

# Scalar mesons and tetraquarks by means of lattice QCD

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# Introduction, motivation (1)

- The nonet of light scalar mesons ( $J^P = 0^+$ )
  - $\sigma \equiv f_0(500)$ ,  $I = 0$ , 400 ... 550 MeV,
  - $\kappa \equiv K_0^*(800)$ ,  $I = 1/2$ ,  $682 \pm 29$  MeV,
  - $a_0(980)$ ,  $f_0(980)$ ,  $I = 1$ ,  $980 \pm 20$  MeV,  $990 \pm 20$  MeV

is poorly understood:

- All nine states are unexpectedly light (should rather be close to the corresponding  $J^P = 1^+, 2^+$  states around 1200 ... 1500 MeV).
- The ordering of states is inverted compared to expectation:
  - \* E.g. in a  $q\bar{q}$  picture the  $I = 1$  states  $a_0(980)$ ,  $f_0(980)$  must necessarily be formed by two  $u/d$  quarks, while the  $I = 1/2$   $\kappa$  states are made from an  $s$  and a  $u/d$  quark; since  $m_s > m_{u/d}$  one would expect  $m(\kappa) > m(a_0(980)), m(f_0(980))$ .

# Introduction, motivation (2)

- \* In a tetraquark picture the quark content could be the following:  
 $\kappa \equiv \bar{s}l\bar{l}l$ , while  $a_0(980), f_0(980) \equiv \bar{s}l\bar{l}s$ ; this would naturally explain the observed ordering.
- Certain decays also support a tetraquark interpretation: e.g.  $a_0(980)$  readily decays to  $K + \bar{K}$ , which indicates that besides the two light quarks required by  $I = 1$  also an  $s\bar{s}$  pair is present.
- Study these states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.
- Examples of heavy mesons, which are tetraquark candidates:
  - $D_{s0}^*(2317)^\pm$  ( $I(J^P) = 0(0^+)$ ),  $D_{s1}(2460)^\pm$  ( $I(J^P) = 0(1^+)$ ),
  - charmonium states  $X(3872)$ ,  $Z(4430)^\pm$ ,  $Z(4050)^\pm$ ,  $Z(4250)^\pm$ , ...
- FAIR, PANDA: charmonium spectroscopy,  $D$  meson spectroscopy (in particular  $D_s$ ), search for gluonic excitations (light scalar mesons).  
[\[http://www-panda.gsi.de/framework/content/physics/phys-spec.php\]](http://www-panda.gsi.de/framework/content/physics/phys-spec.php)

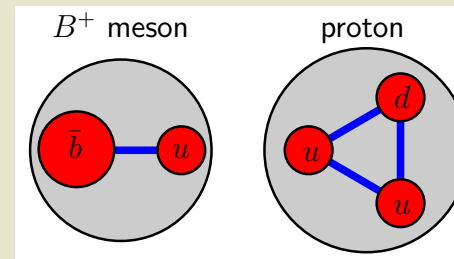
# QCD (quantum chromodynamics)

- Quantum field theory of **quarks** (six flavors  $u, d, s, c, t, b$ , which differ in **mass**) and **gluons**.
- Part of the standard model explaining the formation of hadrons (usually mesons =  $q\bar{q}$  and baryons =  $qqq/\bar{q}\bar{q}\bar{q}$ ) and their masses; essential for decays involving hadrons.
- Definition of QCD by means of an action simple:

$$S = \int d^4x \left( \sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \left( \gamma_\mu (\partial_\mu - iA_\mu) + m^{(f)} \right) \psi^{(f)} + \frac{1}{2g^2} \text{Tr} \left( F_{\mu\nu} F_{\mu\nu} \right) \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).



# Hadron spectroscopy (1)

- Let  $\mathcal{O}$  be a suitable “hadron creation operator”, i.e. an operator such that  $\mathcal{O}|\Omega\rangle$  is a state containing the hadron of interest ( $|\Omega\rangle$ : QCD vacuum).
- More precisely: ... an operator such that  $\mathcal{O}|\Omega\rangle$  has the same quantum numbers ( $J^{PC}$ , flavor) as the hadron of interest.
- Examples:
  - Pion creation operator:  $\mathcal{O} = \int d^3x \bar{u}(\mathbf{x})\gamma_5 d(\mathbf{x})$ .
  - Proton creation operator:  $\mathcal{O} = \int d^3x \epsilon^{abc} u^a(\mathbf{x})(u^{b,T}(\mathbf{x})C\gamma_5 d^c(\mathbf{x}))$ .

