Heavy mesons and tetraquarks from lattice QCD

"3rd Belle II Theory Interface Platform (B2TiP) Workshop", Tsukuba

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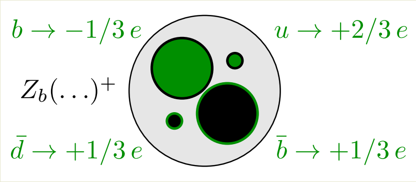
Introduction

- **Goal**:
  - Study mesons, in particular states, which are less well understood (tetraquark candidates [= four-quark states]), from first principles using lattice QCD.
- **Motivation**: Experimentally measured
  - charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states,
  - charged charmonium states, e.g. $Z_c(3900)^\pm$, $Z_c(4020)^\pm$, $Z_c(4050)^\pm$, $Z_c(4250)^\pm$, $Z_c(4430)^\pm$ ... must be four quark states,
  - D_{s0}^*, $D_{s1}(2460)$ are rather light compared to phenomenological expectations, could be tetraquarks,
  - ...
- A very challenging problem in lattice QCD.



Outline

(1) Introduction to QCD, lattice QCD and hadron spectroscopy.

(2) D meson, D_s meson and charmonium masses
(quark-antiquark creation operators).

(3) $\bar{b}\bar{b}qq$ tetraquark candidates
(creation operators with 2 static antiquarks and 2 quarks of finite mass).

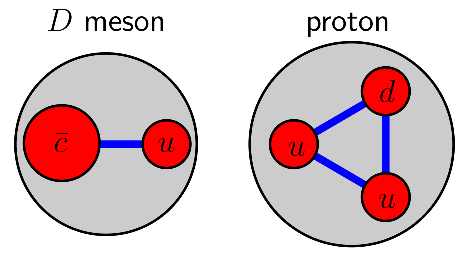
Part 1: Introduction to QCD and lattice QCD

QCD and lattice QCD

- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.
- Part of the standard model explaining the formation of hadrons (mesons with integer spin, usally $q\bar{q}$; baryons with half-integer spin, usually $qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.
- Definition of QCD simple:

$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu \Big) + m^{(f)} \Big) \psi^{(f)} + \frac{1}{2g^2} \mathrm{Tr}\Big(F_{\mu\nu} F_{\mu\nu} \Big) \right)$$

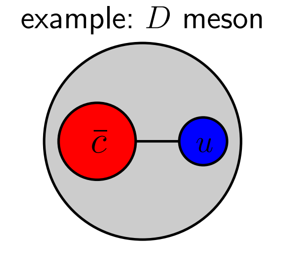
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$



- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of
  - non-linear field equations,
  - the absence of any small parameter/coupling constant (i.e. perturbation theory not applicable).

$\rightarrow$ Solve QCD numerically by means of lattice QCD.

Hadron spectroscopy



- Proceed as follows:

  (1) **Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator O.**

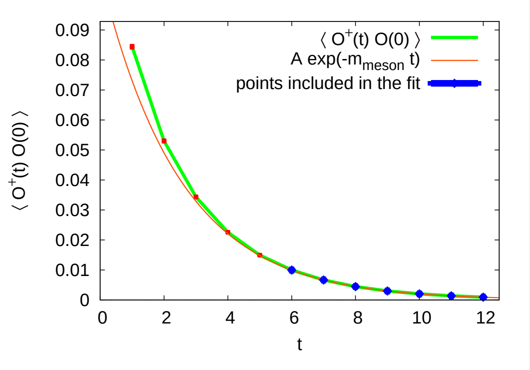
  - An operator O, which generates the quantum numbers (flavor, J^{PC}) of the hadron of interest, when applied to the vacuum $|\Omega\rangle$.
    $\rightarrow$ Easy.
  - An operator O, which crudely generates the hadron of interest (in particular same number of quarks), when applied to the vacuum $|\Omega\rangle$.
    $\rightarrow$ Difficult ... in particular for "exotic mesons", e.g. tetraquark candidates, one might get wrong results, when using quark-antiquark operators.

