Static-light meson masses from twisted mass lattice QCD

and

The $1/2$ versus $3/2$ puzzle

11. Meeting of SFB/TR9 Computational Particle Physics, Aachen

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October 6, 2008

Heavy-light mesons

- Heavy-light meson: a meson made from a heavy quark (b, c) and a light quark (u, d, s), e.g. $B = \{\bar{b}u\,,\,\bar{b}d\}$, $B_s = \bar{b}s$, $D = \{\bar{c}u\,,\,\bar{c}d\}$, $D_s = \bar{c}s$.
- Static limit, i.e. $m_b, m_c \rightarrow \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $\mathcal{P}$ and total angular momentum of the light cloud j.
- m_b, m_c finite, but heavy:
  - Classify states according to parity $\mathcal{P}$ and total angular momentum J.

$j^\mathcal{P}$	$J^\mathcal{P}$
$(1/2)^- \equiv S$	$0^- \equiv H$ $1^- \equiv H^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv H_0^* \equiv H_0^{1/2}$ $1^+ \equiv H_1^* \equiv H_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv H_1 \equiv H_1^{3/2}$ $2^+ \equiv H_2^* \equiv H_2^{3/2}$

Static-light meson masses from twisted mass lattice QCD

in collaboration with

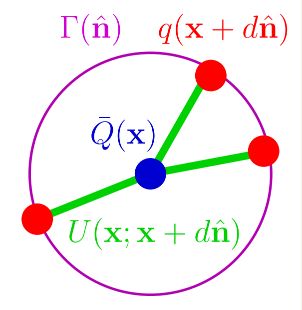
Karl Jansen, Chris Michael, Andrea Shindler

Basic principle (1)

- Let $\mathcal{O}(\mathbf{x})$ be a suitable "static-light meson creation operator", i.e. an operator such that $\mathcal{O}(\mathbf{x})|\Omega\rangle$ is a state containing a static-light meson at position $\mathbf{x}$ ($|\Omega\rangle$: vacuum).
- Determine the mass of the ground state of the corresponding static-light meson from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times T:

$$\begin{aligned} \mathcal{C}(T) \quad &= \quad \langle\Omega|\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger \mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \langle\Omega|e^{+HT}\Big(\mathcal{O}(\mathbf{x},0)\Big)^\dagger e^{-HT}\mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \sum_n \Big|\langle n|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)T\Big) \quad \approx \quad (\text{for } T \gg 1) \\ &\approx \quad \Big|\langle 0|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{\text{meson mass}} T\Big). \end{aligned}$$

Basic principle (2)



- **General form of a static-light meson creation operator**:

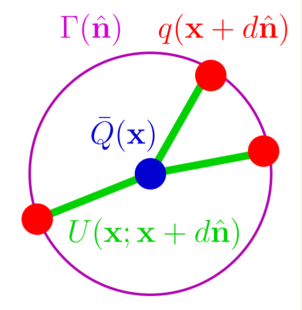
$$\mathcal{O}(\mathbf{x}) = \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

  - $\bar{Q}(\mathbf{x})$ creates an infinitely heavy i.e. static antiquark at position $\mathbf{x}$.
  - $q(\mathbf{x} + d\hat{\mathbf{n}})$ creates a light quark at position $\mathbf{x} + d\hat{\mathbf{n}}$ separated by a distance d from the static antiquark.
  - The spatial parallel transporter

$$U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) = P\left\{\exp\left(+i \int_{\mathbf{x}}^{\mathbf{x}+d\hat{\mathbf{n}}} dz_j\, A_j(\mathbf{z})\right)\right\}$$

    connects the antiquark and the quark in a gauge invariant way via gluons.
  - The integration over the unit sphere $\int d\hat{\mathbf{n}}$ combined with a suitable weight factor $\Gamma(\hat{\mathbf{n}})$ yields well defined total angular momentum J and parity $\mathcal{P}$ ($\Gamma(\hat{\mathbf{n}})$ is a combination of spherical harmonics [$\rightarrow$ angular momentum] and γ-matrices [$\rightarrow$ spin]; Wigner-Eckart theorem).

