

Forces between heavy mesons

ETMC meeting – Bonn

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Ongoing static-light projects

- Continuum limit of the static-light meson spectrum.

[Chris Michael, Andrea Shindler, Marc Wagner]

→ Almost finished, draft circulated last week.

- f_B and f_{B_s} .

[Chris Michael, Andrea Shindler]

- Static-light baryon spectrum.

[Marc Wagner, Christian Wiese]

→ Talk by Christian Wiese.

- Forces between static-light mesons.

[Marc Wagner]

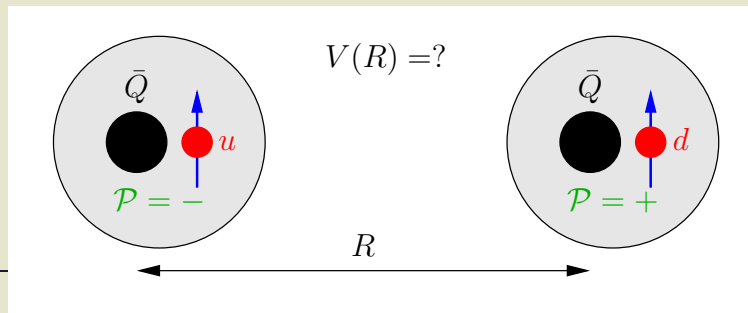
→ This talk.

- String breaking.

[Karl Jansen, Attila Nagy, Marc Wagner]

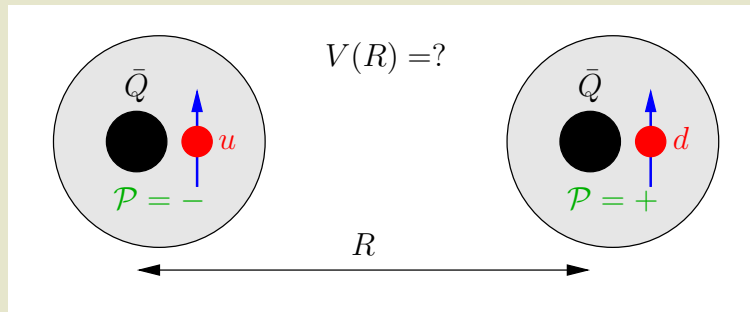
Introduction (1)

- Goal: compute the potential of (or equivalently the force between) two B mesons:
 - Treat the b quark in the static approximation.
 - Consider only pseudoscalar mesons ($j^{\mathcal{P}} = (1/2)^-$, denoted by S) and scalar mesons ($j^{\mathcal{P}} = (1/2)^+$, denoted by P_+), which are among the lightest static-light mesons.
 - Study the dependence of the mesonic potential $V(R)$ on
 - * the light quark flavor u and/or d (isospin),
 - * the light quark spin (the static quark spin is irrelevant),
 - * the type of the meson S and/or P_+ .



Introduction (2)

- Motivation:
 - First principles computation of a hadronic force.
 - Until now it has only been studied in the quenched approximation.



(Pseudo)scalar B mesons

- Symmetries and quantum numbers of static-light mesons:
 - Isospin: $I = 1/2$, $I_z = \pm 1/2$, i.e. $B \equiv \bar{Q}u$ or $B \equiv \bar{Q}d$.
 - Parity: $\mathcal{P} = \pm$.
 - Rotations:
 - * Light cloud angular momentum $j = 1/2$ (for S and P_-), $j_z = \pm 1/2$.
 - * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
- Static-light meson creation operators:
 - $\bar{Q}\gamma_5 q$ (pseudoscalar, i.e. S), $q \in \{u, d\}$,
 - $\bar{Q}q$ (scalar, i.e. P_-)

(j_z is not well-defined, when using these operators).