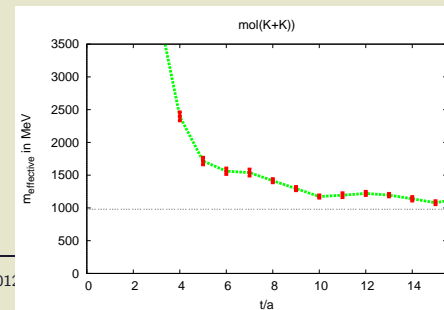
# Hadron spectroscopy (2)

- Determine the mass of the ground state of the hadron of interest from the exponential behavior of the corresponding correlation function  $\mathcal{C}$  at large Euclidean times  $T$ :

$$\begin{aligned}\mathcal{C}(t) &= \langle \Omega | (\mathcal{O}(t))^\dagger \mathcal{O}(0) | \Omega \rangle = \langle \Omega | e^{+Ht} (\mathcal{O}(0))^\dagger e^{-Ht} \mathcal{O}(0) | \Omega \rangle = \\ &= \sum_n \left| \langle n | \mathcal{O}(0) | \Omega \rangle \right|^2 \exp \left( - (E_n - E_\Omega) t \right) \approx \quad (\text{for } t \gg 1) \\ &\approx \left| \langle 0 | \mathcal{O}(0) | \Omega \rangle \right|^2 \exp \left( - \underbrace{(E_0 - E_\Omega)}_{m(\text{hadron})} t \right).\end{aligned}$$

- Usually the exponent is determined by identifying the “plateaux-value” of a so-called effective mass:

$$\begin{aligned}m_{\text{effective}}(t) &= \ln \left( \frac{\mathcal{C}(t)}{\mathcal{C}(t+1)} \right) \approx \quad (\text{for } t \gg 1) \\ &\approx E_0 - E_\Omega = m(\text{hadron}).\end{aligned}$$



# Tetraquark creation operators

- At the moment we study
  - $a_0(980)$ , mass  $980 \pm 20$  MeV, quantum numbers  $I(J^{PC}) = 1(0^{++})$ ;
  - $\kappa \equiv K_0^*(800)$ , mass  $682 \pm 29$  MeV, quantum numbers  $I(J^P) = 1/2(0^+)$ .
- Tetraquark operators for  $a_0(980)$  (quantum numbers  $I(J^{PC}) = 1(0^{++})$ ):

- Needs **two light quarks** due to  $I = 1$ , e.g.  $u\bar{d}$ .
- $a_0(980)$  decays to  $K\bar{K}$  ... suggests an  $s\bar{s}$  component.
- **Molecule type** (models a bound  $K\bar{K}$  state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \int d^3x \left( \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x}) \right) \left( \bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x}) \right).$$

- **Diquark type** (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \int d^3x \left( \epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \right) \left( \epsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \right).$$

# Lattice QCD (1)

- Goal: compute correlation functions  $\mathcal{C}(T)$  of the discussed tetraquark creation operators (corresponding hadron masses can directly be read off from their exponential decays).
- Use the path integral formulation of QCD,

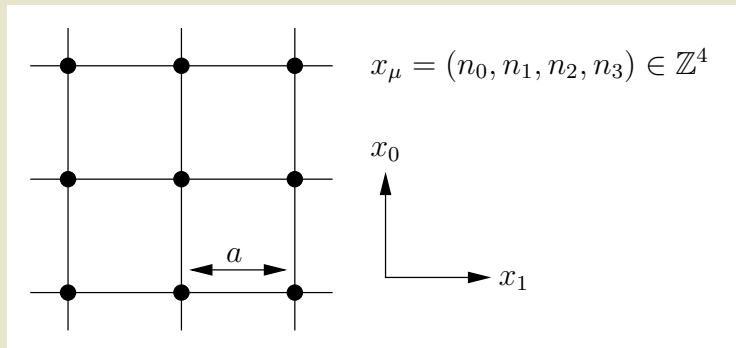
$$\begin{aligned}\mathcal{C}(t) &= \langle \Omega | \left( \mathcal{O}(\mathbf{x}, t) \right)^\dagger \mathcal{O}(\mathbf{x}, 0) | \Omega \rangle = \\ &= \frac{1}{Z} \int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu \left( \mathcal{O}(\mathbf{x}, t) \right)^\dagger \mathcal{O}(\mathbf{x}, 0) e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu]}.\end{aligned}$$

