The SU(2) quark-antiquark potential in the pseudoparticle approach

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30th March 2006

Outline

PP = pseudoparticle

- Basic principle.
- Building blocks of PP ensembles.
- PP ensembles.
- Quark-antiquark potential.
- Quantitative results.
- Summary.
- Outlook.

Basic principle (1)

- Pseudoparticle approach (PP approach):
  - A numerical technique to approximate Euclidean path integrals (in this talk: SU(2) Yang-Mills theory $\approx$ QCD with infinitely heavy quarks):

$$\Big\langle \mathcal{O} \Big\rangle = \frac{1}{Z} \int DA \, \mathcal{O}[A] e^{-S[A]}$$

$$S[A] = \frac{1}{4g^2} \int d^4x \, F^a_{\mu\nu} F^a_{\mu\nu} \quad , \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu.$$

  - A tool to analyze the importance of gauge field configurations with respect to confinement.
  - A method, from which we can get a better understanding of the Yang-Mills path integral.

Basic principle (2)

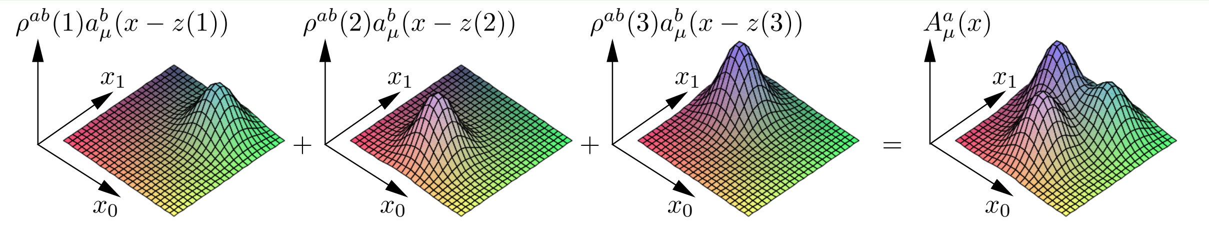
- PP: any gauge field configuration a^a_μ, which is localized in space and time.
- Consider only those gauge field configurations, which can be written as a sum of a fixed number (≈ 400) of PPs:

$$A^a_\mu(x) \quad = \quad \sum_i \rho^{ab}(i) a^b_\mu(x - z(i)).$$

(i: PP index; $\rho^{ab}(i)$: degrees of freedom of the i-th PP, i.e. amplitude and color orientation; $z(i)$: position of the i-th PP).

- Approximate the path integral by an integration over PP degrees of freedom:

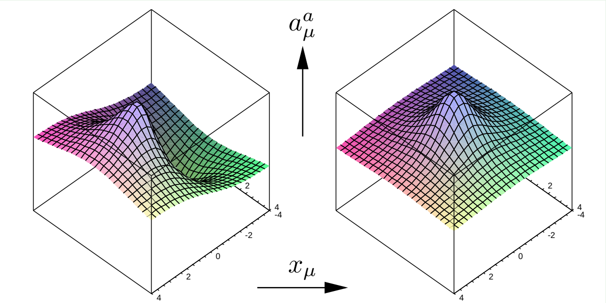
$$\int DA \ldots \quad \rightarrow \quad \int \left(\prod_i d\rho^{ab}(i)\right) \ldots$$



Building blocks of PP ensembles

- Building blocks of PP ensembles: "instantons", "antiinstantons", akyrons (λ: PP size).

$$a^a_{\mu,\text{instanton}}(x) = \eta^a_{\mu\nu}\frac{x_\nu}{x^2+\lambda^2}$$
$$a^a_{\mu,\text{antiinstanton}}(x) = \bar{\eta}^a_{\mu\nu}\frac{x_\nu}{x^2+\lambda^2}$$
$$a^a_{\mu,\text{akyron}}(x) = \delta^{a1}\frac{x_\mu}{x^2+\lambda^2}.$$



- Degrees of freedom: amplitudes $\mathcal{A}(i)$, color orientations $\mathcal{C}^{ab}(i)$, positions $z(i)$.

$$A^a_\mu(x) = \mathcal{A}(i)\mathcal{C}^{ab}(i)a^a_{\mu,\text{instanton}}(x-z(i))$$
$$A^a_\mu(x) = \mathcal{A}(i)\mathcal{C}^{ab}(i)a^a_{\mu,\text{antiinstanton}}(x-z(i))$$
$$A^a_\mu(x) = \mathcal{A}(i)\mathcal{C}^{ab}(i)a^a_{\mu,\text{akyron}}(x-z(i)).$$

