Static-static-light-light tetraquarks in lattice QCD

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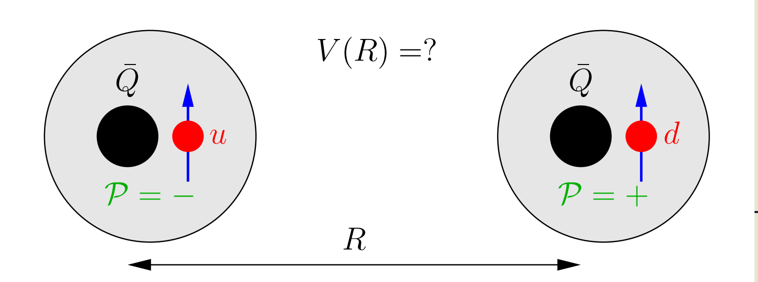
[M. Wagner, PoS **LATTICE2010**, 162 (2010)]

Introduction (1)

- Goal: compute the potential of (or equivalently the force between) two B mesons from first principles by means of lattice QCD:
  - Treat the b quark in the static approximation.
  - Consider only pseudoscalar/pseudovector mesons ($j^{\mathcal{P}} = (1/2)^-$, denoted by S, PDG: B, B^*) and scalar/vector mesons ($j^{\mathcal{P}} = (1/2)^+$, denoted by P_-, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons.
  - Study the dependence of the mesonic potential $V(R)$ on
    * the light quark flavor u and/or d (isospin),
    * the light quark spin (the static quark spin is irrelevant),
    * the type of the meson S and/or P_-.



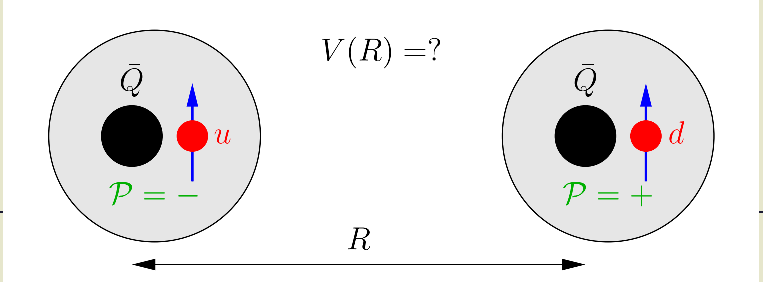
Introduction (2)

- Motivation:
  - First principles computation of a hadronic force.
  - Possible application: determine, whether two B mesons may form bound states (tetraquarks).
  - Until now
    - it has only been studied in the quenched approximation,
    - only pseudoscalar (S), but no scalar (P_-) B mesons have been considered.

[C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]

[W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

[G. Bali and M. Hetzenegger, PoS **LATTICE2010**, 142 (2010)]



Outline

- Symmetries and quantum numbers of B mesons and BB systems.
- Lattice setup.
- Results and their interpretation.

(Pseudo)scalar B mesons

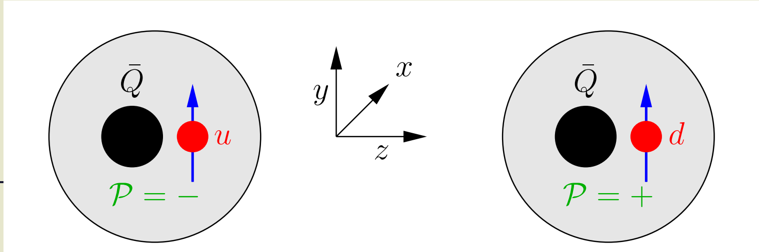
- Symmetries and quantum numbers of static-light mesons:
  - Isospin: $I = 1/2$, $I_z = \pm 1/2$, i.e. $B \equiv \bar{Q}u$ or $B \equiv \bar{Q}d$.
  - Parity: $\mathcal{P} = \pm$.
  - Rotations:
    * Light cloud angular momentum $j = 1/2$ (for S and P_-), $j_z = \pm 1/2$.
    * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
- Examples of static-light meson creation operators:
  - $\bar{Q}\gamma_5 q$ (pseudoscalar, i.e. S), $q \in \{u\,,\,d\}$,
  - $\bar{Q}q$ (scalar, i.e. P_-)

(j_z is not well-defined, when using these operators).

