Studying tetraquark candidates using lattice QCD

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Introduction, motivation (1)

- The nonet of light scalar mesons ($J^P = 0^+$)
  - $\sigma \equiv f_0(500)$, $I = 0$, $400 \ldots 550$ MeV ($\bar{s}s$ ...?),
  - $\kappa \equiv K_0^*(800)$, $I = 1/2$, 682 ± 29 MeV ($\bar{s}u$, $\bar{s}d$, $\bar{u}s$, $\bar{d}s$ ...?),
  - $a_0(980)$, $I = 1$, 980 ± 20 MeV ($\bar{u}d$, $\bar{d}u$, $\bar{u}u - \bar{d}d$ ...?)
    $f_0(980)$, $I = 0$, 990 ± 20 MeV ($\bar{u}u + \bar{d}d$ ...?)

  is poorly understood:
  - All nine states are unexpectedly light (should rather be close to the corresponding $J^P = 1^+, 2^+$ states around $1200 \ldots 1500$ MeV).
  - The ordering of states is inverted compared to expectation:
    * E.g. in a $q\bar{q}$ picture the $I = 1$ $a_0(980)$ states must necessarily be formed by two u/d quarks, while the $I = 1/2$ κ states are made from an s and a u/d quark; since $m_s > m_{u/d}$ one would expect $m(\kappa) > m(a_0(980))$.

Introduction, motivation (2)

- * In a tetraquark picture the quark content could e.g. be the following:
  $\kappa \equiv \bar{s}u(\bar{u}u + \bar{d}d)$ (one s quark, three light quarks)
  $a_0(980) \equiv \bar{s}u\bar{d}s$ (two s quarks, two light quarks);
  this would naturally explain the observed ordering.

– Certain decays also support a tetraquark interpretation: e.g. $a_0(980)$ readily decays to $K + \bar{K}$, which indicates that besides the two light quarks required by $I = 1$ also an $s\bar{s}$ pair is present.

$\rightarrow$ **Study such states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.**

Introduction, motivation (3)

- Examples of heavy mesons, which are tetraquark candidates:
  - $D_{s0}^*(2317)^\pm$, $D_{s1}(2460)^\pm$,
  - charmonium states $X(3872)$, $Z(4430)^\pm$, $Z(4050)^\pm$, $Z(4250)^\pm$, ...
  - $\bar{c}c\bar{c}c$ (experimentally not yet observed, predicted by theory) ...?

    [W. Heupel, G. Eichmann and C. S. Fischer, Phys. Lett. B **718**, 545 (2012) [arXiv:1206.5129 [hep-ph]]]
  - $bb(\bar{u}\bar{d} - \bar{d}\bar{u})$ (experimentally not yet observed, predicted by theory) ...?

    [P. Bicudo and M. Wagner, Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274 [hep-ph]].]

Outline

(1) Wilson twisted mass study of $a_0(980)$:

[C. Alexandrou *et al.* [ETM Collaboration], JHEP **1304**, 137 (2013) [arXiv:1212.1418 [hep-lat]]]

- Wilson twisted mass fermions (generated by the ETM Collaboration).
  [R. Baron *et al.*, JHEP **1006**, 111 (2010) [arXiv:1004.5284 [hep-lat]]]
- Computations at light u/d quark masses corresponding to $m_\pi \gtrsim 280\text{MeV}$.
- No disconnected diagrams/closed fermion loops.

(2) Recent technical advances:

- Wilson + clover fermions (generated by the PACS-CS Collaboration).
  [S. Aoki *et al.* [PACS-CS Collaboration], Phys. Rev. D **79**, 034503 (2009) [arXiv:0807.1661 [hep-lat]]]
- Computations close to physically light u/d quark masses.
- Inclusion of disconnected diagrams/closed fermion loops.

(3) Exploring a possibly existing $\bar{c}c\bar{c}c$ tetraquark.

(4) Static-static-light-light tetraquarks (close to $bb(\bar{u}\bar{d} - \bar{d}\bar{u})$).

Lattice QCD hadron spectroscopy (1)

- Lattice QCD: discretized version of QCD,

$$S = \int d^4x \Big(\sum_{\psi \in \{u,d,s,c,t,b\}} \bar{\psi}\Big(\gamma_\mu\Big(\partial_\mu - iA_\mu\Big) + m^{(\psi)}\Big)\psi + \frac{1}{2g^2}\mathrm{Tr}\Big(F_{\mu\nu}F_{\mu\nu}\Big)\Big)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- Let $\mathcal{O}$ be a suitable "hadron creation operator", i.e. an operator formed by quark fields ψ and gluonic fields A_μ such that $\mathcal{O}|\Omega\rangle$ is a state containing the hadron of interest ($|\Omega\rangle$: QCD vacuum).
- More precisely: ... an operator such that $\mathcal{O}|\Omega\rangle$ has the same quantum numbers ($J^{\mathcal{PC}}$, flavor) as the hadron of interest.
- Examples:
  - Pion creation operator: $\mathcal{O} = \int d^3x\, \bar{u}(\mathbf{x})\gamma_5 d(\mathbf{x})$.
  - Proton creation operator: $\mathcal{O} = \int d^3x\, \epsilon^{abc} u^a(\mathbf{x})(u^{b,T}(\mathbf{x}) C\gamma_5 d^c(\mathbf{x}))$.

