Computation of B mesons and b baryons with lattice QCD

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Outline

(1) Introduction to lattice computations, QCD and lattice QCD ($\approx$ 15 minutes).

(2) Selected research results from the field of B physics ($\approx$ 25 minutes):

- Masses of B and B_s mesons.
- Masses of b baryons.
- Forces between B mesons.
- Semileptonic decays $B \rightarrow D^{**}$.

(3) Further research interests and planned research ($\approx$ 5 minutes).

Part 1: Introduction to lattice computations, QCD and lattice QCD.

Lattice computations in QM (1)

- Introduce the basic principle of lattice computations via a simple example, the 1-dimensional harmonic oscillator in quantum mechanics.

- (Euclidean) action of the harmonic oscillator:

$$S[x] \quad = \quad \int dt \, \Big(\frac{m}{2}\dot{x}(t)^2 + \frac{m\omega^2}{2} x(t)^2\Big).$$

- Goal: compute the average quadratic oscillation x^2 for the ground state $|0\rangle$, i.e. $\langle 0|x^2|0\rangle$, by means of a lattice computation.

- Starting point: path integral formulation (equivalent to Schrödinger's equation),

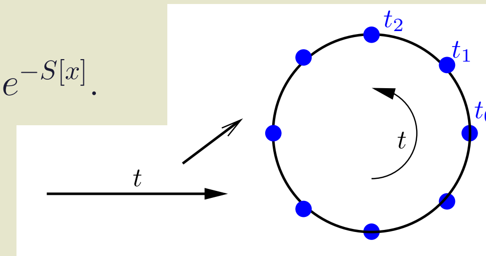
$$\langle 0|x^2|0\rangle \quad = \quad \frac{1}{Z}\int Dx \, x^2 e^{-S[x]} \quad , \quad Z \quad = \quad \int Dx \, e^{-S[x]}.$$

  - $\int Dx$: integral over all possible paths $x(t)$, i.e. an integral over a function space ($=$ "integral over infinitely many variables").
  - $e^{-S[x]}$: weight factor containing the action of the harmonic oscillator.

Lattice computations in QM (2)

- Starting point: path integral formulation (equivalent to Schrödinger's equation),

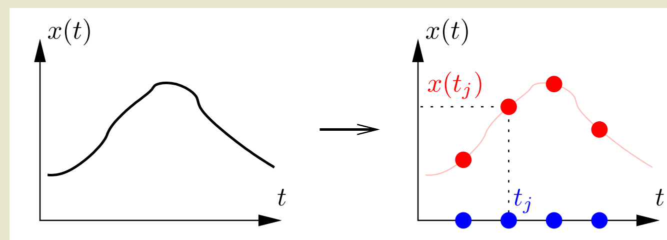
$$\langle 0|x^2|0\rangle \quad = \quad \frac{1}{Z}\int Dx\, x^2 e^{-S[x]} \quad , \quad Z \quad = \quad \int Dx\, e^{-S[x]}.$$



- Discretize and compactify time:
  $t \in \mathbb{R} \quad \rightarrow \quad t_j = j \times \Delta t \quad , \quad j = 0, 1, \ldots, N-1$
  $\rightarrow$ path integral reduced to an ordinary multi-dimensional integral, i.e.

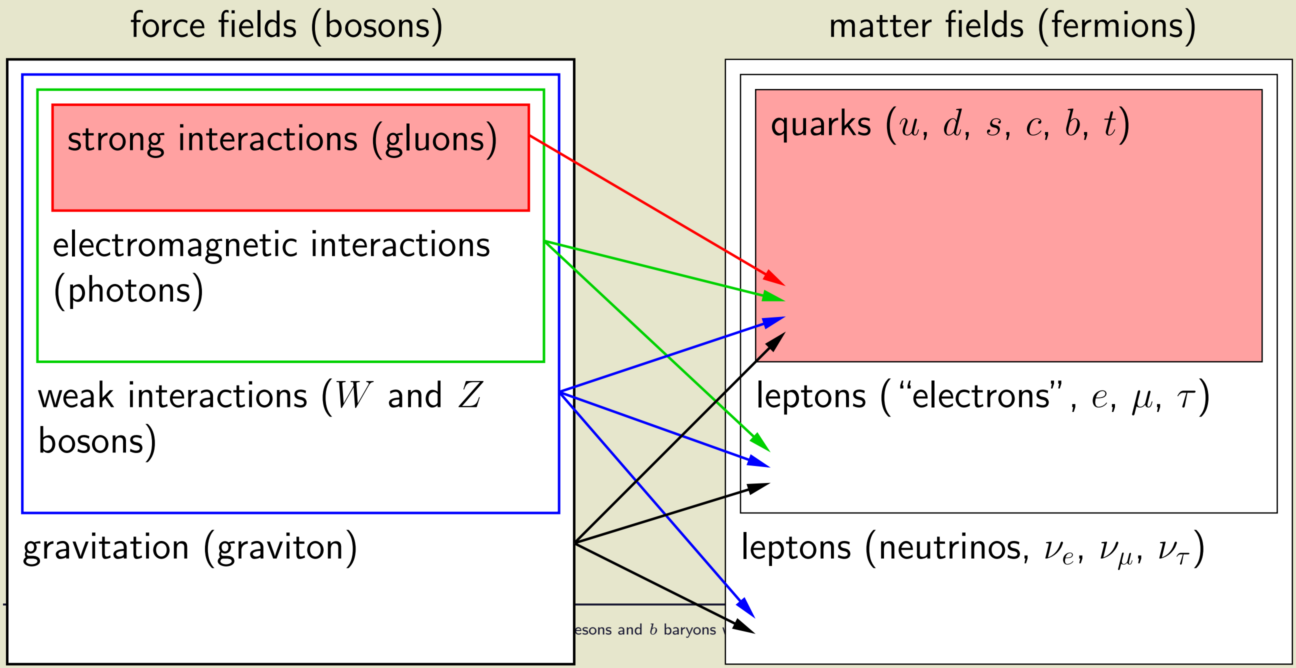
$$\int Dx\, e^{-S[x]} \quad \rightarrow \quad \int \bigg(\prod_{j=0}^{N-1} dx(t_j)\bigg) e^{-S[x(t_0),\ldots,x(t_{N-1})]}.$$

