Static-light mesons (and string breaking)

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Outline

- Simulation setup.
- The static quark antiquark potential.
- Static-light mesons:
  - The static-light pseudoscalar meson.
  - The static-light meson spectrum.
- Smearing techniques:
  - Smearing of links in time direction (fat links, HYP1, HYP2).
  - APE smearing of spatial links.
  - Jacobi smearing of light quark operators.
- Summary and outlook.

Simulation setup

- $N_\mathrm{f} = 2$, $\beta = 3.9$, $24^3 \times 48$ lattice.
- Twisted mass Dirac operator

$$Q^{(\chi)} \quad = \quad \gamma_\mu D_\mu + m + i\mu\gamma_5 \quad , \quad m + 4 \quad = \quad \frac{1}{2\kappa}$$

  with $\kappa = 0.160856$, $\mu = 0.0040$.

- Identifying $r_0 = 5.18$ with $0.5\,\mathrm{fm}$ sets the physical scale yielding a spatial lattice extension of $2.32\,\mathrm{fm}$.
- The pseudoscalar meson mass $m_\mathrm{ps} = 0.136$ amounts to $m_\mathrm{ps} \approx 280\,\mathrm{MeV}$.
- At the moment, I only use 48 gauge configurations (1 out of 50 available) $\rightarrow$ **Statistical errors will be reduced by a factor of ≈ 7.**

The static quark antiquark potential (1)

- Static quark antiquark potential $V(R)$: the energy of the lowest state containing an infinitely heavy quark and an infinitely heavy antiquark, separated by a distance R.

- In principle $V(R)$ can be computed via Wilson loops $W_{(R,T)}$:

$$\begin{aligned}
&\langle 0|\Big(\bar{Q}(\mathbf{x},T)U(\mathbf{x},T;\mathbf{y},T)Q(\mathbf{y},T)\Big)^\dagger \bar{Q}(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)Q(\mathbf{y},0)|0\rangle \quad = \\
&\quad = \quad \ldots \quad = \quad \#\Big\langle W_{(R,T)}\Big\rangle \\
&V(R) \quad = \quad -\frac{1}{a}\lim_{T\to\infty}\Big(\ln\Big\langle W_{(R,T)}\Big\rangle - \ln\Big\langle W_{(R,T-1)}\Big\rangle\Big) .
\end{aligned}$$

The static quark antiquark potential (2)

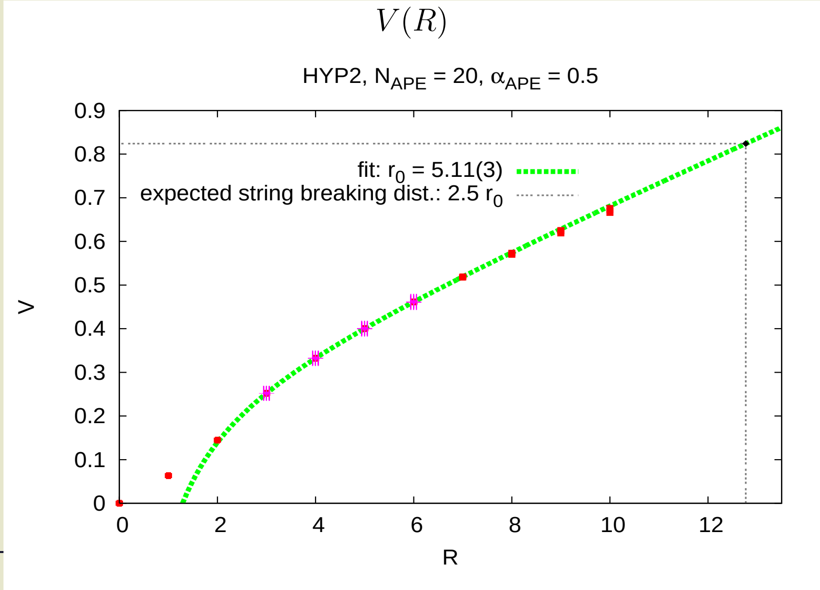
- In practice we have "moderate T separations" instead of $T \rightarrow \infty$:
  - Wilson loops work well for $R < R_{\mathrm{sb}}$ (R_{sb}: string breaking distance).
  - Wilson loops fail for $R > R_{\mathrm{sb}}$:
    * The potential is still linearly rising, i.e. there seems to be no string breaking.
    * Reason: $\bar{Q}(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)Q(\mathbf{y},0)|0\rangle$ has poor overlap to the lowest state for $R > R_{\mathrm{sb}}$ (a "two meson state").
  - Solution:
    * Instead of a single state use a whole set of states containing not only "string states" but also two meson states, i.e. $\bar{Q}(\mathbf{x},0)\gamma_5 q(\mathbf{x},0)\bar{q}(\mathbf{y},0)\gamma_5 Q(\mathbf{y},0)|0\rangle$.
    * Extract the potential from the corresponding correlation matrix.

