Evidence for the existence of $ud\bar{b}\bar{b}$ and the non-existence of $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks from lattice QCD

Physics Day of the HIC for FAIR Expert Group 1

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Goals, motivation (1)

- Study tetraquarks in the following way:

  (1) **Compute potentials of two heavy quarks** (e.g. $\bar{b}\bar{b}$) **in the presence of two lighter quarks** (e.g. ud, ss or cc) **using lattice QCD.**

  (2) **Explore, whether these potentials are sufficiently attractive to host bound states** (rather stable tetraquarks [diquark-antidiquark pairs, mesonic molecules, ...]) **by solving Schrödinger equations.**

- This talk is a summary of

  [M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538]]
  [M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147]]
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [B. Wagenbach, P. Bicudo, M.W., J. Phys.: Conf. Ser. 599, 012006 (2015) [arXiv:1411.2453]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., arXiv:1505.00613]
  [J. Scheunert, P. Bicudo, A. Uenver, M.W., arXiv:1505.03496]

- For recent work from other groups using a similar approach cf. e.g.

  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571 [hep-lat]]
  [Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953 [hep-lat]]

Goals, motivation (2)

- **Why are such investigations important?**
  $\rightarrow$ Quite a number of mesons are only poorly understood.
  - Charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states.
  - Charged charmonium states, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$ ... must be four quark states.
  - $X(3872)$: mass not as expected from quark models; could be a D-D^* molecule, a bound diquark-antidiquark, ...

Outline

- A brief introduction to lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Hadron spectroscopy.
  - Lattice QCD.
- Heavy-heavy-light-light tetraquarks.
- BB static potentials.
- BB tetraquarks.
- $B\bar{B}$ static potentials.
- Inclusion of B/B^* mass splitting.
- Outlook.

QCD (quantum chromodynamics)

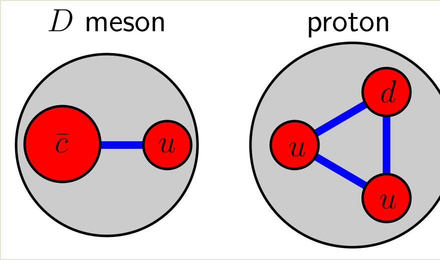
- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (usually mesons $= q\bar{q}$ and baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD simple:

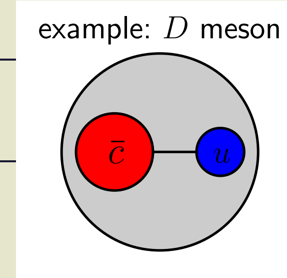
$$S = \int d^4x \Big(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - iA_\mu\Big) + m^{(f)}\Big)\psi^{(f)} + \frac{1}{2g^2}\mathrm{Tr}\Big(F_{\mu\nu}F_{\mu\nu}\Big)\Big)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$



- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).
  $\rightarrow$ Solve QCD numerically by means of lattice QCD.

Hadron spectroscopy



- Proceed as follows:

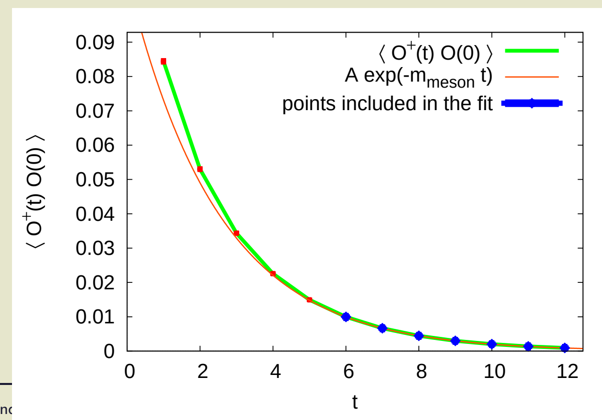
  (1) Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator O (an operator O, which generates the quantum numbers of the hadron of interest, when applied to the vacuum $|\Omega\rangle$).

  (2) Determine the corresponding hadron mass from the asymptotic exponential decay in time.