BB systems (1)

- Symmetries and quantum numbers of a pair of static-light mesons (separated along the z -axis):
 - Isospin: $I = 0, 1$, $I_z = -1, 0, +1$.
 - Rotations around the z -axis:
 - * Angular momentum of the light degrees of freedom $j_z = -1, 0, +1$.
 - * Static quark spin: irrelevant (the static quarks can be treated as spinless color charges).
 - Parity: $\mathcal{P} = \pm$.
 - If $j_z = 0$, reflection along the x -axis: $\mathcal{P}_x = \pm$.
 - Instead of using $j_z = \pm 1$ one can also label states by $|j_z| = 1$, $\mathcal{P}_x = \pm$.
- Label BB states by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$.

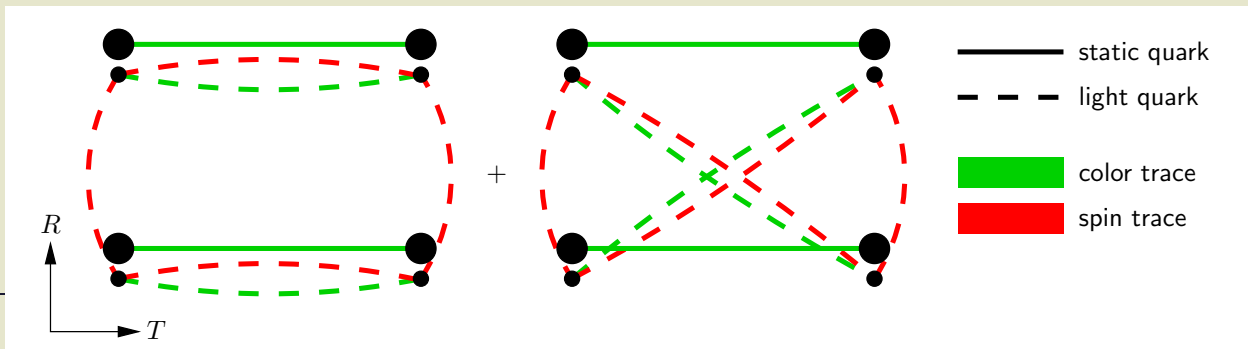
BB systems (2)

- To extract the potential(s) of a given sector (characterized by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$), compute the temporal correlation function of the trial state

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) q_A^{(1)}(-R/2) \right) \left(\bar{Q}_C(+R/2) q_B^{(2)}(+R/2) \right) |\Omega\rangle,$$

where

- $\mathcal{C} = \gamma_0 \gamma_2$ (charge conjugation matrix),
- $q^{(1)} q^{(2)} \in \{ud - du, uu, dd, ud + du\}$ (isospin I, I_z),
- Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}, \mathcal{P}_x$).



BB systems (3)

- Symmetries of twisted mass lattice QCD compared to QCD:
 - SU(2) isospin breaks down to U(1): I_z is still a good quantum number, I is not.
 - Parity \mathcal{P} is replaced by $\mathcal{P}^{(\text{tm})}$, which is parity combined with light flavor exchange.
 - Twisted mass BB sectors:
 - * $I_z = \pm 1$: $(I_z, |j_z|, \underbrace{\mathcal{P}^{(\text{tm})}\mathcal{P}_x^{(\text{tm})}}_{=\mathcal{P}\mathcal{P}_x})$,
 - * $I_z = 0$: $(I_z, |j_z|, \underbrace{\mathcal{P}^{(\text{tm})}}_{=\mathcal{P} \times (2I-1)}, \underbrace{\mathcal{P}_x^{(\text{tm})}}_{=\mathcal{P}_x \times (2I-1)})$.
 - QCD sectors $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$ are pairwise combined.

BB systems (4)

- BB creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) \chi_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) \chi_B^{(u)}(+R/2) \right).$$