The static quark antiquark potential (3)

- Nevertheless, the "pure Wilson loop potential" is a useful quantity:
  - Determine r_0 (set the physical scale).
  - Obtain an estimate of the mass of the lightest static-light meson M_{ps}:
    - Assume a value for the string breaking distance, e.g. $R_{\mathrm{sb}} \approx 1.25\,\mathrm{fm}$.
    - $M_{\mathrm{ps}} \approx V(R_{\mathrm{sb}})/2$.
- To obtain an acceptable signal-to-noise ratio smearing techniques are indispensable:
  - HYP2 smearing of links in time direction to reduce the static quark self energy.
  - APE smearing of spatial links to enhance the ground state overlap of $\bar{Q}(\mathbf{x},0)U(\mathbf{x},0;\mathbf{y},0)Q(\mathbf{y},0)|0\rangle$ (create a state with a flux tube resembling that of the lowest lying state).

The static quark antiquark potential (4)

- Fit with $V(R) = V_0 - \alpha/R + \sigma R$:
  - $r_0 = 5.11(3)$ (consistent with the "official ETMC value" $r_0 = 5.18(6)$).
  - Estimate of the static-light pseudoscalar meson mass, when assuming $R_{\mathrm{sb}} = 1.25\,\mathrm{fm}$: $M_{\mathrm{ps}} \approx 0.412$.



The static-light pseudoscalar meson (1)

- Determine the mass of the static-light pseudoscalar meson from the exponential fall off of the correlation function

$$\mathcal{C}^{(\bar{q}\gamma_5 Q)}(T) \quad = \quad \langle 0|\Big(\bar{q}(\mathbf{x},T)\gamma_5 Q(\mathbf{x},T)\Big)^\dagger \bar{q}(\mathbf{x},0)\gamma_5 Q(\mathbf{x},0)|0\rangle.$$

- The propagator of the static quark is analytically known (essentially a Wilson line):

$$\mathcal{C}^{(\bar{q}\gamma_5 Q)}(T) \quad \propto$$
$$\propto \quad \left\langle \mathrm{Tr}\left(\frac{1+\gamma_0}{2} P\left\{\exp\left(\pm i\int_0^T dz_0\, A_0(\mathbf{x},z_0)\right)\right\}((D^{(q)})^{-1})(\mathbf{x},T;\mathbf{x},0)\right)\right\rangle$$

($\mathrm{Tr}(\ldots)$ denotes the trace in spin and color space).

- **Note that $(D^{(q)})^{-1}$ is a propagator in the physical basis; just using a propagator in the twisted basis yields incorrect results (in contrast to light-light mesons)!**

The static-light pseudoscalar meson (2)

- Use stochastic propagators $\phi^{(\alpha)}(x) = ((D^{(q)})^{-1})(x,y)\xi^{(\alpha)}(y)$, where $\xi^{(\alpha)}(x)$ are $\mathcal{Z}2$-spin diluted timeslice sources:

$$((D^{(\chi,1)})^{-1})(\mathbf{x},T;\mathbf{x},0) \quad \approx \quad \sum_\alpha \phi^{(\alpha)}(\mathbf{x},T)(\xi^{(\alpha)})^\dagger(\mathbf{x},0).$$

  – The information contained in the gauge configurations is exploited more fully, compared to using point sources, when performing the same number of inversions; this in turn significantly reduces statistical errors.