- Example: D meson mass m_D (valence quarks $\bar{c}$ and u, $J^P = 0^-$),

$$O \equiv \int d^3r \, \bar{c}(\mathbf{r})\gamma_5 u(\mathbf{r})$$

$$C(t) \equiv \langle \Omega | O^\dagger(t) O(0) | \Omega \rangle \stackrel{t\to\infty}{\propto} \stackrel{t\to\infty}{\propto} \exp\Big(- m_D t \Big).$$



Lattice QCD (1)

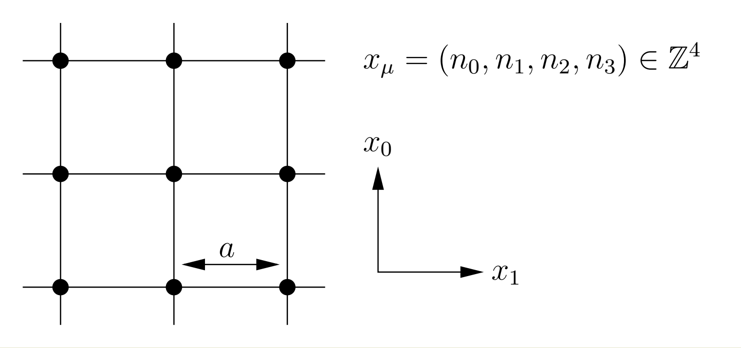
- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle = \\ = \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,O^\dagger(t)O(0)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slides).
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x},t)$ and $A_\mu(\mathbf{x},t)$.
  - $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite volume effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi \, D\bar{\psi} \, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu) \, d\bar{\psi}(x_\mu) \, dU(x_\mu) \bigg) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    * x_μ: $32^4 \approx 10^6$ lattice sites.
    * $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    * $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    * In total: $32^4 \times (2 \times 24 + 32) \approx$ **83** $\times$ **10^6** dimensional integral.

  $\rightarrow$ standard approaches for numerical integration not applicable

  $\rightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

Heavy-heavy-light-light tetraquarks (1)

- Study possibly existing $\bar{Q}\bar{Q}qq$ and $\bar{Q}Q\bar{q}q$ tetraquark states ($q \in \{u, d, s, c\}$):
  - Use the static approximation for the heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$ (reduces the necessary computation time significantly).
  - Most appropriate for $\bar{Q}\bar{Q} \equiv \bar{b}\bar{b}$ and $\bar{Q}Q \equiv \bar{b}b$, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$.
  - Could also provide information about $\bar{Q}\bar{Q} \equiv \bar{c}\bar{c}$ and $\bar{Q}Q \equiv \bar{c}c$, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$.

- Proceed in two steps:

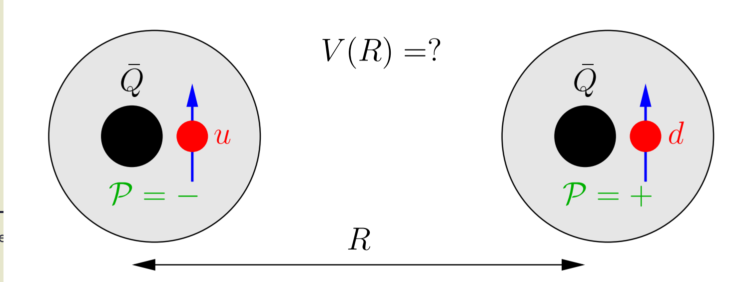
  (1) Compute potentials of two "heavy quarks" $\bar{Q}\bar{Q}$ and $\bar{Q}Q$ in the presence of two "light quarks" qq and $\bar{q}q$ ($q \in \{u, d, s, c\}$) using lattice QCD.

  (2) Solve non-relativistic Schrödinger equations for the relative coordinate of the heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$; the existence of bound states would indicate rather stable tetraquark states.

Heavy-heavy-light-light tetraquarks (2)

- The spins of the two static quarks Q are irrelevant.
- At large $\bar{Q}\bar{Q}$ (or $\bar{Q}Q$) separation R, the four quarks will form two static-light mesons $\bar{Q}q$ and $\bar{Q}q$ (or $\bar{Q}q$ and $\bar{q}Q$).
- Consider only pseudoscalar/vector mesons ($j^\mathcal{P} = (1/2)^-$, PDG: B, B^*) and scalar/pseudovector mesons ($j^\mathcal{P} = (1/2)^+$, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons (j: spin of the light degrees of freedom).
- Study the dependence of the 4-quark/2-meson potential $V(r)$ on
  - the "light" quark flavors u, d, s and/or c (isospin),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^* (parity).

$\rightarrow$ Many different channels/ quantum numbers ... attractive, repulsive ...

