

Heavy mesons and tetraquarks from lattice QCD

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Outline

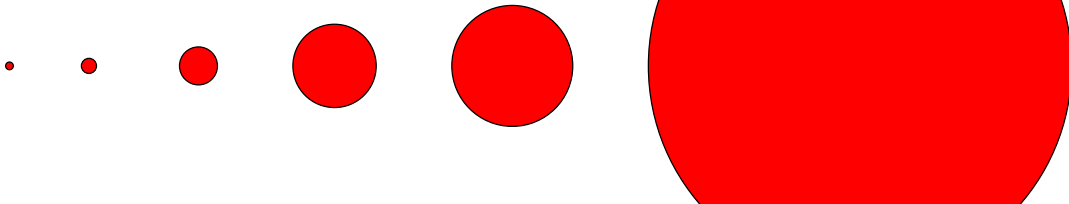
- **Teil 1:** (in Deutsch)
Grundlagen der QCD (Quantenchromodynamik) und Gitter-QCD.
- **Part 2:** (in English)
Heavy mesons and tetraquarks from lattice QCD
(Two of our projects will be discussed).
 - **Part 2a:**
 D mesons, D_s mesons and charmonium
(→ precise lattice QCD results with all sources of error investigated, which can be compared to experiment in a meaningful way).
 - **Part 2b:**
Heavy-heavy-light-light tetraquark candidates
(→ crude results based on approximations, but a lot of interesting physics, e.g. forces between hadrons, formation of tetraquarks).

**Teil 1: Grundlagen der QCD
(Quantenchromodynamik) und
Gitter-QCD**

QCD: Quarks und Gluonen

- **QCD**: Theorie der **Quarks** und **Gluonen** ... beschreibt Aufbau, Masse und mögliche Zerfälle von daraus zusammengesetzten Systemen, z.B. Proton oder Neutron.
- **Quarks und Antiquarks** (Spin 1/2):
 - 6 Flavors ... **up, down, strange, charm, bottom, top** (unterschiedliche Massen).
 - 3 Farben ... **rot, grün, blau** (eine Art Ladung, ähnlich der elektrischen Ladung).
- **Gluonen** (Spin 1):
 - Masselose Austauschteilchen der QCD (ähnlich den Photonen in der Elektrodynamik), vermitteln also Kräfte zwischen Quarks.
 - Tragen selbst (Farb-)Ladung (im Gegensatz zu den Photonen), was zu “eigenartigen” Phänomenen führt, insbesondere **Confinement**.

$u \rightarrow d \rightarrow s \rightarrow c \rightarrow b \rightarrow t$



QCD: Confinement, Hadronen

- Quarks treten niemals isoliert auf, sondern immer in Gruppen, meistens Zweier- oder Dreiergruppen, sogenannten **Hadronen** (\rightarrow **Confinement**).

- **Hadronen:**

- **Mesonen:** Ganzzahliger Spin, i.d.R. gebundene **Quark-Antiquark-Paare** $q\bar{q}$... aber auch **2 Quarks und 2 Antiquarks** $qq\bar{q}\bar{q}$ (**Tetraquarks**) möglich.

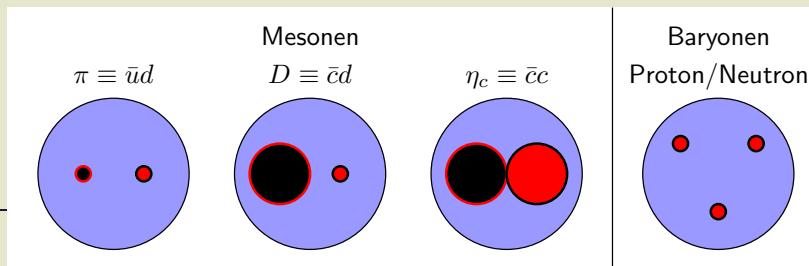
Beispiele: $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$, ..., $ud\bar{b}\bar{b}$, ...

- **Baryonen:** Halbzahliger Spin, i.d.R. 3 gebundene Quarks qqq oder Antiquarks $\bar{q}\bar{q}\bar{q}$.

Beispiele: Proton $\equiv uud$, Neutron $\equiv udd$, ...

- Hunderte von Mesonen und Baryonen wurden in Experimenten beobachtet, unterscheiden sich durch

- * sechs Flavor-Möglichkeiten für jedes Quark/Antiquark (u, d, s, c, b, t),
- * Quantenzahlen ähnlich zum Wasserstoffatom (radiale Quantenzahl, Gesamtdrehimpuls J , Parität P , ...).



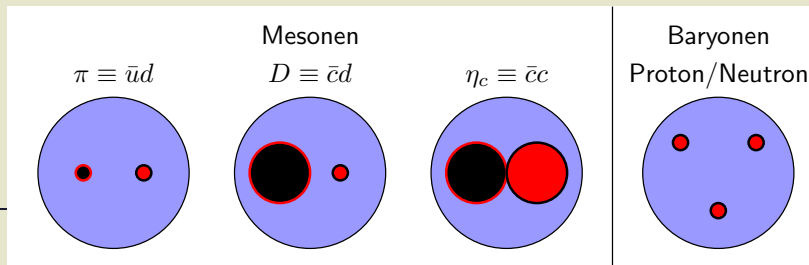
QCD: Definition

- Definition von QCD einfach:

$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \left(\gamma_\mu (\partial_\mu - iA_\mu) + m^{(f)} \right) \psi^{(f)} + \frac{1}{2g^2} \text{Tr} \left(F_{\mu\nu} F_{\mu\nu} \right) \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- $\psi^{(f)}(\mathbf{r}, t), \bar{\psi}^{(f)}(\mathbf{r}, t)$: **Quarkfelder**.
- $A_\mu(\mathbf{r}, t)$: **Gluonfeld**.
- Keine analytischen Lösungen für z.B. Meson- oder Baryonmassen bekannt, da
 - zugehörige Feldgleichungen nicht-linear sind,
 - kein kleiner Parameter (Kopplungskonstante) vorhanden (d.h. Störungstheorie i.A. nicht anwendbar).
- Numerische Methode erforderlich → **Gitter-QCD**.



