Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks based on correlation functions with scattering interpolating operators both at the source and at the sink

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We present first results of a recently started lattice QCD investigation of antiheavy-antiheavy-light-light tetraquark systems including scattering interpolating operators in correlation functions both at the source and at the sink. In particular, we discuss the importance of such scattering interpolating operators for a precise computation of the low-lying energy levels. We focus on the $\bar{b}\bar{b}ud$ four-quark system with quantum numbers $I(J^P) = 0(1^+)$, which has a ground state below the lowest meson-meson threshold. We carry out a scattering analysis using Lüscher’s method to extrapolate the binding energy of the corresponding QCD-stable tetraquark to infinite spatial volume. Our calculation uses clover $u, d$ valence quarks and NRQCD $b$ valence quarks on gauge-link ensembles with HISQ sea quarks that were generated by the MILC collaboration.

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1. Introduction

We report on a recently started lattice QCD project in which we aim to study possibly existing heavy-heavy-light-light tetraquark resonances. In the following, we focus on the $\bar{b}b\bar{d}d$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$, which is theoretically simpler compared to other tetraquark candidates because it is QCD-stable. This tetraquark is the counterpart of the $\bar{c}c\bar{u}d$ tetraquark $T_{cc}$ recently discovered by LHCb [1, 2].

In the past couple of years, several independent lattice QCD studies of $\bar{b}bqq$ and $\bar{b}cqq$ systems ($q$ denotes a light $u$, $d$ or $s$ quark) were published. These computations employed either exclusively local four-quark interpolating operators [3–7] or local and scattering four-quark interpolating operators, but the latter only at the sink [8, 9].

In the work presented here, we include scattering interpolating operators both at the source and at the sink. This allows a more precise determination of finite-volume energy levels not only for bound states, but also for scattering states. This is particularly important for $Q\bar{Q}qq$ systems, where bound states and scattering states are very close, or where bound states do not exist but resonances might exist. An example is a future full lattice QCD investigation of a possibly existing $\bar{b}b\bar{d}d$ tetraquark resonance with $I(J^P) = 0(1^-)$ proposed in Ref. [10].

2. Interpolating operators

To study the $\bar{b}b\bar{d}d$ four-quark system with quantum numbers $I(J^P) = 0(1^+)$, we use local interpolating operators

$$O_1 = O_{[BB^*]_1(0)} = \sum_x \bar{b}\gamma_5 d(x) \bar{b}\gamma_j u(x) - (d \leftrightarrow u),$$

$$O_2 = O_{[B^*B^*]_1(0)} = \epsilon_{jkl} \sum_x \bar{b}\gamma_k d(x) \bar{b}\gamma_l u(x) - (d \leftrightarrow u),$$

$$O_3 = O_{[Dd]_1(0)} = \sum_x \bar{b}^a \gamma_j C \bar{b}^b d^a \gamma^T C \gamma_5 u^b(x) - (d \leftrightarrow u),$$

and scattering interpolating operators

$$O_4 = O_{B(0)B^* (0)} = \left( \sum_x \bar{b}\gamma_5 d(x) \right) \left( \sum_y \bar{b}\gamma_j u(y) \right) - (d \leftrightarrow u),$$

$$O_5 = O_{B^*(0)B^* (0)} = \epsilon_{jkl} \left( \sum_x \bar{b}\gamma_k d(x) \right) \left( \sum_y \bar{b}\gamma_l u(y) \right) - (d \leftrightarrow u).$$

Here, $C$ denotes the charge conjugation matrix, and upper indices $a$ and $b$ are color indices. For more details we refer to our previous work [8].

3. Lattice setup

We use 2 + 1 + 1-flavor HISQ gauge-link ensembles generated by the MILC collaboration [11] as summarized in Table 1.
Table 1: Gauge-link ensembles (a: lattice spacing; \( N_s, N_t \): number of lattice sites in spatial and temporal direction; \( m_\pi^{(\text{sea})}, m_\pi^{(\text{val})} \): pion mass corresponding to light sea and light valence quarks; \( N_{\text{cont}} \): number of gauge-link configurations used for computations).

We use a mixed-action setup with Wilson-clover \( u \) and \( d \) valence quarks [12, 13]. For the \( b \) valence quarks we use lattice NRQCD [14].

Correlation functions are computed with point-to-all propagators if there is a local operator at the source. If there is a scattering operator at the source, we use stochastic timeslice-to-all propagators combined with the one-end trick (see e.g. Ref. [15]). Moreover, we use APE smearing for the gauge links and Gaussian smearing for the quark fields.

