BB and $B\bar{B}$ static potentials and heavy tetraquarks from lattice QCD

Bethe Forum on methods in lattice field theory – Rheinische Friedrich-Wilhelms-Universität Bonn

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Goals, motivation (1)

- **Study tetraquarks/mesonic molecules by combining lattice QCD and phenomenology/model calculations.**
- Basic idea:
  (1) Compute the potential of two heavy quarks in the presence of two light quarks using lattice QCD.
  (2) Explore, whether the potentials are sufficiently attractive to generate a bound state (a rather stable tetraquark/mesonic molecule) using phenomenology/model calculations.

Goals, motivation (2)

- Why are such investigations important?
  Quite a number of mesons are only poorly understood.
  - Charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states.
  - Charged charmonium states, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$ ... must be four quark states.
  - $X(3872)$ ($\bar{c}c$ state): mass not as expected from quark models; could be a D-D^* molecule, a bound diquark-antidiquark, ...

Outline

- A brief introduction to lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Hadron spectroscopy.
  - Lattice QCD.
- Heavy-heavy-light-light tetraquarks.
- BB static potentials.
- BB tetraquarks.
- $B\bar{B}$ static potentials.
- Inclusion of B/B^* mass splitting.
- Outlook.

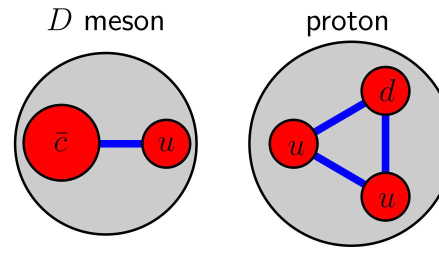
QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.
- Part of the standard model explaining the formation of hadrons (usually mesons $= q\bar{q}$ and baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.
- Definition of QCD simple:

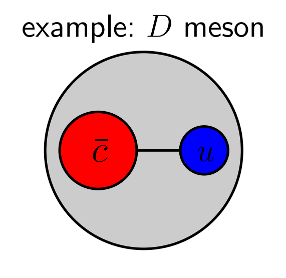
$$S = \int d^4x \Big(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu \Big) + m^{(f)} \Big) \psi^{(f)} + \frac{1}{2g^2} \mathrm{Tr} \Big(F_{\mu\nu} F_{\mu\nu} \Big) \Big)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).
$\rightarrow$ Solve QCD numerically by means of lattice QCD.



Hadron spectroscopy



- Proceed as follows:

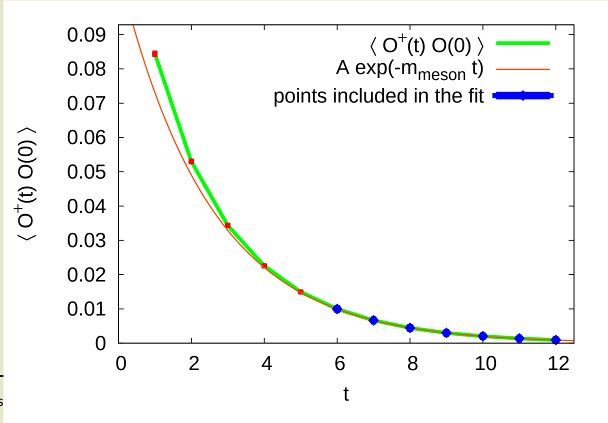
  (1) Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator O (an operator O, which generates the quantum numbers of the hadron of interest, when applied to the vacuum $|\Omega\rangle$).

  (2) Determine the corresponding hadron mass from the asymptotic exponential decay in time.