BB systems (1)

- Symmetries and quantum numbers of a pair of static-light mesons (separated along the z-axis):
  - Isospin: $I = 0, 1$, $I_z = -1, 0, +1$.
  - Rotations around the z-axis:
    - Angular momentum of the light degrees of freedom $j_z = -1, 0, +1$.
    - Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
  - Parity: $\mathcal{P} = \pm$.
  - If $j_z = 0$, reflection along the x-axis: $\mathcal{P}_x = \pm$.
  - Instead of using $j_z = \pm 1$ one can also label states by $|j_z| = 1$, $\mathcal{P}_x = \pm$.

$\longrightarrow$ Label BB states by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$.



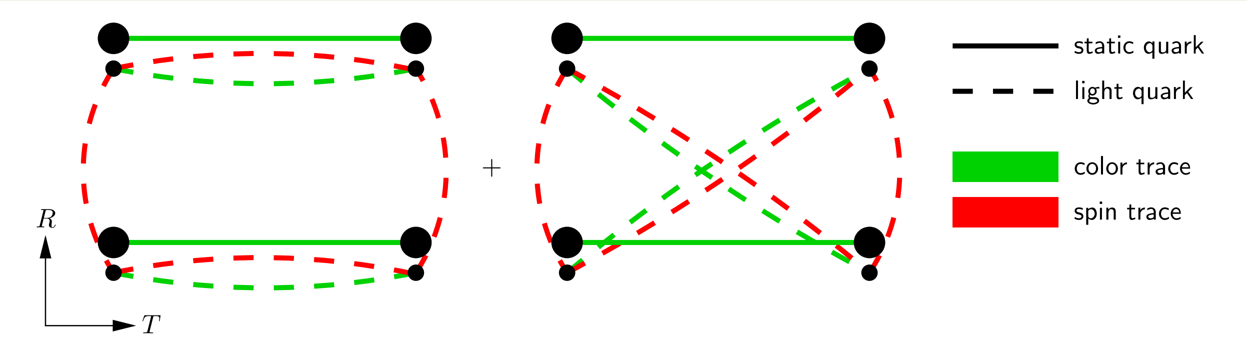
BB systems (2)

- To extract the potential(s) of a given sector (characterized by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$), compute the temporal correlation function of the trial state

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(1)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(2)}(+R/2)\Big)|\Omega\rangle,$$

  where

  - $\mathcal{C} = \gamma_0\gamma_2$ (charge conjugation matrix),
  - $q^{(1)}q^{(2)} \in \{ud - du \quad , \quad uu\, , \, dd\, , \, ud + du\}$ (isospin I, I_z) ,
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}$, $\mathcal{P}_x$).



BB systems (3)

- BB creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big).$$

Γ	$\lvert j_z\rvert$, $\mathcal{P}$, $\mathcal{P}_x$
1	0, $-$, $-$
$\gamma_0\gamma_5$	0, $+$, $+$
γ_5	0, $+$, $+$
γ_0	0, $+$, $-$
γ_3	0, $-$, $-$
$\gamma_0\gamma_3\gamma_5$	0, $+$, $+$
$\gamma_3\gamma_5$	0, $-$, $+$
$\gamma_0\gamma_3$	0, $-$, $-$
γ_1	1, $-$, $+$
$\gamma_0\gamma_1\gamma_5$	1, $+$, $-$
$\gamma_1\gamma_5$	1, $-$, $-$
$\gamma_0\gamma_1$	1, $-$, $+$
$\ldots$	$\ldots$

BB systems (4)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(d)}(+R/2)\Big) \pm (u \leftrightarrow d).$$

Γ, $\pm$	$\lvert j_z \rvert$, I, $\mathcal{P}$, $\mathcal{P}$
γ_5 , $-$	0, 0, $-$, $+$
γ_0 , $-$	0, 0, $-$, $-$
1 , $-$	0, 0, $+$, $-$
$\gamma_0\gamma_5$, $-$	0, 0, $-$, $+$
$\gamma_3\gamma_5$, $-$	0, 0, $+$, $+$
$\gamma_0\gamma_3$, $-$	0, 0, $+$, $-$
γ_3 , $-$	0, 0, $+$, $-$
$\gamma_0\gamma_3\gamma_5$, $-$	0, 0, $-$, $+$
γ_5 , $+$	0, 1, $+$, $+$
γ_0 , $+$	0, 1, $+$, $-$
1 , $+$	0, 1, $-$, $-$
$\gamma_0\gamma_5$, $+$	0, 1, $+$, $+$
$\ldots$	$\ldots$

Lattice setup (1)

- List of criteria a perfect lattice simulation should fulfill (cf. Zoltan Fodor's talk on Tuesday):

  (1) Technically correct:
  I strongly think so ... otherwise I would not give this presentation.