  (2) **Determine the corresponding hadron mass from the asymptotic exponential decay of $C(t)$ in time.**

- Example: D meson mass m_D (valence quarks $\bar{c}$ and u, $J^P = 0^-$),

$$O \equiv \int d^3r\, \bar{c}(\mathbf{r})\gamma_5 u(\mathbf{r})$$

$$C(t) \equiv \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle \overset{t\rightarrow\infty}{\propto} e^{-m_D t}.$$



Part 2: D meson, D_s meson and charmonium masses (quark-antiquark creation operators)

[M. Kalinowski, M.W., accepted by Phys. Rev. D [arXiv:1509.02396]]
[M. Kalinowski, K. Cichy, M.W., arXiv:1510.07862]

D, D_s, charmonium (1)

- In the following masses for D mesons, D_s mesons and charmonium states using quark-antiquark hadron creation operators, e.g. for D

$$\mathcal{O} \equiv \int d^3x \, \bar{c}(\mathbf{x})\gamma_5 u(\mathbf{x}).$$

  - **Accurate and solid results only for rather stable mesons, which are predominantly quark-antiquark states.**
  - Unstable mesons (e.g. D_0^*, $D_1(2430)$) or mesons, which might not predominantly be quark-antiquark states (e.g. the tetraquark candidates D_{s0}^*, D_{s1}), require more sophisticated techniques and computations:
    * The correlation functions computed by means of lattice QCD provide the low-lying energy eigenvalues of the QCD Hamiltonian, which correspond to the masses of stable hadronic states (single or multi-particle).
    * In lattice QCD the hadron creation operators may not be too different from the state, which is investigated.

D, D_s, charmonium (2)

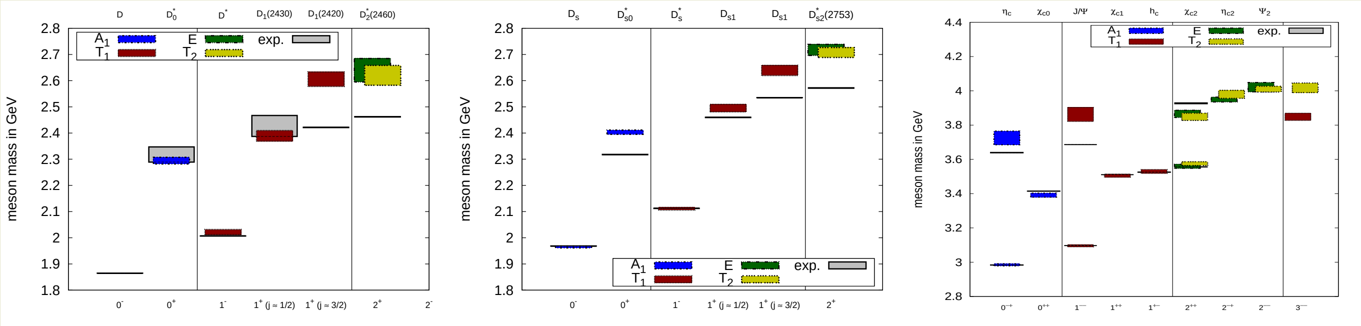
- Summary of current preliminary lattice results (blue, red, green, yellow boxes):
  - D meson masses (left):
    D, D_0^*, D^*, $D_1(2430)$, $D_1(2420)$, D_2^*.
  - D_s meson masses (center):
    D_s, D_{s0}^*, D_s^*, $D_{s1}(2460)$, $D_{s1}(2536)$, D_{s2}^*.
  - Charmonium masses (right):
    η_c, χ_{c0}, J/Ψ, χ_{c1}, h_c, χ_{c2}, η_{c2}, Ψ_2.

(+) Computations with 2+1+1 dynamical quark flavors.

(+) Extrapolated to physically light u/d quark masses.