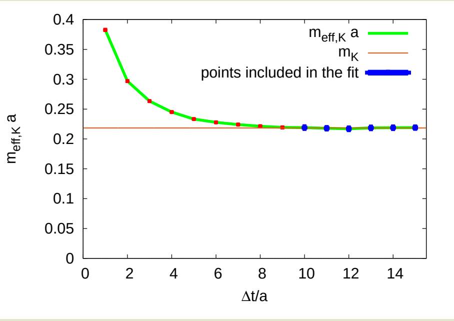
Lattice QCD hadron spectroscopy (2)

- Determine the mass of the ground state of the hadron of interest from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times t:

$$\mathcal{C}(t) \;=\; \langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle \;=\; \langle\Omega|e^{+Ht}\mathcal{O}^\dagger(0)e^{-Ht}\mathcal{O}(0)|\Omega\rangle \;=$$
$$= \sum_n \Big|\langle n|\mathcal{O}(0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)t\Big) \;\approx\; (\text{for "}t \gg 1\text{"})$$
$$\approx \Big|\langle 0|\mathcal{O}(0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{m(\text{hadron})}t\Big).$$

- Usually the exponent is determined by identifying the plateau value of a so-called effective mass:

$$m_{\text{effective}}(t) \;=\; \frac{1}{a}\ln\Big(\frac{\mathcal{C}(t)}{\mathcal{C}(t+a)}\Big) \;\approx\; (\text{for "}t \gg 1\text{"})$$
$$\approx E_0 - E_\Omega \;=\; m(\text{hadron}).$$



Part 1:
Wilson twisted mass study of $a_0(980)$

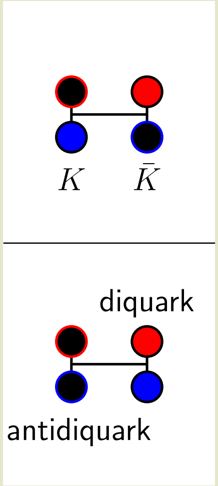
Tetraquark creation operators

- $a_0(980)$:
  - Quantum numbers $I(J^P) = 1(0^+)$.
  - Mass $980 \pm 20\,\text{MeV}$.
- Tetraquark creation operators:
  - Two light quarks needed, due to $I = 1$, e.g. $u\bar{d}$.
  - $a_0(980)$ decays to $K\bar{K}$ ... suggests an additional $s\bar{s}$ pair.
  - $K\bar{K}$ molecule type (models a bound $K\bar{K}$ state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big).$$

  - Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$



Wilson twisted mass lattice setup

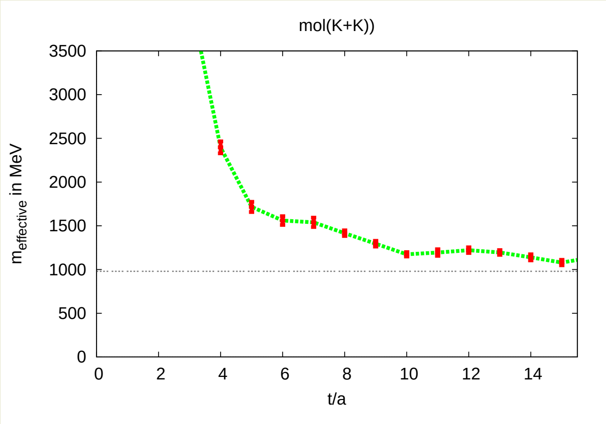
- Gauge link configurations generated by the ETM Collaboration.
  [R. Baron *et al.*, JHEP **1006**, 111 (2010) [arXiv:1004.5284 [hep-lat]]]
- 2+1+1 dynamical Wilson twisted mass quark flavors, i.e. u, d, s and c sea quarks (twisted mass lattice QCD isospin and parity are slightly broken).
- Various light u/d quark masses corresponding pion masses $m_\pi \approx 280 \ldots 460\,\text{MeV}$.
- Singly disconnected contributions/closed fermion loops neglected, i.e. no s quark propagation within the same timeslice ("no quark antiquark pair creation/annihilation").

Numerical results $a_0(980)$ (1)

- Effective mass, molecule type operator:

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big).$$

- The effective mass plateau indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20\,\text{MeV}$.
- Conclusion: $a_0(980)$ is a tetraquark state of $K\bar{K}$ molecule type ...?