QCD: Berechnung von Hadronmassen (1)

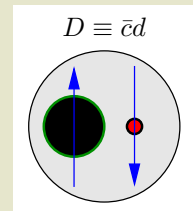
- Gitter-QCD-Berechnung einer Hadronmasse in drei Schritten:

Schritt (1): Konstruiere geeigneten Hadron-Erzeugungsoperator O

- Ein Hadron-Erzeugungsoperator besteht im Wesentlichen aus Quarkfeldoperatoren $\psi^{(f)}(\mathbf{r}) \equiv u(\mathbf{r}), d(\mathbf{r}), s(\mathbf{r}), c(\mathbf{r}), b(\mathbf{r}), t(\mathbf{r})$.
- Der Quarkfeldoperator $u(\mathbf{r})$ platziert ein u -Quark am Raumzeitpunkt \mathbf{r} , $d(\mathbf{r})$ platziert ein d -Quark, usw.
- Ein **geeigneter Hadron-Erzeugungsoperator** O generiert in grober Approximation das entsprechende Hadron:
 - **Feinheiten irrelevant, am Ende berechnete Hadronmasse unabhängig davon.**
 - **Beispiel: D -Meson** ... besteht aus einem Quark-Antiquark-Paar $\bar{c}d$, hat **Gesamtdrehimpuls** $J = 0$ und **Parität** $P = -$; ein möglicher D -Meson-Erzeugungsoperator ist

$$O \equiv \int d^3r \bar{c}(\mathbf{r}) \gamma_5 d(\mathbf{r})$$

(γ_5 sorgt für $J^P = 0^-$, $\int d^3r$ für Impuls $\mathbf{p} = 0$).



QCD: Berechnung von Mesonmassen (2)

Schritt (2): Berechne zeitliche Korrelationsfunktion $C(t)$ des Hadron-Erzeugungsoperators O numerisch mit Hilfe von Gitter-QCD

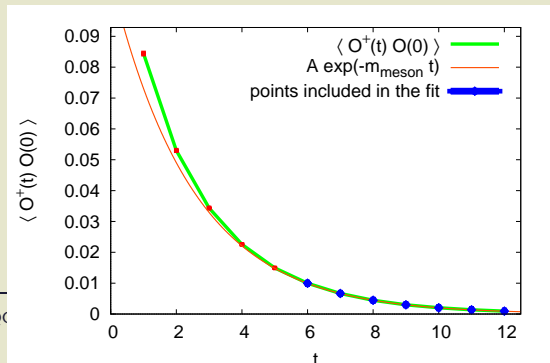
- **Korrelationsfunktion:** $C(t) \equiv \langle \Omega | O^\dagger(t) O(0) | \Omega \rangle$ ($|\Omega\rangle = \text{QCD-Grundzustand} = \text{Vakuum}$).
- Gitter-QCD technisch aufwändig:
 - Anspruchsvolle Computerprogramme müssen geschrieben werden ...
 - ... dann rechnen High-Performance-Computer mehrere Wochen oder Monate ...
 - ... Details dazu auf nächsten Folien.

Schritt (3): Bestimme gesuchte Hadronmasse anhand des exponentiellen Abfalls der Korrelationsfunktion $C(t)$

- Mit elementarer Quantenmechanik ergibt sich

$$C(t) = \langle \Omega | O^\dagger(t) O(0) | \Omega \rangle \stackrel{t \rightarrow \infty}{\propto} e^{-m_D t}.$$

- Fit von $Ae^{-m_D t}$ an Gitter-QCD-Ergebnisse für $C(t)$ liefert gesuchte D -Meson-Masse m_D .

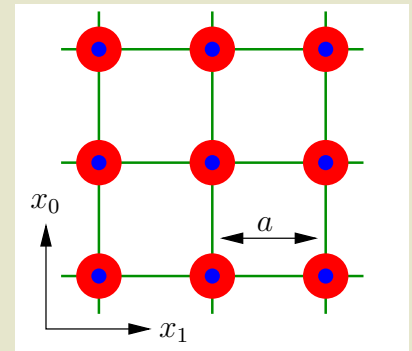


Gitter-QCD

- **Ziel:** Numerische Berechnung von QCD-Observablen, z.B. eine **zeitliche Korrelationsfunktion** und damit eine Hadronmasse.
- Ausgangspunkt ist **Pfadintegralformulierung**,

$$C(t) = \langle \Omega | O^\dagger(t) O(0) | \Omega \rangle = \prod_{x_\nu \in \text{Gitter}} \left(\prod_f d\psi^{(f)}(x_\nu) d\bar{\psi}^{(f)}(x_\nu) \right) dA_\mu(x_\nu) \dots,$$

- an jedem der unendlich vielen Raumzeitpunkte x_ν muss ein “normales Integral” über die Feldwerte $\psi^{(f)}(x_\nu)$ und $A_\mu(x_\nu)$ ausgeführt werden,
 - damit ein unendlich-dimensionales Integral.
- Numerische Umsetzung:
 - Diskretisiere Raumzeit mit kubischem Gitter mit hinreichend kleinem Gitterabstand $a \approx 0.05 \text{ fm} \dots 0.10 \text{ fm}$.
 - Betrachte nur endlichen Ausschnitt der Raumzeit mit Durchmesser $L \approx 2.0 \text{ fm} \dots 4.0 \text{ fm}$.