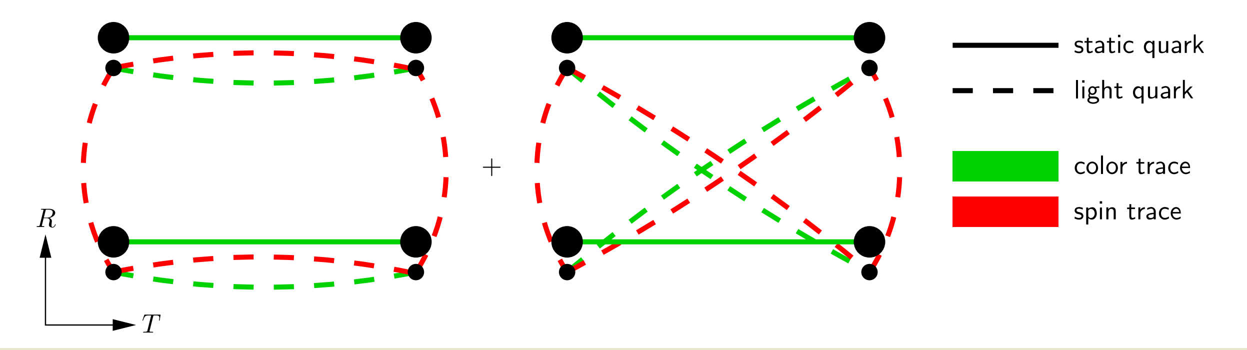


BB static potentials (1)

- In the following $\bar{Q}\bar{Q}qq$, i.e. "BB" (not $\bar{Q}Q\bar{q}q$, i.e. "$B\bar{B}$").
- To extract the potential(s) of a given sector $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$, compute the temporal correlation function of the trial state

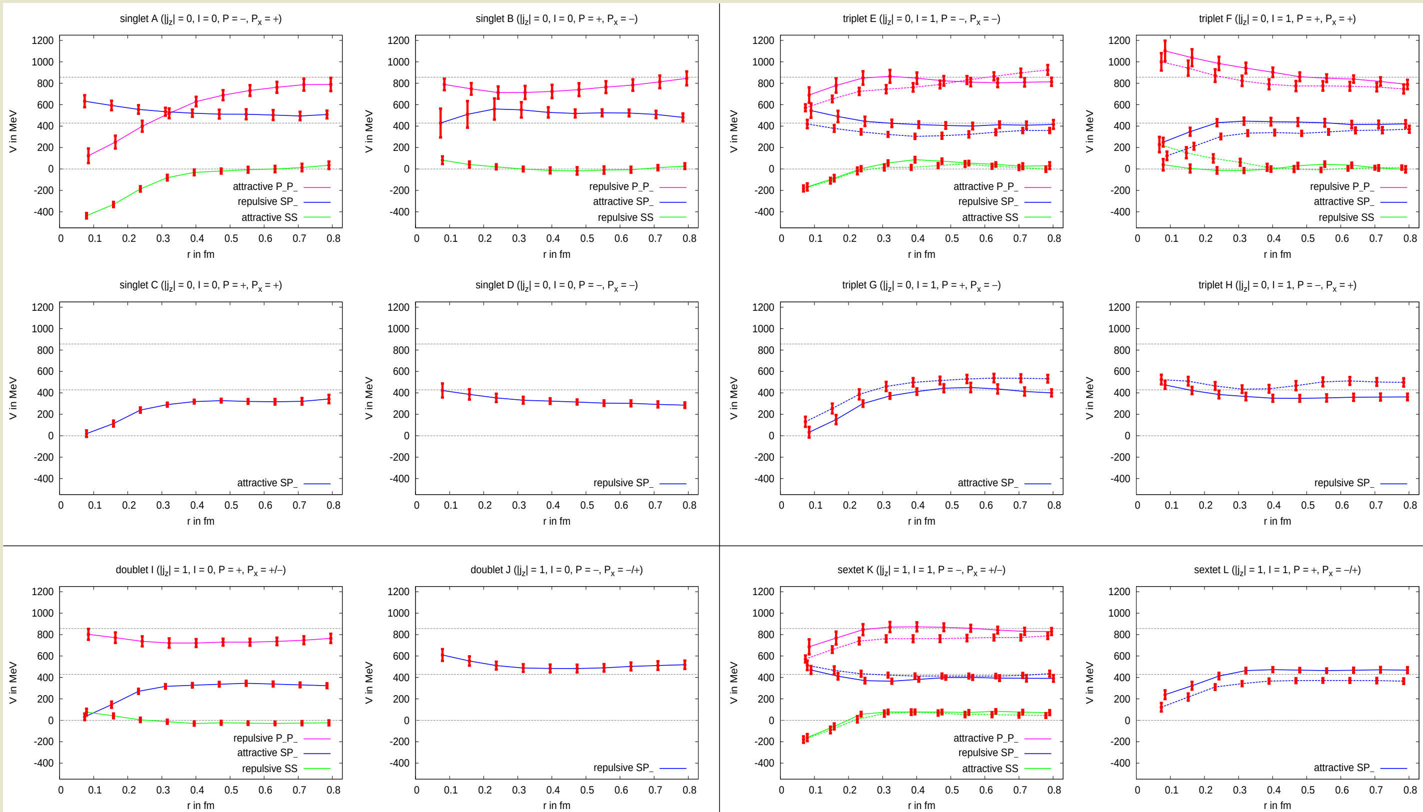
$$(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{Q}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big)|\Omega\rangle.$$

  - $\mathcal{C} = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $q^{(1)}q^{(2)} \in \{ud - du\,,\, uu\,,\, dd\,,\, ud + du\,,\, ss\,,\, cc\}$ (isospin I, I_z).
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}$, $\mathcal{P}_x$).
  - $\tilde{\Gamma} \in \{(1 - \gamma_0)\gamma_5\,,\, (1 - \gamma_0)\gamma_j\}$ (irrelevant).