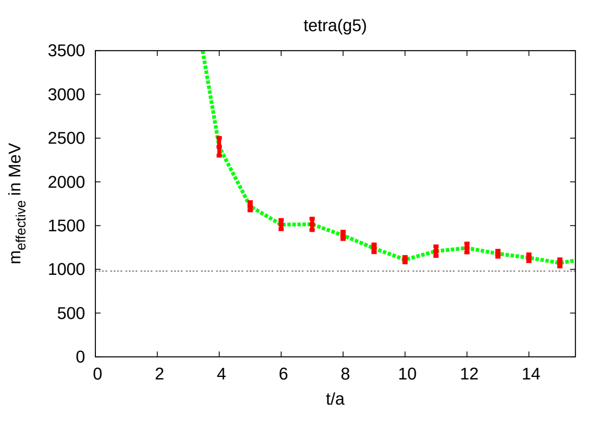


Numerical results $a_0(980)$ (2)

- Effective mass, diquark type operator:

$$\mathcal{O}^{\text{diquark}}_{a_0(980)} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

- The effective mass plateau indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20\,$MeV.
- Conclusion: $a_0(980)$ is a tetraquark state of diquark type ...? Or a mixture of $K\bar{K}$ molecule and a diquark-antidiquark pair?



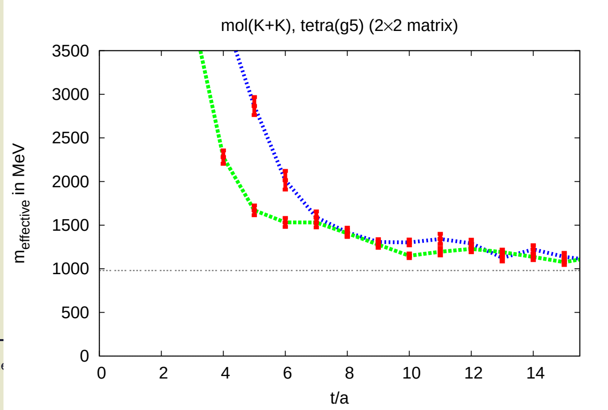
Numerical results $a_0(980)$ (3)

- Study both operators at the same time, extract the two lowest energy eigenstates by diagonalizing a 2×2 correlation matrix ("generalized eigenvalue problem"):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big)$$

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

- Now two orthogonal states roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20\,\text{MeV}$ ...?



Two-particle creation operators (1)

- Explanation: there are two-particle states, which have the same quantum numbers as $a_0(980)$, $I(J^{PC}) = 1(0^{++})$,
  - $K + \bar{K}$ ($m(K) \approx 500\,\text{MeV}$),
  - $\eta_s + \pi$ ($m(\eta_s \equiv \bar{s}\gamma_5 s) \approx 700\,\text{MeV}$, $m(\pi) \approx 300\,\text{MeV}$ in our lattice setup),

  which are both around the expected $a_0(980)$ mass $980 \pm 20\,\text{MeV}$.
- What we have seen in the previous plots might actually be two-particle states (our operators are of tetraquark type, but they nevertheless generate overlap [possibly small, but certainly not vanishing] to two-particle states).
- To determine, whether there is a bound $a_0(980)$ tetraquark state, we need to resolve the above listed two-particle states $K + \bar{K}$ and $\eta_s + \pi$ and check, whether there is an additional 3rd state in the mass region around $980 \pm 20\,\text{MeV}$; to this end we need operators of two-particle type.

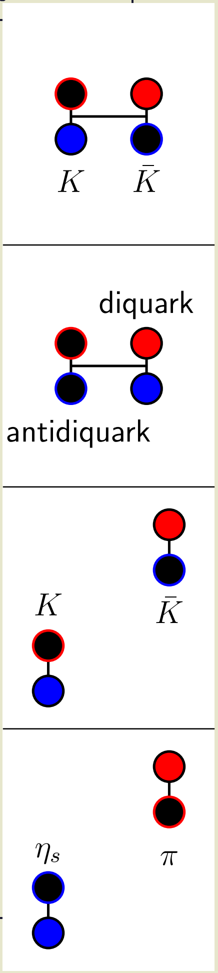
Two-particle creation operators (2)

- Two-particle operators:
  - Two-particle $K + \bar{K}$ type:

$$\mathcal{O}_{a_0(980)}^{K+\bar{K}\text{ two-particle}} = \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\right)\left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y})\gamma_5 s(\mathbf{y})\right).$$

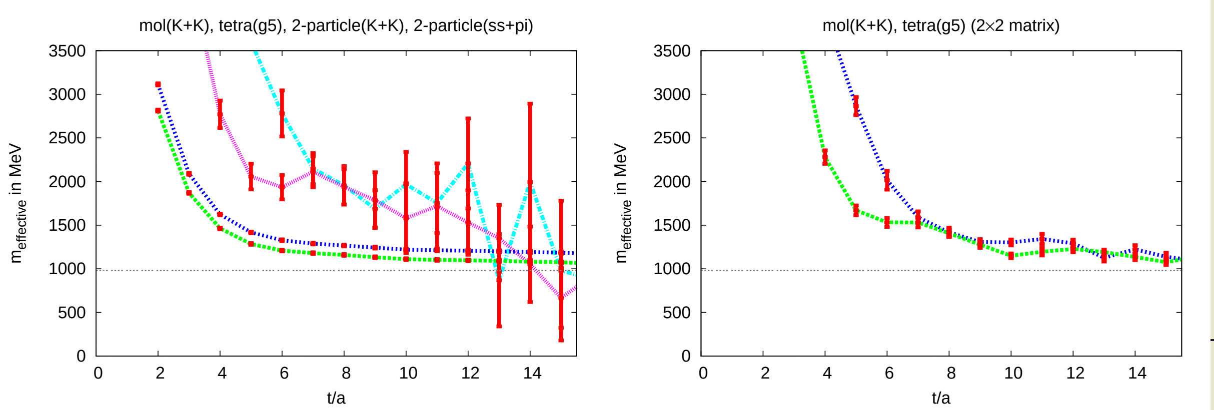
  - Two-particle $\eta_s + \pi$ type:

$$\mathcal{O}_{a_0(980)}^{\eta_s+\pi\text{ two-particle}} = \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x})\gamma_5 s(\mathbf{x})\right)\left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y})\gamma_5 u(\mathbf{y})\right).$$



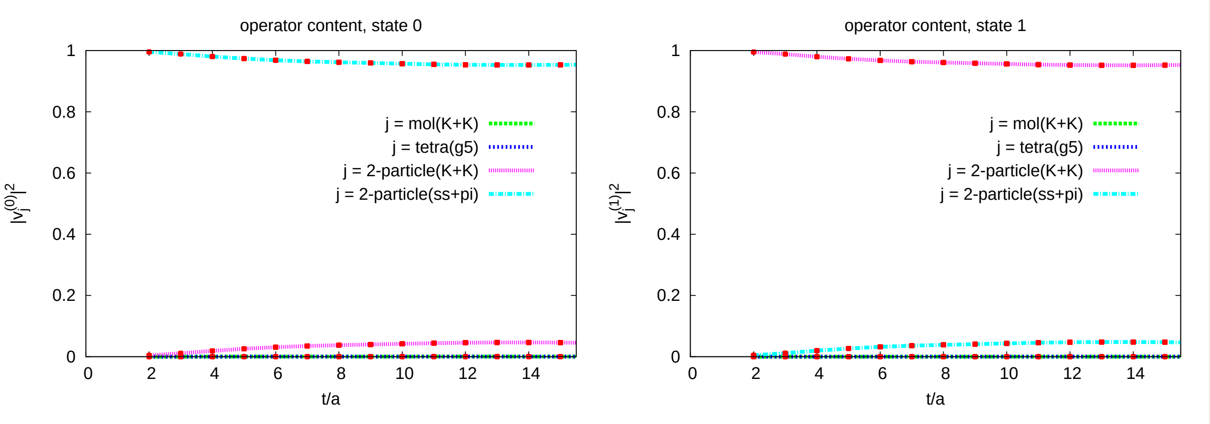
Numerical results $a_0(980)$ (4)

- Study all four operators ($K\bar{K}$ molecule, diquark, $K + \bar{K}$ two-particle, $\eta_s + \pi$ two-particle) at the same time, extract the four lowest energy eigenstates by diagonalizing a 4×4 correlation matrix (left plot).
  - Still only two low-lying states around 980 ± 20 MeV, the 2nd and 3rd excitation are ≈ 750 MeV heavier.
  - The signal of the low-lying states is of much better quality than before (when we only considered tetraquark operators)
    $\rightarrow$ suggests that the observed low-lying states have much better overlap to the two-particle operators and are most likely of two-particle type.



Numerical results $a_0(980)$ (5)

- When determining low-lying eigenstates from a correlation matrix, one does not only obtain their mass, but also information about their operator content, i.e. which percentage of which operator is present in an extracted state:
  - $\rightarrow$ The ground state is a $\eta_s + \pi$ state ($\gtrsim 95\%$ two-particle $\eta_s + \pi$ content).
  - $\rightarrow$ The first excitation is a $K + \bar{K}$ state ($\gtrsim 95\%$ two-particle $K + \bar{K}$ content).

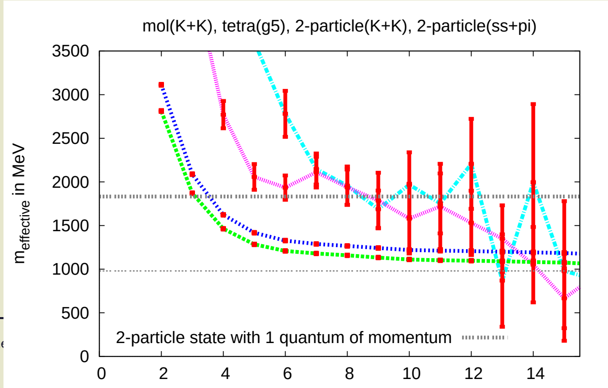


Numerical results $a_0(980)$ (6)

- What about the 2nd and 3rd excitation? ... Are these tetraquark states? ... What is their nature?