  (2) Physical u/d quark masses:
  No ... u/d sea and valence quark masses correspond $m_\pi \approx 340\,\text{MeV}$ ($m_\pi \approx 140\,\text{MeV}$ in nature).
  However,

  * the best existing lattice computation has been done with infinitely heavy u/d sea quark masses and u/d valence quark masses corresponding to $m_\pi \approx 400\,\text{MeV}$.

Lattice setup (2)

- List of criteria a perfect lattice simulation should fulfill (cf. Zoltan Fodor's talk on Tuesday):

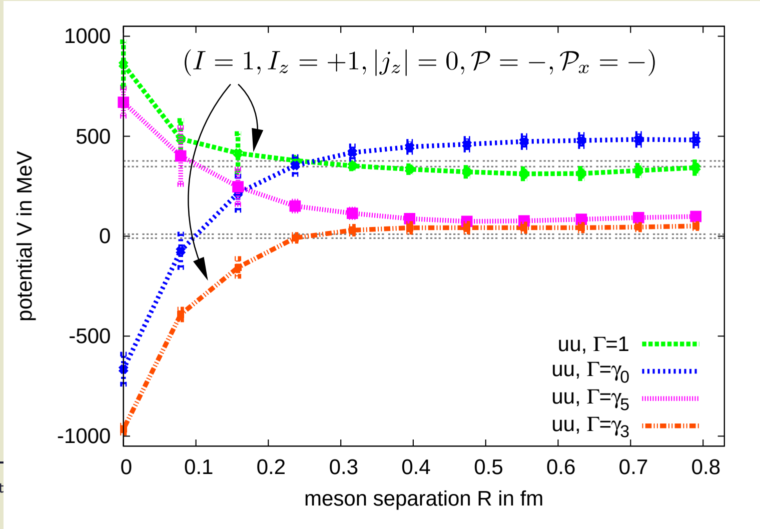
  (3) Continuum extrapolation:
  No ... only a single lattice spacing $a \approx 0.079\,\textrm{fm}$.
  However,

  * in Wilson twisted mass lattice QCD (the lattice discretization I have been using) discretization errors appear only quadratically in the small lattice spacing a,
  * the best existing lattice computation has been done with $a \approx 0.100\,\textrm{fm}$ with linear discretization errors,
  * a recent study of the continuum limit of static-light mesons suggests that discretization errors are significantly smaller than statistical errors ($\rightarrow$ continuum physics within error bars).

    [C. Michael, A. Shindler and M. Wagner [ETM Collaboration], JHEP **1008**, 009 (2010)]

Discussion of results (1)

- Four "types of potentials":
  - Two attractive, two repulsive.
  - Two have asymptotic values, which are larger by $\approx 400\,\mathrm{MeV}$.
- There are cases, where two potentials with identical quantum numbers are completely different (i.e. of different type)
  $\longrightarrow$ at least one of the corresponding trial states must have very small ground state overlap
  $\longrightarrow$ physical understanding, i.e. interpretation of trial states needed.



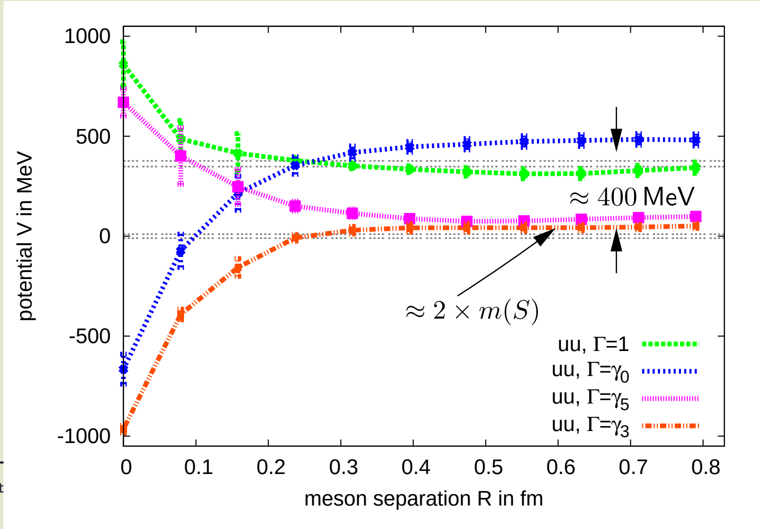
Discussion of results (2)

