$\Lambda_{\overline{\mathrm{MS}}}$ from the static potential for $N_f = 2$

Marc Wagner

Humboldt-Universität zu Berlin, Institut für Physik

Theorie der Elementarteilchen – Phänomenologie/Gittereichtheorie

mcwagner@physik.hu-berlin.de

http://people.physik.hu-berlin.de/~mcwagner/

in collaboration with Karl Jansen, Felix Karbstein, Attila Nagy

ETMC meeting – Barcelona

September 15, 2010

Motivation

- Λ (or $\Lambda_{\overline{\mathrm{MS}}}$) defines the scale for dimensionful perturbative results (analogous to the lattice spacing a for lattice results).
- Λ (or $\Lambda_{\overline{\mathrm{MS}}}$) could in principle be determined by
  - performing an experiment, which can be calculated reliably by means of perturbation theory (i.e. an experiment at high energies),
  - identifying the experimental and the perturbative result,
  - solving for Λ (or $\Lambda_{\overline{\mathrm{MS}}}$).
- Alternatively, one can replace the experiment by a lattice computation of an observable, which can also be calculated reliably by means of perturbation theory (e.g. the static potential $V_{Q\bar{Q}}(r)$, i.e. the potential of a static quark antiquark pair separated by a distance r).

Outline

- Perturbation theory for $V_{Q\bar{Q}}$.
- Setting the scale via $\Lambda_{\overline{\mathrm{MS}}}$.
- Validity of perturbation theory.
- Lattice computation of $V_{Q\bar{Q}}$.
- $\Lambda_{\overline{\mathrm{MS}}}$ fits.
- Quark mass extrapolation.
- Continuum extrapolation.
- Conclusions.

Perturbation theory for $V_{Q\bar{Q}}$ (1)

- Perturbative expansion of the physical coupling constant α ($\alpha = g^2/4\pi$) at momentum p in terms of α at momentum q (valid for $\alpha(p), \alpha(q) \ll 1$):

$$\alpha(p) \quad = \quad \alpha(q) - \#\alpha(q)^2 \ln(p^2/q^2) + \mathcal{O}(\alpha(q)^3) \qquad (1)$$

(expansion known up to $\mathcal{O}(\alpha(q)^5)$).

- Momentum space static potential (delocalized static quarks interacting via gluon exchange, etc. with momentum q):

$$\tilde{V}_{Q\bar{Q}}(p, \alpha(p)) \quad = \quad -\frac{16\pi}{3p^2}\Big(\alpha(p) + \#\alpha(p)^2 + \mathcal{O}(\alpha(p)^3)\Big)$$

(expansion known up to $\mathcal{O}(\alpha(p)^4)$, i.e. NNNLO).

[C. Anzai, Y. Kiyo and Y. Sumino, Phys. Rev. Lett. **104**, 112003 (2010) [arXiv:0911.4335 [hep-ph]]]

[A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **104**, 112002 (2010) [arXiv:0911.4742 [hep-ph]]]

Perturbation theory for $V_{Q\bar{Q}}$ (2)

- Fourier transformation yields the position space static potential (localized static quarks, separation r, interacting via gluon exchange, etc. with arbitrary momenta):
  - Express $\alpha(p)$ in $\tilde{V}_{Q\bar{Q}}(p, \alpha(p))$ in terms of $\alpha(\mu)$ (μ is an arbitrary scale) by means of eq. (1):

    $$\tilde{V}_{Q\bar{Q}}(p, \alpha(p)) \quad \rightarrow \quad \tilde{V}_{Q\bar{Q}}(p, \mu, \alpha(\mu)).$$

  - Perform the Fourier transformation:

    $$\hat{V}_{Q\bar{Q}}(r, \mu, \alpha(\mu)) \quad = \quad \frac{1}{(2\pi)^3} \int d^3p \, e^{+i\mathbf{pr}} \tilde{V}_{Q\bar{Q}}(p, \mu, \alpha(\mu)).$$