- Pfadintegral wird endlich-dimensionales Integral, aber $\mathcal{O}(10^8)$ **Integrationsvariablen**.
- Speziell entwickelte stochastische Algorithmen erforderlich.
- Hochleistungscomputersysteme erforderlich, Zusammenschluss zu Kollaborationen.

Gitter-QCD: Ziele

- Mit Gitter-QCD-Rechnungen verfolgt man eine Vielzahl von Zielen, z.B.:
 - Verifikation (bzw. Falsifikation) von QCD durch Vergleich von Gitter-QCD-Resultaten mit experimentellen Messergebnissen.
 - Part 2a: Präzise Berechnung einiger Mesonmassen, z.B. von D_s , η_c , J/Ψ , ...
 - Vorhersagen von bisher nicht in Experimenten beobachteten Mesonen oder Baryonen (z.B. wertvoller Input für Experimente).
 - Part 2b: Vorhersage eines $\bar{b}bud$ -Tetraquarks mit Quantenzahlen $I(J^P) = 0(1^+)$
 - Untersuchung der Struktur von Mesonen oder Baryonen.
 - Part 2b: Ist ein Meson ein “normales Quark-Antiquark-Paar” oder ein Tetraquark ... falls letzteres, ein mesonisches Molekül oder ein Diquark-Antidiquark-Paar, ...?
 - Auflösen von momentan existierenden Widersprüchen zwischen experimentellen Ergebnissen und theoretischen Modellrechnungen.
 - Part 2a: Separation von $D_1(2430)$ und $D_1(2420)$ zur Untersuchung von $B^{(*)} \rightarrow D^{**}$.
 - Berechnung von experimentell schwer oder nicht zugänglichen QCD-Observablen.
 - Part 2b: Systeme mit zwei schweren \bar{b} Antiquarks, $\bar{b}bud$.

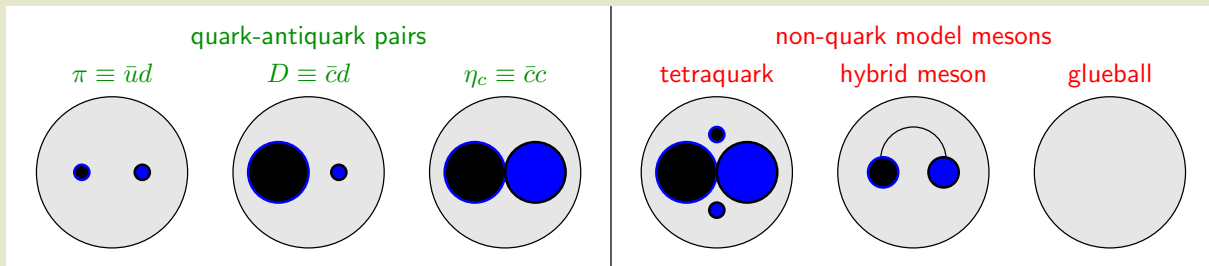
(+) Keine Annahmen. Keine Näherungen. Kein Modell. QCD-Ergebnisse.

(-) Zeitaufwändig ... Gitter-QCD-Projekte benötigen mehrere Jahre.

Part 2: Heavy mesons and tetraquarks from lattice QCD

$\bar{q}q$ mesons and tetraquarks (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \dots$
- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.g. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).
- Certain mesons are poorly understood (significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
 - **2 quarks and 2 antiquarks (tetraquark)**,
 - **a quark-antiquark pair and gluons (hybrid meson)**,
 - **only gluons (glueball)**.



$\bar{q}q$ mesons and tetraquarks (2)

- Indications for tetraquark structures:

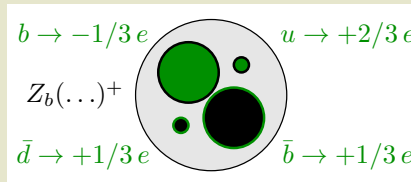
- Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$:

- * Mass suggests a $b\bar{b}$ pair ...

- * ... but $b\bar{b}$ is electrically neutral ...?

- * **Easy to understand, when assuming a tetraquark structure:**

$Z_b(\dots)^+ \equiv b\bar{b}u\bar{d}$ ($u \rightarrow +2/3 e$, $\bar{d} \rightarrow -1/3 e$).



- Electrically charged Z_c states:

- * Similar to Z_b .

- Mass ordering of light scalar mesons:

- * E.g. $m_{\kappa} > m_{a_0(980)}$...?

Several lattice QCD approaches

Goal: Understand masses and structures of mesons using lattice QCD.

- (1) $q\bar{q}$ operators (quark-antiquark) (part 2a)
 - (+) High precision for simple states (uncertainty $\lesssim 0.5\%$).
 - (-) Only for rather stable mesons, which have an ordinary quark-antiquark structure, e.g. D , D^* , D_s , D_s^* , η_c , J/ψ ...
- (2) $qq\bar{q}\bar{q}$ operators (quark-antiquark + tetraquark) (not in this talk)
 - (+) Also for unstable mesons or mesons, which have a complicated structure (tetraquark candidates), e.g. $a_0(980)$, D_{s0}^* , $Z(\dots)^+$, ...
 - (+) Not only masses, but also information about the structure.
 - (-) Technically very challenging.
 - (-) Currently under development, no solid or precise results at the moment.
- (3) $qq\bar{b}\bar{b}$ operators, Born-Oppenheimer approximation (part 2b)
(somewhere between (1) and (2))
 - (+) Allows to investigate specific tetraquark candidates at moderate computational costs, e.g. $Z_b(10610)^+$, $Z_b(10650)^+$.
 - (-) Approximations involved.