- Example: D meson mass m_D (valence quarks $\bar{c}$ and u, $J^P = 0^-$),

$$O \equiv \int d^3r\, \bar{c}(\mathbf{r})\gamma_5 u(\mathbf{r})$$

$$C(t) \equiv \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle \overset{t\to\infty}{\propto} \overset{t\to\infty}{\propto} \exp\Big(-m_D t\Big).$$



Lattice QCD (1)

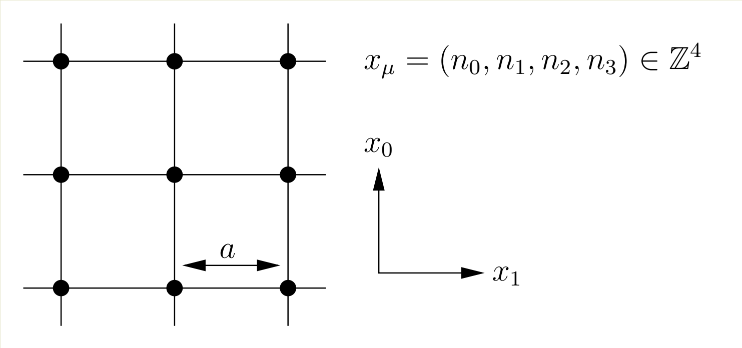
- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle = \\ = \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,O^\dagger(t)O(0)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slides).
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x},t)$ and $A_\mu(\mathbf{x},t)$.
  - $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite volume effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \Big(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \Big) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    * x_μ: $32^4 \approx 10^6$ lattice sites.
    * $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    * $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    * In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

  $\rightarrow$ standard approaches for numerical integration not applicable

  $\rightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

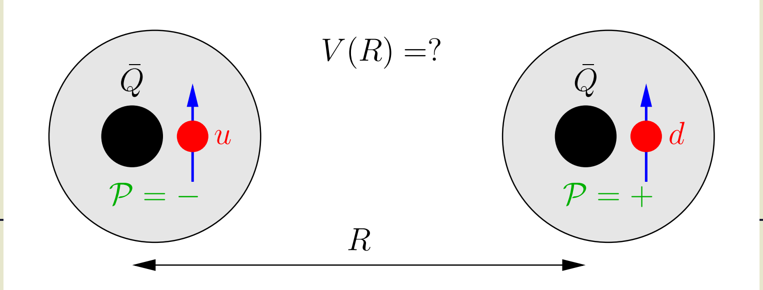
Heavy-heavy-light-light tetraquarks (1)

- Study possibly existing $\bar{Q}\bar{Q}qq$ and $\bar{Q}Q\bar{q}q$ tetraquark states ($q \in \{u, d, s, c\}$):
  - Use the static approximation for the heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$ (reduces the necessary computation time significantly).
  - Most appropriate for $\bar{Q}\bar{Q} \equiv \bar{b}\bar{b}$ and $\bar{Q}Q \equiv \bar{b}b$, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$.
  - Could also provide information about $\bar{Q}\bar{Q} \equiv \bar{c}\bar{c}$ and $\bar{Q}Q \equiv \bar{c}c$, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$.
- Proceed in two steps:
  (1) Compute the potential of two heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$ in the presence of two "light quarks" qq and $\bar{q}q$ ($q \in \{u, d, s, c\}$) using lattice QCD.
  (2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}\bar{Q}$ and $\bar{Q}Q$; the existence of a bound state would indicate a rather stable tetraquark state.

Heavy-heavy-light-light tetraquarks (2)

- Since heavy b quarks are treated in the static approximation, their spins are irrelevant (mesons are labeled by the spin of the light degrees of freedom j).
- Consider only pseudoscalar/vector mesons ($j^\mathcal{P} = (1/2)^-$, PDG: B, B^*) and scalar/pseudovector mesons ($j^\mathcal{P} = (1/2)^+$, PDG: B_0^*, B_1^*), which are among the lightest static-light mesons.
- Study the dependence of the mesonic potential $V(R)$ on
  - the "light" quark flavors u, d, s and/or c (isospin),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson B, B^* and/or B_0^*, B_1^*.

$\rightarrow$ Many different channels/quantum numbers ... attractive, repulsive ...