- Solve this multi-dimensional by means of a (high performance) computer.



“Standard model of particle physics”

- Four fundamental forces, which correspond to gauge bosons.
- Matter: six types of quarks and six types of leptons.
- QCD (quantum chromodynamics): quarks and gluons and their interactions.



QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.

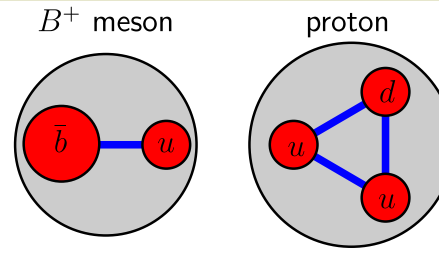
- Part of the standard model explaining the formation of hadrons (mesons $= q\bar{q}$, baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD by means of an action simple:

$$S = \int d^4x \Big(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - iA_\mu\Big) + m^{(f)}\Big)\psi^{(f)} + \frac{1}{2g^2}\mathrm{Tr}\Big(F_{\mu\nu}F_{\mu\nu}\Big)\Big)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).



Lattice QCD (1)

- Goal: compute QCD observables, e.g. hadron masses, from first principles with controllable systematic error.

- Use the path integral formulation of QCD,

$$\left\langle \mathcal{O}(\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu) \right\rangle = \frac{1}{Z} \int \Big(\prod_f D\psi^{(f)} \, D\bar{\psi}^{(f)} \Big) DA_\mu \, \mathcal{O}(\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu) e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu]}.$$

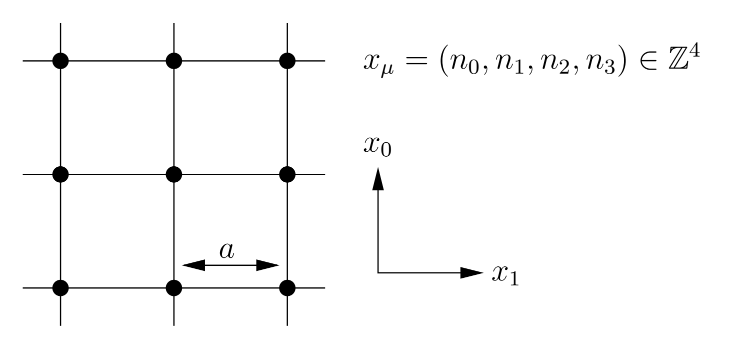
  - $\langle \ldots \rangle$: ground state/vacuum expectation value.
  - $\mathcal{O}(\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu)$: function of the quark and gluon fields, which can be related to an observable, e.g. a specific meson/baryon mass.
  - $\int (\prod_f D\psi^{(f)} \, D\bar{\psi}^{(f)}) DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x}, t)$ and $A_\mu(\mathbf{x}, t)$.
  - $e^{-S[x]}$: weight factor containing the QCD action.

Note that this path integral is analogous to the quantum mechanical example,

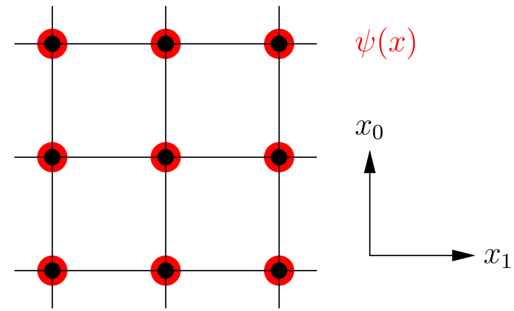
$$\langle 0 | x^2 | 0 \rangle = \frac{1}{Z} \int Dx \, x^2 e^{-S[x]}.$$

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite size effects".



Lattice QCD (3)



- Numerical implementation of the path ...
  - Quark fields:
    * Action (in the continuum):

$$S_{E,\text{quarks}} \quad = \quad \int d^4x \sum_f \bar{\psi}^{(f)} \Big(\gamma_\mu(\partial_\mu - iA_\mu) + m_f\Big)\psi^{(f)}.$$

    * "Direct discretization" of the quark fields $\psi^{(f)}$, i.e. quark fields are defined on the lattice sites.
    * Conceptually more difficult, high performance computer systems needed:
      · Fermion doubling (cf. my previous presentation).
      · Chiral symmetry explicitely broken.
      · Simulations at physically realistic values of the u and d quark masses extremely computer time consuming.
      · Discretization errors proportional to the lattice spacing a, i.e. rather large discretization errors.