Part 2a: D mesons, D_s mesons and charmonium

[M. Kalinowski, M.W. [ETM Collaboration], Phys. Rev. D **92**, 094508 (2015) [arXiv:1509.02396]]

[K. Cichy, M. Kalinowski, M.W. [ETM Collaboration], accepted by Phys. Rev. D (2016) [arXiv:1603.06467]]

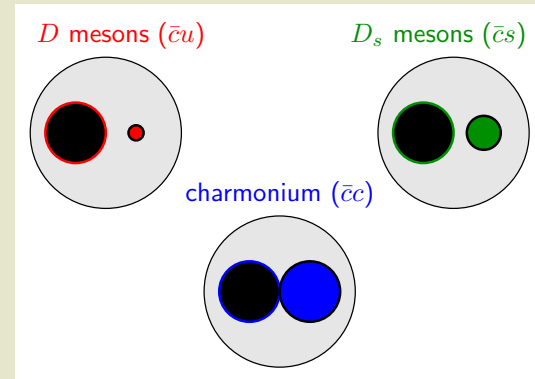
D , D_s , charmonium (1)

- **Goal:** precision lattice QCD computation of several D , D_s and charmonium masses:

- D mesons: D , D^* , D_0^* , D_1 , ...
- D_s mesons: D_s , D_s^* , ...
- Charmonium: η_c , J/ψ , ...

- Computation based on standard quark-antiquark creation operators:

- D mesons $\rightarrow \bar{c}u$, $\bar{c}d$, $\bar{u}c$ or $\bar{d}c$.
- D_s mesons $\rightarrow \bar{c}s$ or $\bar{s}c$.
- Charmonium $\rightarrow \bar{c}c$.

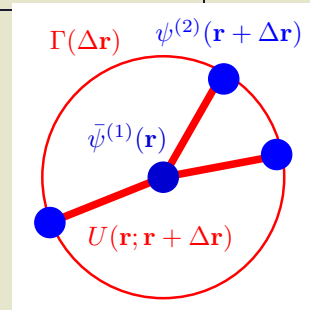


- Successful for rather stable mesons with quark-antiquark structure.
- Not suited e.g. for tetraquark candidates (D_{s0}^* , D_{s1} ...?), but still an important preparatory step.

D, D_s, charmonium (2)

- $\mathcal{O}(100)$ different **meson creation operators**

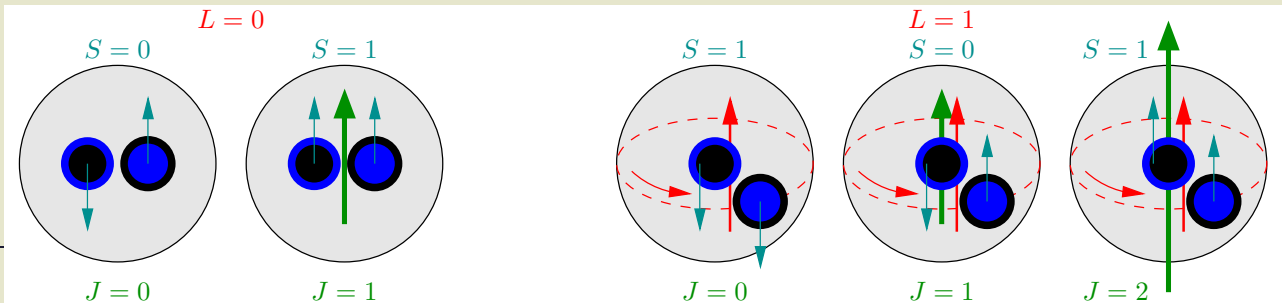
$$O_{\Gamma, \bar{\psi}^{(1)} \psi^{(2)}} \equiv \int d^3r \bar{\psi}^{(1)}(\mathbf{r}) \underbrace{\int_{|\Delta\mathbf{r}|=R} d^3\Delta\mathbf{r} U(\mathbf{r}; \mathbf{r} + \Delta\mathbf{r}) \Gamma(\Delta\mathbf{r}) \psi^{(2)}(\mathbf{r} + \Delta\mathbf{r})}_{= \text{spherical harmonic} \times \text{spin matrix}}$$



generate the following quantum numbers:

- Quark flavor: $\bar{\psi}^{(1)}\psi^{(2)} = \bar{c}u$ for D , $\bar{\psi}^{(1)}\psi^{(2)} = \bar{c}s$ for D_s , $\bar{\psi}^{(1)}\psi^{(2)} = \bar{c}c$ for charmonium.
- Total angular momentum $J = 0, 1, 2, \dots$, parity $P = -, +$.
 Spherical harmonic \rightarrow orbital angular momentum $L = 0, 1, 2, \dots$
 Spin matrix \rightarrow spin $S = 0, 1$.
- $\int d^3x \rightarrow$ vanishing momentum $\mathbf{p} = 0$.

- Mass determination as in part 1 (compute correlation function, fit exponential function).