- Two-particle states with one relative quantum of momentum (one particle has momentum $+p_\text{min} = +2\pi/L$ the other $-p_\text{min}$) also have quantum numbers $I(J^{PC}) = 1(0^{++})$; their masses can easily be estimated:
  - $p_\text{min} = 2\pi/L \approx 715\,\text{MeV}$ (the results presented correspond to a small lattice with spatial extension $L = 1.73\,\text{fm}$);
  - $m(K(+p_\text{min}) + \bar{K}(-p_\text{min})) \approx 2\sqrt{m(K)^2 + p_\text{min}^2} \approx 1750\,\text{MeV}$;
  - $m(\eta(+p_\text{min}) + \pi(-p_\text{min})) \approx \sqrt{m(\eta)^2 + p_\text{min}^2} + \sqrt{m(\pi)^2 + p_\text{min}^2} \approx$
    $\approx 1780\,\text{MeV}$;

  these estimated mass values are consistent with the observed mass values of the 2nd and 3rd excitation
  $\rightarrow$ suggests to interpret these states as two-particle states.



Numerical results $a_0(980)$ (7)

- Summary:
  - In the $a_0(980)$ sector (quantum numbers $I(J^{PC}) = 1(0^{++})$) we do not observe any low-lying (mass $\lesssim 1750$ MeV) tetraquark state, even though we employed operators of tetraquark structure ($K\bar{K}$ molecule, diquark).
  - The experimentally measured mass for $a_0(980)$ is 980 ± 20 MeV.
  - Conclusion: $a_0(980)$ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state or a rather unstable resonance.

Part 2:
Recent technical advances

Wilson + clover lattice setup

- Gauge link configurations generated by the PACS-CS Collaboration.
  [S. Aoki *et al.* [PACS-CS Collaboration], Phys. Rev. D **79**, 034503 (2009) [arXiv:0807.1661 [hep-lat]]]

- 2+1 dynamical Wilson + clover quark flavors, i.e. u, d and s sea quarks.
  $\rightarrow$ In contrast to twisted mass parity and isospin are exact symmetries, i.e. no pion and kaon mass splitting, easy separation of $P = +, -$ states, ...

- Light u/d quark masses corresponding to pion masses $m_\pi \approx 150\,\text{MeV}$ and $m_\pi \approx 300\,\text{MeV}$.
  $\rightarrow$ Computations close to physically light u/d quark masses possible.

- Singly disconnected contributions/closed fermion loops included.
  $\rightarrow$ s quark propagation within the same timeslice ("quark antiquark pair creation/annihilation taken into account").

Closed fermion loops (1)

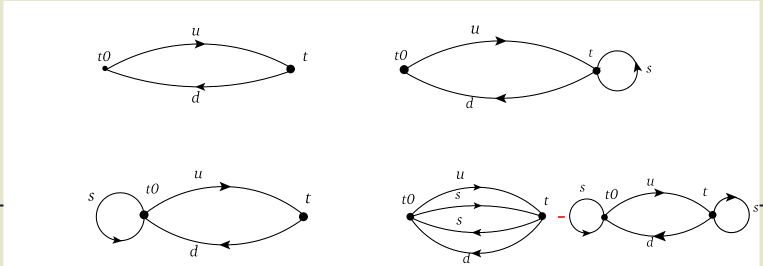
- In our previous Wilson twisted mass study of $a_0(980)$ we neglected singly disconnected contributions/closed fermion loops:

  $\rightarrow$ We could not consider a $q\bar{q}$ operator,

  $$\mathcal{O}^{q\bar{q}}_{a_0(980)} \; = \; \sum_{\mathbf{x}} \Big(\bar{d}(\mathbf{x}) u(\mathbf{x}) \Big),$$

  because cross correlations between this operator and any of the four-quark operators $\mathcal{O}^{K\bar{K}\text{ molecule}}_{a_0(980)}$, $\mathcal{O}^{\text{diquark}}_{a_0(980)}$, $\mathcal{O}^{K+\bar{K}\text{ two-particle}}_{a_0(980)}$ or $\mathcal{O}^{\eta_s+\pi\text{ two-particle}}_{a_0(980)}$ correspond to closed fermion loops.

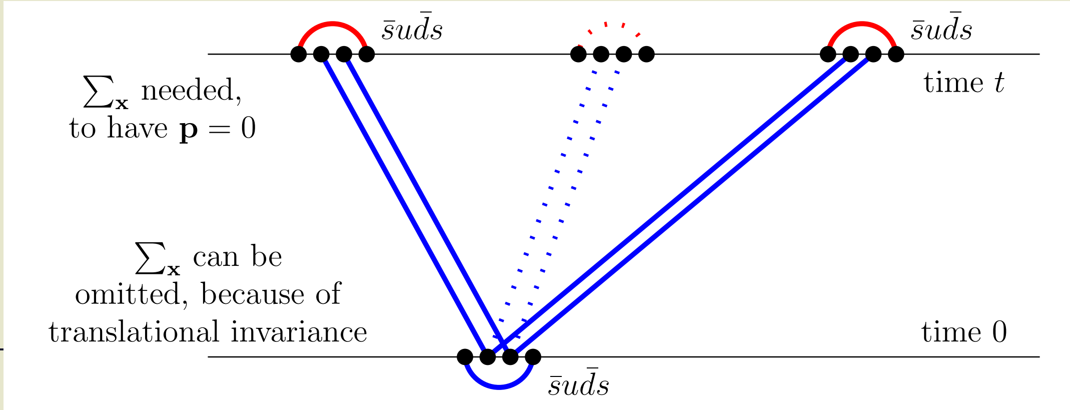
  $\rightarrow$ Also correlations between the four-quark operators include closed fermion loops; therefore, we introduced a source of systematic error, which is difficult to estimate or to control.