Γ twisted	$ j_z , \mathcal{P}^{(\text{tm}, \text{light})} \mathcal{P}_x^{(\text{tm}, \text{light})}$	Γ pseudo physical	$ j_z , \mathcal{P}^{(\text{light})}, \mathcal{P}_x^{(\text{light})}$
γ_5	0, +	$\mp i$	0, -, -
$\gamma_0 \gamma_5$	0, +	$+\gamma_0 \gamma_5$	0, +, +
1	0, +	$\mp i \gamma_5$	0, +, +
γ_0	0, -	$+\gamma_0$	0, +, -
γ_3	0, +	$+\gamma_3$	0, -, -
$\gamma_0 \gamma_3$	0, +	$\mp i \gamma_0 \gamma_3 \gamma_5$	0, +, +
$\gamma_3 \gamma_5$	0, -	$+\gamma_3 \gamma_5$	0, -, +
$\gamma_0 \gamma_3 \gamma_5$	0, +	$\mp i \gamma_0 \gamma_3$	0, -, -
γ_1	1, -	$+\gamma_1$	1, -, +
$\gamma_0 \gamma_1$	1, -	$\mp i \gamma_0 \gamma_1 \gamma_5$	1, +, -
$\gamma_1 \gamma_5$	1, +	$+\gamma_1 \gamma_5$	1, -, -
$\gamma_0 \gamma_1 \gamma_5$	1, -	$\mp i \gamma_0 \gamma_1$	1, -, +
...

BB systems (5)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB} \left(\bar{Q}_C(-R/2) \chi_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2) \chi_B^{(d)}(+R/2) \right) \pm (u \leftrightarrow d).$$

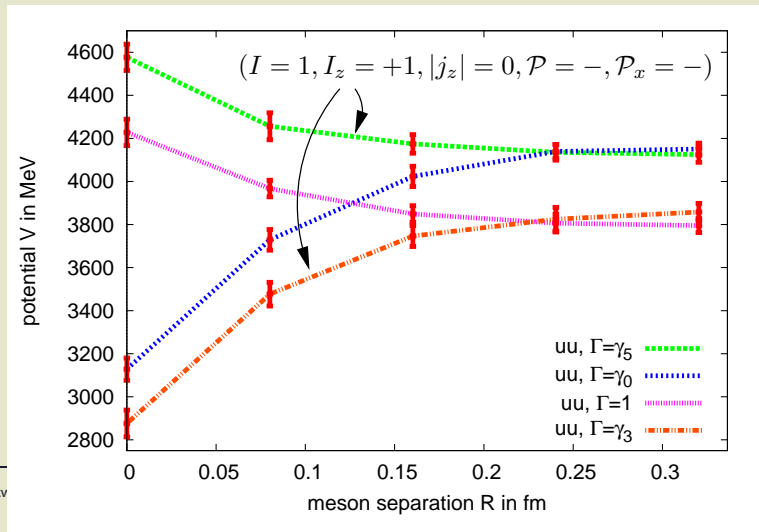
Γ twisted, \pm	$ j_z , \mathcal{P}^{(\text{tm}, \text{light})}, \mathcal{P}_x^{(\text{tm}, \text{light})}$	Γ pseudo physical, \pm	$ j_z , I, \mathcal{P}^{(\text{light})}, \mathcal{P}_x^{(\text{light})}$
$\gamma_5, +$	0, +, +	$+\gamma_5, +$	0, 1, +, +
$\gamma_0\gamma_5, +$	0, +, +	$+i\gamma_0, -$	0, 0, -, -
1, -	0, -, +	$+1, -$	0, 0, +, -
$\gamma_0, -$	0, +, +	$+i\gamma_0\gamma_5, +$	0, 1, +, +
$\gamma_5, -$	0, +, -	$+\gamma_5, -$	0, 0, -, +
$\gamma_0\gamma_5, -$	0, +, -	$+i\gamma_0, +$	0, 1, +, -
1, +	0, -, -	$+1, +$	0, 1, -, -
$\gamma_0, +$	0, +, -	$+i\gamma_0\gamma_5, -$	0, 0, -, +
$\gamma_3, +$	1, -, -	$+i\gamma_3\gamma_5, -$	1, 0, +, +
$\gamma_0\gamma_3, +$	1, -, -	$+\gamma_0\gamma_3, +$	1, 1, -, -
$\gamma_3\gamma_5, -$	1, -, -	$+i\gamma_3, +$	1, 1, -, -
$\gamma_0\gamma_3\gamma_5, -$	1, +, -	$+\gamma_0\gamma_3\gamma_5, -$	1, 0, -, +
...