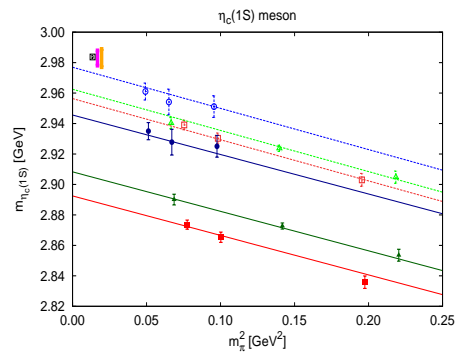
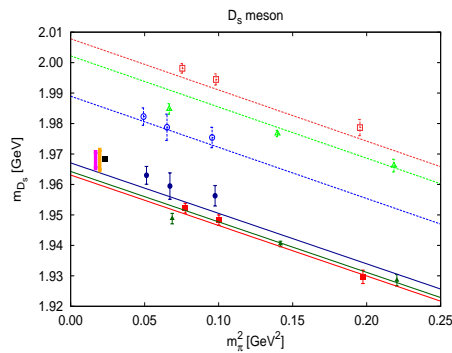


D, D_s , charmonium (3)

- Computations at unphysically heavy u/d quark masses and at finite lattice spacing, then **combined linear extrapolation**
 - in the u/d quark mass $m_{u,d} \propto m_\pi^2$ to $m_\pi = 135$ MeV (horizontal axes) and
 - in the squared lattice spacing a^2 to the continuum, i.e. $a = 0$ (different colors)
 - using two different Wilson twisted mass lattice discretizations (same colors).

→ Results in pink/yellow.

- Examples: D_s meson (left plot), η_c meson (right plot), both $J^P = 0^-$.
- **Perfect agreement between experimental results (black) and the quark mass and continuum extrapolated lattice QCD results (pink/yellow) (errors on the per mille level).**



$D, D_s, \text{charmonium (4)}$

- **Summary of lattice QCD results** (blue [$P = -$] and red [$P = +$] boxes):

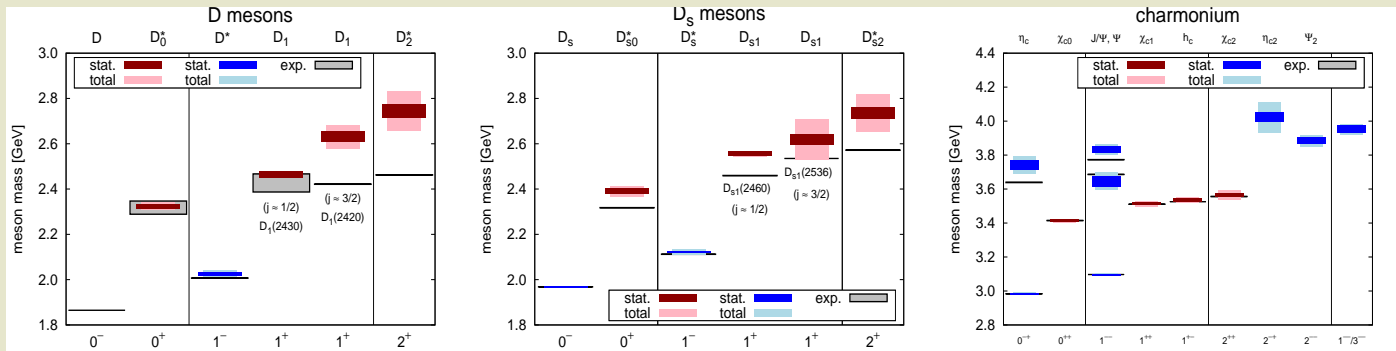
- D meson masses (left):
 $D, D_0^*, D^*, D_1(2430), D_1(2420), D_2^*$.
- D_s meson masses (center):
 $D_s, D_{s0}^*, D_s^*, D_{s1}(2460), D_{s1}(2536), D_{s2}^*$.
- Charmonium masses (right):
 $\eta_c, \chi_{c0}, J/\Psi, \chi_{c1}, h_c, \chi_{c2}, \eta_{c2}, \Psi_2$.

(+) Computations with 2+1+1 dynamical quark flavors (ETMC gauge link configurations).

(+) Extrapolated to physically light u/d quark masses.

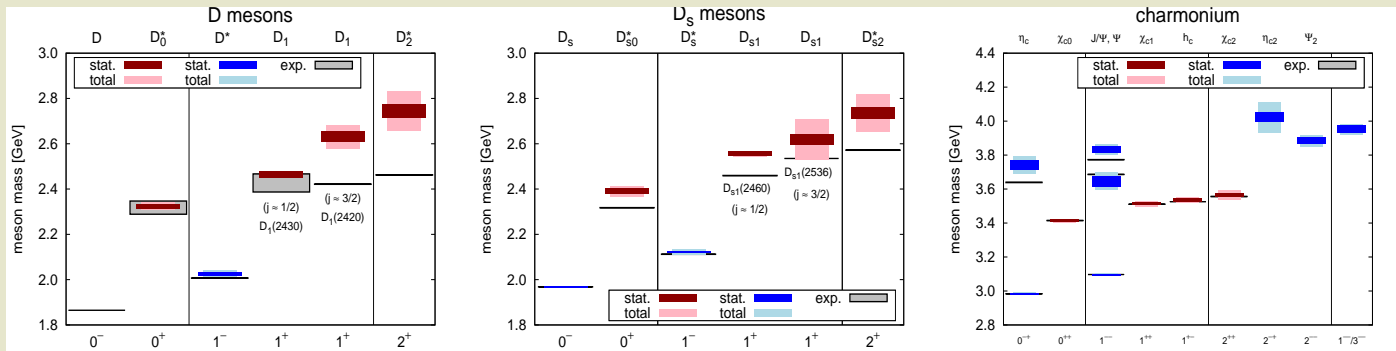
(+) Extrapolated to the continuum.