For the analysis of the correlation matrices we employ two independent methods: solving standard generalized eigenvalue problems (GEVP) as well as the Athens Model Independent Analysis Scheme (AMIAS) [16].

4. Effective masses from “exclusively local” versus “local and scattering” interpolating operators

Based on previous lattice QCD computations [3, 5, 7, 8] we expect the ground state around 100 MeV below the \( BB^* \) threshold (the lowest meson-meson threshold in this channel) representing the QCD-stable tetraquark. The first and second excitations in the finite spatial volume should be meson-meson scattering states resembling \( BB^* \) and \( B^*B^* \) close to the respective thresholds.

The left plot in Figure 1 shows effective masses from a GEVP using only local interpolating operators \( O_1, O_2 \) and \( O_3 \) (i.e. corresponding to a 3 \( \times \) 3 matrix). The plateaus exhibit a strong discrepancy with the expectation discussed in the previous paragraph. The right plot in Figure 1 shows effective masses from a GEVP using both local interpolating operators \( O_1, O_2 \) and \( O_3 \) as well as scattering interpolating operators \( O_4 \) and \( O_5 \) (i.e. corresponding to a 5 \( \times \) 5 matrix). These effective masses are consistent with the expectation. Thus, Figure 1 demonstrates that scattering operators are essential for a precise determination of scattering states.

5. Scattering analysis

To determine the mass of the \( \bar{b}\bar{b}ud \) tetraquark in infinite volume (for a given ensemble, i.e. at given \( m_\pi \) and nonzero \( a \)), we proceed as in our previous work [8]:

(1) Compute the two lowest energy levels in the finite spatial volume (see section 4).
(2) Compute the corresponding phase shifts $\delta_0(k_0), \delta_0(k_1)$ using Lüscher’s finite-volume method [17].

(3) Parameterize $\delta_0(k_0), \delta_0(k_1)$ using the effective-range expansion,

$$ k \cot(\delta_0(k)) = \frac{1}{a_0} + \frac{r_0}{2} k^2 $$

with fit parameters $a_0$ and $r_0$.

(4) The mass of the $\bar{b}\bar{b}ud$ tetraquark (and the energy of the first excitation) in infinite spatial volume corresponds to a pole in the scattering amplitude

$$ T_0(k) = \frac{1}{\cot(\delta_0(k)) - i}.$$  

The position of the pole can be obtained via Eqs. (6) and (7).

The dark-gray data points in the left plot of Figure 2 represent lattice QCD finite-volume energy levels (ground state and first excitation) for three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but identical $a \approx 0.12$ fm and $m_\pi \approx 220$ MeV (ensembles a12m220S, a12m220, a12m220L). The orange curves correspond to the two lowest finite volume energy levels as functions of the spatial extent $L$, computed with Lüscher’s finite-volume method using the effective-range expansion (6). There are rather small differences between the finite-volume and infinite-volume energy levels. We attribute this to the large binding energy, $\Delta E_0 \approx O(100$ MeV). Scattering analyses are, however, expected to be more important for smaller binding energies (e.g. for the $\bar{b}\bar{b}su$ system with $J^P = 1^+$) and essential for tetraquark resonances (e.g. for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$ [10]).

The right plot of Figure 2 shows an extrapolation of the tetraquark binding energy in the light $u/d$ quark mass based on the six ensembles listed in Table 1. The preliminary result at the physical pion mass $m_{\pi,\text{phys}} = 135$ MeV is

$$ \Delta E_0(m_{\pi,\text{phys}}) \approx (-103 \pm 8) \text{ MeV},$$  

**Figure 1**: Effective energies from a GEVP for ensemble a09m310. (left) 3×3 matrix, only local interpolating operators $O_1, O_2$ and $O_3$. (right) 5×5 matrix, both local interpolating operators $O_1, O_2$ and $O_3$ and scattering interpolating operators $O_4$ and $O_5$. 

**Figure 2**: Extrapolation of the tetraquark binding energy in the light $u/d$ quark mass based on the six ensembles listed in Table 1. The preliminary result at the physical pion mass $m_{\pi,\text{phys}} = 135$ MeV is
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Figure 2: (left) Lattice QCD finite volume energy levels (dark gray: ground state and first excitation) for three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but identical $a \approx 0.12 \text{ fm}$ and $m_\pi \approx 220 \text{ MeV}$ (ensembles a12m220S, a12m220, a12m220L) together with a fit based on the effective range expansion (6). (right) Extrapolation of the tetraquark binding energy in the light $u/d$ quark mass to the physical point.

where only the statistical uncertainty is shown. This binding energy is slightly smaller than, but consistent with, previous lattice results [3, 5–8].

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References