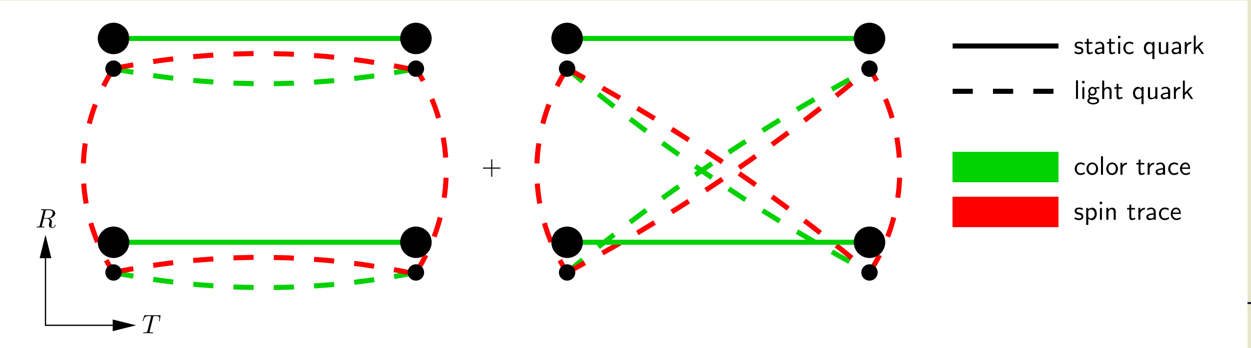


BB static potentials (1)

- In the following $\bar{Q}\bar{Q}qq$, i.e. "BB" (not $\bar{Q}Q\bar{q}q$, i.e. "$B\bar{B}$").
- To extract the potential(s) of a given sector $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$, compute the temporal correlation function of the trial state

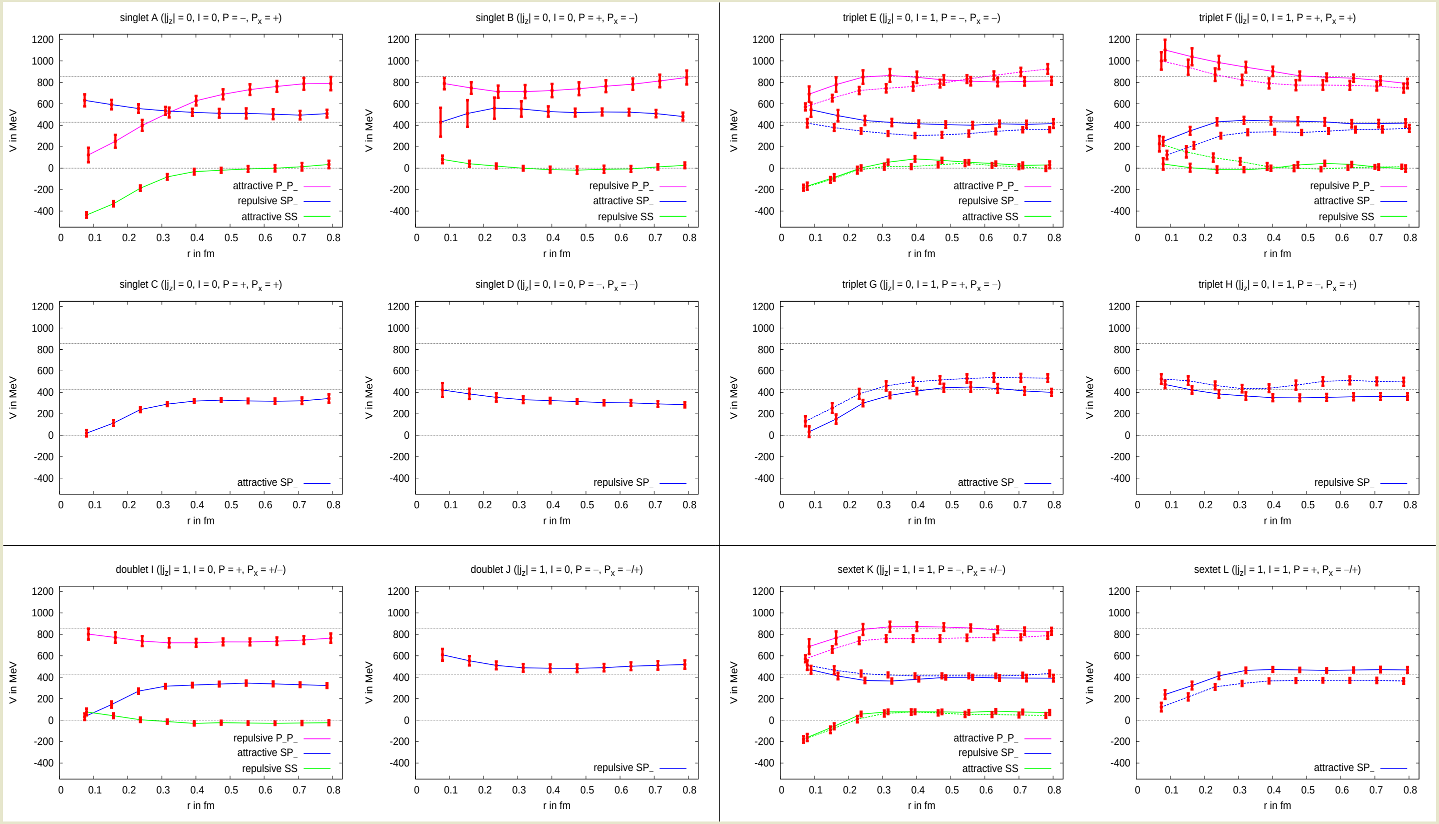
$$(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}_C(-R/2)q_A^{(1)}(-R/2)\Big)\Big(\bar{Q}_D(+R/2)q_B^{(2)}(+R/2)\Big)|\Omega\rangle.$$