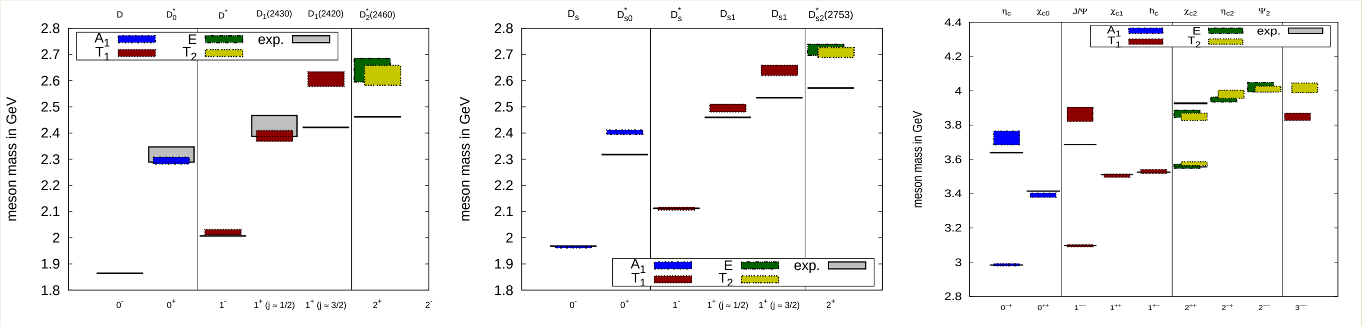
(+) Extrapolated to the continuum.

(−) Only quark-antiquark creation operators, no four-quark operators at the moment.



D, D_s, charmonium (3)

- Comparison with existing experimental results (black boxes):
  - Agreement for the majority of states.
  - Tension/disagreement:
    * D_{s0}^*, $D_{s1}(2460)$: tetraquark candidates
      ... might require four-quark creation operators.
      [D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, R. M. Woloshyn, Phys. Rev. Lett. **111**, 222001 (2013) [arXiv:1308.3175]]
      [C. B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, R. M. Woloshyn, Phys. Rev. D **90**, 034510 (2014) [arXiv:1403.8103]]
    * Radial excitations and higher orbital excitations (e.g. $\eta_c(2S)$, $J/\Psi(2S)$, D_2^*, D_{s2}^*)
      ... such excitations exhibit poor signals in lattice QCD, better statistics, i.e. longer computations required.



D, D_s, charmonium (4)

- Prediction of a couple of states, which have not yet been observed experimentally.
- Preparatory step for more advanced computations using also four-quark creation operators.
- Clear separation of the two $J^P = 1^+$ states
  $D_1(2430)$ (light spin $j \approx 1/2$)
  $D_1(2420)$ (light spin $j \approx 3/2$).
  [D. Mohler, S. Prelovsek, R. M. Woloshyn, Phys. Rev. D **87**, 034501 (2013) [arXiv:1208.4059]]
  [M. Kalinowski, M.W., accepted by Phys. Rev. D [arXiv:1509.02396]]
  - Important to study semileptonic decays $B^{(*)} \rightarrow D^{**} + l + \nu$
    (D^{**}: $J^P = 0^+, 1^+, 2^+$ D mesons).
  - Persistent conflict between experiment and theory ("$1/2$ versus $3/2$ puzzle")
    ... experiment: decays to $j \approx 1/2$ more likely
    ... theory (QCD sum rules, quark models, lattice QCD): decays to $j \approx 3/2$ more likely
    ... both experiment and theory have problems (vague data, assumptions, etc.).
    [I. I. Bigi *et al.*, Eur. Phys. J. C **52**, 975 (2007) [arXiv:0708.1621 [hep-ph]]]
  - $B^{(*)} \rightarrow D^{**} + l + \nu$ **can be studied at Belle ...**

Part 3: $\bar{b}\bar{b}qq$ tetraquark candidates (creation operators with 2 static antiquarks and 2 quarks of finite mass)

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
[P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
[J. Scheunert, P. Bicudo, A. Uenver, M.W., Acta Phys. Polon. Supp. **8**, 363 (2015) [arXiv:1505.03496]
[P. Bicudo, K. Cichy, A. Peters, M.W., arXiv:1510.03441]