Lattice QCD (4)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \left(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \right) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    - x_μ: $32^4 \approx 10^6$ lattice sites.
    - $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    - $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    - In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

  $\longrightarrow$ standard approaches for numerical integration not applicable

  $\longrightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

Twisted mass lattice QCD, ETMC

- Discretizing the QCD action is not unique.
- **Twisted mass lattice QCD** is a specific lattice discretization of QCD:
  - Advantages of twisted mass lattice QCD:

    (+) Automatic $\mathcal{O}(a)$ improvement of physical observables
    $\rightarrow$ lattice discretization errors due to fermions (quarks) do not appear linearly in the small lattice spacing a, only quadratically.

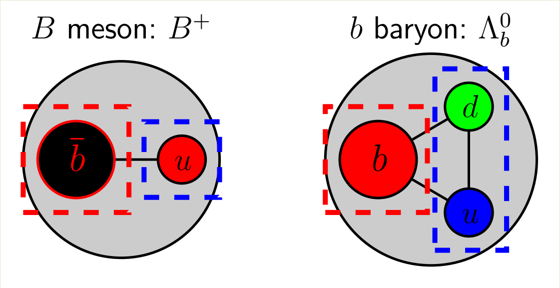
    (+) Compared to certain other lattice discretizations rather cheap
    $\rightarrow$ large lattice extensions and small lattice spacings feasible.
- The **European Twisted Mass Collaboration** (ETMC), a collaboration of more than twenty European universities and research institutes, of which I am a member, successfully uses this this discretization for already a couple of years to perform large scale computations of QCD with 2 quark flavors; recently we have started a similar major project to simulate 2+1+1 quark flavors.

Part 2: Selected research results from the field of B physics.

B physics, B mesons, b baryons (1)

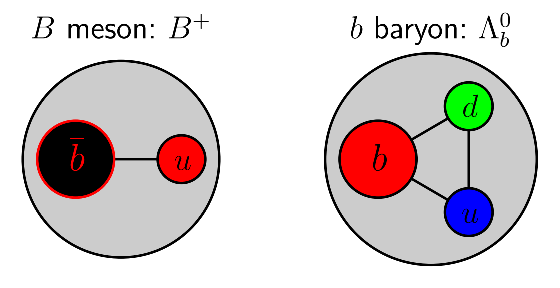
- B physics, typical questions: properties of B mesons and b baryons, e.g. masses, investigations of their decays; ...
  - B **mesons**: bound quark antiquark pairs, where one of the quarks is a heavy "bottom" oder b quark, the other is a light u, d or s quark.
  - b **baryons**: bound systems of three quarks/antiquarks, where one of the quarks is a heavy b quark, the remaining two are light u, d or s quarks.

quark	mass in MeV/c^2	electrical charge
up	$1.5 \ldots 3.3$	$+2/3e$
down	$3.5 \ldots 6$	$-1/3e$
strange	104^{+26}_{-34}	$-1/3e$
charm	1270^{+70}_{-11}	$+2/3e$
bottom	4200^{+170}_{-70}	$-1/3e$
top	170900 ± 1800	$+2/3e$

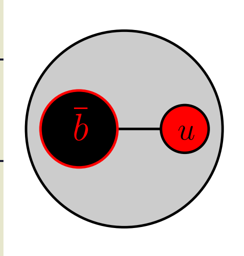


B physics, B mesons, b baryons (2)

- Discovery of the b quark 1977.
- Present-day experiments:
  - BaBar experiment, SLAC (USA).
  - Belle experiment, KEK (Japan).
  - LHCb experiment, CERN (Switzerland).
  - ...



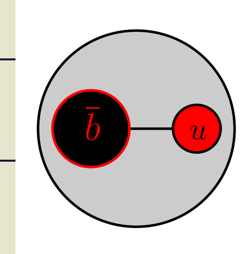
Masses of B and B_s mesons (1)



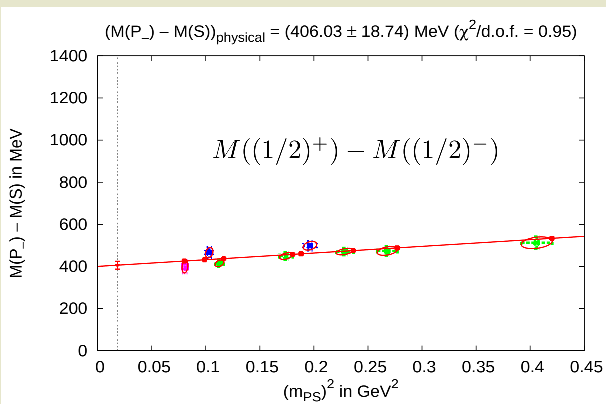
- B/B_s meson:
  - Bound quark antiquark pair (a heavy b quark and a light u, d/s quark).
  - "Hydrogen atom of QCD": a light particle (u, d, or s) "orbits" a heavy particle ($\bar{b}$).
  - States are characterized by:
    - Total angular momentum/spin of the light degrees of freedom j (light quarks and gluons); we perform computations in the limit $m_B \rightarrow \infty$ (static limit), which amounts to neglecting hyperfine splitting; hyperfine splitting is "reincluded" at the end.
    - Parity $\mathcal{P}$.
    - Radial quantum number; in the following, however, mostly ground states in the corresponding $j^{\mathcal{P}}$ sectors.