  - Get rid of μ and $\alpha(\mu)$ by expressing $\alpha(\mu)$ in $\hat{V}_{Q\bar{Q}}(r, \mu, \alpha(\mu))$ in terms of $\alpha(1/r)$ by means of eq. (1):

    $$\hat{V}_{Q\bar{Q}}(r, \alpha(1/r)) \quad = \quad -\frac{4\pi}{3r}\Big(\alpha(1/r) + \#\alpha(1/r)^2 + \mathcal{O}(\alpha(1/r)^3)\Big).$$

Perturbation theory for $V_{Q\bar{Q}}$ (3)

- However, Fourier transform $\hat{V}_{Q\bar{Q}}(r, \alpha(1/r))$ of a perturbative expansion of $\tilde{V}_{Q\bar{Q}}(p, \alpha(p))$ is physically questionable/meaningless, because of integrating over all momenta p:
  - As an illustration consider e.g.

$$\begin{aligned} \tilde{V}_{Q\bar{Q}}(p, \alpha(p)) &= -\frac{16\pi\alpha(p)}{3p^2} + \ldots \\ \alpha(p) &= \alpha(\mu) - \#\alpha(\mu)^2 \ln(p^2/\mu^2) + \ldots , \end{aligned}$$

    which is obviously valid only for small $\alpha(p)$ and $\alpha(\mu)$, i.e. large p and μ, and for $p^2/\mu^2 \approx 1$.

Perturbation theory for $V_{Q\bar{Q}}$ (4)

- Remove the error introduced by Fourier transforming a perturbative expansion of the static potential by means of a technique called "renormalon subtraction":

  [M. Beneke, Phys. Rept. **317**, 1 (1999) [arXiv:hep-ph/9807443]]

  [A. Pineda, JHEP **0106**, 022 (2001) [arXiv:hep-ph/0105008]]

  [A. Pineda, J. Phys. G **29**, 371 (2003) [arXiv:hep-ph/0208031]]

  – Replace

  $$\hat{V}_{Q\bar{Q}}(r, \alpha(1/r)) \quad \rightarrow$$
  $$\rightarrow \quad V_{Q\bar{Q}}(r, \alpha(1/r), \rho, \alpha(\rho)) \quad = \quad \hat{V}_{Q\bar{Q}}(r, \alpha(1/r)) - V_{Q\bar{Q}}^{\mathrm{ren}}(\rho, \alpha(\rho)),$$

  where

  $$V_{Q\bar{Q}}^{\mathrm{ren}}(\rho, \alpha(\rho)) \quad = \quad \rho\Big(\#\alpha(\rho)^2 + \#\alpha(\rho)^3 + \ldots\Big).$$

  – Renormalon subtraction introduces a new scale ρ.

  – No renormalon subtraction at $\mathcal{O}(\alpha)$, i.e. LO.

Perturbation theory for $V_{Q\bar{Q}}$ (5)

- $V_{Q\bar{Q}}(r, \alpha(1/r), \rho, \alpha(\rho))$ depends on "two coupling constants".
- To obtain a result, which depends on a "single coupling constant" only, one can again use

$$\alpha(1/r) \quad = \quad \alpha(\rho) - \#\alpha(\rho)^2 \ln(1/r^2\rho^2) + \mathcal{O}(\alpha(\rho)^3).$$

- Final result:

$$V_{Q\bar{Q}}(r, \rho, \alpha(\rho)) \quad = \quad \sum_{n=1}^{\infty} \alpha(\rho)^n f_n(r, \rho) \quad =$$

$$= \quad -\alpha(\rho)\frac{4}{3r} + \alpha(\rho)^2\bigg(- \frac{2.04\ldots + 2.05\ldots \ln(r\rho)}{r}\bigg) + \mathcal{O}(\alpha(\rho)^3) + V_0.$$

- The renormalon scale is chosen according to $1/\rho = (r_\mathrm{min} + r_\mathrm{max})/2$, where $r_\mathrm{min} \leq r \leq r_\mathrm{max}$ is the region, where the perturbative static potential will be fitted to the lattice static potential.

Perturbation theory for $V_{Q\bar{Q}}$ (6)

- Potential NRQCD (pNRQCD):

  [A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64**, 428 (1998) [arXiv:hep-ph/9707481]]

  [N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. B **566**, 275 (2000) [arXiv:hep-ph/9907240]]

  [N. Brambilla, A. Vairo, X. Garcia i Tormo and J. Soto, Phys. Rev. D **80**, 034016 (2009) [arXiv:0906.1390 [hep-ph]]]

  – Effective field theory for QCD, specifically designed for bound states of heavy quarks.