- Expectation at large meson separation R: $V(R) \approx 2 \times$ meson mass.
  - Potentials with smaller asymptotic value at $\approx 2 \times m(S)$.
  - $m(P_-) - m(S) \approx 400\,\mathrm{MeV}$: approximately the observed difference between different types of potentials.

$\longrightarrow$ Two types correspond to $S \leftrightarrow S$ potentials.

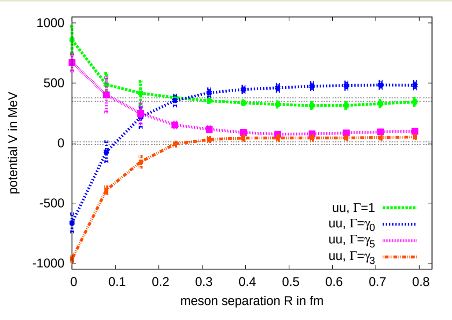
$\longrightarrow$ Two types correspond to $S \leftrightarrow P_-$ potentials.

- Can this be understood in detail on the level of the used BB creation operators?

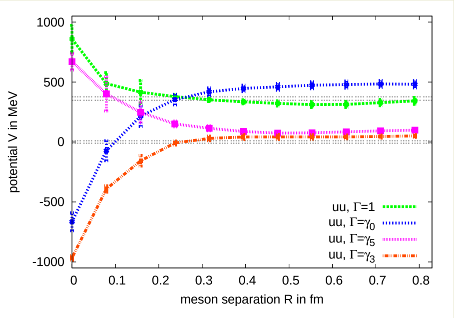


Discussion of results (3)

- Express the BB creation operators in terms of static-light meson creation operators (use suitable spin and parity projectors for the light quarks).
  - Examples:
    - uu, $\Gamma = 1 \quad \rightarrow \quad \mathcal{P} = -, \mathcal{P}_x = -$:
      $$(\mathcal{C}1)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \propto$$
      $$\propto \quad S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow.$$
    - uu, $\Gamma = \gamma_3 \quad \rightarrow \quad \mathcal{P} = -, \mathcal{P}_x = -$:
      $$(\mathcal{C}\gamma_3)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \propto$$
      $$\propto \quad S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow.$$
  - SS/SP_- content and asymptotic values in agreement for all 64 correlation functions/ potentials
    $\rightarrow$ asymptotic differences understood.

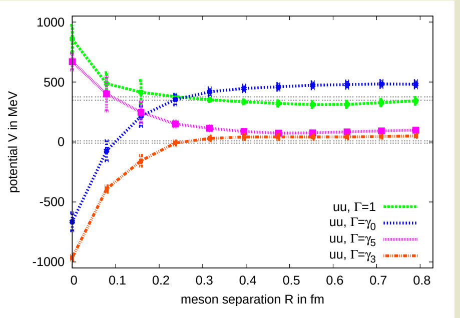


Discussion of results (4)



- Is there a general rule, about when a potential is repulsive and when attractive?
  - $S \leftrightarrow S$ potentials:
    * $(I = 0\ ,\ s = 0)$ or $(I = 1\ ,\ s = 1)$, i.e. $I = s \quad \rightarrow \quad$ attractive
      $(I = 0\ ,\ s = 1)$ or $(I = 1\ ,\ s = 0)$, i.e. $I \neq s \quad \rightarrow \quad$ repulsive
      (s: combined angular momentum of the two mesons).
    * Example: uu, $\Gamma = \gamma_3 \quad \rightarrow \quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
      $$(\mathcal{C}\gamma_3)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \propto$$
      $$\propto \quad S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow.$$
      i.e. $I = 1$, $s = 1$; the numerically obtained potential is attractive, i.e. in agreement with the above stated rule.
    * All 32 $S \leftrightarrow S$ correlation functions/potentials fulfill the rule.
    * Agreement with similar quenched lattice studies.
      [C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]
      [W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

Discussion of results (5)



– $S \leftrightarrow P_-$ potentials:

∗ Do not obey the above stated rule.