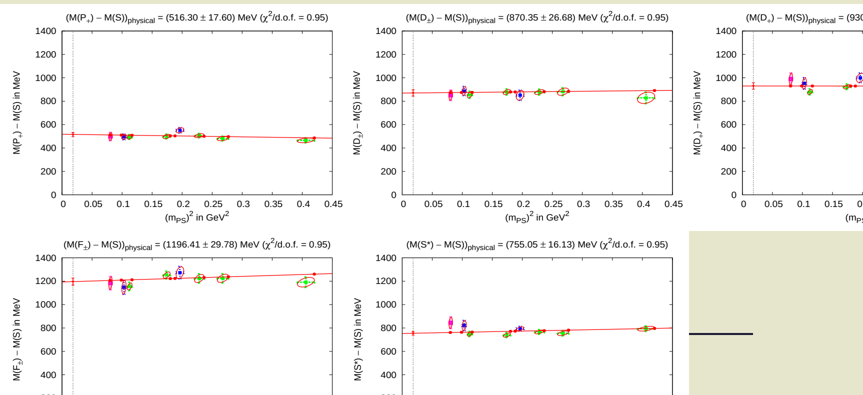
  Consequently, states are labeled by $j^{\mathcal{P}}$.

Masses of B and B_s mesons (2)

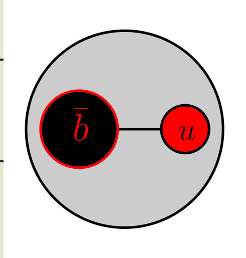


- Compute static-light meson masses (B/B_s mesons with $m_b \to \infty$) for different light u/d quark masses and different lattice spacings:
  - Different u/d quark masses to extrapolate to the physical u/d quark mass (due to technical reasons $m_\pi^{\text{(lattice)}} \gtrsim 300\,\text{MeV}$, $m_\pi^{\text{physical}} \approx 135\,\text{MeV}$).
  - Different lattice spacings to extrapolate to the continuum.
  - Horizontal axis: pion mass $(m_\pi^{\text{(lattice)}})^2$.
  - Vertical axis: $M(j^\mathcal{P}) - m_B$ mass difference between radially and orbitally excited B mesons (B_0^*, B_1^*, B_1, B_2^*, ...) and the ground state B meson ($B^\pm/B^0/B^* \equiv j^\mathcal{P} = (1/2)^-$) ... analogous for B_s mesons.





Masses of B and B_s mesons (3)

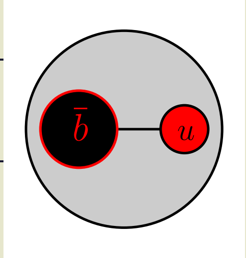


- Summary of the computed static-light meson spectrum:

$j^{\mathcal{P}}$	alternative notation	B mesons ($\bar{b}u$ or $\bar{b}d$): $M(\text{meson}) - M(B)$ in MeV	B_s mesons ($\bar{b}s$) $M(\text{meson}) - M(B_s)$ in MeV
$(1/2)^+$	P_-	$406(19)$	$413(12)$
$(3/2)^+$	P_+	$516(18)$	$504(12)$
$(3/2)^-$, $(5/2)^-$	$D_\pm$	$870(27)$	$770(26)$
$(5/2)^-$	D_+	$930(28)$	$960(24)$
$(5/2)^+$, $(7/2)^+$	$F_\pm$	$1196(30)$	$1179(37)$
$(1/2)^-$	S^*	$755(16)$	$751(26)$

- Motivation/achievements:
  - Continuum limit (among the first).
  - Dependence on the light u/d sea quark mass (for the first time).
  - Valuable input for model builders (e.g. no reversal of $M(P_-)$ and $M(P_+)$, ...).

Masses of B and B_s mesons (4)



- Comparison to experimental results:
  - Extrapolation to the physical (finite) b quark mass $m_B \approx 4200$ MeV by means of rather precise experimental results for c quarks, i.e. D mesons (amounts to "reincluding" hyperfine splitting):

	$M - M(B)$ in MeV			$M - M(B_s)$ in MeV	
name	lattice	experiment	name	lattice	experiment
B_0^*	$443(21)$		B_{s0}^*	$391(8)$	
B_1^*	$460(22)$		B_{s1}^*	$440(8)$	
B_1	$530(12)$	$444(2)$	B_{s1}	$526(8)$	$463(1)$
B_2^*	$543(12)$	$464(5)$	B_{s2}^*	$539(8)$	$473(1)$
B_J^*		$418(8)$	B_{sJ}^*		$487(15)$

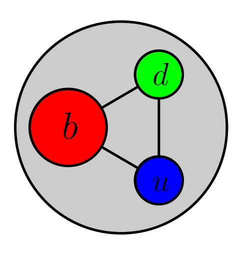
- Difference between lattice and experimental results: scale setting problem?

[K. Jansen, C. Michael, A. Shindler and M.W. [ETM Collaboration], PoS **LATTICE2008**, 122 (2008)]

[K. Jansen, C. Michael, A. Shindler and M.W. [ETM Collaboration], JHEP **0812**, 058 (2008)]

[C. Michael, A. Shindler and M.W. [ETM Collaboration], JHEP **1008**, 009 (2010)]

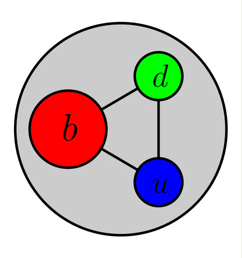
Masses of b baryons (1)



- b baryon: bound system of three quarks/antiquarks (a heavy b quark and two light u, d or s quarks).
- Computation of masses similar to that of B and B_s mesons.
- Summary of the computed static-light baryon spectrum:

$j^{\mathcal{P}}$	I	S	name	$m(\text{baryon}) - m(B)$ in MeV
0^+	0	0	Λ_b^0	$434(46)$
1^+	1	0	Σ_b/Σ_b^*	$671(46)/632(39)$
0^-	0	0	$-$	$1389(113)$
1^-	1	0	$-$	$1008(92)/1014(79)$
0^+	$1/2$	-1	Ξ_b^-	$630(41)/677(36)$
1^+	$1/2$	-1	$-$	$789(45)/798(49)$
0^-	$1/2$	-1	$-$	$1200(90)/1262(77)$
1^-	$1/2$	-1	$-$	$1233(58)/1285(69)$
1^+	0	-2	Ω_b^-	$903(42)$
1^-	0	-2	$-$	$1315(79)$

Masses of b baryons (2)



- Comparison to experimental results:
  - Extrapolation to the physical (finite) b quark mass $m_B \approx 4200$ MeV by means of rather precise experimental results for c quarks, i.e. charm baryons.

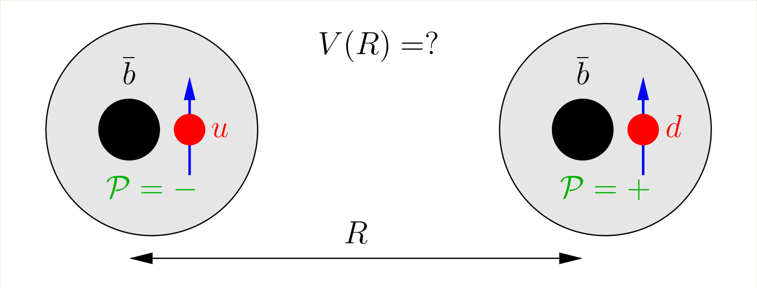
	$M - M(B)$ in MeV	
name	lattice	experiment
Λ_b^0	$429(30)$	$341(2)$
Σ_b	$629(28)$	$528(3)$
Σ_b^*	$651(28)$	$550(3)$
Ξ_b^-	$635(25)$	$513(3)$
Ω_b^-	$877(27)$	$775(7)$

- Differences between lattice and experimental results similar as for B/B_s.
- In progress: continuum limit.

[M.W. and C. Wiese [ETM Collaboration], PoS **LATTICE2010**, 130 (2010)]

Forces between B mesons (1)

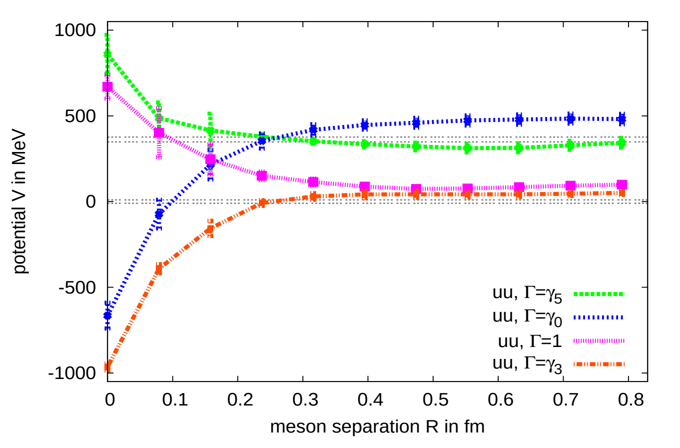
- Goal: compute the potential of (or equivalently the force between) two B mesons:
  - Treat the b quark in the static approximation.
  - Consider only pseudoscalar mesons ($j^{\mathcal{P}} = (1/2)^-$) and scalar mesons ($j^{\mathcal{P}} = (1/2)^+$), which are among the lightest static-light mesons.
  - Study the dependence of the mesonic potential $V(R)$ on
    * the light quark flavor u and/or d (isospin),
    * the light quark spin (the static quark spin is irrelevant),
    * the type of the meson, i.e. $j^{\mathcal{P}} = (1/2)^-$ or $j^{\mathcal{P}} = (1/2)^+$.



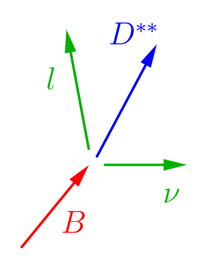
Forces between B mesons (2)

- Motivation/achievements:
  - First principles computation of a hadronic force, i.e. nuclear physics from elementary particles their interactions.
  - For the first time with dynamical quarks (until now only quenched results).
  - For the first time also $j^{\mathcal{P}} = (1/2)^+$ mesons (until now only $j^{\mathcal{P}} = (1/2)^-$ mesons).

[M.W. [ETM Collaboration], PoS **LATTICE2010**, 162 (2010)]



Semileptonic decays $B \to D^{**}$ (1)

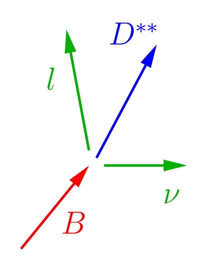


- The weak interactions change quark flavor, e.g. $b \to c + l + \nu$.
- Consider the specific weak decays

$$B \quad \to \quad D^{**} + l + \nu.$$

  - B: $j = (1/2)^-$ B meson.
  - D^{**}:
    - ∗ Orbitally excited D meson (e.g. $\bar{c}u$) with parity $\mathcal{P} = +$.
    - ∗ Coupling of angular momentum $L = 1$ ("P wave") and the light and the heavy quark spin yields four possible states:
      - · Two $1/2$ D^{**} ($L = 1$ and light quark spin $1/2$ are coupled to total angular momentum $j = 1/2$).
      - · Two $3/2$ D^{**} ($L = 1$ and light quark spin $1/2$ are coupled to total angular momentum $j = 3/2$).
  - $l + \nu$: lepton and corresponding neutrino.