  – Allows renormalization group improvement, i.e. summation of a specific class of logarithms (NNLO $\rightarrow$ NNLL, NNNLO $\rightarrow$ NNNLL).

  – At NNNLL a new unknown constant appears.

  – At the moment we investigate, whether it is advantageous with respect to a $\Lambda_{\overline{\mathrm{MS}}}$ determination to use the leading logarithmic orders NNLL and NNNLL.

Perturbation theory for $V_{Q\bar{Q}}$ (7)

- Summary of perturbative orders at our disposal:
  - Leading order (LO), i.e. $\mathcal{O}(\alpha(\rho))$.
  - Leading order (NLO), i.e. $\mathcal{O}(\alpha(\rho)^2)$.
  - Leading order (NNLO), i.e. $\mathcal{O}(\alpha(\rho)^3)$.
  - Leading order (NNLL), i.e. $\mathcal{O}(\alpha(\rho)^3)$ and logarithmic contributions.
  - Leading order (NNNLO), i.e. $\mathcal{O}(\alpha(\rho)^4)$.
  - Leading order (NNNLL), i.e. $\mathcal{O}(\alpha(\rho)^4)$ and logarithmic contributions, but an additional unknown constant.

Setting the scale via $\Lambda_{\overline{\mathrm{MS}}}$

- $\Lambda = \Lambda_{\overline{\mathrm{MS}}}$ at 1-loop:
  - Resummed 1-loop relation between $\alpha(q)$ and $\alpha(\rho)$:

$$\alpha(q) \quad = \quad \frac{\alpha(\rho)}{1 + \alpha(\rho)\beta_0 \ln(q^2/\rho^2)/4\pi} \quad , \quad \beta_0 \quad = \quad 29/3.$$

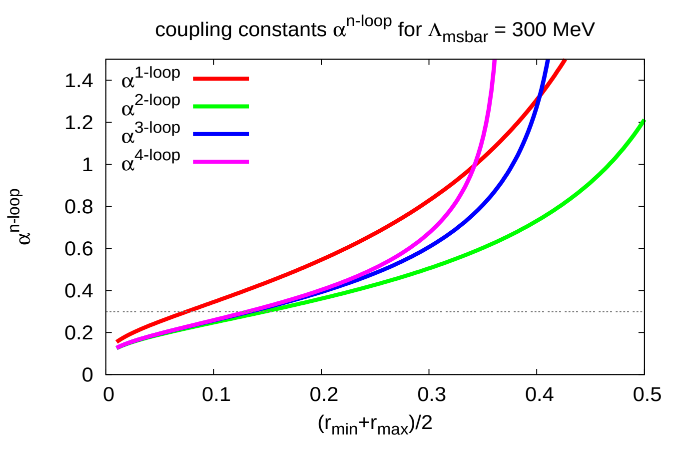
  - Define Λ via $\alpha(\Lambda) = \infty$; consequently

$$\Lambda \quad = \quad \rho e^{-2\pi/\alpha(\rho)\beta_0} \quad \leftrightarrow \quad \alpha(\rho) \quad = \quad \frac{2\pi}{\beta_0 \ln(\rho/\Lambda)}.$$

  - Λ independent from ρ.
- Definition can be generalized to n-loop ... however, lengthy.
- 3-loop, 4-loop, ...: Λ depends on the renormalization scheme, i.e. $\Lambda \rightarrow \Lambda_{\overline{\mathrm{MS}}}$.

Validity of perturbation theory

- 1-loop, 2-loop, 3-loop and 4-loop result for α as functions of $1/\rho = (r_{\mathrm{min}} + r_{\mathrm{max}})/2$ for $\Lambda_{\overline{\mathrm{MS}}} = 300\,\mathrm{MeV}$.



- Perturbation theory expected to be valid up to $r \lesssim 0.25\,\mathrm{fm}$.