- $|\Omega\rangle$ : ground state/vacuum.
- $(\mathcal{O}(\mathbf{x}, t))^\dagger \mathcal{O}(\mathbf{x}, 0)$ : function of the quark and gluon fields (cf. previous slides).
- $\int (\prod_f D\psi^{(f)} D\bar{\psi}^{(f)}) DA_\mu$ : integral over all possible quark and gluon field configurations  $\psi^{(f)}(\mathbf{x}, t)$  and  $A_\mu(\mathbf{x}, t)$ .
- $e^{-S[x]}$ : weight factor containing the QCD action.



# Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing  
 $a \approx 0.05 \text{ fm} \dots 0.10 \text{ fm}$   
→ “continuum physics”.
  - “Make spacetime periodic” with sufficiently large extension  
 $L \approx 2.0 \text{ fm} \dots 4.0 \text{ fm}$  (4-dimensional torus)  
→ “no finite size effects”.



# Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi D\bar{\psi} DA \dots \rightarrow \prod_{x_\mu} \left( \int d\psi(x_\mu) d\bar{\psi}(x_\mu) dU(x_\mu) \right) \dots$$

- Typical present-day dimensionality of a discretized QCD path integral:
  - \*  $x_\mu$ :  $32^4 \approx 10^6$  lattice sites.
  - \*  $\psi = \psi_A^{a,(f)}$ : 24 quark degrees of freedom for every flavor ( $\times 2$  particle/antiparticle,  $\times 3$  color,  $\times 4$  spin), 2 flavors.
  - \*  $U = U_\mu^{ab}$ : 32 gluon degrees of freedom ( $\times 8$  color,  $\times 4$  spin).
  - \* In total:  $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$  dimensional integral.
- standard approaches for numerical integration not applicable
- sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

# Lattice setup (1)

- 2+1+1 dynamical quark flavors, i.e.  $u$ ,  $d$ ,  $s$  and  $c$  sea quarks.
- Lattice spacing  $a = 0.086$  fm (rather fine, computations at even finer lattice spacings planned).
- Various lattice volumes:
  - Small volume  $L^3 \times T = 20^3 \times 48$  lattice sites, spatial extension 1.73 fm  
→ rather easy to identify momentum excitations.  
(Most of the numerical results shown in the following were obtained with this volume.)
  - ...
  - Large volume  $L^3 \times T = 32^3 \times 64$  lattice sites, spatial extension 2.75 fm  
→ less finite size effects.
  - Different volumes needed to study resonances in a rigorous way.  
(Not done yet ... will be one of our next steps.)

# Lattice setup (2)

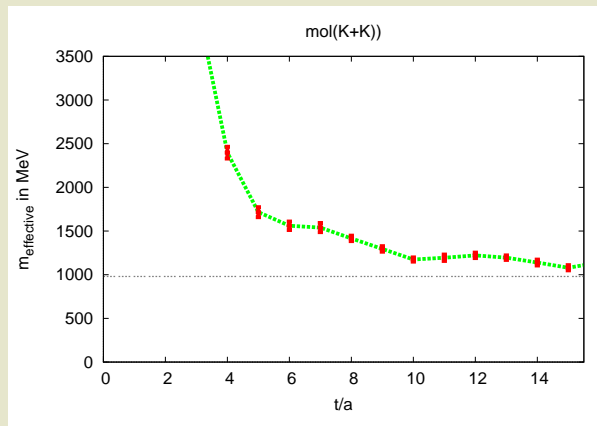
- Various light  $u/d$  quark masses, corresponding pion masses  $m_{\text{PS}} \approx 280 \dots 460 \text{ MeV}$  (physical light  $u/d$  quark masses [ $m_{\text{PS}} = m_{\pi} \approx 140 \text{ MeV}$ ] are technically extremely challenging; because of that in lattice QCD one usually studies several heavier quark masses and extrapolates to the “physical point”).