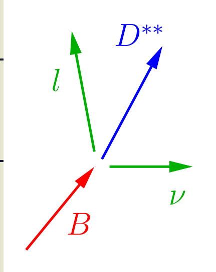
Semileptonic decays $B \to D^{**}$ (2)



- There is a conflict between theory and experiment:
  - **Theory** (operator product expansion, sum rules):
    - Decay of B to $3/2$ D^{**} is more likely.
    - However:
      - Statements only hold in the limit $m_B \to \infty$.
      - Assumption: excited states can be neglected in sum rules.
      - Statements apply only for the "zero recoil situation".
  - **Experiment**:
    - Decay of B to $1/2$ D^{**} is more likely.
    - However:
      - The measured signal for $1/2$ D^{**} is extremely weak.
      - Assumption: no contributions of states "above D^{**}".

$\to$ Lattice computations can help, to resolve this conflict.

Semileptonic decays $B \to D^{**}$ (3)

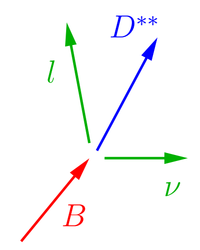


- Computation of the decay probabilities:
  - Based on time-dependent perturbation theory (Fermi's golden rule).
    - "Unperturbed" theory: QCD.
    - "Perturbing Hamiltonian": weak interactions.
    - One has to compute the matrix elements

      $$\mathcal{M}_{fi} = \langle D^{**}_{1/2|3/2}\, l\, \nu | \mathcal{H}_{\text{weak}} | B \rangle .$$

      - $\mathcal{M}_{fi}$ can be splitted into a leptonic part (contains l, ν) and a hadronic part (contains $D^{**}_{1/2|3/2}$, B).
      - The leptonic part can be calculated analytically ("kinematical factors" in differential decay rates).
      - The hadronic part can be computed by means of lattice QCD ("Isgur-Wise functions" $\tau_{1/2}$ and $\tau_{3/2}$ in differential decay rates).

Semileptonic decays $B \rightarrow D^{**}$ (4)



- Lattice result:

$$\tau_{1/2} \ = \ 0.30(3) \quad , \quad \tau_{3/2} \ = \ 0.53(2)$$

("$|\tau_{1/2}|^2$ is proportional to the decay probability in $1/2$ D^{**}"; ...).

[B. Blossier, M.W. and O. Pene [ETM Collaboration], JHEP **0906**, 022 (2009)]

[B. Blossier, M.W. and O. Pene [ETM Collaboration], PoS **LATTICE2009**, 253 (2009)]

- Theory result (sum rules):

$$\Big|\tau_{3/2}\Big|^2 - \Big|\tau_{1/2}\Big|^2 \ \approx \ \frac{1}{4}$$

(comparison with the lattice result: $0.53^2 - 0.30^2 = 0.19 \approx 1/4$; sum rule fulfilled by around 80%).

- Experimental result (Belle):

$$\tau_{1/2} \ \approx \ 1.28 \quad , \quad \tau_{3/2} \ \approx \ 0.75.$$

Summary

- Lattice QCD is a method to compute QCD observables from first principles; systematic errors can be controlled and removed by suitable extrapolations.
- Lattice QCD is a powerful tool in the field of B physics.
- Goals:
  - Precision computations followed by comparisons to experimental results ("search for new physics")
    $\longrightarrow$ necessary accuracy for B physics observables not yet reached.
  - Prediction/computation of observables, which are difficult/impossible to access experimentally
    $\longrightarrow$ example: masses of excited B mesons beyond $j = (3/2)^+$;
    $\longrightarrow$ example: isospin, spin and parity dependent forces between B mesons.
  - Qualitative understanding
    $\longrightarrow$ example: level ordering of states, e.g. no reversal for P_- and P_+;
    $\longrightarrow$ example: decay of B into $3/2$-D^{**} more likely than into $1/2$-D^{**}.

Part 3: Further research interests and planned research.

Further research interests (1)

- During the past four years I roughly invested $1/3$ of my research time for questions related to B **physics** (the projects and results I have been reporting).
- Roughly another $1/3$ went into a major project of the European Twisted Mass Collaboration concerned with **simulations of QCD with 2+1+1 dynamical quark flavors** (we are worldwide among the first to perform such simulations).

Further research interests (2)

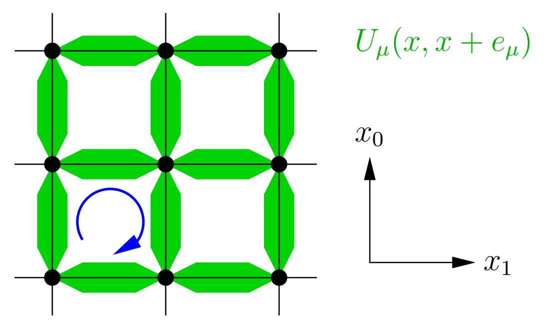
- The remaining time I used for a variety of smaller projects:
  - **Determination of** $\Lambda_{\overline{\mathrm{MS}}}$ (establish contact between lattice QCD and perturbative QCD).
  - **String breaking** in adjoint Yang-Mills theory and in QCD (theoretical aspects and numerical demonstration).
  - **Topology on the lattice**, **simulations at fixed topology** (preparatory steps for mixed action setups [overlap on twisted mass]).
  - (SU(2) four flavor) **QCD under extreme conditions** (finite temperature, strong external magnetic fields).
  - **Models based on topological excitations** for Yang-Mills theory/QCD both at zero temperature (merons, instantons, dimerons) and finite temperature (dyons) (qualitative understanding in particular regarding the phenomenon of confinement).