Simulation setup

- $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0040$
 - lattice spacing $a \approx 0.080$ fm
 - pion mass $m_{\text{PS}} \approx 336$ MeV.
- Preliminary results on 18 gauge configurations:
 - Effective mass plateaus are fitted at rather small temporal separation
 - contamination by excited states.
- 12 u and 12 d inversions per gauge configuration (stochastic timeslice sources located on the same timeslice).
- APE smearing of spatial links and Gaussian smearing of light quark fields to “optimize” the ground state overlap of trial states.
- Wilson lines of static quarks are discretized by path ordered products of ordinary links, i.e. no HYP static action.

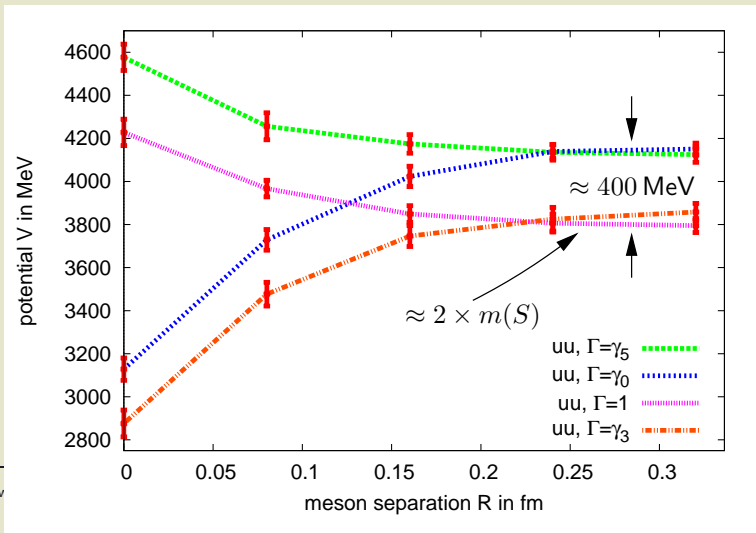
Discussion of results (1)

- Four “types of potentials”:
 - Two attractive, two repulsive.
 - Two have asymptotic values, which are larger by ≈ 400 MeV.
- There are cases, where two potentials with identical quantum numbers are completely different (i.e. of different type)
 - at least one of the corresponding trial states must have very small ground state overlap
 - physical understanding, i.e. interpretation of trial states needed.



Discussion of results (2)

- Expectation at large meson separation R : $V(R) \approx 2 \times \text{meson mass}$.
 - Potentials with smaller asymptotic value at $\approx 2 \times m(S)$.
 - $m(P_-) - m(S) \approx 400 \text{ MeV}$: approximately the observed difference between different types of potentials.
- Two types correspond to $S-S$ potentials.
- Two types correspond to $S-P_-$ potentials.
- Can this be understood in detail on the level of the used BB creation operators?



Discussion of results (3)

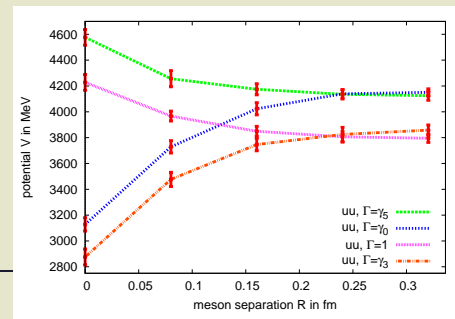
- Rotate the BB creation operators to the pseudo physical basis and express them in terms of static-light meson creation operators (use suitable spin and parity projectors for the light quarks).

– Examples:

$$\begin{aligned}
 * uu, \Gamma = \gamma_5 &\rightarrow \Gamma^{(\text{ppb})} = -i \rightarrow \mathcal{P} = -, \mathcal{P}_x = -: \\
 (\mathcal{C}\gamma_5)_{AB} \left(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2) \right) &= \\
 = +i \left(S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow \right). &
 \end{aligned}$$