  - $\mathcal{C} = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $q^{(1)}q^{(2)} \in \{ud - du\,,\, uu\,,\, dd\,,\, ud + du\,,\, ss\,,\, cc\}$ (isospin I, I_z).
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}$, $\mathcal{P}_x$).



BB static potentials (2)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



BB static potentials (3)

Why are certain channels attractive and others repulsive? (1)

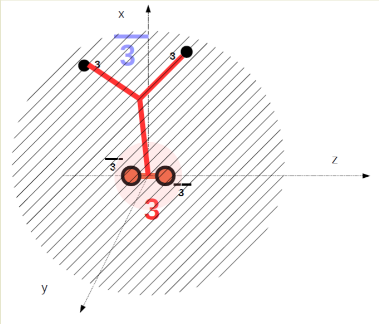
- Wave function of two identical fermions (light quarks $q^{(1)}q^{(2)}$) must be antisymmetric (Pauli principle); in qft/QCD automatically realized on the level of states.
- $q^{(1)}q^{(2)}$ isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- $q^{(1)}q^{(2)}$ spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- $q^{(1)}q^{(2)}$ color:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e. a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e. a sextet 6.
- The four quarks $\bar{Q}\bar{Q}q^{(1)}q^{(2)}$ must form a color singlet:
  - $q^{(1)}q^{(2)}$ in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{Q}\bar{Q}$ also in a triplet 3.
  - $q^{(1)}q^{(2)}$ in a color sextet 6 $\rightarrow$ static quarks $\bar{Q}\bar{Q}$ also in a sextet $\bar{6}$.

BB static potentials (4)

Why are certain channels attractive and others repulsive? (2)

- Attractive/repulsive behavior of $\bar{Q}\bar{Q}$ at small separations r is mainly due to 1-gluon exchange (the static quarks $\bar{Q}\bar{Q}$ are rather close, inside a large light quark cloud formed by $q^{(1)}q^{(2)}$, i.e. no color screening of the color charges $\bar{Q}\bar{Q}$ due to $q^{(1)}q^{(2)}$):
  - color triplet 3 is attractive, $V(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V(r) = +\alpha_s/3r$

  (easy to calculate in LO perturbation theory).

