r) - A^e_{2,\Lambda^{e}} a^2.$$
Hybrid static potentials: results (6)
Hybrid static potentials: results (7)

- Exclusion of the following types of systematic errors:
  
  - **Topological freezing:**
    * Monte Carlo algorithms have difficulties changing the topological charge $Q$ for lattice spacings $a \lesssim 0.05$ fm.
    * Monte Carlo histories of $Q$ need to be checked.
    * Lengthy simulations necessary for small $a$.
  
  - **Finite volume corrections:**
    * Negative energy shifts, because of virtual glueballs traveling around the far side of the periodic spacetime volume.
    * Positive energy shifts, because of squeezed wave functions.
    * Corrections turned out to be negligible for $L \gtrsim 1.2$ fm.
  
  - **Glueball decays:**
    * At small $r$ hybrid flux tubes can decay into $\Sigma_g^+$ flux tubes and glueballs.
    * $\Sigma_u^-$ flux tube: protected by symmetries from decays into a $0^{++}$ glueball.
    * $\Pi_u$: decays into a $0^{++}$ glueball possible for $r \lesssim 0.11$ fm; however, no indication that $V_{\Pi_u}^c(r)$ is contaminated by such decays (the $\Pi_u$ and $\Sigma_u^-$ potentials approach each other for small $r$).
Hybrid static potentials: results (8)

- Summary of improvements:
  - Separations as small as 0.08 fm.
  - Self energies subtracted.
  - Lattice discretization errors removed to a large extent.
  - Various systematic errors checked and excluded.

- Improvements important: Born-Oppenheimer predictions of heavy hybrid meson masses change by $O(10 \text{ MeV} \ldots 45 \text{ MeV})$.

- Bare lattice data, improved lattice data and parameterizations provided. [C. Schlosser, M.W., arXiv:2111.00741]
**bb** and **cc** hybrid meson masses: BO

- Born-Oppenheimer approximation: Compute $\bar{b}b$ and $\bar{c}c$ hybrid meson masses in two steps.

  1. **Compute potentials of two static quarks** ($\bar{b}b$ or $\bar{c}c$) in the presence of excited gluons generating quantum numbers $\Lambda^\epsilon_\eta$ using lattice gauge theory.

  2. **Solve Schrödinger equations** for the relative coordinate of $\bar{b}b$ or $\bar{c}c$ using the potentials from (1) and the mass of either the $b$ or the $c$ quark, e.g.

\[
\left(-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{L(L+1)-2\Lambda^2 + J_{\Lambda^\epsilon_\eta}(J_{\Lambda^\epsilon_\eta}+1)}{2\mu r^2} + V_{\Lambda^\epsilon_\eta}(r)\right)u_{\Lambda^\epsilon_\eta;L,n}(r) = E_{\Lambda^\epsilon_\eta;L,n}u_{\Lambda^\epsilon_\eta;L,n}(r).
\]

Energy eigenvalues $E_{\Lambda^\epsilon_\eta;L,n}$ correspond to masses of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.

Hybrid flux tubes: computation

- We are interested in
  \[ \Delta F_{\mu
u,\Lambda_\eta}^2(r; x) = \langle 0_{\Lambda_\eta}(r) | F_{\mu
u}^2(x) | 0_{\Lambda_\eta}(r) \rangle - \langle \Omega | F_{\mu
u}^2 | \Omega \rangle. \]

- \( F_{\mu
u}^2(x), F_{\mu\nu}^2 \): squared chromoelectric/chromomagnetic field strength.
- \( |0_{\Lambda_\eta}(r)\rangle \): “hybrid static potential (ground) state” (\( r \) denotes the \( \bar{Q}Q \) separation).
- \( |\Omega\rangle \): vacuum state.

- The sum over the six independent \( \Delta F_{\mu\nu,\Lambda_\eta}^2(r; x) \) is proportional to the chromoelectric and -magnetic energy density of hybrid flux tubes.

- With lattice gauge theory \( \Delta F_{\mu\nu,\Lambda_\eta}^2(r; x) \) can be computed via
  \[ \Delta F_{\mu\nu,\Lambda_\eta}^2(r; x) = \pm \frac{\langle \bar{W}(r, t_2, t_0) P_{\mu\nu}(x, t_1) \rangle_U}{\langle \bar{W}(r, t_2, t_0) \rangle_U} \mp \langle P_{\mu\nu} \rangle_U. \]

- \( P_{\mu\nu}(x), P_{\mu\nu} \): plaquette, i.e. lattice gauge theory expression for \( F_{\mu\nu}^2(x) \).
- \( \bar{W}(r, t_2, t_0) \): “Wilson loop” (spatial extent \( r \), temporal extent \( t_2 - t_1 \)), with spatial parallel transporters as in the “hybrid static potential trial states”.

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Hybrid flux tubes: results (1)

- $\Delta F^2_{\mu \nu, \Lambda_\eta}(r; x)$, SU(2), mediator plane ($x$-$y$ plane with $Q$, $\bar{Q}$ at $(0, 0, \pm r/2)$), $r \approx 0.8$ fm.

- See also [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D 98, 114507 (2018) [arXiv:1808.08815 [hep-lat]]] for results for $\Lambda_\eta^\epsilon = \Sigma_{g}^+, \Sigma_{u}^+, \Pi_{u}$.

$\Delta E^2_x | \Delta E^2_y | \Delta E^2_z$

$\Delta B^2_x | \Delta B^2_y | \Delta B^2_z$

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Hybrid flux tubes: results (2)

- $\Delta F_{\mu\nu,\Lambda_1}(r; x) - \Delta F_{\mu\nu,\Sigma_1}(r; x)$, SU(2), mediator axis ($x$ axis with $Q, \bar{Q}$ at $(0, 0, \pm r/2)$), $r \approx 0.8$ fm.

- Chromoelectric and chromomagnetic field strengths reflect typical operators used to study hybrid static potentials, e.g. in pNRQCD.
- $\Delta F_{\mu\nu,\Lambda^\epsilon}(r;x)$, SU(2), separation plane ($x$-$z$ plane with $Q$, $\bar{Q}$ at $(0, 0, \pm r/2)$), $r \approx 0.8$ fm. [L. Müller, O. Philipson, C. Reisinger, M. Wagner, Phys. Rev. D 100, 054503 (2019) [arXiv:1907.014820]]

- See also [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D 98, 114507 (2018) [arXiv:1808.08815 [hep-lat]]] for results for $\Lambda^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$.