- To double statistics, compute the correlation function both in positive and in negative time direction.

The static-light pseudoscalar meson (3)

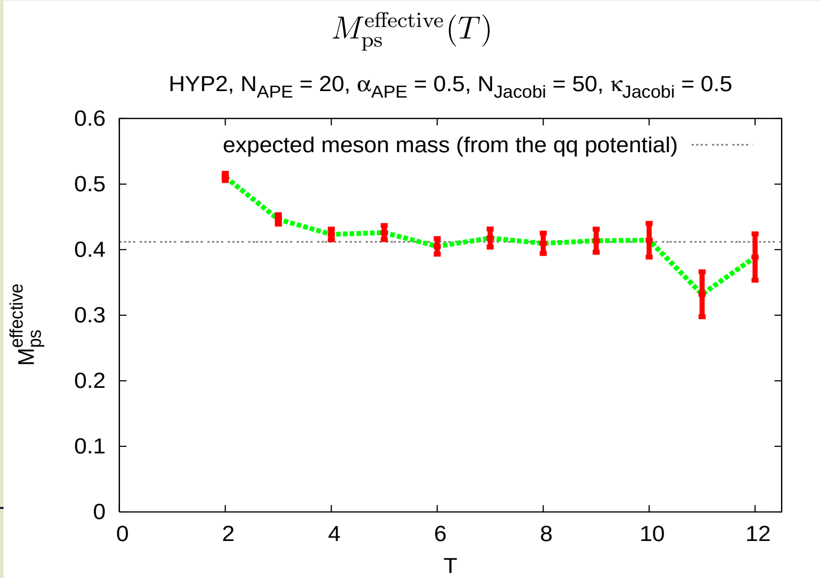
- Again, smearing is indispensable:
  - HYP2 smearing of links in time direction to reduce the static quark self energy.
  - Jacobi smearing of light quark operators and APE smearing of spatial links to enhance the ground state overlap of $\bar{q}(\mathbf{x},0)\gamma_5 Q(\mathbf{x},0)|0\rangle$ (create a state resembling the lightest static-light pseudoscalar meson).

The static-light pseudoscalar meson (4)

- Effective static-light pseudoscalar meson mass:

$$M_{\mathrm{ps}}^{\mathrm{effective}}(T) \quad = \quad -\frac{1}{a}\bigg(\ln\Big(\mathcal{C}^{(\bar{q}\gamma_5 Q)}(T)\Big) - \ln\Big(\mathcal{C}^{(\bar{q}\gamma_5 Q)}(T-1)\Big)\bigg).$$

- Excellent agreement with the estimate obtained by considering the pure Wilson loop potential and assuming $R_{\mathrm{sb}} \approx 1.25\,\mathrm{fm}$.



The static-light meson spectrum (1)

- To compute the static light spectrum, consider extended "meson creation operators" with different spatial structure and different spin structure, yielding "well defined" total angular momentum.
- Static-light meson masses are degenerate with respect to the "heavy spin".
- Therefore, it is common to label static-light mesons by $L_\pm$, where L is the angular momentum quantum number and $\pm$ describes the coupling of the "light spin", i.e. $J = L \pm 1/2$.
- Of course, parity is also a good quantum number.
- Since static-light mesons are made from non-identical quarks, charge conjugation is not a useful quantum number (static-light meson masses are degenerate with respect to charge conjugation).

The static-light meson spectrum (2)

- General form of meson creation operators (continuum version):

$$\bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}),$$

  where Γ is a combination of spherical harmonics ($\rightarrow$ angular momentum) and γ-matrices ($\rightarrow$ spin).

- For any given Γ the total angular momentum quantum number(s) can be obtained by applying the Wigner-Eckart theorem.