Basic principle (3)



- **General form of a static-light meson creation operator**:

$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x};\mathbf{x}+d\hat{\mathbf{n}}) q(\mathbf{x}+d\hat{\mathbf{n}}).$$

- **List of operators** (J: total angular momentum; j: total angular momentum of the light cloud; $\mathcal{P}$: parity):

$\Gamma(\hat{\mathbf{n}})$	$J^{\mathcal{P}}$	$j^{\mathcal{P}}$	$\mathrm{O_h}$	lattice $j^{\mathcal{P}}$	notation
γ_5 , $\gamma_5\gamma_j\hat{n}_j$	0^- $[1^-]$	$(1/2)^-$	A_1	$(1/2)^-$, $(7/2)^-$, ...	S
1 , $\gamma_j\hat{n}_j$	0^+ $[1^+]$	$(1/2)^+$		$(1/2)^+$, $(7/2)^+$, ...	P_-
$\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2$ (and cyclic)	2^+ $[1^+]$	$(3/2)^+$	E	$(3/2)^+$, $(5/2)^+$, ...	P_+
$\gamma_5(\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2)$ (and cyclic)	2^- $[1^-]$	$(3/2)^-$		$(3/2)^-$, $(5/2)^-$, ...	$D_\pm$
$\gamma_1\hat{n}_2\hat{n}_3 + \gamma_2\hat{n}_3\hat{n}_1 + \gamma_3\hat{n}_1\hat{n}_2$	3^- $[2^-]$	$(5/2)^-$	A_2	$(5/2)^-$, $(7/2)^-$, ...	D_+
$\gamma_5(\gamma_1\hat{n}_2\hat{n}_3 + \gamma_2\hat{n}_3\hat{n}_1 + \gamma_3\hat{n}_1\hat{n}_2)$	3^+ $[2^+]$	$(5/2)^+$		$(5/2)^+$, $(7/2)^+$, ...	$F_\pm$

Twisted mass lattice QCD

- Twisted mass action (two degenerate flavors, "continuum version"):

$$S_{\mathrm{fermionic}} = \int d^4x\,\bar{\chi}\Big(\gamma_\mu D_\mu + m + \underbrace{i\mu\gamma_5\tau_3}_{\text{twisted mass term}} - \underbrace{\frac{a}{2}\Box}_{\text{Wilson term}}\Big)\chi$$

$$\psi = e^{i\omega\gamma_5\tau_3/2}\chi$$

(ψ: physical basis quark fields; χ: twisted basis quark fields; μ: twisted mass; τ_3: third Pauli matrix acting in flavor space; a: lattice spacing).

  – Wilson term: removes fermionic doublers.

  – Twisted mass term: automatic $\mathcal{O}(a)$ improvement, when tuned to maximal twist ($\omega = \pi/2$).

\+ Automatic $\mathcal{O}(a)$ improvement.

\+ Numerically cheap, i.e. large lattices and small lattice spacings possible.

– Explicit breaking of parity and flavor symmetry.

Simulation setup

- $24^3 \times 48$ lattices.
- Twisted mass Dirac operator with two degenerate flavors:

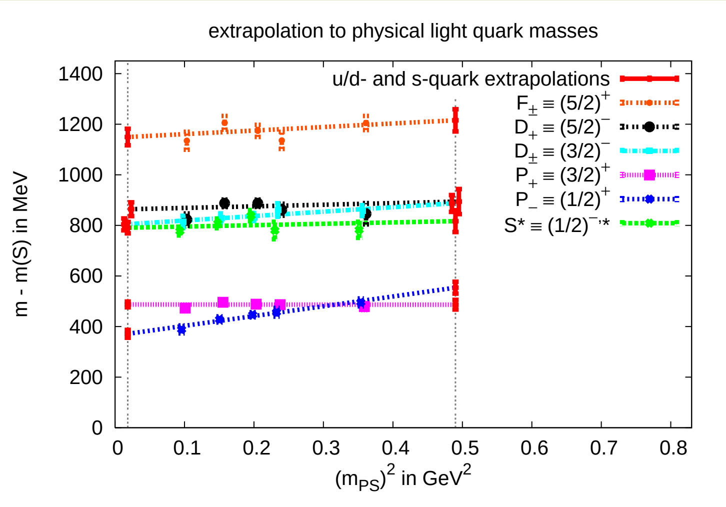
$$Q^{(\chi)} \quad = \quad \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\Box \quad , \quad m + 4 \quad = \quad \frac{1}{2\kappa}$$

  with $\kappa = 0.160856$.
- Tree-level Symanzik improved gauge action with $\beta = 3.9$.
- Lattice spacing $a \approx 0.0855(5)\,\mathrm{fm}$, spatial lattice extension $24 \times a \approx 2.05\,\mathrm{fm}$.