BB static potentials (2)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



BB static potentials (3)

Why are certain channels attractive and others repulsive? (1)

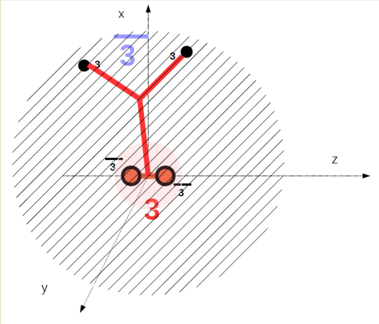
- Wave function of two identical fermions (light quarks $q^{(1)}q^{(2)}$) must be antisymmetric (Pauli principle); in quantum field theory/QCD automatically realized on the level of states.
- $q^{(1)}q^{(2)}$ isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- $q^{(1)}q^{(2)}$ spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- $q^{(1)}q^{(2)}$ color:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e. a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e. a sextet 6.
- The four quarks $\bar{Q}\bar{Q}q^{(1)}q^{(2)}$ must form a color singlet:
  - $q^{(1)}q^{(2)}$ in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{Q}\bar{Q}$ also in a triplet 3.
  - $q^{(1)}q^{(2)}$ in a color sextet 6 $\rightarrow$ static quarks $\bar{Q}\bar{Q}$ also in a sextet $\bar{6}$.

BB static potentials (4)

Why are certain channels attractive and others repulsive? (2)

- Assumption: attractive/repulsive behavior of $\bar{Q}\bar{Q}$ at small separations r is mainly due to 1-gluon exchange (the static quarks $\bar{Q}\bar{Q}$ are rather close, inside a large light quark cloud formed by $q^{(1)}q^{(2)}$, i.e. no color screening of the color charges $\bar{Q}\bar{Q}$ due to $q^{(1)}q^{(2)}$),
  - color triplet 3 is attractive, $V(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V(r) = +\alpha_s/3r$

  (easy to calculate in LO perturbation theory).



- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{Q}\bar{Q}$ potential $V(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{Q}\bar{Q}$ potential $V(r)$.

This expectation is consistent with the obtained lattice results (for ground state potentials $[B, B^*]$; can be extended to excitations $[B_0^*, B_1^*]$).

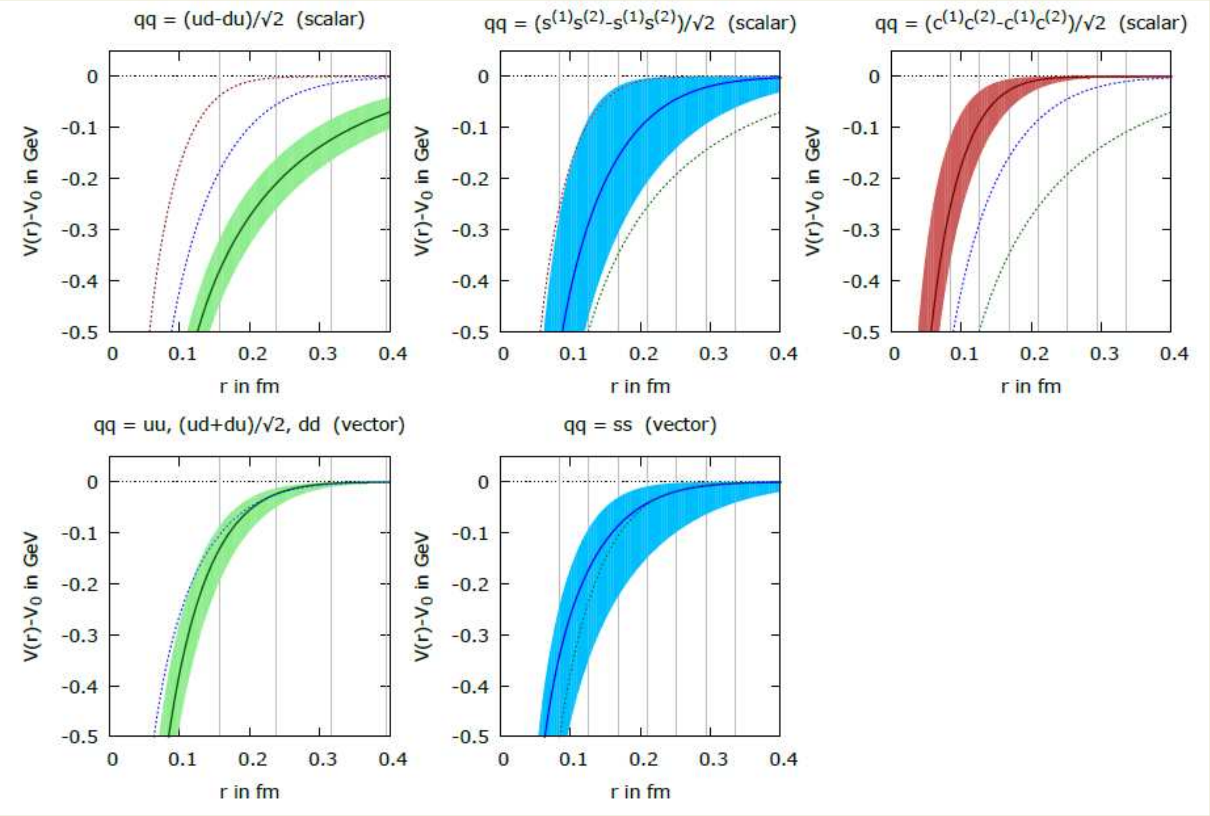
BB static potentials (5)