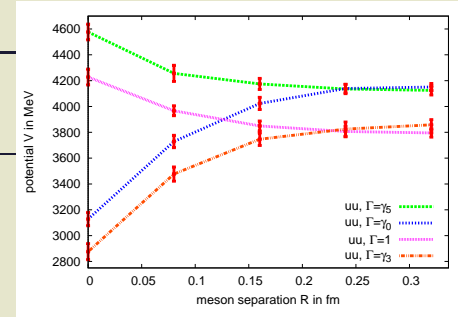
$$\begin{aligned}
 * uu, \Gamma = \gamma_3 &\rightarrow \Gamma^{(\text{ppb})} = \gamma_3 \rightarrow \mathcal{P} = -, \mathcal{P}_x = -: \\
 (\mathcal{C}\gamma_3)_{AB} \left(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2) \right) \left(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2) \right) &= \\
 = -i \left(S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow \right). &
 \end{aligned}$$

- SS/SP_- content and asymptotic values in agreement for all 12 + 24 independent potentials
 \rightarrow asymptotic differences understood.



Discussion of results (4)

- Is there a general rule, about when a potential is repulsive and when attractive?



– S - S potentials:

- * $(I = 0, s = 0)$ or $(I = 1, s = 1)$, i.e. $I = s \rightarrow$ attractive
 - * $(I = 0, s = 1)$ or $(I = 1, s = 0)$, i.e. $I \neq s \rightarrow$ repulsive
- (s : combined angular momentum of the two mesons).
- * **Example:** $uu, \Gamma = \gamma_3 \rightarrow \Gamma^{(ppb)} = \gamma_3 \rightarrow \mathcal{P} = -, \mathcal{P}_x = -:$
 $-i(S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow),$
 i.e. $I = 1, s = 1$; the numerically obtained potential is attractive, i.e. in agreement with the above stated rule.
 - * All 6 + 12 independent S - S potentials fulfill the rule.
 - * Agreement with similar quenched lattice studies.

[C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]

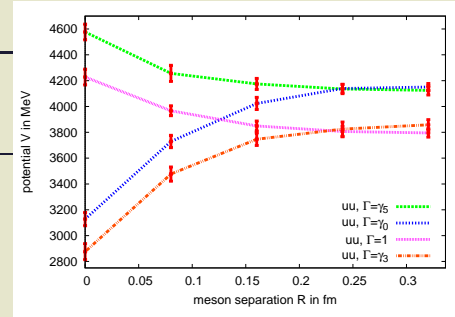
[W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

Discussion of results (5)

– S - P_- potentials:

- * Do not obey the above stated rule.
- * It can, however, easily be generalized by including parity, i.e. symmetry or antisymmetry under exchange of S and P_- :
 - trial state symmetric under meson exchange \rightarrow attractive
 - trial state antisymmetric under meson exchange \rightarrow repulsive (meson exchange \equiv exchange of flavor, spin and parity).
- * Example: $uu, \Gamma = \gamma_0 \rightarrow \Gamma^{(ppb)} = \gamma_0 \rightarrow \mathcal{P} = +, \mathcal{P}_x = -$:

$$-(S_\uparrow P_\downarrow - S_\downarrow P_\uparrow - P_\uparrow S_\downarrow + P_\downarrow S_\uparrow),$$
 i.e. $I = 1$ (symmetric), $s = 0$ (antisymmetric), antisymmetric with respect to $S \leftrightarrow P_-$; the numerically obtained potential is attractive, i.e. in agreement with the above stated general rule.
- * All 6 + 12 independent S - P_- potentials (and all 6 + 12 independent S - S potentials) fulfill the general rule.



Summary and future plans

- Contraction and analysis code for computation of BB potentials (arbitrary flavor, spin and parity) ready.
- Preliminary results promising:
 - Qualitative agreement with existing quenched results for S - S potentials.
 - Computation of S - P potentials seems feasible.
- To do list:
 - Implement off-axis separations.
 - Increase statistics by a factor of ≈ 50 .
 - Other β , $L^3 \times T$, μ values.
 - Partially quenched computations, to obtain $B_s B_s$ and/or $B_s B$ potentials.
 - Improve lattice meson potentials at small separations (where the suppression of UV fluctuations due to the lattice cutoff yields wrong results) with corresponding perturbative potentials.