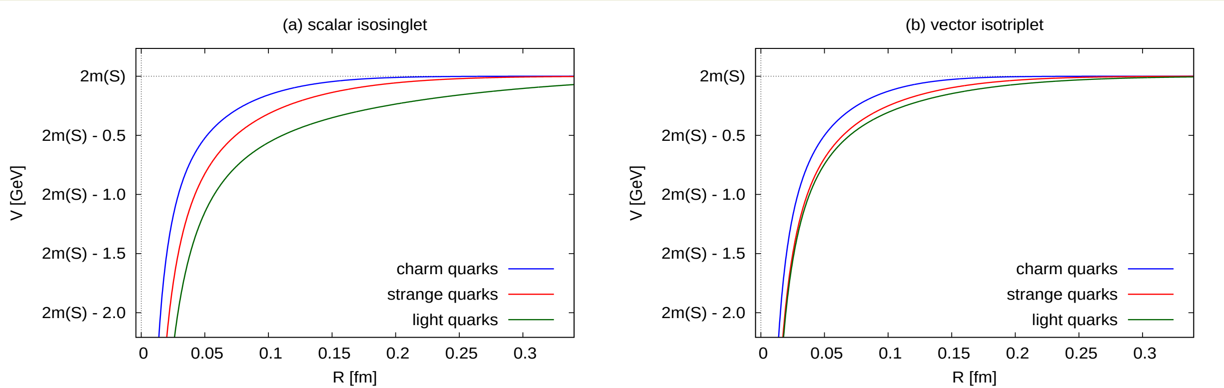


- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ $\rightarrow$ attractive $\bar{Q}\bar{Q}$ potential $V(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ $\rightarrow$ repulsive $\bar{Q}\bar{Q}$ potential $V(r)$.

This expectation is consistent with the obtained lattice results.

BB static potentials (5)

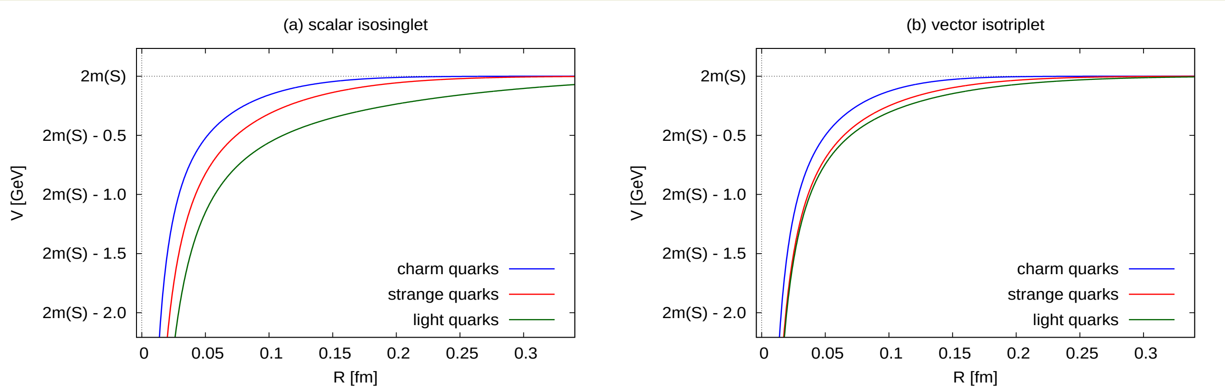
- Focus on the two attractive channels between ground state static-light mesons "B and B^*" (probably the best candidates to find a tetraquark):
  - Scalar isosinglet (more attractive):
    $qq = (ud - du)/\sqrt{2}$, $\Gamma = \gamma_5 + \gamma_0\gamma_5$,
    quantum numbers $(I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (0, 0, -, +)$.
  - Vector isotriplet (less attractive):
    $qq \in \{uu, (ud + du)/\sqrt{2}, dd\}$, $\Gamma = \gamma_j + \gamma_0\gamma_j$,
    quantum numbers $(I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (1, \{0, 1\}, -, \pm)$.
- Computations for $qq = ll, ss, cc$ ($l \in \{u, d\}$) to study the mass dependence.