∗ It can, however, easily be generalized by including parity, i.e. symmetry or antisymmetry under exchange of S and P_-:
trial state symmetric under meson exchange $\longrightarrow$ attractive
trial state antisymmetric under meson exchange $\longrightarrow$ repulsive
(meson exchange $\equiv$ exchange of flavor, spin and parity).

∗ Example: uu, $\Gamma = \gamma_0 \quad \longrightarrow \quad \mathcal{P} = +$, $\mathcal{P}_x = -$:

$$(\mathcal{C}\gamma_0)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(u)}(+R/2)\Big) \quad \propto$$

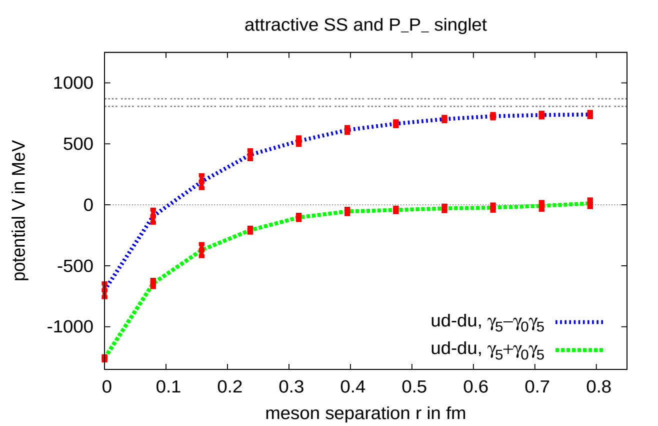
$$\propto \quad S_\uparrow P_\downarrow - S_\downarrow P_\uparrow - P_\uparrow S_\downarrow + P_\downarrow S_\uparrow,$$

i.e. $I = 1$ (symmetric), $s = 0$ (antisymmetric), antisymmetric with respect to $S \leftrightarrow P_-$; the numerically obtained potential is attractive, i.e. in agreement with the above stated general rule.

∗ All 32 $S \leftrightarrow P_-$ correlation functions/potentials (and all 32 $S \leftrightarrow S$ correlation functions/potentials) fulfill the generalized rule.

Discussion of results (6)

- Improvements after having understood the extraction and interpretation of BB potentials from single correlation functions:
  - Linearly combine BB operators to either eliminate $P_- \leftrightarrow P_-$ or $S \leftrightarrow S$ combinations.
  - Example:
    $ud - du$, $\Gamma = \gamma_5 \quad \rightarrow \quad -S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow + P_\downarrow P_\uparrow$
    $ud - du$, $\Gamma = \gamma_0\gamma_5 \quad \rightarrow \quad -S_\uparrow S_\downarrow + S_\downarrow S_\uparrow + P_\uparrow P_\downarrow - P_\downarrow P_\uparrow$
    $\rightarrow$ use $\gamma_5 + \gamma_0\gamma_5$ to obtain a better signal for the $S \leftrightarrow S$ potential
    $\rightarrow$ use $\gamma_5 - \gamma_0\gamma_5$ to extract the $P_- \leftrightarrow P_-$ potential.



Discussion of results (7)

- Improvements after having understood the extraction and interpretation of BB potentials from single correlation functions:
  - Use correlation matrices instead of single correlation functions to avoid mixing with BB states of lower energy, which is present, because
    - although the product of two specific B meson creation operators closely resembles the corresponding BB state, it will still have a non-vanishing overlap to BB states corresponding to B mesons with different isospin, spin and/or parity,
    - twisted mass lattice QCD explicitely breaks isospin and parity (the breaking is proportional to the lattice spacing a; isospin and parity will be restored in the continuum limit).

Summary of BB states and degeneracies

- Two B mesons, each can have $I_z = \pm 1/2$, $j_z = \pm 1/2$, $\mathcal{P} = \pm$ $\rightarrow 8 \times 8 = 64$ states.

- $S \leftrightarrow S$ potentials:
  - Attractive: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{6}_{I=1,|j_z|=1}$ (10 states).
  - Repulsive: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{2}_{I=0,|j_z|=1}$ (6 states).