Closed fermion loops (2)

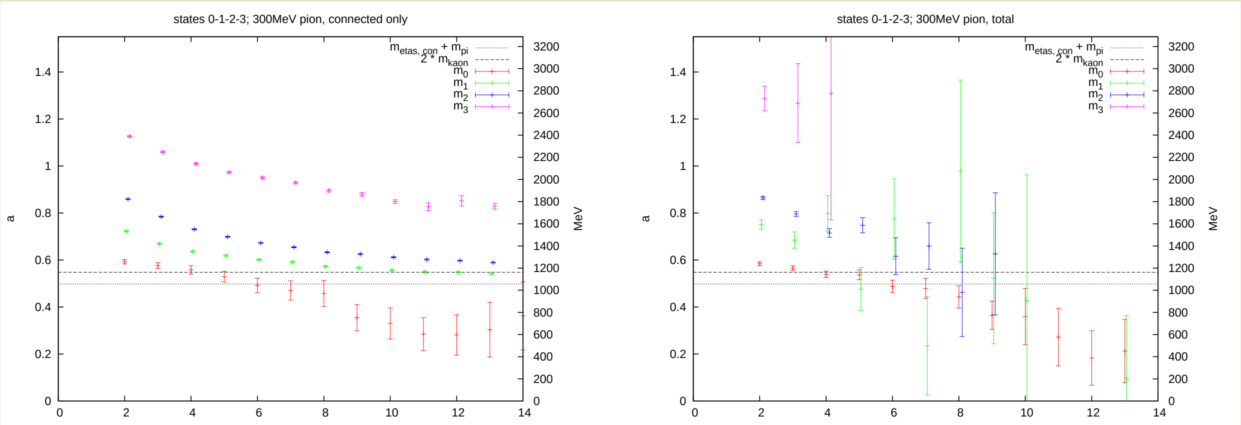
- Technical aspects of disconnected diagrams/closed fermion loops:
  - Blue: point-to-all propagators applicable.
  - Red: due to $\sum_{\mathbf{x}}$, timeslice-to-all propagators needed.
  - Timeslice-to-all propagators can be estimated stochastically.
  - Using several stochastic timeslice-to-all propagators results in a poor signal-to-noise ratio.

  → Combine three point-to-all (blue) and one stochastic timeslice-to-all (red) propagator.



Closed fermion loops (3)

- Effective masses from a 4×4 correlation matrix ($\mathcal{O}_{a_0(980)}^{q\bar{q}}$, $\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}}$, $\mathcal{O}_{a_0(980)}^{\eta_s\pi\text{ molecule}}$, $\mathcal{O}_{a_0(980)}^{\text{diquark}}$) at $m_\pi \approx 300\,\text{MeV}$:
  - Lowest (two) energy level(s) roughly consistent with $K + \bar{K}$, $\eta + \pi$ and a possibly existing additional $a_0(980)$ state.
  - For physically interesting statements we need smaller errors and to include $\mathcal{O}_{a_0(980)}^{K+\bar{K}\text{ two-particle}}$ and $\mathcal{O}_{a_0(980)}^{\eta_s+\pi\text{ two-particle}}$ (work in progress).



Work in progress, outlook

- Enlarge correlation matrices such that
  - $q\bar{q}$ operators,
  - tetraquark operators (mesonic molecules, diquark-antidiquark pairs),
  - two-meson operators

  are included.
- Perform computations at pion mass $m_\pi \approx 150\,\text{MeV}$.
- Address various physical questions/systems (tetraquark candidates with different flavor structure, search for additional bound states, ...).

Part 3:
Exploring a possibly existing $\bar{c}c\bar{c}c$ tetraquark

$\bar{c}c\bar{c}c$ tetraquark ...? (1)

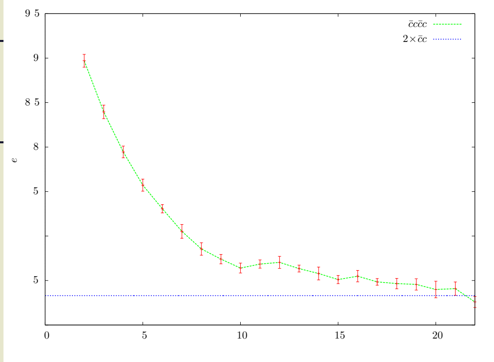
- Recently a $\bar{c}c\bar{c}c$ tetraquark has been predicted
  - using a coupled system of covariant Bethe-Salpeter equations,
  - mass $m(\bar{c}c\bar{c}c) = (5.3 \pm 0.5)$ GeV,
  - predominantly of mesonic molecule type (two η_c mesons),
  - rather strongly bound ($2 \times m(\eta_c) = 6.0$ GeV), binding energy $\Delta E = m(\bar{c}c\bar{c}c) - 2 \times m(\eta_c) \approx -(0.7 \pm 0.5)$ GeV.