# Numerical results $a_0(980)$ (1)

- Effective mass, molecule type operator:

$$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \sum_{\mathbf{x}} \left( \bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right) \left( \bar{d}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \right).$$

- The effective mass plateau indicates a state, which is roughly consistent with the experimentally measured  $a_0(980)$  mass  $980 \pm 20$  MeV.
- Conclusion:  $a_0(980)$  is a tetraquark state of  $K\bar{K}$  molecule type ...?

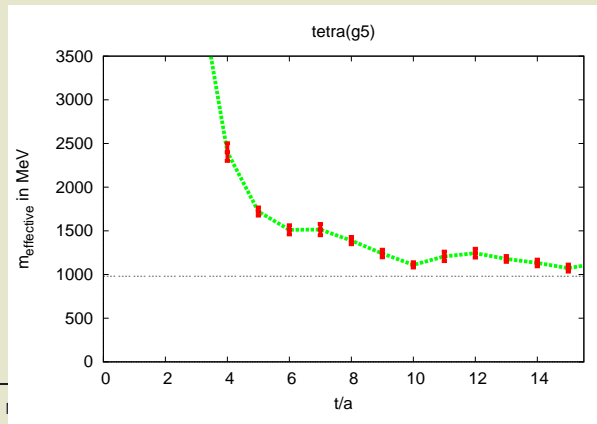


# Numerical results $a_0(980)$ (2)

- Effective mass, diquark type operator:

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \left( \epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \right) \left( \epsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \right).$$

- The effective mass plateaux indicates a state, which is roughly consistent with the experimentally measured  $a_0(980)$  mass  $980 \pm 20$  MeV.
- Conclusion:  $a_0(980)$  is a tetraquark state of diquark type ...? Or a mixture of  $K\bar{K}$  molecule and tetraquark?



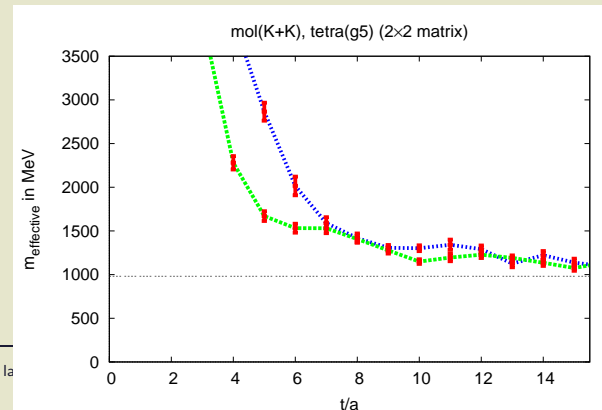
# Numerical results $a_0(980)$ (3)

- Study both operators at the same time, extract the two lowest energy eigenstates by diagonalizing a  $2 \times 2$  correlation matrix (“generalized eigenvalue problem”):

$$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \sum_{\mathbf{x}} \left( \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x}) \right) \left( \bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x}) \right)$$

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \left( \epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \right) \left( \epsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \right).$$

- Now two orthogonal states roughly consistent with the experimentally measured  $a_0(980)$  mass  $980 \pm 20$  MeV ...?



# Two-particle creation operators (1)

- Explanation: there are two-particle states, which have the same quantum numbers as  $a_0(980)$ ,  $I(J^{PC}) = 1(0^{++})$ ,

- $K + \bar{K}$  ( $m(K) \approx 500$  MeV),

- $\eta + \pi$  ( $m(\eta) \approx 700$  MeV,  $m(\pi) \approx 300$  MeV in our lattice setup),

which are both around the expected  $a_0(980)$  mass  $980 \pm 20$  MeV.

- What we have seen in the previous plots might actually be two-particle states (our operators are of tetraquark type, but they nevertheless generate overlap [possibly small, but certainly not vanishing] to two-particle states).
- To determine, whether there is a bound  $a_0(980)$  tetraquark state, we need to resolve the above listed two-particle states  $K + \bar{K}$  and  $\eta + \pi$  and check, whether there is an additional 3rd state in the mass region around  $980 \pm 20$  MeV; to this end we need operators of two-particle type.