μ	m_{PS} in MeV	number of gauges
0.0040	314(2)	1400
0.0064	391(1)	1450
0.0085	448(1)	1350
0.0100	485(1)	900
0.0150	597(2)	1000

Results (1)

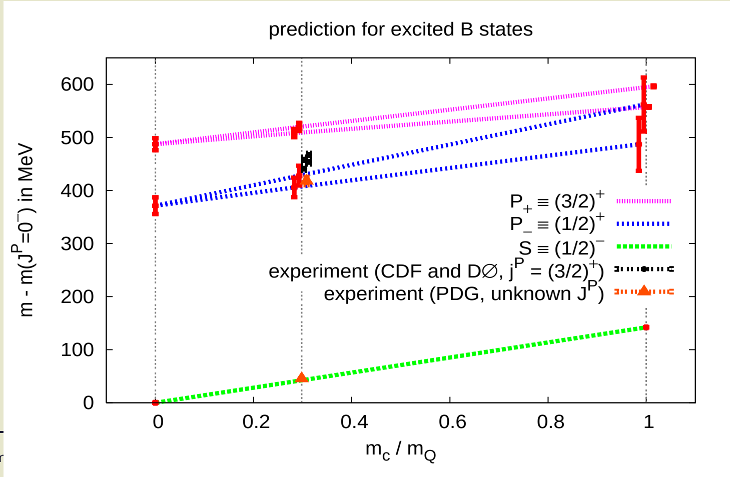
- Linear extrapolation in $(m_\pi)^2$ to physical light quark masses:
  - "B mesons": u/d quark extrapolation ($m_{\mathrm{PS}} = 139.6\,\mathrm{MeV}$).
  - "B_s mesons": s quark extrapolation ($m_{\mathrm{PS}} = 700.0\,\mathrm{MeV}$).
    - $*$ However: sea of two degenerate s instead of u and d.



Results (2)

- Prediction for excited B states B_0^*, B_1^*, B_1 and B_2^* (P wave states):
  - Linear interpolation in m_c/m_Q to physical b quark mass (input: u/d extrapolated lattice data for $m_Q = \infty$, experimental data for $m_Q = m_c$).
- Experimental results:
  - CDF and DØ (both $j^{\mathcal{P}} = (3/2)^+$ states, i.e. B_1 and B_2^*).
  - PDG (unknown $J^{\mathcal{P}}$, denoted by B_J^*).

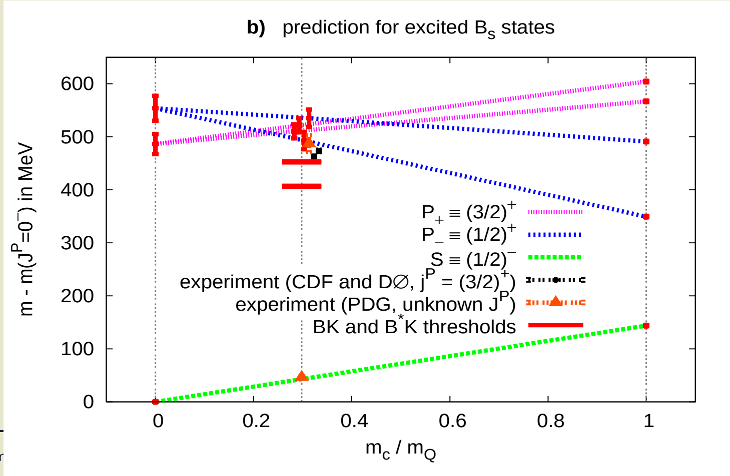
	$m - m(B^0)$ in MeV			
state	lattice	CDF	DØ	PDG
B_0^*	$406(19)$			
B_1^*	$428(19)$			
B_1	$508(8)$	$454(5)$	$441(4)$	
B_2^*	$520(8)$	$458(6)$	$467(4)$	
B_J^*				$418(8)$



Results (3)

- Prediction for excited B_s states B^*_{s0}, B^*_{s1}, B_{s1} and B^*_{s2} (P wave states):
  - Linear interpolation in m_c/m_Q to physical b quark mass (input: s extrapolated lattice data for $m_Q = \infty$, experimental data for $m_Q = m_c$).
- Experimental results:
  - CDF and DØ (both $j^{\mathcal{P}} = (3/2)^+$ states, i.e. B_{s1} and B^*_{s2}).
  - PDG (unknown $J^{\mathcal{P}}$, denoted by B^*_{sJ}).