Planned research

- Continue ongoing projects, in particular ETMC simulations with 2+1+1 dynamical twisted mass quarks.
- Strange and charm meson spectroscopy (kaons, D mesons, D_s mesons, charmonium) with 2+1+1 dynamical twisted mass quarks.
- QCD at finite temperature (part of the Twisted Mass Finite Temperature Collaboration is located at the Johann Wolfgang Goethe University Frankfurt).
- QCD simulations at fixed topology, dependence of physical observables on the topological sector (relevant for mixed action setups including overlap quarks and for fully dynamical overlap simulations).
- ...

some additional slides

Lattice QCD (A1)



- Numerical implementation of the path ...
  - Gluon field:
    * Action (in the continuum):

$$S_{E,\mathrm{gluons}} = \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\Big(F_{\mu\nu} F_{\mu\nu}\Big).$$

    * To preserve gauge invariance, "indirect discretization" of the gluon field A_μ via so-called gauge links (parallel transporters, which connect neighboring lattice sites):

$$U_\mu(x) = P\bigg\{\exp\bigg(-i\int_x^{x+ae_\mu} dz_\mu \, A_\mu(z)\bigg)\bigg\} \approx \exp\Big(-iaA_\mu(x)\Big).$$

    * Example: (no sum over μ and ν):

$$a^4\Big(\mathrm{Tr}\Big(F_{\mu\nu}F_{\mu\nu}\Big) + \mathcal{O}(a^2)\Big) =$$

$$= 6\bigg(1 - \frac{1}{3}\mathrm{Re}\Big(\mathrm{Tr}\Big(U_\mu(x)U_\nu(x+ae_\mu)U_\mu^\dagger(x+ae_\nu)U_\nu^\dagger(x)\Big)\Big)\bigg)$$

(in electrodynamics this expression would be $\propto (E_x)^2$ or $(B_x)^2$ or ...).

b quarks with lattice QCD (A1)

- In contrast to light u, d and s quarks, heavy b quarks cannot be treated by means of the lattice techniques explained:
  - Field configurations close to classical solutions with small action are only weakly suppressed by e^{-S_E}, i.e. play an important role in the path integral.
  - The classical equation of motion for quarks is the Dirac equation ("relativistic version of Schrödinger's equation"):

    $$\Big(\gamma_\mu(\partial_\mu - iA_\mu) + m_f\Big)\psi^{(f)} = 0$$

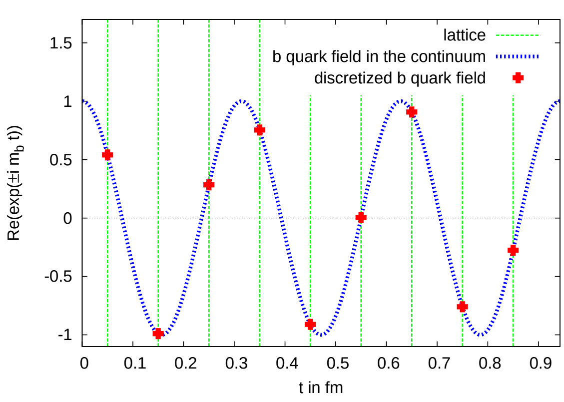
    ($\psi^{(f)}$ has four spin components; γ_μ: 4×4 matrices).
  - Solutions in the free case ($A_\mu = 0$) are plane waves:

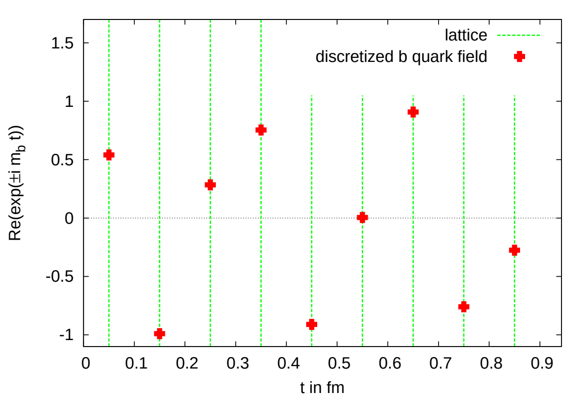
    $$\psi^{(f)}_{-,s} = e^{-i(E(\mathbf{p})t - \mathbf{p}\mathbf{x})} u^s(\mathbf{p}) \quad , \quad \psi^{(f)}_{+,s} = e^{+i(E(\mathbf{p})t - \mathbf{p}\mathbf{x})} (v^s(\mathbf{p}))^\dagger$$

    with $E(\mathbf{p}) = \sqrt{m_f^2 + \mathbf{p}^2} \approx m_f$ for small momenta $\mathbf{p}$ and $s = 1, 2$, i.e. two solutions $\psi^{(f)}_{-,s} \propto e^{-im_f t}$ and two solutions $\psi^{(f)}_{+,s} \propto e^{+im_f t}$.

b quarks with lattice QCD (A2)