[W. Heupel, G. Eichmann and C. S. Fischer, Phys. Lett. B **718**, 545 (2012) [arXiv:1206.5129 [hep-ph]]]

- Should be within experimental reach (PANDA experiment).

$\rightarrow$ Investigate the existence of this $\bar{c}c\bar{c}c$ state using lattice QCD.

$\bar{c}c\bar{c}c$ tetraquark ...? (2)

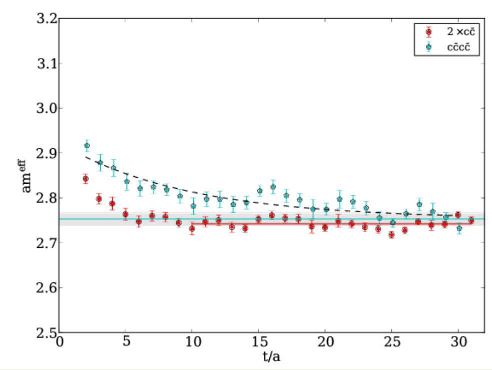


- Use the same techniques and setup as discussed for the $a_0(980)$ meson.
- First attempt:
  - Molecule type $\bar{c}c\bar{c}c$ creation operator (models a bound $\eta_c\eta_c$ state):

$$\mathcal{O}_{\bar{c}c\bar{c}c}^{\eta_c\eta_c \text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{c}(\mathbf{x})\gamma_5 c(\mathbf{x})\Big)\Big(\bar{c}(\mathbf{x})\gamma_5 c(\mathbf{x})\Big).$$

  - Inconclusive results:
    - Neither an indication for a $\bar{c}c\bar{c}c$ state significantly below $2 \times m(\eta_c)$ ...
    - ... nor can the existence of such a state be ruled out

  (the effective mass still decreases at large temporal separations t, which signals a trial state $\mathcal{O}_{\bar{c}c\bar{c}c}^{\eta_c\eta_c \text{ molecule}}|\Omega\rangle$, which has a poor ground state overlap; the ground state could be $|\eta_c + \eta_c\rangle$ or $|\bar{c}c\bar{c}c\rangle$ of different structure).



$\bar{c}c\bar{c}c$ tetraquark ...? (3)

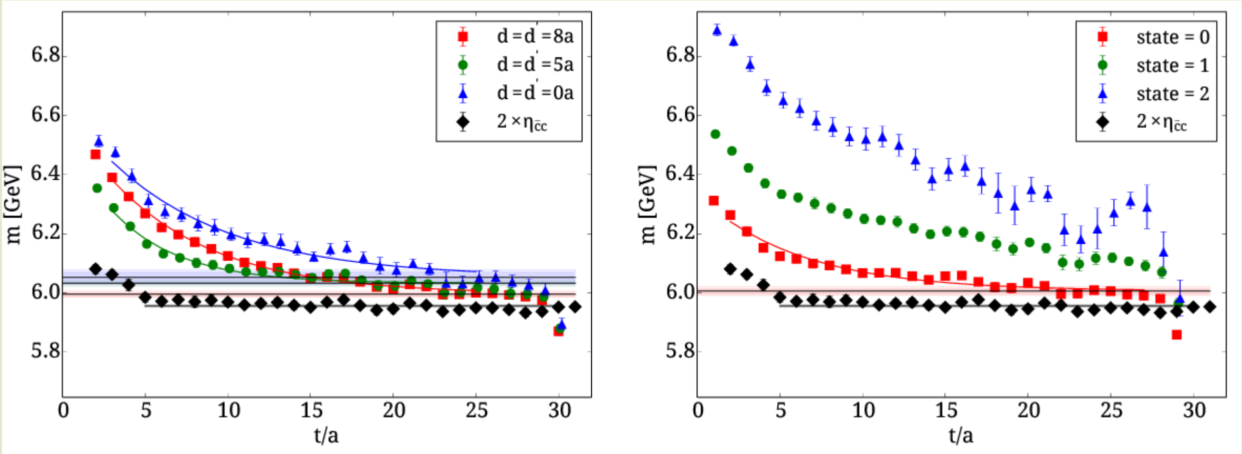
- The molecule type $\bar{c}c\bar{c}c$ creation operator used generates a trial state with the two η_c mesons essentially on top of each other.
- In a possibly existing $\bar{c}c\bar{c}c$ tetraquark state the two η_c mesons could be quite far separated, which would imply a poor overlap of the above trial state with the $\bar{c}c\bar{c}c$ state.
- Therefore, we also employed an improved molecule type $\bar{c}c\bar{c}c$ creation operator:

$$\mathcal{O}_{\bar{c}c\bar{c}c}^{\eta_c\eta_c\text{ molecule}}(d) \quad = \quad \sum_{\mathbf{x}} \Big(\bar{c}(\mathbf{x})\gamma_5 c(\mathbf{x})\Big) \sum_{\mathbf{n}=\pm\mathbf{e}_x,\pm\mathbf{e}_y,\pm\mathbf{e}_z} \Big(\bar{c}(\mathbf{x}+d\mathbf{n})\gamma_5 c(\mathbf{x}+r\mathbf{n})\Big)$$

(d models the size of the mesonic molecule, the separation of the two η_c mesons).

$\bar{c}c\bar{c}c$ tetraquark ...? (4)

- Still no sign of a $\bar{c}c\bar{c}c$ state significantly below $2 \times m(\eta_c)$ ...
  - Left plot: $d \approx 0.00$ fm , 0.45 fm , 0.72 fm.
  - Right plot: solving a generalized eigenvalue problem.

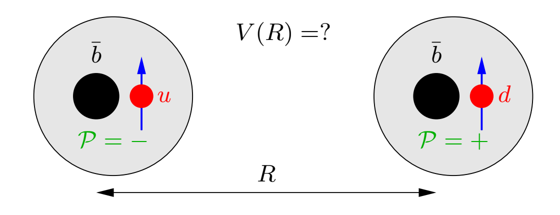


- We plan to explore the dependence of the results on the quark masses, in particular the existence of a bound four-quark state (lattice results strongly indicate that two B mesons can form a bound $bb(\bar{u}\bar{d} - \bar{d}\bar{u})$ state) ...

Part 4:
Static-static-light-light tetraquarks

Static-static-light-light tetraquarks (1)

- Study possibly existing $QQ\bar{q}\bar{q}$ (heavy-heavy-light-light) tetraquark states:
  - Use the static approximation for the heavy quarks QQ (reduces the necessary computation time significantly).
  - Most appropriate for $QQ \equiv bb$.
  - Could also yield information for $QQ \equiv cc$.



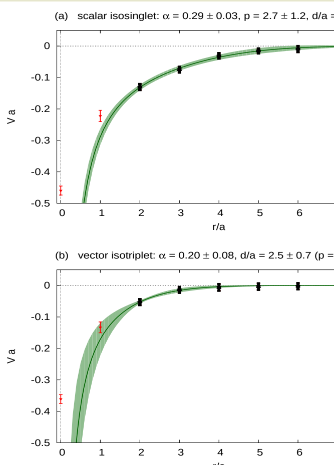
- Proceed in two steps:

  (1) Compute the potential of two heavy quarks QQ in the background of two light antiquarks $\bar{q}\bar{q}$ by means of lattice QCD

$$\mathcal{O}_{QQ\bar{q}\bar{q}} = (C\Gamma)_{AB}\Big(Q_C(\mathbf{x}_1)\bar{q}_A^{(1)}(\mathbf{x}_1)\Big)\Big(Q_C(\mathbf{x}_2)\bar{q}_B^{(2)}(\mathbf{x}_2)\Big)$$

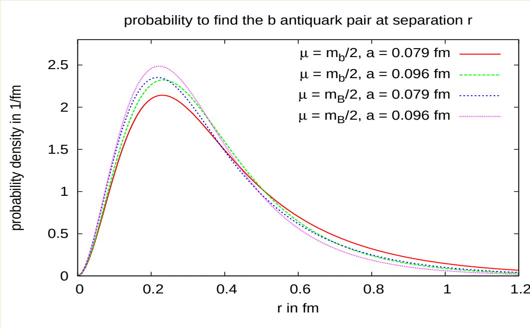
$(R = |\mathbf{x}_1 - \mathbf{x}_2|$, $\bar{q}^{(1)}\bar{q}^{(2)} \in \{ud - du \quad , \quad uu, dd, ud + du\}$,
$C =$ charge conjugation matrix, $\Gamma =$ any γ combination)
$\rightarrow$ many different channels/quantum numbers.

[M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538 [hep-lat]]]
[M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147 [hep-lat]]]



Static-static-light-light tetraquarks (2)

(2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks QQ.



- Clear indication for a bound state for $QQ \equiv bb$ in a specific channel:
  - Quantum numbers: $I(J^P) = 0(0^+)\,,\,0(1^+)$ (degeneracy with respect to the heavy spin).
  - Binding energy: $E \approx -50\,\mathrm{MeV}$.

[P. Bicudo and M. Wagner, Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274 [hep-ph]]]

- No four-quark binding in other channels.
- Next steps:
  - Extend from $QQ\bar{q}\bar{q}$ to $Q\bar{Q}q\bar{q}$ (experimentally more realistic/interesting).
  - Establish connection to computations with four quarks of finite mass.