	$m - m(B_s)$ in MeV			
state	lattice	CDF	DØ	PDG
B^*_{s0}	$493(16)$			
B^*_{s1}	$535(16)$			
B_{s1}	$510(13)$	$463(1)$		
B^*_{s2}	$521(13)$	$473(1)$	$473(2)$	
B^*_{sJ}				$487(16)$



Summary

- Static-light meson masses have been computed via twisted mass lattice QCD at a small value of the lattice spacing ($a = 0.0855\,\mathrm{fm}$) and at small values of the pion mass ($300\,\mathrm{MeV} \lesssim m_{\mathrm{PS}} \lesssim 600\,\mathrm{MeV}$):
  - Total angular momentum of the light cloud $j = 1/2\,,\,3/2\,,\,5/2$.
  - Parity $\mathcal{P} = +\,,\,-$.
- Interpolation/extrapolation to physical quark masses allows predictions for the spectrum of B mesons and B_s mesons:
  - Agreement up to 15% with experimental P wave B meson results.
  - Agreement up to 10% with experimental P wave B_s meson results.
- Outlook:
  - Extrapolate to the continuum.
  - Include a sea of u/d quarks for B_s computations by using 2+1+1 flavor twisted mass lattice QCD.

The $1/2$ versus $3/2$ puzzle

in collaboration with

Benoit Blossier, Karl Jansen, Olivier Pène

$1/2$ versus $3/2$: experimental side

- Consider the semileptonic decay $B \rightarrow X_c \, l \, \nu$.
- Experiments, which have studied this decay: ALEPH, BaBar, BELLE, CDF, DELPHI, DØ.
- What is X_c?
  - $\approx 75\%$ D and D^*, i.e. S wave states (agreement with theory).
  - $\approx 10\%$ $D_1^{3/2}$ and $D_2^{3/2}$, i.e. $j = 3/2$ P wave states (agreement with theory).
  - For the remaining $\approx 15\%$ the situation is not clear:
    * A "natural candidate" would be $D_0^{1/2}$ and $D_1^{1/2}$, i.e. $j = 1/2$ P wave states.
    * This would imply $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) > \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$, which is in "conflict" with theory.
    * This "conflict" between experiment and theory is called the "$1/2$ versus $3/2$ puzzle".

$1/2$ **versus** $3/2$**: theory side (1)**

- Static limit $(m_b, m_c \rightarrow \infty)$ with both b and c quark at rest:

$$\begin{aligned} \langle D_0^{1/2} | \bar{c} \gamma_5 \gamma_j D_k b | B \rangle &= -i g_{jk} \Big(m(D_0^{1/2}) - m(B) \Big) \tau_{1/2} \\ \langle D_2^{3/2} | \bar{c} \gamma_5 \gamma_j D_k b | B \rangle &= +i \sqrt{3} \epsilon_{jk} \Big(m(D_2^{3/2}) - m(B) \Big) \tau_{3/2} \end{aligned}$$

and

$$\frac{\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu)}{\Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)} \quad \text{“} = \text{”} \quad \frac{|\tau_{1/2}|^2}{|\tau_{3/2}|^2}.$$

($\tau_{1/2}$, $\tau_{3/2}$: Isgur-Wise form factors).

$1/2$ versus $3/2$: theory side (2)

- Phenomenological models:
  - $|\tau_{1/2}| < |\tau_{3/2}|$, which is in "conflict" with experiment.
- OPE:
  - Uraltsev sum rule:

    $$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 = \frac{1}{4}$$

    ($\tau_{1/2} \equiv \tau_{1/2}^{(0)}$ and $\tau_{3/2} \equiv \tau_{3/2}^{(0)}$).
  - From experience with sum rules one would expect approximate saturation from the ground states, i.e.

    $$|\tau_{3/2}^{(0)}|^2 - |\tau_{1/2}^{(0)}|^2 \approx \frac{1}{4},$$

    which also implies $|\tau_{1/2}| < |\tau_{3/2}|$, which is in "conflict" with experiment.

$1/2$ versus $3/2$: possible explanations

- **Experiment:**
  - The signal for the remaining 15% of X_c is rather vague; therefore, only a small part might be $D_{0,1}^{1/2}$.
- **Phenomenological models:**
  - Models might give a wrong answer.
- **OPE:**
  - Sum rules hold in the static limit and might change significantly for finite quark masses.
  - Sum rules might not be saturated by the ground states.
- A lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ could shed some light on this puzzle.