- Focus on the two attractive channels between B and B^*:
  - Scalar isosinglet (more attractive, $(I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (0, 0, -, +)$): $qq = (ud - du)/\sqrt{2}$, $\Gamma = (1 + \gamma_0)\gamma_5$.
  - Vector isotriplet (less attractive, $(I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (1, \{0, 1\}, -, \pm)$): $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$, $\Gamma = (1 + \gamma_0)\gamma_j$.
- Computations for $qq = ll, ss, cc$ ($l \in \{u, d\}$) to study the mass dependence.
- Describe the lattice potential results by contiuous functions obtained by χ^2 minimizing fits of

$$V(r) \quad = \quad -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0 :$$

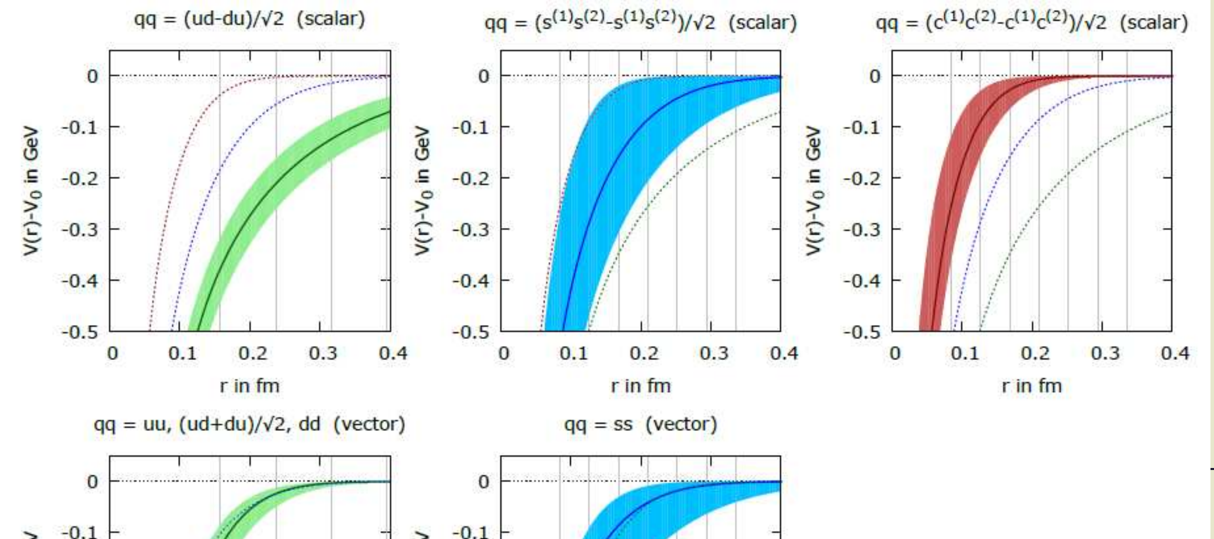
  - $1/r$ term: 1-gluon exchange at small $\bar{Q}\bar{Q}$ separations.
  - $\exp(-(r/d)^p)$ term: color screening at large $\bar{Q}\bar{Q}$ separations due to meson formation.
  - Fit parameters α, d and V_0; $p = 2$ from quark models.