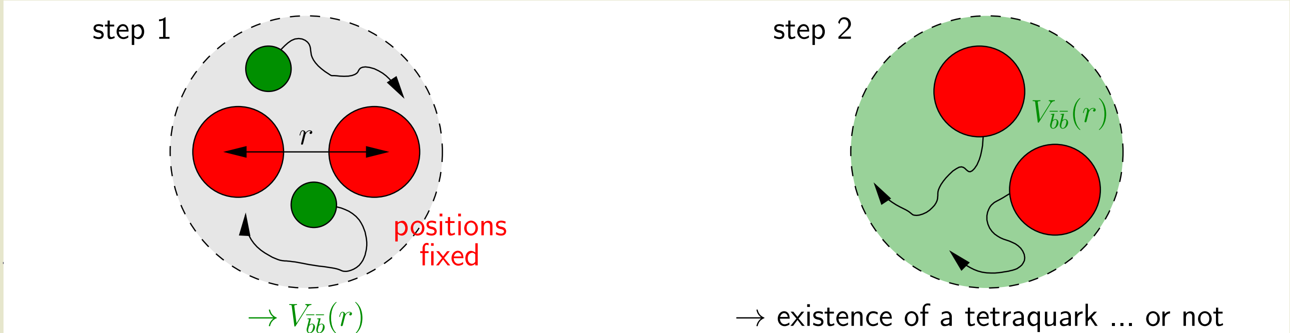
$\bar{b}\bar{b}qq$ tetraquarks (1)

- **Basic idea**: Investigate the existence of heavy tetraquarks $\bar{b}\bar{b}qq$ in two steps.

  (1) **Compute potentials of two static antiquarks ($\bar{b}\bar{b}$) in the presence of two lighter quarks ($qq \in \{ud, ss, cc\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive, to host a bound state by solving a corresponding Schrödinger equation.** ($\rightarrow$ This would indicate a stable $\bar{b}\bar{b}qq$ tetraquark.)

- (1) + (2) $\rightarrow$ **Born-Oppenheimer approximation**:
  - Proposed in 1927 for molecular and solid state calculations.
    [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Annalen der Physik 389, Nr. 20, 1927]
  - In our computations step (1) not quantum mechanics, but lattice QCD.
  - Approximation valid, if $m_q \ll m_b$ (most appropriate for $qq = ud$).



$\bar{b}\bar{b}qq$ tetraquarks (2)

Born-Oppenheimer approximation, step (1)

- Lattice QCD computation of $\bar{b}\bar{b}$ potentials $V_{\bar{b}\bar{b}}(r)$.

  (1) Use $\bar{b}\bar{b}qq$ creation operators

  $$O_{\bar{b}\bar{b}qq} \equiv (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{b}_C(-\mathbf{r}/2)q_A^{(1)}(-\mathbf{r}/2)\Big)\Big(\bar{b}_D(+\mathbf{r}/2)q_B^{(2)}(+\mathbf{r}/2)\Big).$$

  * Different light quark flavors $qq \in \{ud, ss, cc\}$.
  * Different quark spin/parity.