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (1)

- Simulation setup:
  - As before, but only a single value of the light quark mass ($\mu = 0.0040$, corresponding to $m_{\mathrm{PS}} \approx 300\,\mathrm{MeV}$).
  - Preliminary results (computations have been performed on ≈ 100 gauge configurations only [1400 available]).

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- "Effective form factors",

$$\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) =$$

$$= \left| \frac{N(\tilde{P}_-) \ N(\tilde{S}) \ \langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \ \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \ \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right|$$

$$\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2) =$$

$$= \sqrt{\frac{1}{6}} \left| \frac{N(\tilde{P}_+) \ N(\tilde{S}) \ \langle \tilde{P}_+(T_0) | (\bar{Q}\gamma_5(\gamma_1 D_1 - \gamma_2 D_2) Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_+) - m(\tilde{S})) \ \langle \tilde{P}_+(T_0) | \tilde{P}_+(T_1) \rangle \ \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right| :$$

  - $N(X)$: norm of state $|X\rangle$.
  - $m(X)$: mass of state $|X\rangle$.
  - Three-point functions (T_0, T_1 and T_2).
  - Two-point functions (T_0 and T_1 or T_1 and T_2).

- $\tau_{1/2} = \lim_{T_0 - T_1, T_1 - T_2 \to \infty} \tau_{1/2,\text{effective}}$, $\tau_{3/2} = \lim_{T_0 - T_1, T_1 - T_2 \to \infty} \tau_{3/2,\text{effective}}$.

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (3)

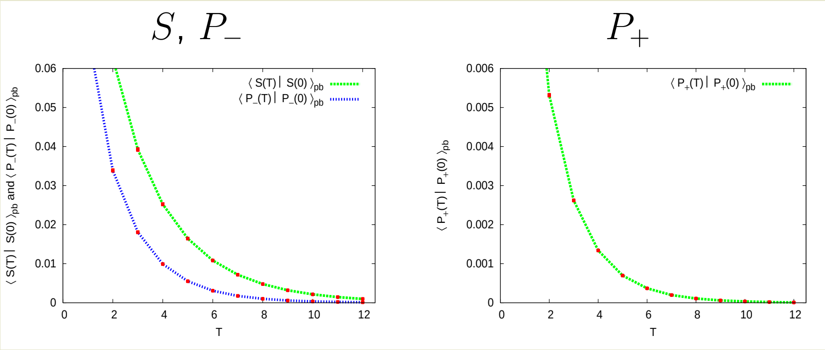
- $$\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad = \quad = \left| \frac{N(\tilde{P}_-) \quad N(\tilde{S}) \quad \langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \quad \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \quad \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right| \quad , \quad \ldots$$

- Two-point function $\langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle$: a standard lattice computation.
- Determine the norm of $|\tilde{S}\rangle$, $N(\tilde{S})$, by performing a χ^2 minimizing fit with

$$f(T) \quad = \quad N(\tilde{S})^2 e^{-m(S)T}$$

to $\langle \tilde{S}(T) | \tilde{S}(0) \rangle$ at large T.

- Analogously for the others.

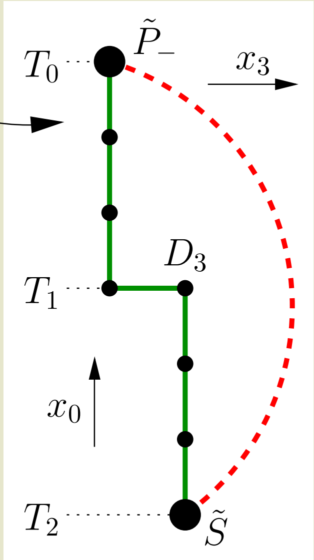


Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (4)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad =$

$$= \left| \frac{N(\tilde{P}_-) \quad N(\tilde{S}) \quad \langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \quad \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \quad \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right| \quad , \quad \ldots$$

- Three-point functions $\langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle$:



- Analogously for the other three-point functions.

- Mass differences $m(P_-) - m(S)$ and $m(P_+) - m(S)$: cf. the first part of this talk.

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (5)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2)$ and $\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2)$ exhibit nice plateaus due to "optimized" trial states $|\tilde{S}\rangle$, $|\tilde{P}_-\rangle$ and $|\tilde{P}_+\rangle$.

- $T_0 - T_2 = 8$:
  - $\tau_{1/2} = 0.32$, $\tau_{3/2} = 0.47$.
  - $(\tau_{3/2})^2 - (\tau_{1/2})^2 = 0.12$.