BB static potentials (6)



BB static potentials (7)

- Two competing effects:
  - The potential for light quarks is wider/deeper, i.e. favors the existence of a bound state (a tetraquark).
  - Heavier quarks correspond to heavier mesons ($m(B) < m(B_s) < m(B_c)$), which form more readily a bound state (a tetraquark), i.e. require a less wide/deep potential for a bound state.



BB tetraquarks (1)

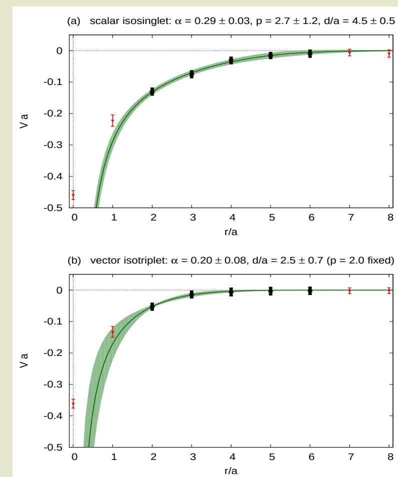
- Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}\bar{Q}$,

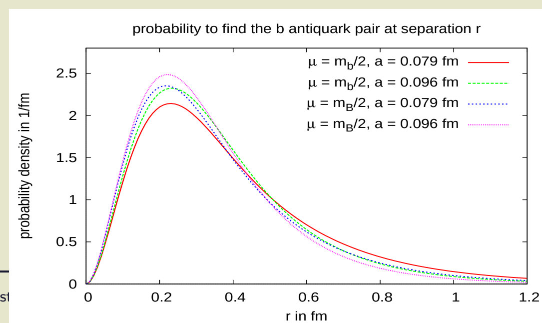
$$\left(-\frac{1}{2\mu}\triangle + V(r)\right)\underbrace{\psi(\mathbf{r})}_{=R(r)/r} = E\psi(\mathbf{r}) \quad , \quad \mu = m(B_{(s,c)})/2;$$

  a bound state, i.e. $E < 0$, would be an indication for a tetraquark state.

- There is a bound state for the scalar isosinglet and $qq = ll$ (i.e. BB), binding energy $E = -90^{+46}_{-42}\,\mathrm{MeV}$, i.e. confidence level $\approx 2\,\sigma$.

- No binding for the vector isotriplet or for $qq = ss, cc$ (i.e. $B_s B_s$, $B_c B_c$).



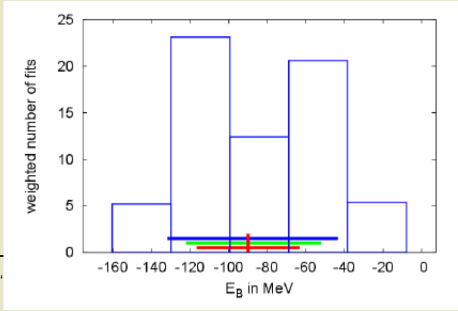


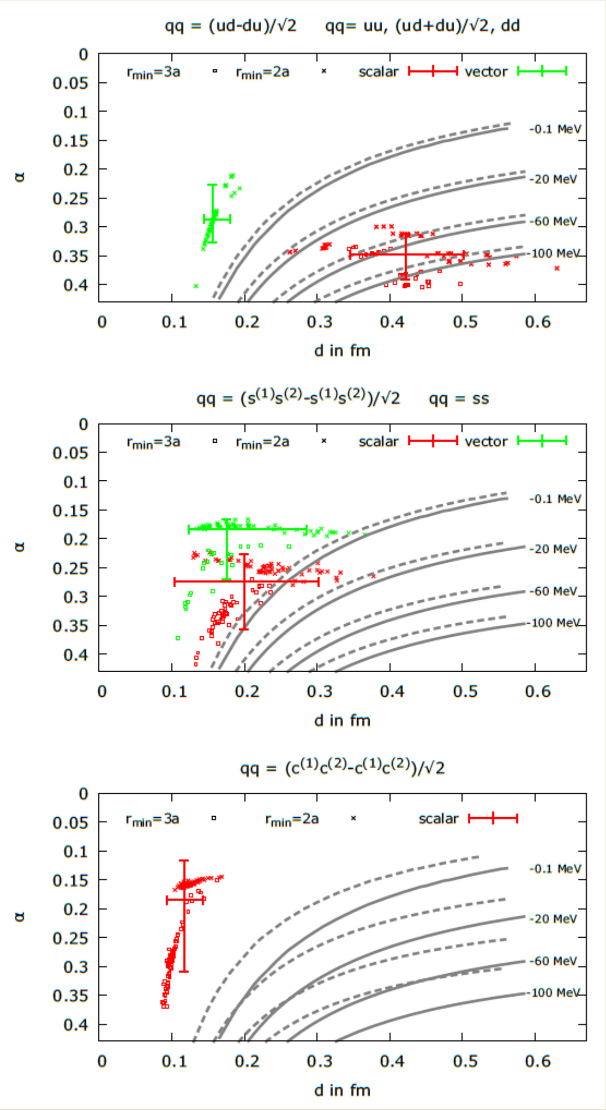
BB tetraquarks (2)