- In this talk:

common notation	$\Gamma(\mathbf{x})$	L^P	spin	total angular momentum	J^P
S_+	γ_5	0^+	0^-	0^-	$(1/2)^-$
P_-	$\gamma_j x_j$	1^-	1^-	0^+	$(1/2)^+$
P_+	$\gamma_1 x_1 - \gamma_2 x_2$	1^-	1^-	2^+	$(3/2)^+$
$D_\pm$	$\gamma_5(x^2 - y^2)$	2^+	0^-	2^-	$(3/2)^-$, $(5/2)^-$

The static-light meson spectrum (3)

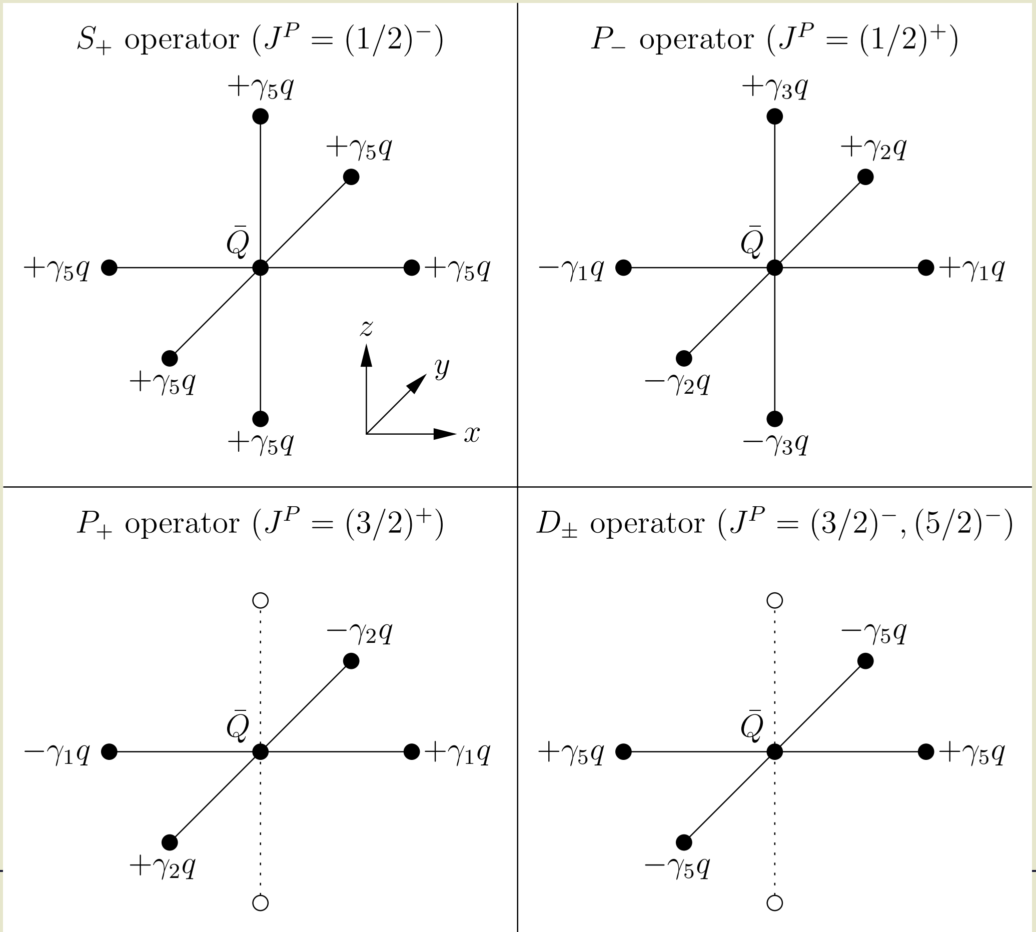
- General form of meson creation operators (lattice version):

$$\bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}}=\pm\mathbf{e}_1,\pm\mathbf{e}_2,\pm\mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x};\mathbf{x}+d\hat{\mathbf{n}}) q(\mathbf{x}+d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+ .$$

- To determine the total angular momentum quantum numbers, Γ has to be expanded in terms of spherical harmonics, which is always an infinite sum.