(-) Only quark-antiquark creation operators, no four-quark operators at the moment.



$D, D_s, \text{charmonium (5)}$

- **Summary of lattice QCD results** (blue [$P = -$] and red [$P = +$] boxes):
 - Comparison with existing experimental results (black boxes):
 - * Agreement for the majority of states.
 - * **Tension/disagreement:**
 - $D_{s0}^*, D_{s1}(2460)$: tetraquark candidates
... might require four-quark creation operators.
 - Radial excitations and higher orbital excitations (e.g. $\eta_c(2S), J/\Psi(2S), D_2^*, D_{s2}^*$)
... such excitations exhibit poor signals in lattice QCD (reflected by large errors), better statistics, i.e. longer computations required.



D , D_s , charmonium (6)

Summary of part 2a

- First principles QCD computation of several states, all sources of systematic error investigated and quantified, agreement with corresponding experimental results on the per mille level.
- Preparatory step for more advanced computations using also four-quark creation operators (investigation of tetraquark candidates and unstable mesons).
- Clear separation of the two $J^P = 1^+$ states
 - $D_1(2430)$ (light spin $j \approx 1/2$)
 - $D_1(2420)$ (light spin $j \approx 3/2$).
- Important to study semileptonic decays $B^{(*)} \rightarrow D^{**} + l + \nu$.
- Persistent conflict between experiment and theory (“ $1/2$ versus $3/2$ puzzle”).

Part 2b: Heavy-heavy-light-light tetraquark candidates

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]

[P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]

[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]

[A. Peters, P. Bicudo, K. Cichy, M.W., arXiv:1602.07621]

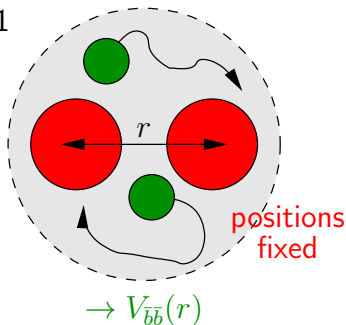
[A. Peters, P. Bicudo, L. Leskovec, S. Meinel, M.W., arXiv: 1609.00181]

[P. Bicudo, J. Scheunert, M.W., arXiv:1609.00548]

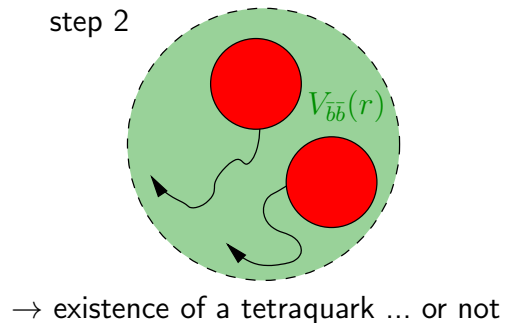
$\bar{b}\bar{b}qq$ tetraquarks (1)

- **Basic idea:** Investigate existence of heavy tetraquarks $\bar{b}\bar{b}qq$ in two steps.
 - (1) **Compute potentials of two static antiquarks ($\bar{b}\bar{b}$) in the presence of two lighter quarks ($qq \in \{ud, ss, cc\}$) using lattice QCD.**
 - (2) **Check, whether these potentials are sufficiently attractive, to host a bound state by solving a corresponding Schrödinger equation.**
(\rightarrow This would indicate a stable $\bar{b}\bar{b}qq$ tetraquark.)
- ((1) + (2) \rightarrow Born-Oppenheimer approximation).

step 1



step 2



$\bar{b}\bar{b}qq$ tetraquarks (2)

Born-Oppenheimer approximation, step (1)

- Lattice QCD computation of $\bar{b}\bar{b}$ potentials $V_{\bar{b}\bar{b}}(r)$ (2 flavor ETMC gauge link configurations).

(1) Use $\bar{b}\bar{b}qq$ creation operators

$$O_{\bar{b}\bar{b}qq} \equiv (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD} \left(\bar{b}_C(-\mathbf{r}/2) q_A^{(1)}(-\mathbf{r}/2) \right) \left(\bar{b}_D(+\mathbf{r}/2) q_B^{(2)}(+\mathbf{r}/2) \right).$$

- * Different light quark flavors $qq \in \{ud, ss, cc\}$.
- * Different light quark spin/parity.
- * Different heavy quark spin/parity (no effect on $V_{\bar{b}\bar{b}}(r)$).

→ Many different channels

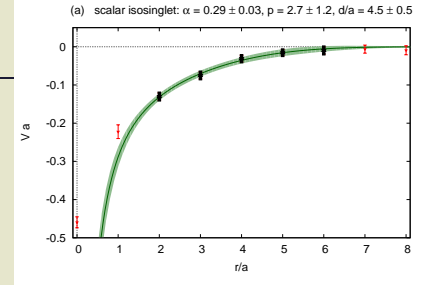
... some attractive, some repulsive

... some correspond for large $\bar{b}\bar{b}$ separations to pairs of ground state mesons (B and/or B^*), some to excited mesons (one or two B_0^* and/or B_1^*).

(2) Compute temporal correlation functions.