- $S \leftrightarrow P_-$ potentials:
  - Attractive: $\underbrace{1 \oplus 1 \oplus 3 \oplus 3}_{|j_z|=0} \oplus \underbrace{2 \oplus 6}_{|j_z|=1}$ (16 states).
  - Repulsive: $\underbrace{1 \oplus 1 \oplus 3 \oplus 3}_{|j_z|=0} \oplus \underbrace{2 \oplus 6}_{|j_z|=1}$ (16 states).

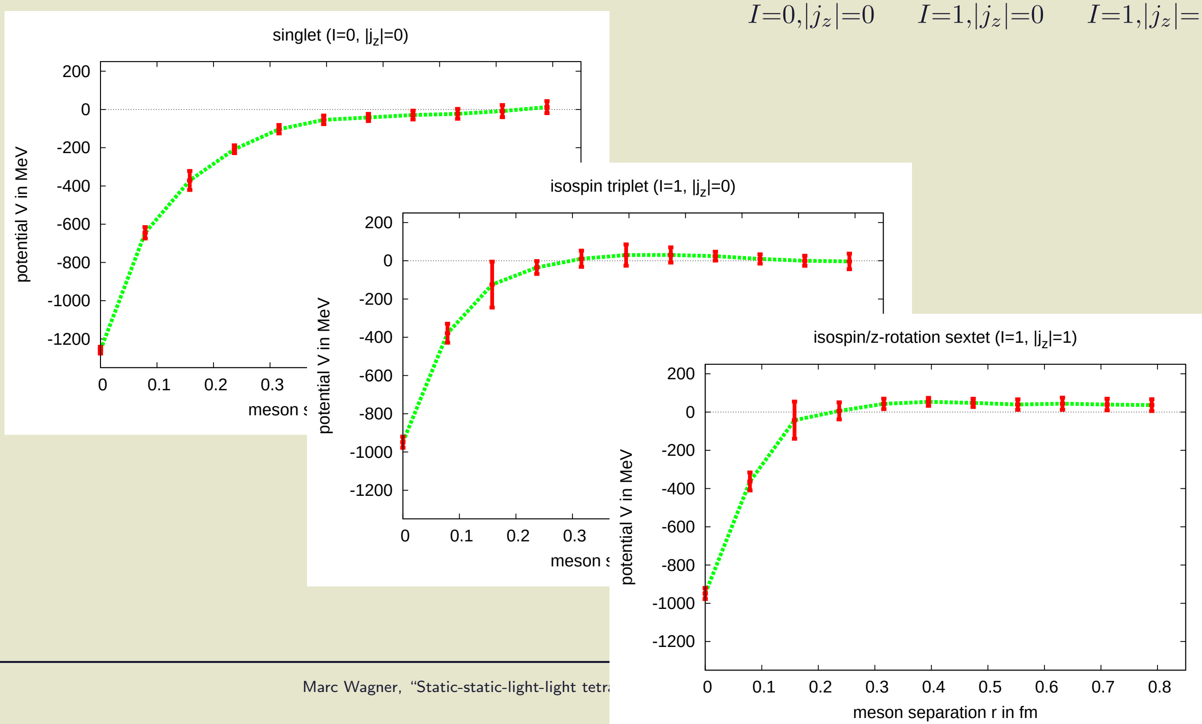
- $P_- \leftrightarrow P_-$ potentials: identical to $S \leftrightarrow S$ potentials.

- In total 24 different potentials.

Attractive $S \leftrightarrow S$ potentials

- Attractive $S \leftrightarrow S$ potentials are relevant, when trying to determine, whether BB may form a bound state.

- Three different attractive $S \leftrightarrow S$ potentials: $\underbrace{1}_{I=0,|j_z|=0} \oplus \underbrace{3}_{I=1,|j_z|=0} \oplus \underbrace{6}_{I=1,|j_z|=1}$.



Summary, conclusions, future plans (1)

- Computation of BB potentials (arbitrary flavor, spin and parity) with light dynamical quarks ($m_{\mathrm{PS}} \approx 340\,\mathrm{MeV}$).
  - Qualitative agreement with existing quenched results for $S \leftrightarrow S$ potentials.
  - First lattice computation of $S \leftrightarrow P_-$ and $P_- \leftrightarrow P_-$ potentials.
  - Clear statements about whether a potential of a given channel is attractive or repulsive.
- Statistical accuracy problematic (exponentially decaying correlation functions are quickly lost in statistical noise):
  - Reasonable accuracy for attractive $S \leftrightarrow S$ potentials (interesting, when trying to determine, whether BB may form a bound state).
  - Other (higher) potentials:
    $\rightarrow$ BB potentials are extracted at rather small temporal separations
    $\rightarrow$ slight contamination from excited states cannot be excluded.