- $T_0 - T_2 = 10$:
  - $\tau_{1/2} = 0.30$, $\tau_{3/2} = 0.54$.
  - $(\tau_{3/2})^2 - (\tau_{1/2})^2 = 0.20$.

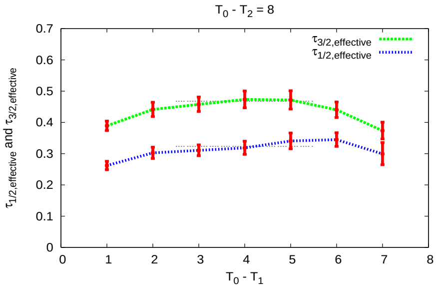
- $\tau_{3/2} > \tau_{1/2}$, i.e. theoretical expectation confirmed.

- "Consistent" with Uraltsev sum rule:

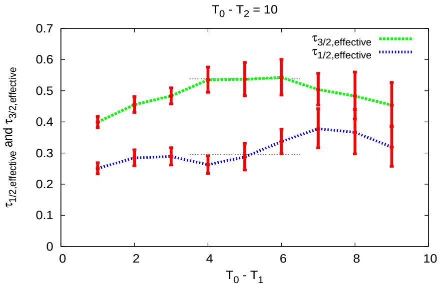
$$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 \quad = \quad \frac{1}{4}.$$

$\tau_{1/2}$ and $\tau_{3/2}$ as functions of $T_0 - T_1$

$T_0 - T_2 = 8$

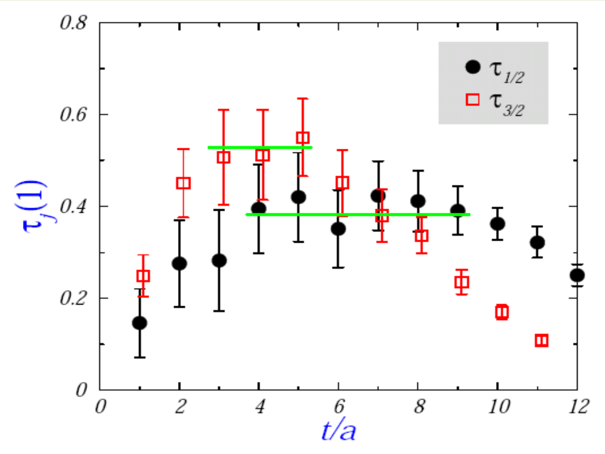


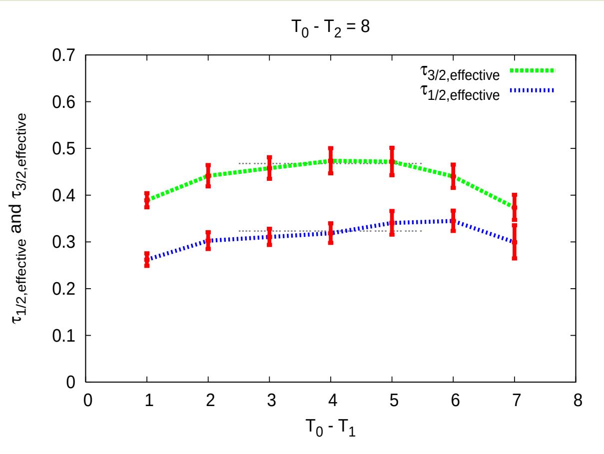
$T_0 - T_2 = 10$



Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (6)

- Comparison with the only existing lattice study (quenched, exploratory):
  - D. Becirevic *et al.*, "Lattice measurement of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$," Phys. Lett. B **609**, 298 (2005) [arXiv:hep-lat/0406031].
  - $16^3 \times 40$ lattice, $m_{\mathrm{sea}} = \infty$, $m_{\mathrm{PS}} = 800\,\mathrm{MeV}$.
  - $\tau_{1/2} = 0.38(4)$, $\tau_{3/2} = 0.53(8)$.





Conclusions

- $\tau_{1/2}$ and $\tau_{3/2}$ have been computed on dynamical ETMC gauge field configurations.
- Preliminary results indicate that in the static limit

  $$\Gamma(B \rightarrow D_{0,1}^{1/2}\, l\, \nu) \quad < \quad \Gamma(B \rightarrow D_{1,2}^{3/2}\, l\, \nu)$$

  (as expected from OPE and phenomenological models).
- "To do list":
  - Improve statistics.
  - Consider different light quark masses to extrapolate to u/d masses.
  - Perform the continuum limit.
  - Compute HQET $1/m_Q$ corrections.