- Therefore, lattice operators have no well defined total angular momentum quantum numbers; they always create an infinite superposition of total angular momentum eigenstates (literature: discrete rotation group O_h, representations A_1, A_2, E, T_1, T_2).

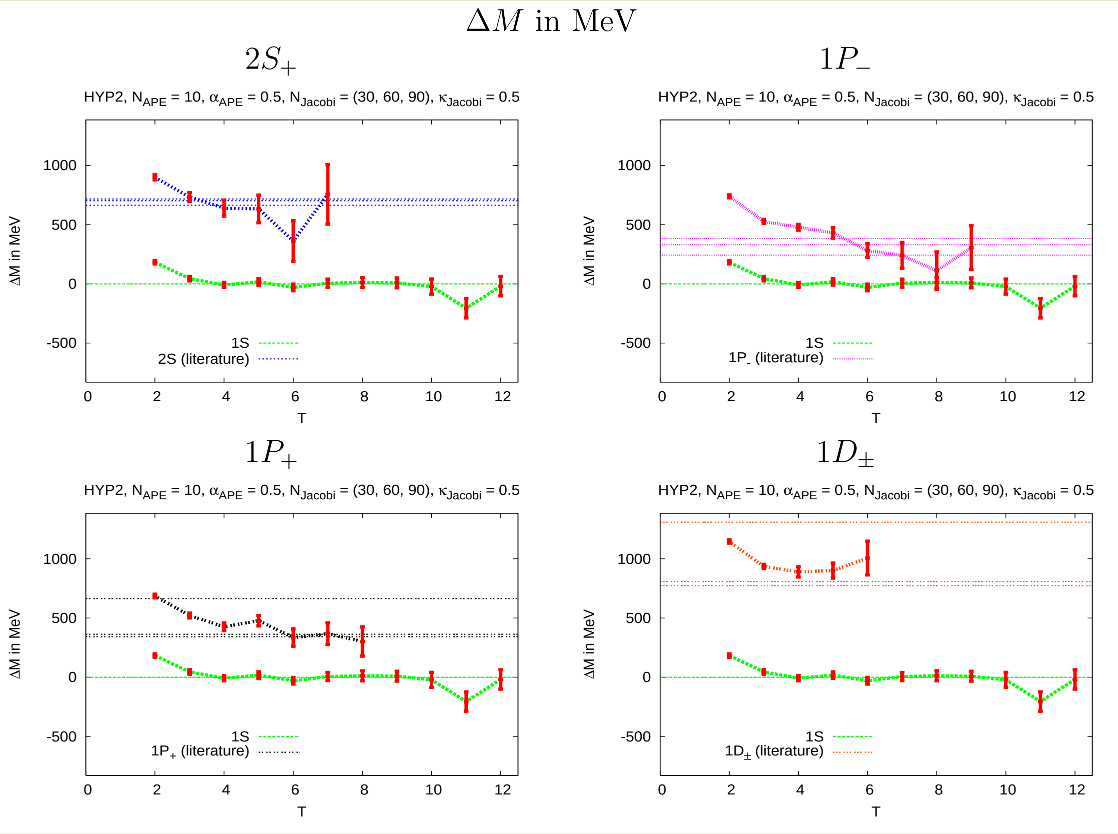
The static-light meson spectrum (4)



The static-light meson spectrum (5)

- To increase the ground state overlap and to compute excited states, use a whole set of states (usually three states obtained by applying the same operator with different smearing levels).
- Effective static-light meson masses are in qualitative agreement with results from literature:
  - A. M. Green, J. Koponen, C. McNeile, C. Michael and G. Thompson [UKQCD Collaboration], Phys. Rev. D **69**, 094505 (2004) [arXiv:hep-lat/0312007].
  - T. Burch and C. Hagen, Comput. Phys. Commun. **176**, 137 (2007) [arXiv:hep-lat/0607029].