Lattice computation of $V_{Q\bar{Q}}$

- $N_f = 2$, $\beta = 4.20$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0020$ (21 gauge configurations) $\rightarrow a = 0.051\,\mathrm{fm}$, $L = 1.23\,\mathrm{fm}$, $m_{\mathrm{PS}} \approx 284\,\mathrm{MeV}$.
- The static potential is computed from Wilson loops:

$$V_{Q\bar{Q}}(r) \quad = \quad \lim_{t \rightarrow \infty} \frac{1}{a} \ln \left(\frac{W(r,t)}{W(r,t+1)} \right).$$

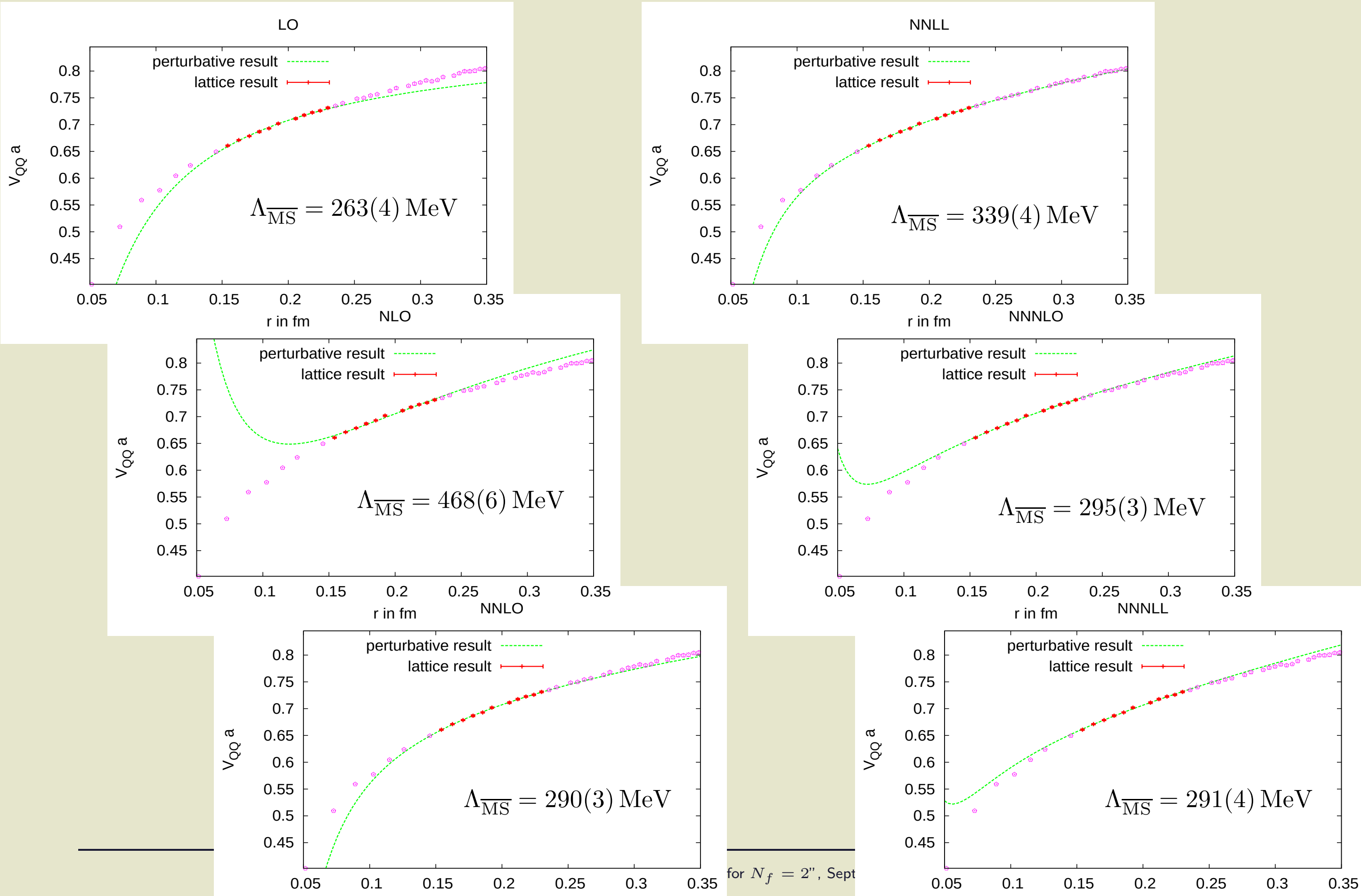
  - APE smearing of spatial links to increase the ground state overlap.
  - No HYP smearing of temporal links (UV fluctuations are important for the static potential at small $Q\bar{Q}$ separations).
  - Off-axis Wilson loops to increase the resolution of the static potential.