# Two-particle creation operators (2)

- Two particle operators with quantum numbers  $I(J^{PC}) = 1(0^{++})$ :

– Two particle  $K + \bar{K}$  type:

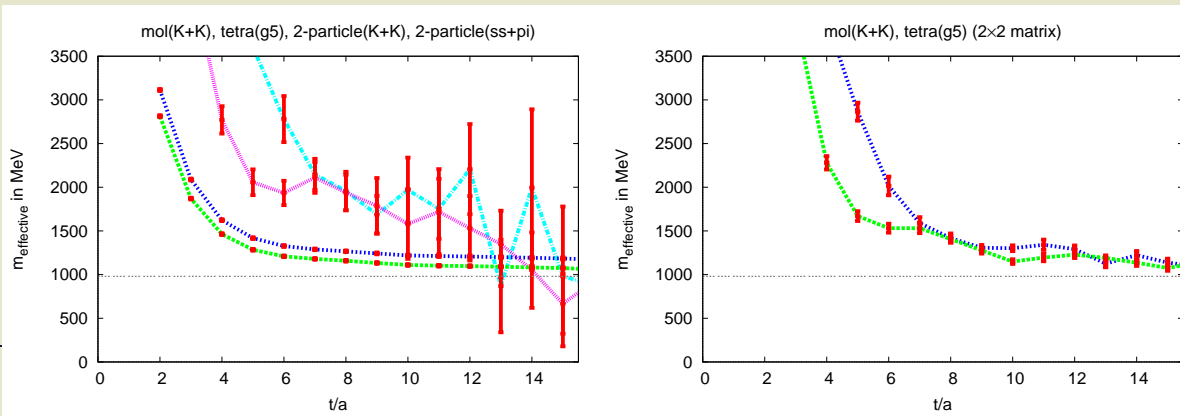
$$\mathcal{O}_{a_0(980)}^{K+\bar{K} \text{ two particle}} = \left( \sum_{\mathbf{x}} \bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right) \left( \sum_{\mathbf{y}} \bar{d}(\mathbf{y}) \gamma_5 s(\mathbf{y}) \right).$$

– Two particle  $\eta + \pi$  type:

$$\mathcal{O}_{a_0(980)}^{\eta+\pi \text{ two particle}} = \left( \sum_{\mathbf{x}} \bar{s}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \right) \left( \sum_{\mathbf{y}} \bar{d}(\mathbf{y}) \gamma_5 u(\mathbf{y}) \right).$$

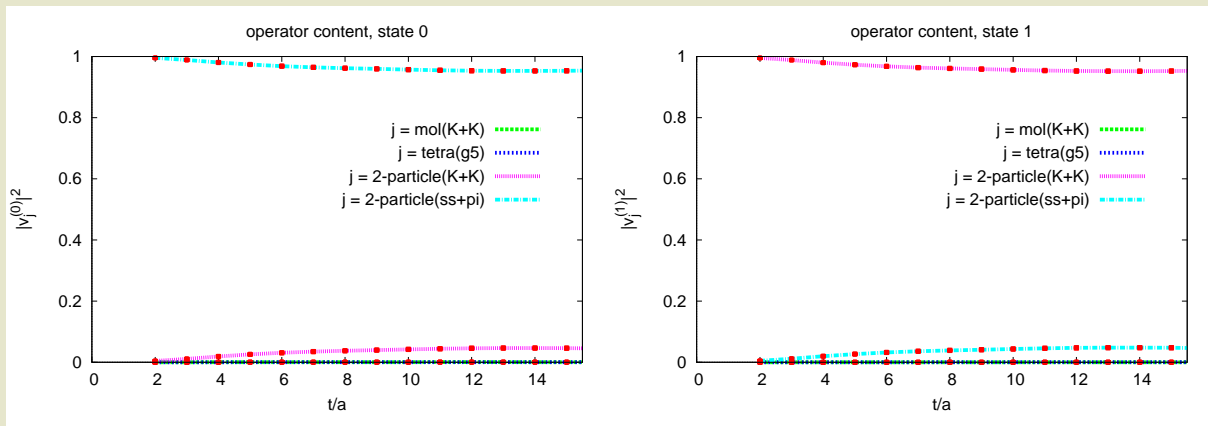
# Numerical results $a_0(980)$ (4)

- Study all four operators ( $K\bar{K}$  molecule, diquark,  $K + \bar{K}$  two particle,  $\eta + \pi$  two particle) at the same time, extract the four lowest energy eigenstates by diagonalizing a  $4 \times 4$  correlation matrix (left plot).
  - Still only two low lying states around  $980 \pm 20$  MeV, the 2nd and 3rd excitation are  $\approx 750$  MeV heavier.
  - The signal of the low lying states is of much better quality than before (when we only considered tetraquark operators)
    - suggests that the observed low lying states have much better overlap to the two-particle operators and are most likely of two-particle type.