The static-light meson spectrum (6)



Smearing of links in time direction (1)

- There is some freedom regarding the choice of the static quark action.
- Exploit this freedom to reduce the static quark self energy.
- In the following two different techniques will be compared:
  - Fat links in time direction:

$$U(x, x+e_0) \quad \rightarrow \quad P_{\mathrm{SU}(3)}\bigg(\epsilon U(x, x+e_0) + \sum_{j=\pm 1,\pm 2,\pm 3} U(x, x+e_j) \\ U(x+e_j, x+e_j+e_0) U(x+e_j+e_0, x+e_0)\bigg).$$

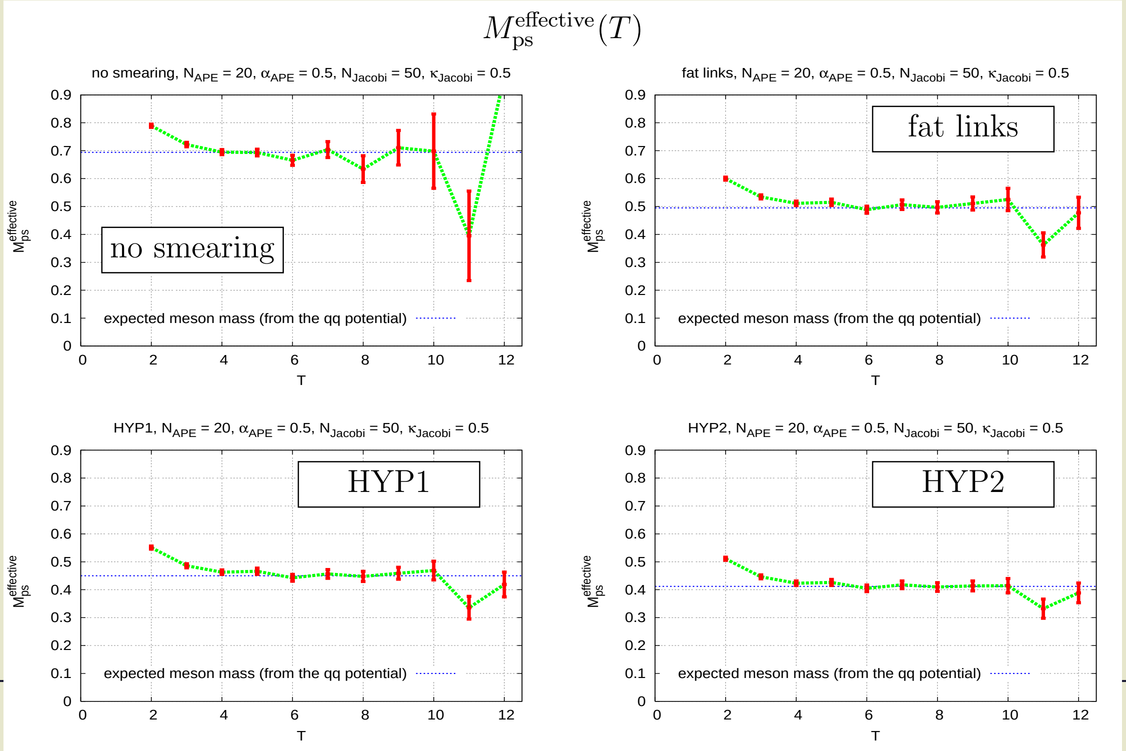
    * Optimal weight (from lattice perturbation theory): $\epsilon = 0.0$.

Smearing of links in time direction (2)

– HYP links in time direction:
  - Rather "complicated formulas".
  - Essentially three smearing steps similar to the fat link case, but do not allow links, which are separated more than one lattice site from the original link (HYP $\rightarrow$ smearing inside a HYPercube).
  - Three parameters $(\alpha_1, \alpha_2, \alpha_3)$ (weights of the three smearing steps):
    - HYP1 ($\alpha_1 = 0.75$, $\alpha_2 = 0.60$, $\alpha_3 = 0.30$): approximate maximization of the average of the smallest plaquette.
    - HYP2 ($\alpha_1 = 1.00$, $\alpha_2 = 1.00$, $\alpha_3 = 0.50$): approximate minimization of the noise-to-signal ratio of the static-light pseudoscalar corelation function.