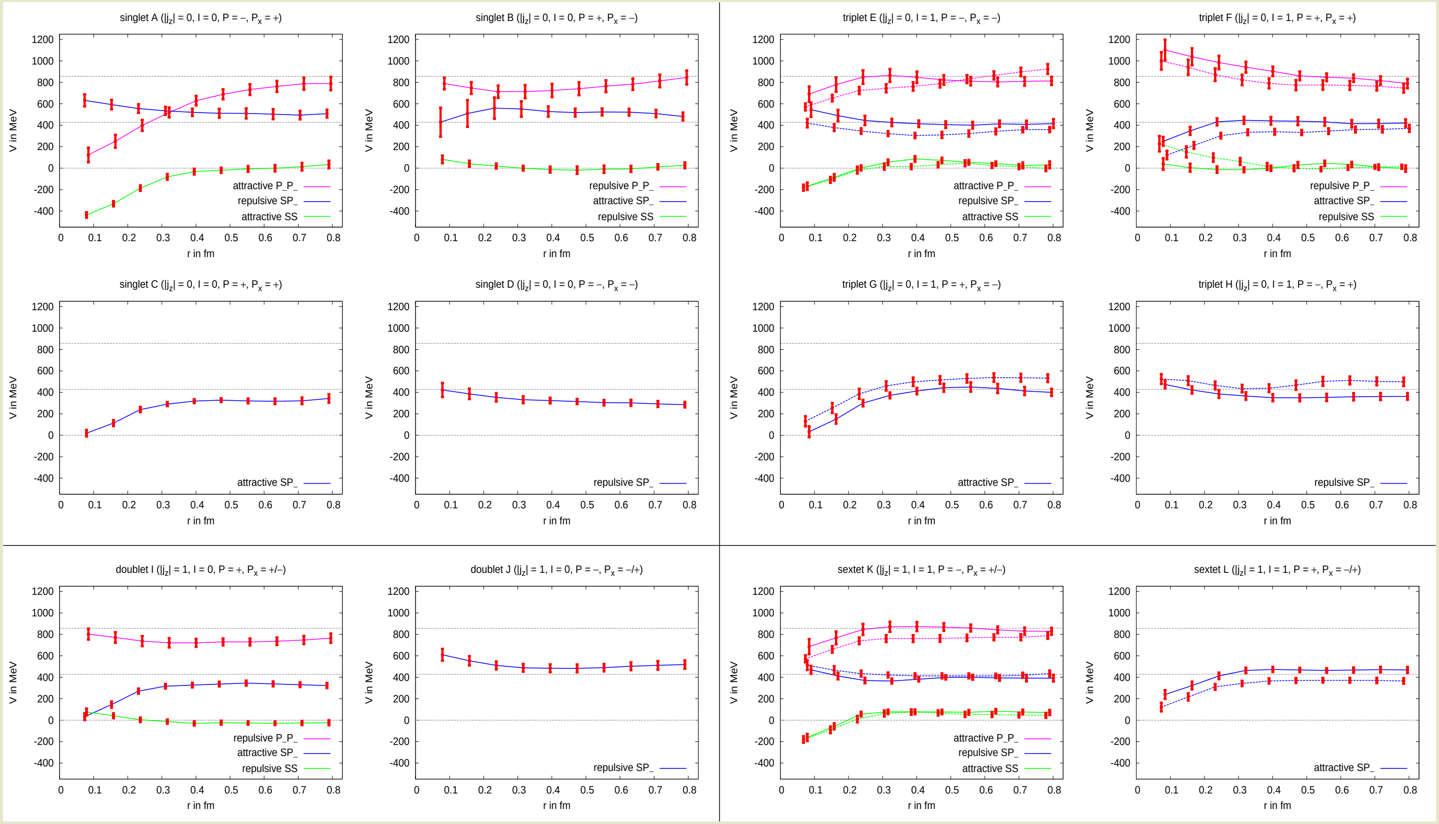
  $\rightarrow$ Many different channels
  ... some attractive, some repulsive
  ... some correspond for large $\bar{b}\bar{b}$ separations to pairs of ground state mesons, some to excited mesons.

  (2) Compute temporal correlation functions.

  (3) Determine $V_{\bar{b}\bar{b}}(r)$ from the exponential decays of the correlation functions.

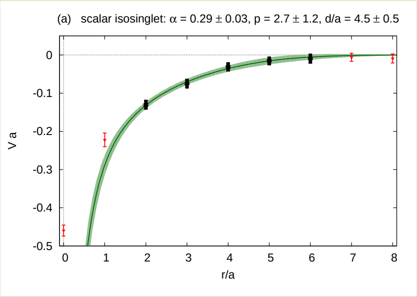
$\bar{b}\bar{b}qq$ tetraquarks (3)

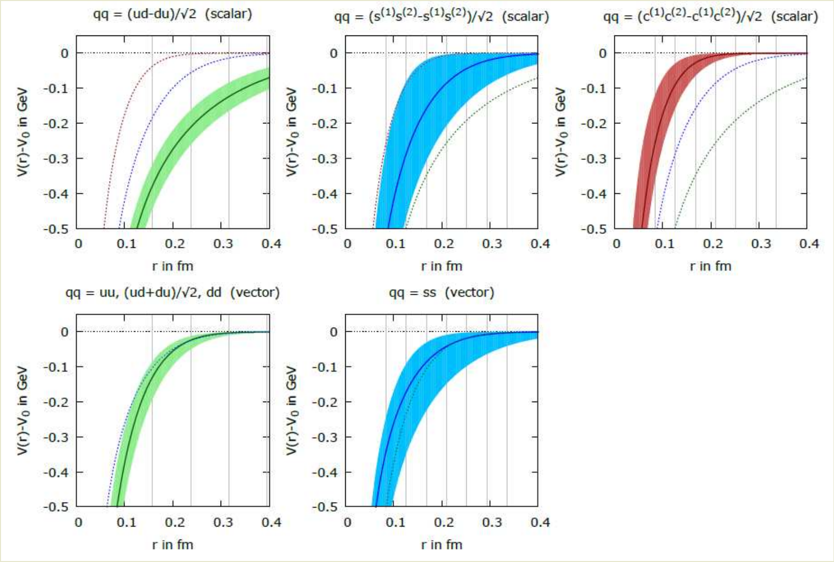
- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).



