Numerical computation of phase diagrams of QCD-inspired models in the large-N limit

Project A03: Inhomogeneous phases at high density

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Outline

Introduction

- **Goal**: calculate phase diagrams of QCD-inspired models in the large-N ($=$ mean field) limit numerically (N: number of flavors).
  - Particular focus on possibly existing inhomogeneous phases (compute also the shapes of the corresponding condensates).
  - (1) **No assumptions, no ansatz** (e.g. no restriction to a chiral density wave etc.).
  - (2) **Precise computation** ("no statistical errors").
  - (1) + (2) $\rightarrow$ correct and precise field theoretical results.

- At the moment we study the Gross-Neveu (GN) model in 1+1 dimensions.
  - Phase diagram analytically known.
  - Ideal to explore and test numerical/lattice field theory methods.
- Action:

$$S = \int d^2x \left(\sum_{j=1}^{N} \bar{\psi}_j \Big(\gamma_0(\partial_0 + \mu) + \gamma_1 \partial_1\Big)\psi_j - \frac{g^2}{2}\left(\sum_{j=1}^{N} \bar{\psi}_j \psi_j\right)^2\right).$$

- After introducing a scalar field σ (= condensate) and performing the integration over fermionic fields

$$S_{\text{eff}} = N\left(\frac{1}{2\lambda}\int d^2x\, \sigma^2 - \ln\Big(\det(\underbrace{\gamma_0(\partial_0 + \mu) + \gamma_1\partial_1 + \sigma}_{=Q})\Big)\right)$$

$$Z = \int D\sigma\, e^{-S_{\text{eff}}},$$

where $\lambda = Ng^2$.

GN model in 1+1 dimensions (2)

- For $N \to \infty$ only that σ minimizing S_{eff} contributes to the partition function.
- To study a field theory numerically, the infinite number of degrees of freedom has to be reduced to a finite number
  $\rightarrow$ discretization needed.
  - For example Lattice field theory.
  - There are other possibilities to discretize, e.g. finite mode discretization, discretization by piecewise polynomials, etc.
- Challenges, problems:
  - **Discretization of the fermionic determinant** (various problems, e.g. fermion doubling problem, explicit breaking of chiral symmetry, unphysical zero modes, ...).
  - **Efficient computation of** $\det(Q)$ (after discretization, Q is a large matrix).

Discretization of the fermionic determinant (1)

- Various possibilities tested:
  - Expansion in a set of basis functions, e.g. plane waves,

$$\psi(x,t) \quad \rightarrow \quad \sum_{m_t,m_x} c_{m_t,m_x} e^{i(p_{m_t} t + p_{m_x} x)}$$

    with $p_{m_t} = 2\pi(m_t - 1/2)/L_t$, $p_{m_x} = 2\pi m_x / L_x$ and similar for the scalar field σ. (requires $\det(Q) = \det(Q^\dagger)$, not the case for $\mu_I \neq 0$ or $\mu_s \neq 0$)
  - Lattice discretization:
    - Naively discretized fermions. (fermion doubling)

$$\psi(x,t) \quad \rightarrow \quad \psi_{x,t} \quad , \quad \partial_x \psi(x,t) \quad \rightarrow \quad \frac{\psi_{x+a,t} - \psi_{x-a,t}}{2a} \quad , \quad \ldots$$

      ($x, t = 0, a, 2a, \ldots$; a: lattice spacing).
    - Naively discretized with non-symmetric derivatives. (no fermion doubling, but other severe problems)
    - Staggered fermions. (fermion doubling still present)
    - ...

Discretization of the fermionic determinant (2)

- Most promising seems to be a combination of two approaches:
  - Plane wave expansion in t direction.
    $\rightarrow$ **Rather easy analytical simplifications possible**, e.g. $\det(Q)$ factorizes in several smaller determinants.
  - Naive lattice discretization in x direction.
    $\rightarrow$ $\det(Q) = \det(Q^\dagger)$ **not required**, e.g. computations for non-vanishing isospin and strangeness chemical potential might be possible.
    $\rightarrow$ **Fermion doubling not a problem**, since we consider the large-N limit ("$2 \times \infty = \infty$").

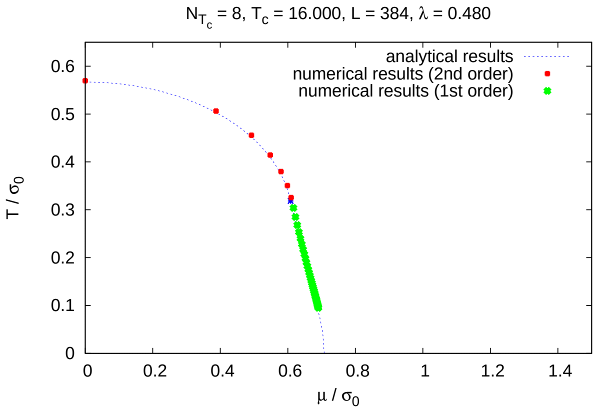
$$\psi(x,t) \quad \rightarrow \quad \sum_m \psi_{x,m} e^{i p_m t}$$

with $p_m = 2\pi(m - 1/2)/L_t$.

Efficient computation of $\det(Q)$

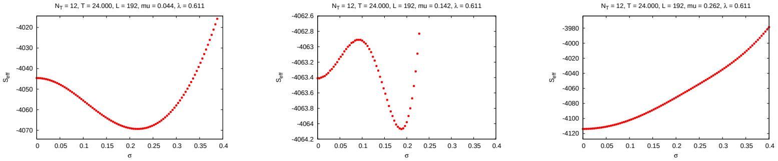
- Q is a large matrix, e.g. $\mathcal{O}(10^5) \times \mathcal{O}(10^5)$ entries.
- Needs
  - preparatory analytical simplifications, e.g. to factorize $\det(Q)$,
  - efficient algorithms and codes.
- Work in progress.
- Details are beyond the scope of this presentation.

- Phase diagram with restriction to homogeneous condensate σ.



Numerical results (2)

- $S_{\text{eff}}(\sigma)$ for homogeneous condensate σ.
  - Left: far inside the chirally broken phase ($\mu/\sigma_0 = 0.20$, $T = T_c/3$).
  - Center: in the chirally broken phase close to the 1st order phase boundary ($\mu/\sigma_0 = 0.65$, $T = T_c/3$).
  - Right: in the chirally restored phase ($\mu/\sigma_0 = 1.20$, $T = T_c/3$).



- First tests for inhomogeneous condensate $\sigma = \sigma(x)$ successful, i.e. analytically known phase boundaries for inhomogeneous condensate reproduced.