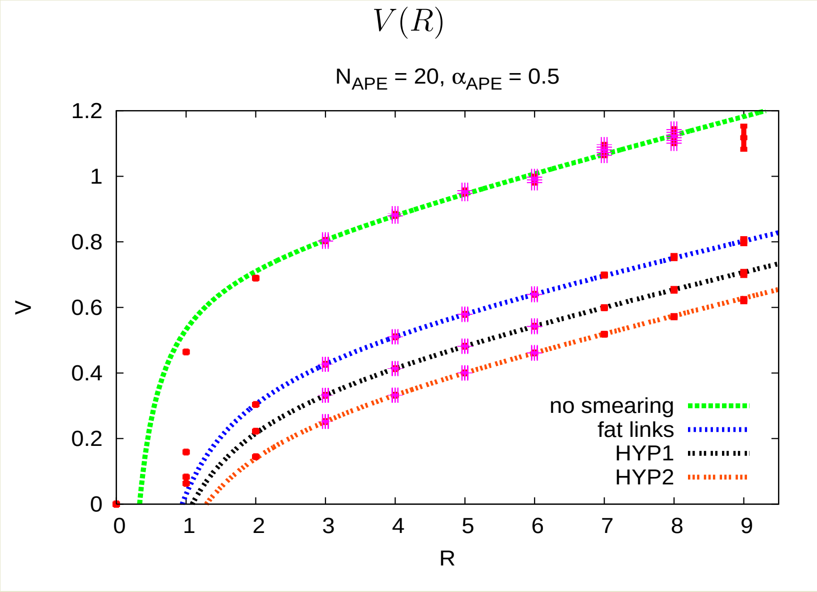
Smearing of links in time direction (3)

- The static quark self energy is significantly reduced, when using smeared links in time direction; this in turn reduces statistical errors for effective meson masses.



Smearing of links in time direction (4)

- The static quark self energy is significantly reduced, when using smeared links in time direction; this in turn reduces statistical errors for the static quark antiquark potential.

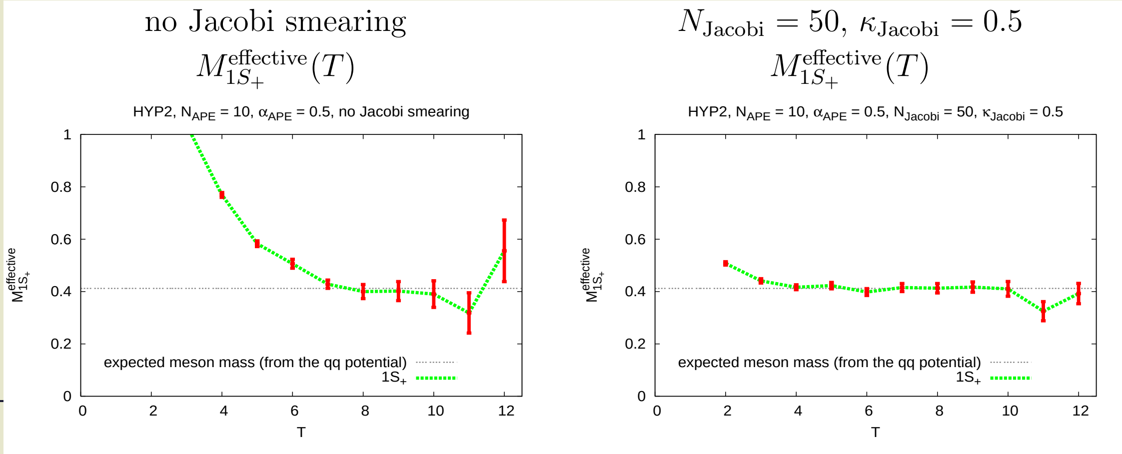


Jacobi smearing of light quark operators

- Enhance the ground state overlap by using Jacobi smeared light quark operators, i.e. create a state with a certain extension, which is similar to the corresponding meson:

$$q^{(N)}(x) \quad = \quad q^{(N-1)}(x) + \kappa \sum_{j=\pm 1,\pm 2,\pm 3} U(x, x+e_j) q^{(N-1)}(x+e_j).$$

- Effective meson masses reach their plateaux for significantly smaller T, when Jacobi smearing is applied.