$\Lambda_{\overline{\mathrm{MS}}}$ fits (1)

- Perform an uncorrelated χ^2-minimizing fit of the perturbative static potentials to the lattice results, fitting range $[r_\mathrm{min}, r_\mathrm{max}]$:
  - r_min restricted by lattice results.
  - r_max restricted by perturbative results.
  - In the following: $[r_\mathrm{min}, r_\mathrm{max}] = [0.15\,\mathrm{fm}\,,\ 0.23\,\mathrm{fm}]$.
  - Results for $\Lambda_{\overline{\mathrm{MS}}}$ are rather stable with respect to a moderate variation of the fitting range: $r_\mathrm{min} \in \{0.12\,\mathrm{fm}\,,\ 0.15\,\mathrm{fm}\}$, $r_\mathrm{max} \in \{0.23\,\mathrm{fm}\,,\ 0.26\,\mathrm{fm}\}$.
- NNNLL contains a new unknown constant K_2, which cannot reliably be determined by a fit; we use the value $K_2 = -1.25/r_0$.

  [N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo, arXiv:1006.2066 [hep-ph]].

$\Lambda_{\overline{MS}}$ fits (2)



$\Lambda_{\overline{\mathrm{MS}}}$ fits (3)

- Summary of fitting results:

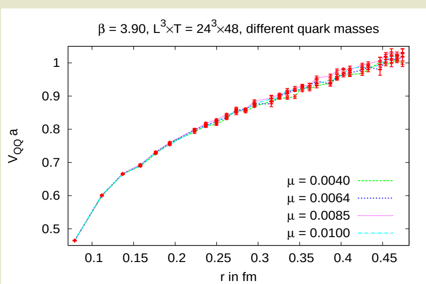
order	$\Lambda_{\overline{\mathrm{MS}}}$ in MeV	χ^2/dof	$\alpha^{\text{1-loop}}$	$\alpha^{\text{2-loop}}$	$\alpha^{\text{3-loop}}$	$\alpha^{\text{4-loop}}$
LO	$263(4)$	1.02	0.48	0.32	0.35	0.35
NLO	$468(6)$	2.50	0.83	0.51	0.61	0.68
NNLO	$290(3)$	0.39	0.52	0.35	0.37	0.38
NNLL	$339(4)$	0.34	0.59	0.38	0.42	0.44
NNNLO	$295(3)$	0.51	0.52	0.35	0.38	0.39
NNNLL	$291(4)$	0.53	0.52	0.35	0.37	0.38

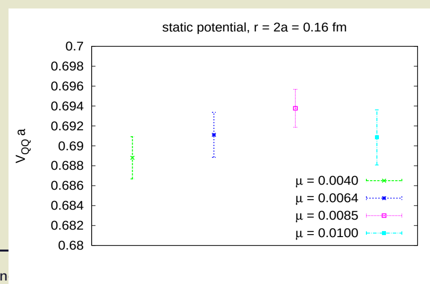
- Preliminary estimate: $\Lambda_{\overline{\mathrm{MS}}} = 290\,\mathrm{MeV} \ldots 330\,\mathrm{MeV}$.

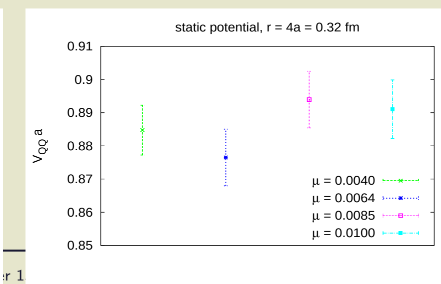
- In agreement with the $N_f = 2$ ETMC $\Lambda_{\overline{\mathrm{MS}}}$ from the ghost-gluon running QCD coupling constant: $\Lambda_{\overline{\mathrm{MS}}}^{\text{ghost-gluon}} = 330(23)(22)_{-33}\,\mathrm{MeV}$ (massless quarks) and/or $\Lambda_{\overline{\mathrm{MS}}}^{\text{ghost-gluon}} = 294(10)\,\mathrm{MeV}$ (massive quarks).
  [B. Blossier, Ph. Boucaud, F. De soto, V. Morenas, M. Gravina, O. Pene and J. Rodriguez-Quintero [ETM Collaboration], Phys. Rev. D **82**, 034510 (2010) [arXiv:1005.5290 [hep-lat]].