BB static potentials (6)

- Two competing effects:
  - The potential for light quarks is wider/deeper, i.e. favors the existence of a bound state (a tetraquark).
  - Heavier quarks correspond to heavier mesons ($m(B) < m(B_s) < m(B_c)$), which form more readily a bound state (a tetraquark), i.e. require a less wide/deep potential for a bound state.

[M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538]]
[M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147]]
[B. Wagenbach, P. Bicudo, M.W., arXiv:1411.2453]



BB tetraquarks (1)

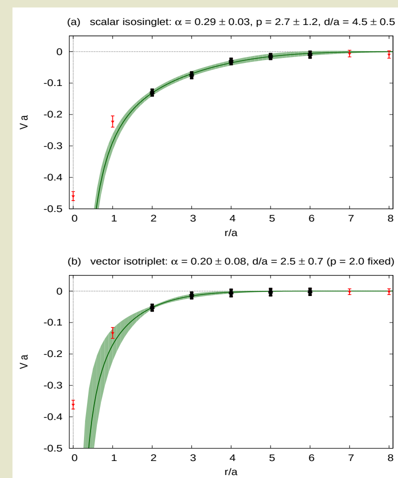
- Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}\bar{Q}$,

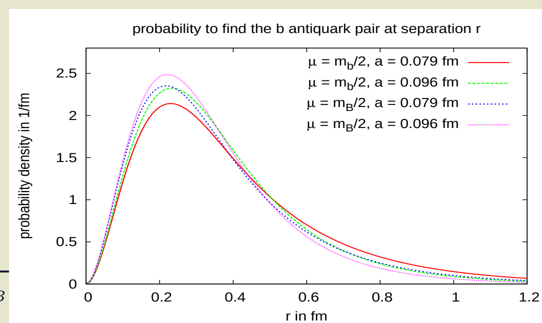
$$\left(-\frac{1}{2\mu}\triangle + V(r)\right)\underbrace{\psi(\mathbf{r})}_{=R(r)/r} = E\psi(\mathbf{r}) \quad , \quad \mu = m(B_{(s,c)})/2;$$

  a bound state, i.e. $E_0 < 0$, would be an indication for a tetraquark state.

- There is a bound state for the scalar isosinglet and $qq = ll$ (i.e. BB), binding energy $E \approx -50\,\mathrm{MeV}$, confidence level $\approx 2\,\sigma$.

- No binding for the vector isotriplet or for $qq = ss, cc$ (i.e. $B_s B_s$, $B_c B_c$).





BB tetraquarks (2)

- To quantify "no binding", we list for each channel the factor, by which the effective mass μ in Schrödinger's equation has to be multiplied, to obtain binding with confidence level $1\,\sigma$ and $2\,\sigma$ (the potential is not changed).

flavor	light		strange		charm	
confidence level for binding	$1\,\sigma$	$2\,\sigma$	$1\,\sigma$	$2\,\sigma$	$1\,\sigma$	$2\,\sigma$
scalar isosinglet	0.8	1.0	1.9	2.2	3.1	3.2
vector isotriplet	1.9	2.1	2.5	2.7	3.4	3.5

  - Factors ≤ 1.0 indicate binding.
  - Light quarks (u/d) are unphysically heavy (correspond to $m_\pi \approx 340\,$MeV); physically light u/d quarks are expected to yield stronger binding for the scalar isosinglet, might lead to binding also for the vector isotriplet (computations in progress).
  - Mass splitting $m(B^*) - m(B) \approx 50\,$MeV, neglected at the moment, is expected to weaken binding (coupled channel analysis; see later slides).