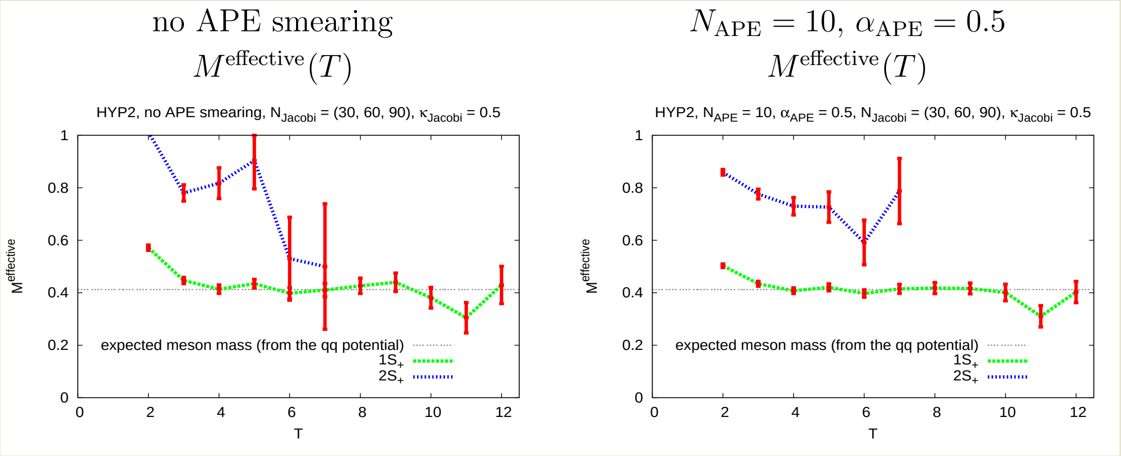
APE smearing of spatial links (1)

- Enhance the ground state overlap by using APE smeared spatial links, i.e. create a state with a certain extension, which is similar to the corresponding meson:

$$\begin{aligned} U^{(N)}(x,x+e_k) &= P_{\mathrm{SU}(3)}\Big(U^{(N-1)}(x,x+e_k) \\ &+\alpha \sum_{j=\pm 1,\pm 2,\pm 3,j\neq\pm k} U^{(N-1)}(x,x+e_j)U^{(N-1)}(x+e_j,x+e_j+e_k) \\ &U^{(N-1)}(x+e_j+e_k,x+e_k)\Big). \end{aligned}$$

APE smearing of spatial links (2)

- Effective meson masses reach their plateaux for smaller T, when APE smearing is applied; moreover, statistical errors are significantly reduced.



Summary

- There is consistency between the pure Wilson loop static quark antiquark potential and the mass of the static-light pseudoscalar meson, i.e. a string breaking distance of $\approx 1.25\,\mathrm{fm}$ can be expected.
- The static-light meson spectrum is in qualitative agreement with results from literature.
- Extensive use of smearing techniques is necessary, to obtain an acceptable signal-to-noise ratio.

Outlook

- Reduce statistical errors by a factor of ≈ 7 by considering all available gauge configurations:
  - Computing a rather precise low lying static-light meson spectrum should be possible.
- Static-light meson spectrum:
  - Compute further static-light meson masses by considering operators with a more complex spatial structure (L-shaped and/or U-shaped paths instead of straight paths).
  - Apply D^2, $(D^2)^2$, ... to the light quark operators to create nodes in the corresponding trial states; this should increase the overlap to low lying excited states.
- String breaking:
  - Compute correlation matrices both from string states and from two meson states and extract the "full" static quark antiquark potential.