# Numerical results $a_0(980)$ (5)

- When determining low lying eigenstates from a correlation matrix one does not only obtain their mass, but also information about their operator content, i.e. which percentage of which operator is present in an extracted state:
  - The ground state is a  $\eta + \pi$  state ( $\gtrsim 95\%$  two particle  $\eta + \pi$  content).
  - The first excitation is a  $K + \bar{K}$  state ( $\gtrsim 95\%$  two particle  $K + \bar{K}$  content).

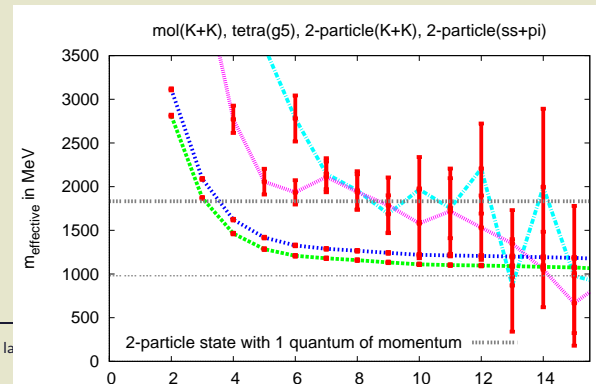


# Numerical results $a_0(980)$ (6)

- What about the 2nd and 3rd excitation? ... Are these tetraquark states? ... What is their nature?
- Two particle states with one relative quantum of momentum (one particle has momentum  $+p_{\min} = +2\pi/L$  the other  $-p_{\min}$ ) also have quantum numbers  $I(J^{PC}) = 1(0^{++})$ ; their masses can easily be estimated:
  - $p_{\min} = 2\pi/L \approx 715$  MeV (the results presented correspond to the small lattice with spatial extension  $L = 1.73$  fm);
  - $m(K(+p_{\min}) + \bar{K}(-p_{\min})) \approx 2\sqrt{m(K)^2 + p_{\min}^2} \approx 1750$  MeV;
  - $m(\eta(+p_{\min}) + \pi(-p_{\min})) \approx \sqrt{m(\eta)^2 + p_{\min}^2} + \sqrt{m(\pi)^2 + p_{\min}^2} \approx 1780$  MeV;

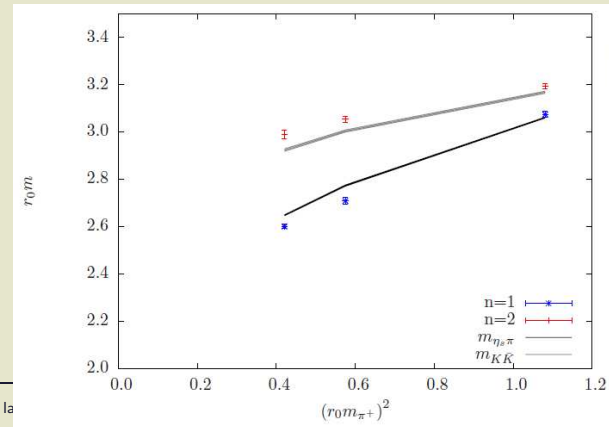
these estimated mass values are consistent with the observed mass values of the 2nd and 3rd excitation

→ suggests to interpret these states as two-particle states.