Summary, conclusions, future plans (2)

- Further plans and possibilities:
  - Other values of the lattice spacing, the spacetime volume and/or the u/d quark mass.
  - Partially quenched computations, to obtain $B_s B_s$ and/or $B_s B$ potentials.
  - Improve lattice meson potentials at small separations (where the suppression of UV fluctuations due to the lattice cutoff yields wrong results) with corresponding perturbative potentials.

BB systems (A)

- Wilson twisted mass action:

$$S_\mathrm{F}[\chi, \bar{\chi}, U] \quad = \quad a^4 \sum_x \bar{\chi}(x) \Big(D_\mathrm{W} + i \mu_\mathrm{q} \gamma_5 \tau_3 \Big) \chi(x) \quad , \quad \psi(x) \quad = \quad e^{i \gamma_5 \tau_3 \omega / 2} \chi(x).$$

- Symmetries of Wilson twisted mass lattice QCD compared to QCD:
  - SU(2) isospin breaks down to U(1): I_z is still a good quantum number, I is not.
  - Parity $\mathcal{P}$ is replaced by $\mathcal{P}^{(\mathrm{tm})}$, which is parity combined with light flavor exchange.
  - Twisted mass BB sectors:
    * $I_z = \pm 1$: $(I_z \ , \ |j_z| \ , \ \underbrace{\mathcal{P}^{(\mathrm{tm})} \mathcal{P}_x^{(\mathrm{tm})}}_{= \mathcal{P} \mathcal{P}_x})$,
    * $I_z = 0$: $(I_z \ , \ |j_z| \ , \ \underbrace{\mathcal{P}^{(\mathrm{tm})}}_{= \mathcal{P} \times (2I - 1)} \ , \ \underbrace{\mathcal{P}_x^{(\mathrm{tm})}}_{= \mathcal{P}_x \times (2I - 1)})$.

  $\rightarrow$ QCD sectors $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$ are pairwise combined.

BB systems (B)

- BB creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2)\Big).$$

Γ twisted	$\lvert j_z\rvert$, $\mathcal{P}^{(\mathrm{tm,light})}\mathcal{P}_x^{(\mathrm{tm,light})}$	Γ pseudo physical	$\lvert j_z\rvert$, $\mathcal{P}^{(\mathrm{light})}$, $\mathcal{P}_x^{(\mathrm{light})}$
γ_5	$0, +$	$\mp i$	$0, -, -$
$\gamma_0\gamma_5$	$0, +$	$+\gamma_0\gamma_5$	$0, +, +$
1	$0, +$	$\mp i\gamma_5$	$0, +, +$
γ_0	$0, -$	$+\gamma_0$	$0, +, -$
γ_3	$0, +$	$+\gamma_3$	$0, -, -$
$\gamma_0\gamma_3$	$0, +$	$\mp i\gamma_0\gamma_3\gamma_5$	$0, +, +$
$\gamma_3\gamma_5$	$0, -$	$+\gamma_3\gamma_5$	$0, -, +$
$\gamma_0\gamma_3\gamma_5$	$0, +$	$\mp i\gamma_0\gamma_3$	$0, -, -$
γ_1	$1, -$	$+\gamma_1$	$1, -, +$
$\gamma_0\gamma_1$	$1, -$	$\mp i\gamma_0\gamma_1\gamma_5$	$1, +, -$
$\gamma_1\gamma_5$	$1, +$	$+\gamma_1\gamma_5$	$1, -, -$
$\gamma_0\gamma_1\gamma_5$	$1, -$	$\mp i\gamma_0\gamma_1$	$1, -, +$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

BB systems (C)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)\chi_B^{(d)}(+R/2)\Big) \pm (u \leftrightarrow d).$$