- In contrast to light u, d and s quarks, heavy b quarks cannot be treated by means of the lattice techniques just explained:
  - Lattice representation of these solutions for b quarks:
    - $m_b \approx 4000\,\text{MeV} \approx 20/\text{fm}$.
    - Typical lattice spacing: $a = (1/10)\,\text{fm}$.
    - Oscillations of the b quark field $\psi_{\pm,s}^{(b)} \propto e^{\pm i m_b t}$ cannot be resolved at typical present-day lattice spacings.
    - No such problems with light quarks (larger wave length of $e^{\pm i m_f t}$).





b quarks with lattice QCD (A3)

- Solution: HQET (Heavy Quark Effective Theory).
  - Rewrite the b quark field in terms of a new field according to

    $$\psi^{(b)} \quad \rightarrow \quad \psi'^{(b)} \quad = \quad e^{+im_b t}\psi^{(b)},$$

    i.e. perform a simple change of variables.
  - Two of the four solutions "loose" the strongly oscillating phase factor:

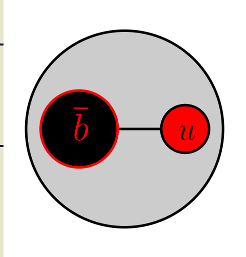
    $$\psi'^{(f)}_{-,s} \quad \propto \quad 1.$$

  - Two of the four solutions oscillate even stronger:

    $$\psi'^{(f)}_{+,s} \quad \propto \quad e^{-i2m_b t}.$$

    However, one can analytically perform the integration over field configurations corresponding to these strongly oscillating solutions.
  - Result: power series in $1/m_b$, where the leading order describes static (= infinitely heavy) b quarks.

Masses of B and B_s mesons (A1)

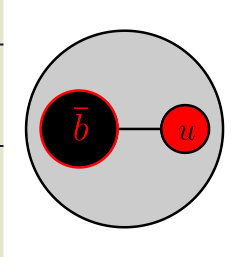


- Construction of a B meson state:
  - The quark field operator $\psi^{(u)}(\mathbf{x})$ creates a u quark at position $\mathbf{x}$.
  - The quark field operator $\bar{\psi}^{(b)}(\mathbf{x})$ generates a b antiquark at position $\mathbf{x}$.
  - The following state contains a B meson at position $\mathbf{x}$:

    $$B^{(\Gamma)}|\Omega\rangle \quad = \quad \bar{\psi}^{(b)}(\mathbf{x})\Gamma\psi^{(u)}(\mathbf{x})|\Omega\rangle$$ ($B^{(\Gamma)}$: meson creation operator).

    * Γ: suitably chosen 4×4 matrix.
      - Acts on the spin indices of the quarks; realizes the desired angular momentum J and/or j as well as parity $\mathcal{P}$.
      - A combination of the γ matrices from the Dirac equation, e.g. $\Gamma = \gamma_5$ corresponds to $J^{\mathcal{P}} = 0^-$ and $j^{\mathcal{P}} = (1/2)^-$, the lightest B meson.
    * $B^{(\Gamma)}|\Omega\rangle$ is a superposition of b meson states, which have the same quantum numbers $j^{\mathcal{P}}$, but which differ in their radial quantum number.

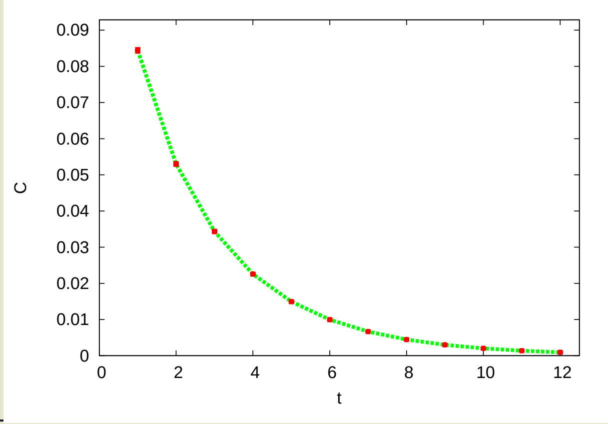
Masses of B and B_s mesons (A2)



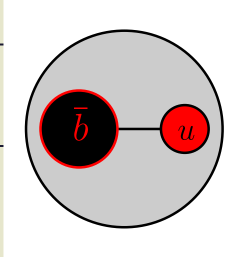
- Computation of the B meson mass ($J = 0^-$, $j = (1/2)^-$):
  - Compute the vacuum expectation value

$$C(t) \quad = \quad \langle \Omega | \underbrace{\Big(\bar{\psi}^{(b)}(\mathbf{x},t)\gamma_5 \psi^{(u)}(\mathbf{x},t)\Big)^\dagger}_{=(B^{(\gamma_5)})^\dagger(t)} \underbrace{\bar{\psi}^{(b)}(\mathbf{x},0)\gamma_5 \psi^{(u)}(\mathbf{x},0)}_{=B^{(\gamma_5)}(0)} |\Omega\rangle$$

    as a function of t by means of lattice QCD.
  - $C(t)$: "meson correlation function".



Masses of B and B_s mesons (A3)



- Computation of the B meson mass ($J = 0^-$, $j = (1/2)^-$):
  - Insert an identity in terms of energy eigenstates:

$$C(t) \quad = \quad \langle\Omega|\Big(B^{(\Gamma)}(t)\Big)^\dagger B^{(\Gamma)}(0)|\Omega\rangle \quad = \quad \ldots \quad \approx_{t \gg 1} \quad \text{const} \times e^{-m_B t}$$

    (m_B: mass of the B meson; const: an irrelevant constant).
  - Extract m_B e.g. by fitting $Ae^{-m_B t}$ to the computed points of $C(t)$ with A and m_B being the fit parameters.