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
[B. Wagenbach, P. Bicudo, M.W., arXiv:1411.2453]

BB tetraquarks (3)

What are the quantum numbers of the $\bar{b}\bar{b}ll$ tetraquark (light scalar isosinglet)?

- Light scalar isosinglet: $I = 0$, $j = 0$, ll in a color $\bar{3}$, $\bar{b}\bar{b}$ in a color 3 (antisymmetric) ... as discussed above.
- Wave function of $\bar{b}\bar{b}$ must also be antisymmetric (Pauli principle); in the lattice QCD computation not automatically realized (static quarks are spinless color charges, which can be distinguished by their positions).
  - $\bar{b}\bar{b}$ is flavor symmetric.
  - $\bar{b}\bar{b}$ spin must also be symmetric, i.e. $j_b = 1$.
- **The $\bar{b}\bar{b}ll$ tetraquark has quantum numbers isospin $I = 0$, spin $J = 1$, parity $\mathcal{P} = +$ (parity not obvious).**

$B\bar{B}$ static potentials

- Experimentally more interesting case: $\bar{Q}Q\bar{q}q$, i.e. "$B\bar{B}$", trial states

$$\Gamma_{AB}\tilde{\Gamma}_{CD}\Big(\bar{Q}_C(-R/2)q_B^{(1)}(-R/2)\Big)\Big(\bar{q}_A^{(2)}(+R/2)Q_D(+R/2)\Big)|\Omega\rangle.$$

- At the moment only preliminary results for $\bar{q}q = \bar{c}c$, "$I = 1$".
- Qualitative difference to $\bar{Q}\bar{Q}qq$: all channels are attractive (for $\bar{Q}\bar{Q}qq$ half of them are attractive, half of them are repulsive).
- Can again be understood by the 1-gluon exchange potential of $\bar{Q}Q$:
  - No Pauli principle for $\bar{q}^{(1)}q^{(2)}$ (particle and antiparticle are not identical).
  - $\bar{q}^{(1)}q^{(2)}$ can be in a symmetric color singlet 1 for any isospin/spin orientation.
  - $\bar{q}^{(1)}q^{(2)}$ in a color singlet 1 $\quad\rightarrow\quad$ static quarks $\bar{Q}Q$ also in a singlet 1.
  - Color singlet is attractive, $V(r) = -4\alpha_s/3r$ (LO perturbation theory).

Inclusion of B/B^* mass splitting (1)

- Mass splitting $m_{B^*} - m_B \approx 50\,\text{MeV}$ has been neglected so far.
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $E \approx -50\,\mathrm{MeV}$.
- Moreover, two competing effects:
  - An attractive $\bar{Q}\bar{Q}qq$ channel correspond to a linear combination of BB, BB^* and/or B^*B^*, e.g.

    $$\text{scalar isosinglet} \quad \equiv \quad BB + B^*_x B^*_x + B^*_y B^*_y + B^*_z B^*_z .$$

  - The BB interaction is a superposition of attractive and repulsive $\bar{Q}\bar{Q}qq$ potentials.
- Goal: take mass splitting $m_{B^*} - m_B \approx 50\,\text{MeV}$ into account
  $\rightarrow$ refined model calculation with the computed $\bar{Q}\bar{Q}qq$ potentials.
- **Will there still be a bound state?**

Inclusion of B/B^* mass splitting (2)

Solve a coupled channel Schrödinger equation (1)