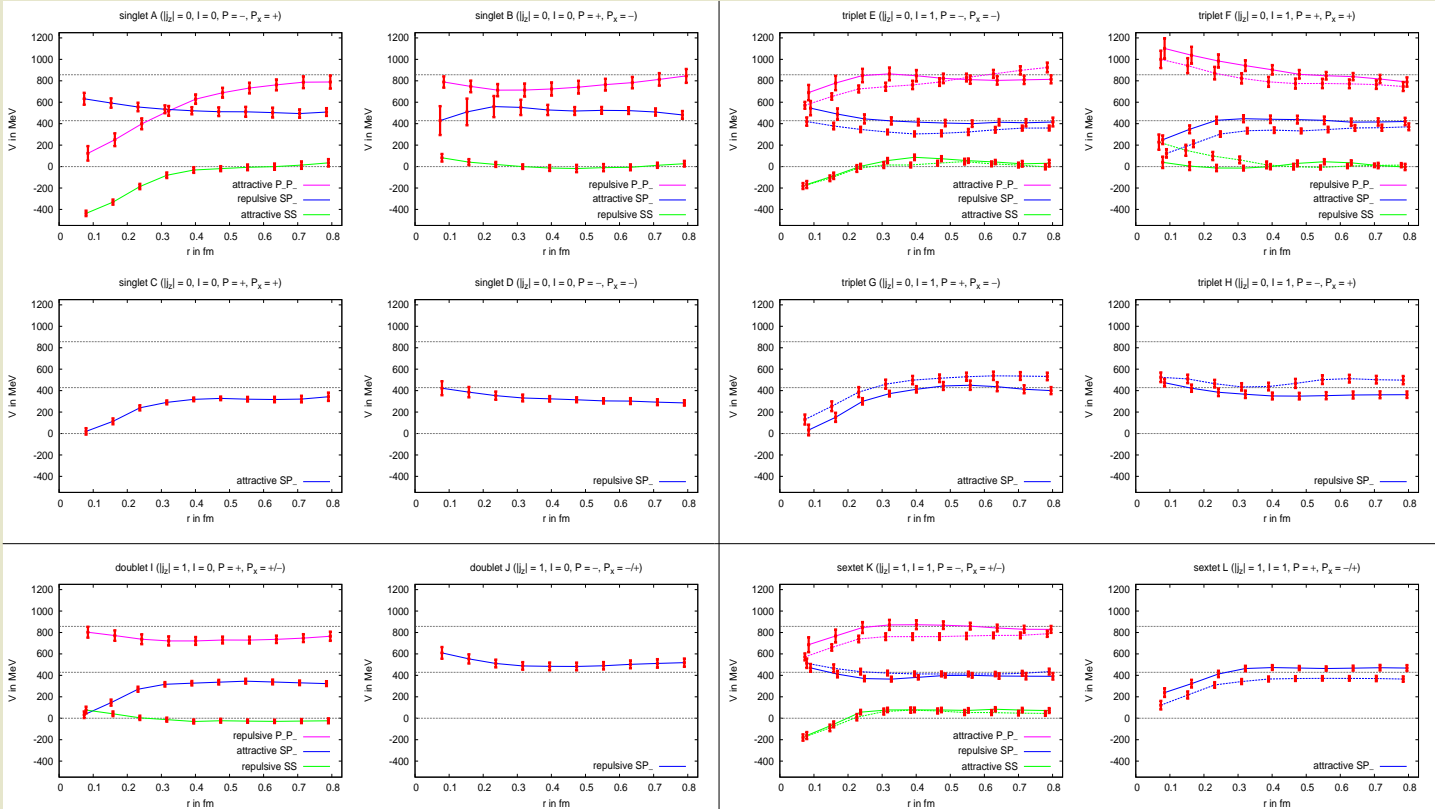
(3) Determine $V_{\bar{b}\bar{b}}(r)$ from the exponential decays of the correlation functions.

- First principles QCD computation of forces between hadrons.**



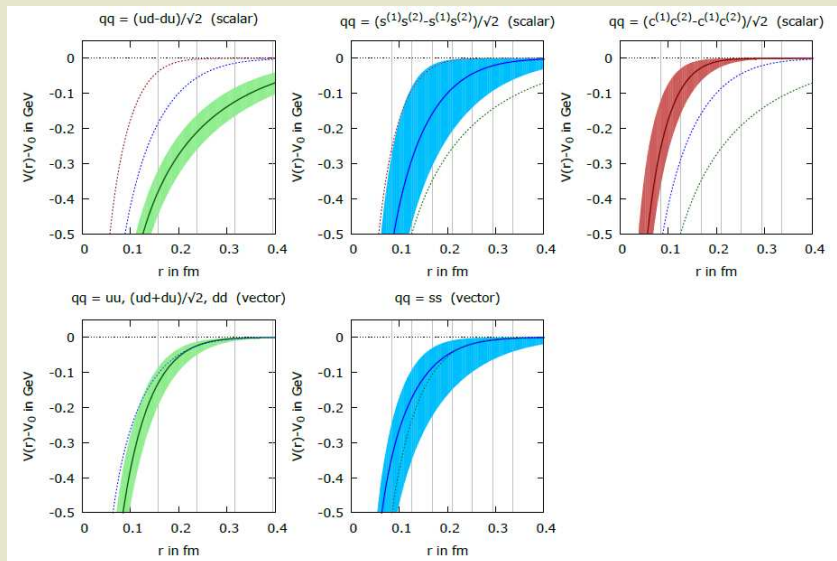
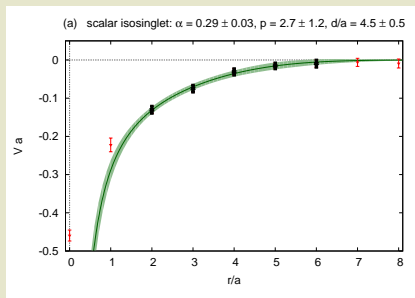
$\bar{b}\bar{b}qq$ tetraquarks (3)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