$\bar{b}\bar{b}qq$ tetraquarks (4)

- Two attractive channels corresponding to pairs of ground state mesons.
- Light quark mass dependence of these channels:
  wider and deeper for $qq = ud$ compared to $qq = ss$ compared to $qq = cc$.





$\bar{b}\bar{b}qq$ tetraquarks (5)

Born-Oppenheimer approximation, step (2)

- Solve the Schrödinger equation for the relative coordinate $\mathbf{r}$ of the two $\bar{b}$ quarks,

$$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\right)\underbrace{\psi(\mathbf{r})}_{=R(r)/r} \quad = \quad E\psi(\mathbf{r}) \quad , \quad \mu \quad = \quad m_b/2;$$

  possibly existing bound states, i.e. $E < 0$, indicate $\bar{b}\bar{b}qq$ tetraquarks.

- A single bound state for one specific potential $V_{\bar{b}\bar{b}}(r)$ and light quarks $qq = ud$:
  - Binding energy $E = -93^{+47}_{-43}\,\mathrm{MeV}$, i.e. confidence level $\approx 2\,\sigma$.
  - Quantum numbers of the $\bar{b}\bar{b}ud$ tetraquark: $I(J^P) = 0(1^+)$.

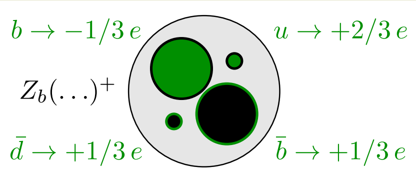
  $\rightarrow$ **Prediction of a tetraquarks.**

- No further bound states, in particular not for $qq = ss$ or $qq = cc$.

- **Experimental results for $\bar{b}\bar{b}qq$ would be very interesting ...**

$\bar{b}\bar{b}qq$ tetraquarks (6)

- Work in Progress:
  - Including effects due to the $\bar{b}\bar{b}$ spins (static quark spins are irrelevant).
    $\rightarrow$ Binding energy reduced, $\bar{b}\bar{b}ud$ tetraquark persists (preliminary).
  - Investigation of the structure of the $\bar{b}\bar{b}ud$ tetraquark
    ... is it a mesonic molecule ... or a diquark-antidiquark pair?
  - The experimentally simpler/theoretically harder case $\bar{b}b\bar{q}q$ (e.g. $Z_b(10610)^+$, $Z_b(10650)^+$).



Lattice QCD (A)

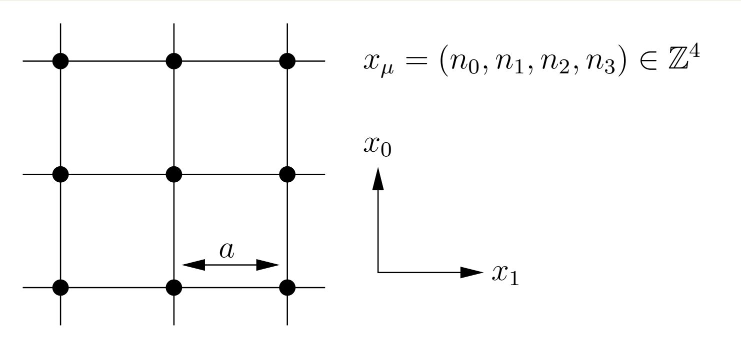
- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle\Omega|O^\dagger(t)O(0)|\Omega\rangle =$$
$$= \frac{1}{Z}\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu\,O^\dagger(t)O(0)e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slide).
  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{r},t)$ and $A_\mu(\mathbf{r},t)$.
  - $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.

Lattice QCD (B)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ "no finite volume effects".



Lattice QCD (C)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi \, D\bar{\psi} \, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu) \, d\bar{\psi}(x_\mu) \, dU(x_\mu) \bigg) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    * x_μ: $32^4 \approx 10^6$ lattice sites.
    * $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    * $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    * In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

  $\rightarrow$ Standard approaches for numerical integration not applicable.

  $\rightarrow$ Sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).