- Previously:
  - A wave function ψ with 1 component corresponding to BB ($B \equiv B^*$).
- Now:
  - A static light meson can correspond to B or $B^* = (B^*_x, B^*_y, B^*_z)$.
  - Therefore, a wave function $\vec{\psi}$ with 16 components corresponding to $(BB, BB^*_x, BB^*_y, BB^*_z, B^*_xB, B^*_xB^*_x, B^*_xB^*_y, B^*_xB^*_z, \ldots, B^*_zB^*_z)$.
- Coupled channel Schrödinger equation $H\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2) \ = \ E\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2)$,

$$H \ = \ M \otimes 1 + 1 \otimes M + \frac{\mathbf{p}_1^2}{2}(M \otimes 1)^{-1} + \frac{\mathbf{p}_2^2}{2}(1 \otimes M)^{-1} + V(|\mathbf{r}_1 - \mathbf{r}_2|),$$

where $M = \mathrm{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$ and V is a 16×16 non-diagonal matrix containing the $\bar{Q}\bar{Q}qq$ potentials (both attractive and repulsive).

Inclusion of B/B^* mass splitting (3)

Solve a coupled channel Schrödinger equation (2)

- Coupled channel Schrödinger equation $H\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2) \; = \; E\vec{\psi}(\mathbf{r}_1, \mathbf{r}_2)$,

  $$H \; = \; M \otimes 1 + 1 \otimes M + \frac{\mathbf{p}_1^2}{2}(M \otimes 1)^{-1} + \frac{\mathbf{p}_2^2}{2}(1 \otimes M)^{-1} + V(|\mathbf{r}_1 - \mathbf{r}_2|),$$

  where $M = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$ and V is a 16×16 non-diagonal matrix containing the $\bar{Q}\bar{Q}qq$ potentials (both attractive and repulsive).

- Specific limits:
  - $V = 0$, i.e. no interactions:

    $$E \; = \; m_B + m_B + \frac{\mathbf{p}_1^2}{2m_B} + \frac{\mathbf{p}_2^2}{2m_B} \; , \; m_B + m_{B^*} + \frac{\mathbf{p}_1^2}{2m_B} + \frac{\mathbf{p}_2^2}{2m_{B^*}} \; , \ldots$$

  - $m_{B^*} = m_B$, i.e. "old 1-component SE calculation":

    $$E \; \approx \; 2m_B - 50\,\text{MeV}.$$

Inclusion of B/B^* mass splitting (4)

Solve a coupled channel Schrödinger equation (3)

- Transform the 16×16 Schrödinger equation to block diagonal structure:
  - Total spin $J = 0$: 2×2 structure.
  - Total spin $J = 1$: 3×3 structure ($3\times$ due to J_z degeneracy).
  - Total spin $J = 2$: 1×1 structure ($5\times$ due to J_z degeneracy).
- Work in progress ...
  - First very preliminary results indicate that for $J = 0$ the bound state does not exist anymore (however, still very close to a bound state).
  - However:
    * More realistic/relevant $J = 1$ equation not yet investigated.
    * Unphysically heavy u/d quarks ($m_\pi \approx 340\,\text{MeV}$) ... physically light quarks will lead to more attractive $\bar{Q}\bar{Q}qq$ potentials.
    * $M = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$; for small $\bar{Q}\bar{Q}$ separation $M = m_b$ would be more appropriate ... should enhance binding.

Outlook (1)

- Future plans for BB and $B\bar{B}$:
  - Computations with light u/d quarks of physical mass ($m_\pi \approx 140\,\textrm{MeV}$ instead of $m_\pi \approx 340\,\textrm{MeV}$).
  - Light quarks of different mass: BB_s, BB_c and $B_s B_c$ potentials.

Outlook (2)

- Future plans for BB and $B\bar{B}$:
  - Study the structure of the states corresponding to the computed potentials:
    - In a lattice computation two different creation operators generating the same quantum numbers yield the same potential.
    - At the moment exclusively creation operators of mesonic molecule type.
    - For BB use also
      - creation operators of diquark-antidiquark type.
    - For $B\bar{B}$ use also
      - creation operators of diquark-antidiquark type,
      - creation operators of bottomonium + pion type ($Q\bar{Q}$ string $+ \pi$),
      - for $I = 0$ creation operators of bottomonium type ($Q\bar{Q}$ string).
    - Resulting correlation matrices provide information about the structure.