$\bar{b}\bar{b}qq$ tetraquarks (4)

- Two attractive channels corresponding to pairs of ground state mesons (B and/or B^*).
- Light quark mass dependence of these channels:
 $V_{\bar{b}\bar{b}}(r)$ wider and deeper for $qq = ud$ compared to $qq = ss$ compared to $qq = cc$.
 → **Good candidates to find tetraquarks are systems of two very heavy and two very light quarks, i.e. $\bar{b}\bar{b}ud$.**



$\bar{b}\bar{b}qq$ tetraquarks (5)

Born-Oppenheimer approximation, step (2)

- Solve the Schrödinger equation for the relative coordinate \mathbf{r} of the two \bar{b} quarks,

$$\left(-\frac{1}{2\mu}\Delta + V_{\bar{b}\bar{b}}(r) \right) \underbrace{\psi(\mathbf{r})}_{=R(r)/r} = E\psi(\mathbf{r}) \quad , \quad \mu = m_b/2;$$

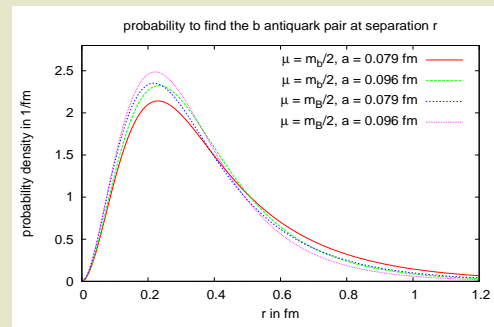
possibly existing bound states, i.e. $E < 0$, indicate $\bar{b}\bar{b}qq$ tetraquarks.

- A single bound state for one specific potential $V_{\bar{b}\bar{b}}(r)$ and light quarks $qq = ud$:
 - Binding energy $-E = 90^{+43}_{-36}$ MeV, i.e. confidence level $\approx 2\sigma$.
 - Quantum numbers of the $\bar{b}\bar{b}ud$ tetraquark: $I(J^P) = 0(1^+)$.
 - Average separation of $\bar{b}\bar{b}$: ≈ 0.25 fm.

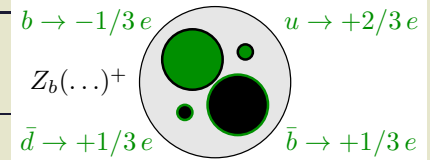
→ **Prediction of a tetraquark.**

(No experimental results for $\bar{b}\bar{b}ud$ yet.)

- No further bound states, in particular not for $qq = ss$ or $qq = cc$.



$\bar{b}\bar{b}qq$ tetraquarks (6)



Ongoing research

(1) Including the spin of the \bar{b} quarks: (Static quarks are spinless.)

- Consider many channels at the same time via a coupled channel Schrödinger equation.
- Result: $I(J^P) = 0(1^+)$ tetraquark persists, binding energy $-E = 59_{-38}^{+30}$ MeV.

(2) Structure of the tetraquark: (Mesonic molecule or diquark-antidiquark?)

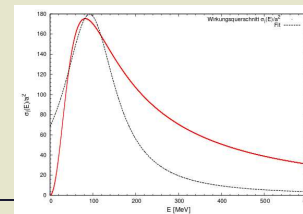
- Several hadron creation operators and correlation matrices required ...

(3) $\bar{b}b\bar{q}q$ instead of $\bar{b}\bar{b}qq$: (i.e. $Z_b(10610)^+$, $Z_b(10650)^+$)

- Theoretically harder, because light quarks can form a far separated light meson, e.g. a pion ... result would then be about “bottomonium + pion”, not about a tetraquark.
- Crude result: $I(J^P) = 1(1^+)$ $\bar{b}b\bar{d}u$ tetraquark, binding energy $-E = (58 \pm 71)$ MeV.

(4) Tetraquark resonances:

- Resonance parameters (phase shifts, resonance masses and widths) can be determined numerically using quantum mechanics/scattering theory.
- Indication of a resonance for $I(J^P) = 0(1^-)$.



Recent research, future plans, potential synergies

($\approx 60\%$) Hadron spectrum, structure, decays (in particular exotic mesons):

- Tetraquark candidates with four finite mass quarks: $a_0(980)$, κ , D_{s0}^* , $D_{s1}(2460)$, ...
- $\bar{b}\bar{b}qq$ and $\bar{b}\bar{b}\bar{q}q$ tetraquarks with **static quarks** (HQET) and with NRQCD. (part 2b)
- Hybrid static potentials and hybrid mesons.
- Standard quark-antiquark mesons (orbital, radial and parity excitations): D , D_s , charmonium, B , B_s , bottomonium ... $B \rightarrow D^{**}$... (part 2a)

(FAIR, Emmy Noether research group, DFG grant)

($\approx 20\%$) Lattice QCD simulations at fixed or frozen topology.

(Emmy Noether research group)

($\approx 15\%$) Static potential:

- Matching of lattice QCD and **perturbative QCD**, determination of $\Lambda_{\overline{\text{MS}}}$.
- Non-perturbative definition and interpretation of color-octet potentials.
- Non string-like trial states, efficient computation of off-axis separations.

($\approx 5\%$) Phase structure of QCD-inspired models for $\mu > 0$, inhomogeneous condensates.