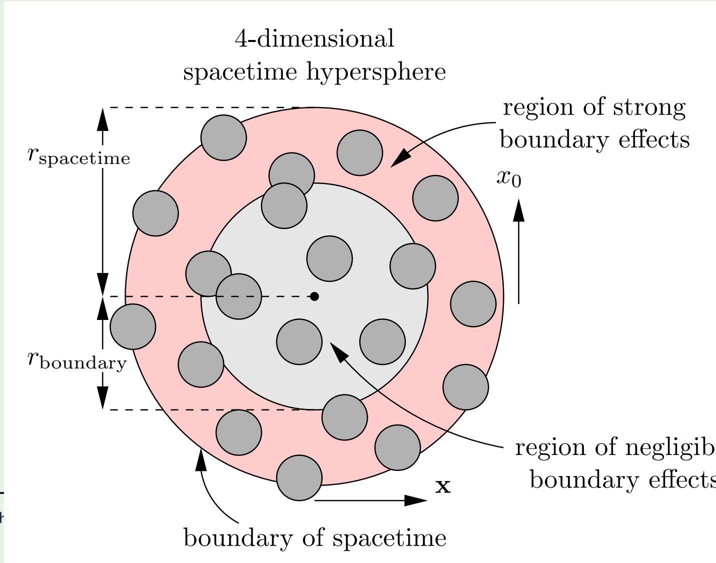
- Instantons, antiinstantons and akyrons form a basis of all gauge field configurations in the "continuum limit".

PP ensembles (1)

- PP ensemble: a fixed number of PPs inside a "spacetime hypersphere".
- Gauge field:

$$\begin{aligned} A_\mu^a(x) \; = \; & \sum_i \mathcal{A}(i)\mathcal{C}^{ab}(i) a^b_{\mu,\text{instanton}}(x - z(i)) + \\ & \sum_j \mathcal{A}(j)\mathcal{C}^{ab}(j) a^b_{\mu,\text{antiinstanton}}(x - z(j)) + \\ & \sum_k \mathcal{A}(k)\mathcal{C}^{ab}(k) a^b_{\mu,\text{akyron}}(x - z(k)). \end{aligned}$$

- Choose color orientations $\mathcal{C}^{ab}(i)$ and positions $z(i)$ randomly.
- A_μ^a is no classical solution (not even close to a classical solution)!!!
- Long range interactions between PPs.

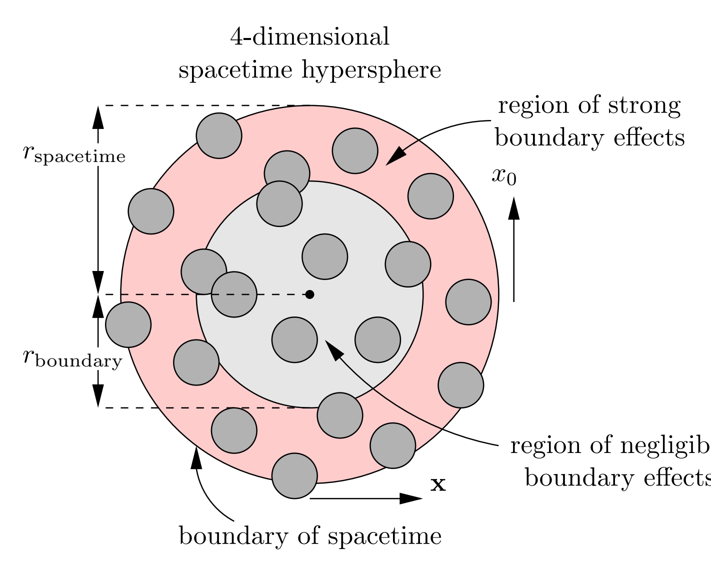


PP ensembles (2)

- Approximation of the path integral:

$$\left\langle \mathcal{O} \right\rangle = \frac{1}{Z} \int \left(\prod_i d\mathcal{A}(i)\right) \mathcal{O}(\mathcal{A}(i)) e^{-S(\mathcal{A}(i))}.$$

- Solve this multidimensional integral via Monte-Carlo simulations.
- Exclude boundary effects: observables have to be "measured" sufficiently far away from the boundary.



Quark-antiquark potential (1)

- Common tool to determine the potential of a static quark-antiquark pair: Wilson loops.