Quark mass extrapolation

- Perturbation theory for the static potential is for $m_u = m_d = 0$, while lattice computations are performed at $m_u = m_d \neq 0$
  $\rightarrow$ extrapolation needed.
- $N_f = 2$, $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$,
  $\mu \in \{0.0040\,,\, 0.0064\,,\, 0.0085\,,\, 0.0100\}$ (21 gauge configurations)
  $\rightarrow a = 0.079\,\mathrm{fm}$, $L = 1.90\,\mathrm{fm}$, $m_{\mathrm{PS}} = 340\,\mathrm{MeV}\ \ldots\ 525\,\mathrm{MeV}$:
  - At this level of statistical accuracy there is no indication of a quark mass dependence of the static potential.
  - Determining $\Lambda_{\overline{\mathrm{MS}}}$ at these four different quark mass values is not possible, because of the rather coarse lattice spacing ($\beta = 4.05$ might help).

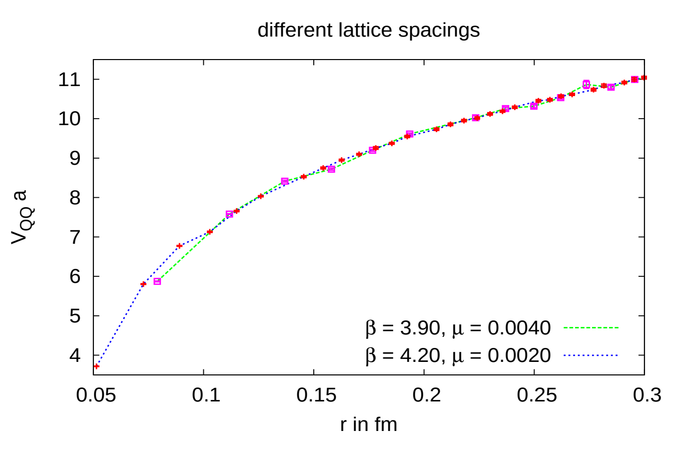






Continuum extrapolation

- $N_f = 2$, $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0040$ (21 gauge configurations) $\rightarrow a = 0.079\,\mathrm{fm}$, $L = 1.90\,\mathrm{fm}$, $m_{\mathrm{PS}} = 340\,\mathrm{MeV}$
  and
  $N_f = 2$, $\beta = 4.20$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0020$ (21 gauge configurations) $\rightarrow a = 0.051\,\mathrm{fm}$, $L = 1.23\,\mathrm{fm}$, $m_{\mathrm{PS}} \approx 284\,\mathrm{MeV}$:
  - Lattice discretization effects seem to be mild in the relevant region $0.12\,\mathrm{fm} \lesssim r \lesssim 0.26\,\mathrm{fm}$.



Conclusions

- Reliably extracting $N_f = 2$ $\Lambda_{\overline{\mathrm{MS}}}$ by means of the static potential from ETMC gauge configurations seems to be feasible.
- Preliminary estimate: $\Lambda_{\overline{\mathrm{MS}}} = 290\,\mathrm{MeV} \ldots 330\,\mathrm{MeV}$.
- At the moment the error of $\Lambda_{\overline{\mathrm{MS}}}$ is dominated by systematic differences between different perturbative orders
  $\longrightarrow$ simulations at lattice spacing $a < 0.051\,\mathrm{fm}$ might help ($N_f = 2$,
  $\beta = 4.35$, $L^3 \times T = 32 \times 64$ will be available soon).
- A more thorough quark mass and continuum investigation/extrapolation is necessary ($\beta = 4.05$ might help).
- A finite volume investigation/extrapolation is needed ($\beta = 4.20$, $L^3 \times T = 48^3 \times 96$ might help).