- Estimate the systematic error by varying input parameters:
  - the t fitting range to extract the potential from effective masses,
  - the r fitting range for

$$V(r) = -\frac{\alpha}{r} \exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0.$$

- Left: isoline plots of the binding energy E.
- Bottom: histogram for the binding energy E of the scalar isosinglet with $qq = ll$.





BB tetraquarks (3)

- To quantify "no binding", we list for each channel the factor, by which the effective mass μ in Schrödinger's equation has to be multiplied, to obtain a tiny but negative energy E.

qq	spin	factor
$(ud - du)/\sqrt{2}$	scalar	0.46
uu, $(ud + du)/\sqrt{2}$, dd	vector	1.49
$(s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}$	scalar	1.20
ss	vector	2.01
$(c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}$	scalar	2.57

  - Factors $\ll 1$ indicate strongly bound states, while for values $\gg 1$ bound states are essentially excluded.
  - Light quarks (u/d) are unphysically heavy (correspond to $m_\pi \approx 340\,\text{MeV}$); physically light u/d quarks are expected to yield stronger binding for the scalar isosinglet, might lead to binding also for the vector isotriplet (computations in progress).
  - Mass splitting $m(B^*) - m(B) \approx 50\,\text{MeV}$, neglected at the moment, is expected to weaken binding (coupled channel analysis; see later slides).

BB tetraquarks (4)

What are the quantum numbers of the $\bar{b}\bar{b}ll$ tetraquark (light scalar isosinglet)?

- Light scalar isosinglet: $I = 0$, $j = 0$, ll in a color $\bar{3}$, $\bar{b}\bar{b}$ in a color 3 (antisymmetric) ... as discussed above.
- Wave function of $\bar{b}\bar{b}$ must also be antisymmetric (Pauli principle); in the lattice QCD computation not automatically realized (static quarks are spinless color charges, which can be distinguished by their positions).
  - $\bar{b}\bar{b}$ is flavor symmetric.
  - $\bar{b}\bar{b}$ spin must also be symmetric, i.e. $j_b = 1$.
- **The $\bar{b}\bar{b}ll$ tetraquark has isospin $I = 0$, spin $J = 1$.**
- We study states, which correspond for large $\bar{b}\bar{b}$ separations to pairs of $B^{(*)}_{(s,c)}$ mesons in a spatially symmetric s-wave. **Therefore, the $\bar{b}\bar{b}ll$ tetraquark has parity $P = +$** (the product of the parity quantum numbers of the two mesons, which are both negative).

$B\bar{B}$ static potentials

- Experimentally more interesting case: $\bar{Q}Q\bar{q}q$, i.e. "$B\bar{B}$", trial states

$$\Gamma_{AB}\tilde{\Gamma}_{CD}\Big(\bar{Q}_C(-\mathbf{r}/2)q_B^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{q}_A^{(2)}(+\mathbf{r}/2)Q_D(+\mathbf{r}/2)\Big)|\Omega\rangle.$$

- At the moment only preliminary results for $\bar{q}q = \bar{c}c$, "$I = 1$".
- Qualitative difference to $\bar{Q}\bar{Q}qq$: all channels are attractive (for $\bar{Q}\bar{Q}qq$ half of them are attractive, half of them are repulsive).
- Can again be understood by the 1-gluon exchange potential of $\bar{Q}Q$:
  - No Pauli principle for $\bar{q}^{(1)}q^{(2)}$ (particle and antiparticle are not identical).
  - $\bar{q}^{(1)}q^{(2)}$ can be in a symmetric color singlet 1 for any isospin/spin orientation.
  - $\bar{q}^{(1)}q^{(2)}$ in a color singlet 1 $\rightarrow$ static quarks $\bar{Q}Q$ also in a singlet 1.
  - Color singlet is attractive, $V(r) = -4\alpha_s/3r$ (LO perturbation theory).