- Wilson loop (z: closed spacetime curve):

$$W_z[A] \quad = \quad \frac{1}{2}\mathrm{Tr}\bigg(P\bigg\{\exp\bigg(i\oint dz_\mu\, A_\mu(z)\bigg)\bigg\}\bigg).$$

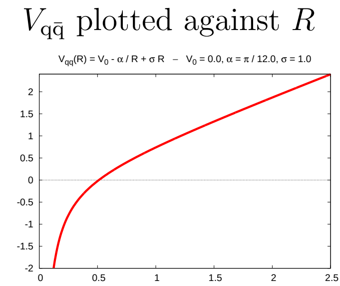
- Rectangular Wilson loop (R, T: spatial and temporal extension): $W_{(R,T)}$.

- Wilson loops $\leftrightarrow$ quark-antiquark potential (R: quark-antiquark separation):

$$V_{\mathrm{q\bar{q}}}(R) \quad = \quad -\lim_{T\to\infty}\frac{1}{T}\ln\Big\langle W_{(R,T)}\Big\rangle.$$

- Assumption: the potential can be parameterized according to

$$V_{\mathrm{q\bar{q}}}(R) \quad = \quad V_0 - \frac{\alpha}{R} + \sigma R.$$



Quark-antiquark potential (2)

Method 1: Determine the string tension σ and the Coulomb coefficient α

- "Guess" the functional dependence of ensemble averages of Wilson loops:

$$-\ln\left\langle W_{(R,T)}\right\rangle \quad = \quad V_0\Big(R+T\Big) - \alpha\left(\frac{R}{T}+\frac{T}{R}\right) + \beta + \sigma RT.$$

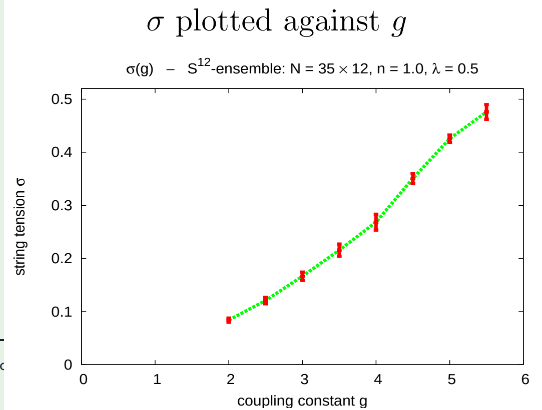
- Determine the string tension σ and the Coulomb coefficient α by fitting the "Wilson loop ansatz" to Monte-Carlo data for $-\ln\langle W_{(R,T)}\rangle$.

- Several approaches:
  - Area perimeter fits.
  - Creutz ratios.
  - Generalized Creutz ratios.
  - ...

Quark-antiquark potential (3)

Method 1: Determine the string tension σ and the Coulomb coefficient α

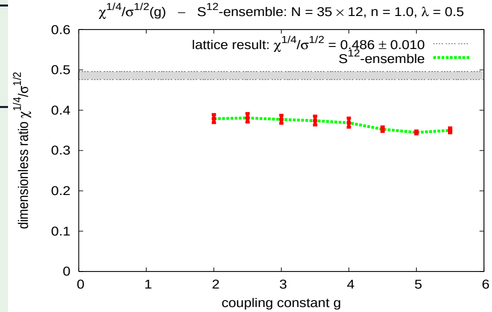
- Results for PP ensembles containing ≈ 400 PPs:
  - Coulomb coefficient $\alpha > 0$
    $\longrightarrow$ attractive "Coulomb-like" interaction at small quark-antiquark separations.
  - String tension $\sigma > 0$
    $\longrightarrow$ linear potential for large quark-antiquark separations, confinement.
  - σ is an increasing function of the coupling constant g
    $\longrightarrow$ adjust the physical scale by choosing appropriate values for g.