# Numerical results $a_0(980)$ (7)

- Summary regarding the presented “ $a_0(980)$  results”:
  - In the  $a_0(980)$  sector (quantum numbers  $I(J^{PC}) = 1(0^{++})$ ) we do not observe any low lying (mass  $\lesssim 1750$  MeV) tetraquark state, even though we employed operators of tetraquark structure ( $K\bar{K}$  molecule, diquark).
  - The experimentally measured mass for  $a_0(980)$  is  $980 \pm 20$  MeV.
  - Conclusion:  $a_0(980)$  does not seem to be a bound tetraquark state ... maybe an ordinary quark-antiquark state (unlikely, lattice results indicate the opposite) or a rather unstable resonance.
- Similar results for the range of light quark masses investigated ( $m_{\text{PS}} \approx 280 \dots 460$  MeV).



# Numerical results $\kappa$

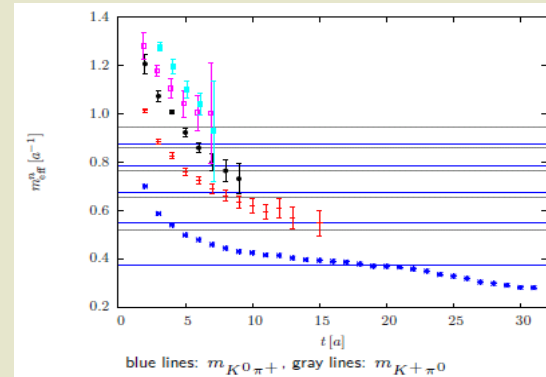
- Tetraquark operators for  $\kappa$  (quantum numbers  $I(J^P) = 1/2(0^+)$ ):
  - Molecule type (models a bound  $K\pi$  state):

$$\mathcal{O}_{\kappa}^{K\pi \text{ molecule}} = \sum_{\mathbf{x}} \left( \bar{s}(\mathbf{x}) \gamma_5 q(\mathbf{x}) \right) \left( \bar{q}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right), \quad q\bar{q} = u\bar{u} + d\bar{d}$$

- Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{\kappa}^{\text{diquark}} = \sum_{\mathbf{x}} \left( \epsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{q}^{c,T}(\mathbf{x}) \right) \left( \epsilon^{ade} q^{d,T}(\mathbf{x}) C \gamma_5 u^e(\mathbf{x}) \right).$$

- Two particle operator of  $K + \pi$  type.
- An analysis yields only a single low lying state, which is of two particle  $K + \pi$  nature.
- Conclusions:  $\kappa$  does not seem to be a bound tetraquark state ... maybe an ordinary quark-antiquark state (unlikely, lattice results indicate the opposite) or a rather unstable resonance.



# Conflict with existing lattice results

- In a similar recent lattice study of  $\sigma \equiv f_0(500)$  and  $\kappa \equiv K_0^*(800)$  bound tetraquark states have been observed in both sectors.

[S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. -F. Liu, N. Mathur and D. Mohler,  
Phys. Rev. D **82**, 094507 (2010) [arXiv:1005.0948 [hep-lat]]

- In particular for  $\kappa$  this conflict has to be resolved.

# $a_0(980)$ and $\kappa$ as resonances

- A lattice study of  $a_0(980)$  and  $\kappa$  as resonances requires rather precise computations of the masses of the two particle states  $K + \bar{K}$ ,  $\eta + \pi$  and  $K + \pi$  for various spatial volumes.
- Technically very challenging.
- No results yet.



# Sources of systematic error, outlook (1)

- The computations presented are technically rather challenging; there are several possible sources of systematic error, which have not yet been studied, but which need to be addressed in the future:

- Inclusion of disconnected and partially disconnected diagrams.
- Include also  $q\bar{q}$  creation operators (implies partially disconnected diagrams), e.g. for  $a_0(980)$

$$\mathcal{O}_{a_0(980)}^{q\bar{q}} = \sum_{\mathbf{x}} \bar{d}(\mathbf{x})u(\mathbf{x}).$$

- Continuum limit (at the moment only a single value of the lattice spacing,  $a = 0.086$  fm, has been considered).
- Finite volume studies (extrapolate the here presented results to infinite spatial volume, determine resonance properties).

# Sources of systematic error, outlook (2)

- The techniques and codes developed can be used with only minor modifications to study other tetraquark candidates, e.g.
  - $\sigma \equiv f_0(500), f_0(980),$
  - $D_{s0}^*(2317)^\pm, D_{s1}(2460)^\pm,$
  - charmonium states  $X(3872), Z(4430)^\pm, Z(4050)^\pm, Z(4250)^\pm, \dots$