Inclusion of B/B^* mass splitting (1)

- Mass splitting $m_{B^*} - m_B \approx 46\,\text{MeV}$ has been neglected so far.
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $E = -90^{+46}_{-42}\,\text{MeV}$.
- Moreover, two competing effects:
  - An attractive $\bar{Q}\bar{Q}qq$ channel corresponds to a linear combination of BB, BB^* and/or B^*B^*, e.g.

    $$\text{scalar isosinglet} \quad \equiv \quad BB + B^*_x B^*_x + B^*_y B^*_y + B^*_z B^*_z.$$
  - The BB interaction is a superposition of attractive and repulsive $\bar{Q}\bar{Q}qq$ potentials.
- Goal: take mass splitting $m_{B^*} - m_B \approx 46\,\text{MeV}$ into account
  $\rightarrow$ refined "Schrödinger calculation" with the computed $\bar{Q}\bar{Q}qq$ potentials.
- **Will there still be a bound state?**

Inclusion of B/B^* mass splitting (2)

Solve a coupled channel Schrödinger equation (1)

- Previously:
  - A wave function ψ with 1 component corresponding to BB ($B \equiv B^*$).
- Now:
  - A static light meson can correspond to B or $B^* = (B^*_x, B^*_y, B^*_z)$.
  - Therefore, a wave function $\vec{\psi}$ with 16 components corresponding to $(BB, BB^*_x, BB^*_y, BB^*_z, B^*_x B, B^*_x B^*_x, B^*_x B^*_y, B^*_x B^*_z, \ldots, B^*_z B^*_z)$.
- Coupled channel Schrödinger equation $H\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2) \quad = \quad E\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2)$,

$$H \quad = \quad \frac{\mathbf{p}_1^2}{2m_b} + \frac{\mathbf{p}_2^2}{2m_b} + M \otimes 1 + 1 \otimes M + V(|\mathbf{r}_1 - \mathbf{r}_2|),$$

where $M = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$ and V is a 16×16 non-diagonal matrix containing the $\bar{Q}\bar{Q}qq$ potentials (both attractive and repulsive).

Inclusion of B/B^* mass splitting (3)

Solve a coupled channel Schrödinger equation (2)

- Coupled channel Schrödinger equation $H\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2) \quad = \quad E\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2)$,

  $$H \quad = \quad \frac{\mathbf{p}_1^2}{2m_b} + \frac{\mathbf{p}_2^2}{2m_b} + M \otimes 1 + 1 \otimes M + V(|\mathbf{r}_1 - \mathbf{r}_2|),$$

  where $M = \mathrm{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$ and V is a 16×16 non-diagonal matrix containing the $\bar{Q}\bar{Q}qq$ potentials (both attractive and repulsive).

- Specific limits:
  - $V = 0$, i.e. no interactions:

    $$E \quad = \quad m_B + m_B \ , \ m_B + m_{B^*} \ , \ \ldots$$

  - $m_{B^*} = m_B$, i.e. "old 1-component SE calculation":

    $$E \quad = \quad 2m_B - 93^{+47}_{-43}\,\text{MeV}.$$

Inclusion of B/B^* mass splitting (4)

Solve a coupled channel Schrödinger equation (3)

- Transform the 16×16 Schrödinger equation to block diagonal structure:
  - Total spin $J = 0$: 2×2 structure.
  - Total spin $J = 1$: 3×3 structure ($3\times$ due to J_z degeneracy).
  - Total spin $J = 2$: 1×1 structure ($5\times$ due to J_z degeneracy).
- Work in progress ...
  - **Rather preliminary results indicate that for $I(J^P) = 0(1^+)$ the bound state does still exist** (however, with significantly reduced binding energy, $E \approx 2m_B - 15\,\text{MeV}$)**.**
  - Persistence of the bound state additionally supported by unphysically heavy u/d quarks ($m_\pi \approx 340\,\mathrm{MeV}$) ... physically light quarks are rexpected to lead to more attractive $\bar{Q}\bar{Q}qq$ potentials.

Outlook (1)

- **Study BB, which is experimentally more relevant** $(Z_b(10610)^+$, $Z_b(10650)^+$, ...)**.**

- Future plans for BB and $B\bar{B}$:
  - Computations with light u/d quarks of physical mass ($m_\pi \approx 140\,$MeV instead of $m_\pi \approx 340\,$MeV).
  - Light quarks of different mass: BB_s, BB_c and B_sB_c potentials.

Outlook (2)

- Future plans for BB and $B\bar{B}$:
  - Study the structure of the states corresponding to the computed potentials:
    * In a lattice computation two different creation operators generating the same quantum numbers yield the same potential.
    * At the moment exclusively creation operators of mesonic molecule type.
    * For BB use also
      - creation operators of diquark-antidiquark type.
    * For $B\bar{B}$ use also
      - creation operators of diquark-antidiquark type,
      - creation operators of bottomonium + pion type ($Q\bar{Q}$ string $+ \pi$),
      - for $I = 0$ creation operators of bottomonium type ($Q\bar{Q}$ string).
    * Resulting correlation matrices provide information about the structure.