Γ twisted, $\pm$	$\lvert j_z \rvert$, $\mathcal{P}^{(\mathrm{tm,light})}$, $\mathcal{P}_x^{(\mathrm{tm,light})}$	Γ pseudo physical, $\pm$	$\lvert j_z \rvert$, I, $\mathcal{P}^{(\mathrm{light})}$, $\mathcal{P}_x^{(\mathrm{light})}$
γ_5 , $+$	$0, +, +$	$+\gamma_5$, $+$	$0, 1, +, +$
$\gamma_0\gamma_5$, $+$	$0, +, +$	$+i\gamma_0$, $-$	$0, 0, -, -$
1 , $-$	$0, -, +$	$+1$, $-$	$0, 0, +, -$
γ_0 , $-$	$0, +, +$	$+i\gamma_0\gamma_5$, $+$	$0, 1, +, +$
γ_5 , $-$	$0, +, -$	$+\gamma_5$, $-$	$0, 0, -, +$
$\gamma_0\gamma_5$, $-$	$0, +, -$	$+i\gamma_0$, $+$	$0, 1, +, -$
1 , $+$	$0, -, -$	$+1$, $+$	$0, 1, -, -$
γ_0 , $+$	$0, +, -$	$+i\gamma_0\gamma_5$, $-$	$0, 0, -, +$
γ_3 , $+$	$1, -, -$	$+i\gamma_3\gamma_5$, $-$	$0, 0, +, +$
$\gamma_0\gamma_3$, $+$	$1, -, -$	$+\gamma_0\gamma_3$, $+$	$0, 1, -, -$
$\gamma_3\gamma_5$, $-$	$1, -, -$	$+i\gamma_3$, $+$	$0, 1, -, -$
$\gamma_0\gamma_3\gamma_5$, $-$	$1, +, -$	$+\gamma_0\gamma_3\gamma_5$, $-$	$0, 0, -, +$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

Simulation setup (A)

- Fermionic action: Wilson twisted mass, $N_f = 2$ degenerate flavors,

$$S_{\mathrm{F}}[\chi, \bar{\chi}, U] \;=\; a^4 \sum_x \bar{\chi}(x) \Big(D_{\mathrm{W}} + i\mu_{\mathrm{q}}\gamma_5\tau_3\Big)\chi(x)$$

$$D_{\mathrm{W}} \;=\; \frac{1}{2}\Big(\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - a\nabla^*_\mu\nabla_\mu\Big) + m_0$$

(m_0: untwisted mass; μ_{q}: twisted mass; τ_3: third Pauli matrix acting in flavor space).

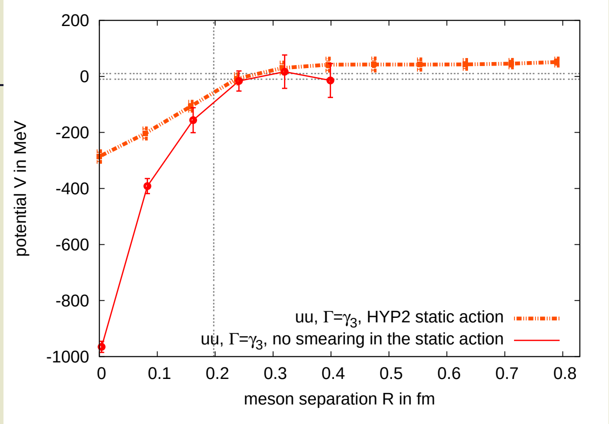
- Relation between the physical basis ψ and the twisted basis χ (in the continuum):

$$\psi \;=\; \frac{1}{\sqrt{2}}\Big(\cos(\omega/2) + i\sin(\omega/2)\gamma_5\tau_3\Big)\chi$$

$$\bar{\psi} \;=\; \frac{1}{\sqrt{2}}\bar{\chi}\Big(\cos(\omega/2) + i\sin(\omega/2)\gamma_5\tau_3\Big)$$

(ω: twist angle; $\omega = \pi/2$: maximal twist).

Simulation setup (B)



- $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0040$
  $\longrightarrow$ lattice spacing $a \approx 0.079\,\mathrm{fm}$
  $\longrightarrow$ lattice extension $L \approx 1.90\,\mathrm{fm}$
  $\longrightarrow$ pion mass $m_{\mathrm{PS}} \approx 340\,\mathrm{MeV}$.
- Inversions/contractions on 210 gauge configurations for light u/d quarks.
- 12 u and 12 d inversions per gauge configuration (stochastic timeslice sources located on the same timeslice).
- APE smearing of spatial links and Gaussian smearing of light quark fields to "optimize" the ground state overlap of trial states.
- Wilson lines of static quarks are discretized by path ordered products of ordinary links (small separations) and HYP2 smeared links (large separations).