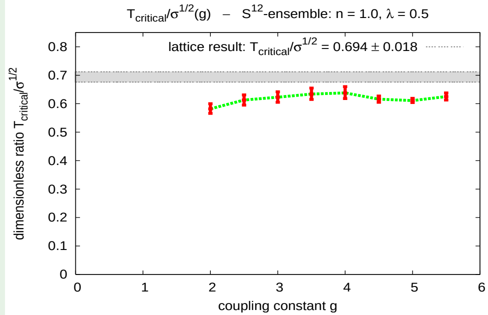
Quantitative results

- For quantitative results, including the string tension, we need other dimensionful quantities:
  - Topological susceptibility $\chi = \langle Q_V^2 \rangle / V$.
  - Critical temperature T_{critical}.
- Dimensionless quantities (physically meaningful): $\chi^{1/4}/\sigma^{1/2}$, $T_{\mathrm{critical}}/\sigma^{1/2}$, α.
- Consider different $g = 2.0 \ldots 5.5$ (diameter of the spacetime hypersphere $0.9\,\mathrm{fm} \ldots 2.0\,\mathrm{fm}$).
- Results are in qualitative agreement with results from lattice calculations.
- Consistent scaling behavior of σ, χ and T_{critical}.
- α should be constant.

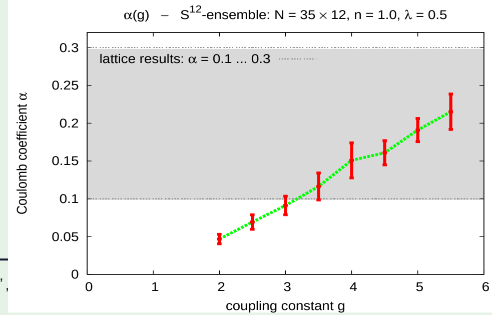
$\chi^{1/4}/\sigma^{1/2}$ plotted against g



$T_{\mathrm{critical}}/\sigma^{1/2}$ plotted against g



α plotted against g



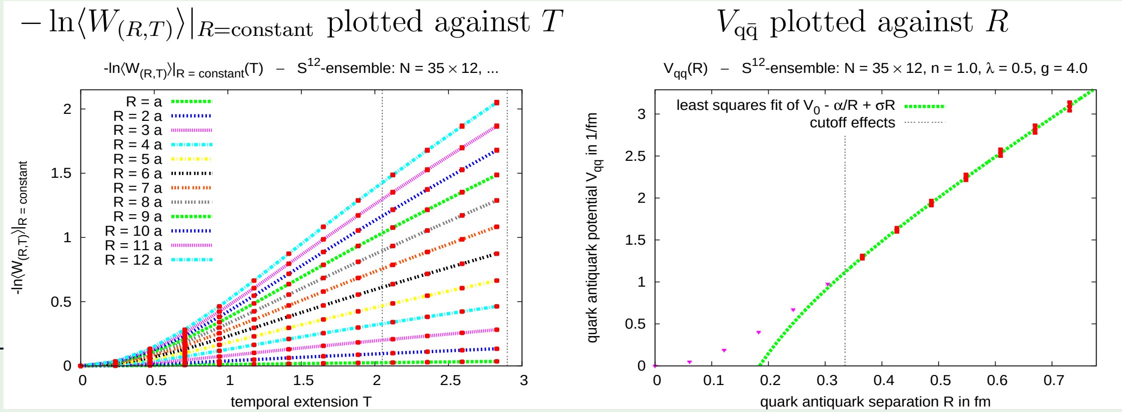
Quark-antiquark potential (4)

Method 2: Calculate the quark-antiquark potential directly

- For large T:

$$V_{q\bar{q}}(R)T \quad \approx \quad -\ln\left\langle W_{(R,T)}\right\rangle.$$

- From the slope of $-\ln\langle W_{(R,T)}\rangle|_{R=\text{constant}}$ we can read off $V_{q\bar{q}}(R)$.
- Results are in agreement with our previous results.



Summary

- The PP approach with ≈ 400 instantons, antiinstantons and akyrons is able to reproduce many essential features of SU(2) Yang-Mills theory:
  - Quark-antiquark potential:
    * Linear potential for large quark-antiquark separations (confinement).
    * "Coulomb-like" attractive force for small quark-antiquark separations.
  - Consistent scaling behavior of σ, χ and T_{critical}.
  - Dimensionless quantities $\chi^{1/4}/\sigma^{1/2}$, $T_{\mathrm{critical}}/\sigma^{1/2}$ and α are in qualitative agreement with results from lattice calculations.

Outlook

- Compare different PP ensembles to analyze, which gauge field configurations are responsible for confinement:
  - Pure akyron ensembles (no topological charge density) $\longrightarrow$ deconfinement.
  - Gaussian localized PPs (PPs of limited size) $\longrightarrow$ deconfinement for small PP size, confinement for large PP size.
- Overall picture: topological charge and long range interactions between PPs are important for confinement.