

Structure of hybrid static potential flux tubes in SU(2) lattice Yang-Mills theory*

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We study the structure of the hybrid static potential flux tube in the Π_u sector in SU(2) lattice Yang-Mills theory. To this end, we compute the squares of the chromoelectric and chromomagnetic field strengths in the presence of a static quark-antiquark pair. We show clear evidence that the gluon distribution is significantly different compared to that of the ordinary static potential with quantum numbers Σ_g^+ .

1. Introduction

Hybrid mesons are systems composed of a quark-antiquark pair and excited gluons, where the latter allow quantum numbers J^{PC} not possible in the quark model. Such hybrid mesons are subject of current investigations both in experimental and theoretical physics. On the theoretical side lattice field theory is particularly suited to study hybrid mesons with two very heavy quarks, i.e. a quark-antiquark pair in the static limit. While there are several existing computations of hybrid static potentials (cf. e.g. [1, 2, 3, 4, 5] and references therein), the corresponding flux tubes have been studied for the first time only recently [6] (for completeness we also mention an earlier paper [7] by the same authors, where hybrid static potential flux tubes have been studied using not only static quarks, but also static gluons).

In the following we present the status of our ongoing computations of hybrid static potential flux tubes. We use methods quite similar to those used in [6]. We find, however, results, which are significantly different from the results presented in [6] as we will detail at the end of section 4.

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2. Hybrid static potentials and flux tubes

A hybrid meson is a meson, where not only the quark Q and the antiquark \bar{Q} , but also the gluons contribute to the quantum numbers J^{PC} . When studying hybrid mesons in the limit of infinitely heavy quarks, i.e. static quarks Q and \bar{Q} , symmetries are somewhat different, resulting in the following three quantum numbers (cf. e.g. [3, 5] for a detailed discussion).

- The absolute value of angular momentum with respect to the $Q\bar{Q}$ separation axis (here the z axis): $\Lambda \in \{\Sigma \doteq 0, \Pi \doteq 1, \Delta \doteq 2\}$.
- The eigenvalue of parity combined with charge conjugation $P \circ C$: $\eta \in \{g \doteq +, u \doteq -\}$.
- The eigenvalue of the spatial reflection along an axis perpendicular to the $Q\bar{Q}$ separation axis (here the x axis): $\epsilon \in \{+, -\}$.

Computing the energy of the groundstate in a given Λ_η^ϵ sector as a function of the $Q\bar{Q}$ separation r yields a static potential denoted by $V_{\Lambda_\eta^\epsilon}(r)$. While $V_{\Sigma_g^+}(r)$ is the well known ordinary static potential, all other potentials $V_{\Lambda_\eta^\epsilon}(r)$ exhibit significantly larger energies and are referred to as hybrid static potentials (for a summary plot cf. e.g. [3], Fig. 2).

$V_{\Sigma_g^+}(r)$ can be computed from standard rectangular Wilson loops. To access Λ_η^ϵ sectors different from Σ_g^+ , one has to replace the two spatial sides of the Wilson loops by more complicated structures. We focus on the Π_u hybrid static potential¹ by considering Wilson loop-like correlation functions with spatial parts as shown in Fig. 1. With quark and antiquark positions at $\mathbf{0}$ and $\mathbf{r} = r\mathbf{e}_z$, respectively, the resulting Wilson loop reads $W_{\Pi_u} = \mathcal{O}(0, \mathbf{0})S_t(0, \mathbf{r})O^\dagger(t, \mathbf{0})S_t^\dagger(0, \mathbf{0})$, where S_t denotes a straight path of

¹ We omit the quantum number ϵ , since hybrid static potentials with $\Lambda \geq 1$ are degenerate with respect to ϵ .

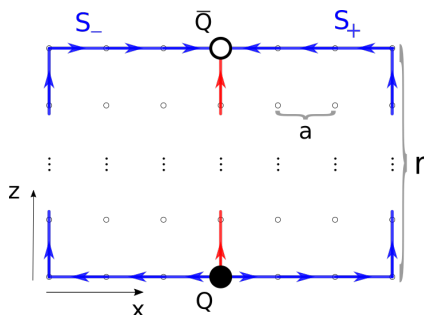


Fig. 1: Spatial part of a Wilson loop suited to study the Π_u sector.

t/a links in temporal direction, $\mathcal{O}(t, \mathbf{r}) = S_+(t, \mathbf{r}) - S_-(t, \mathbf{r})$ and S_+ and S_- as defined in Fig. 1.

The main goal of our work is to study hybrid static potential flux tubes by computing expectation values of squares of chromoelectric and chromomagnetic field strength components, E_j^2 and B_j^2 ($j = x, y, z$ and the square implies a sum over color indices). We have carried out such computations in the presence of a quark-antiquark pair in the Σ_g^+ and Π_u sectors as functions of the spatial coordinate $\mathbf{x} = (x, y, z)$ with respect to the vacuum using the equations

$$\langle E_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle E_j^2 \rangle_{\text{vac}} \propto \left(\frac{\langle W \cdot P_{0j}(t/2, \mathbf{x}) \rangle}{\langle W \rangle} - \langle P_{0j} \rangle \right) \quad (2.1)$$

$$\langle B_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle B_j^2 \rangle_{\text{vac}} \propto \left(\langle P_{kl} \rangle - \frac{\langle W \cdot P_{kl}(t/2, \mathbf{x}) \rangle}{\langle W \rangle} \right), \quad (2.2)$$

where P_{0j} denotes a spacetime plaquette and P_{kl} a spatial plaquette in the plane perpendicular to the j direction.

3. Lattice setup

At the moment all computations have been done using an 18^4 lattice and the standard Wilson plaquette action with coupling constant $\beta = 2.5$.

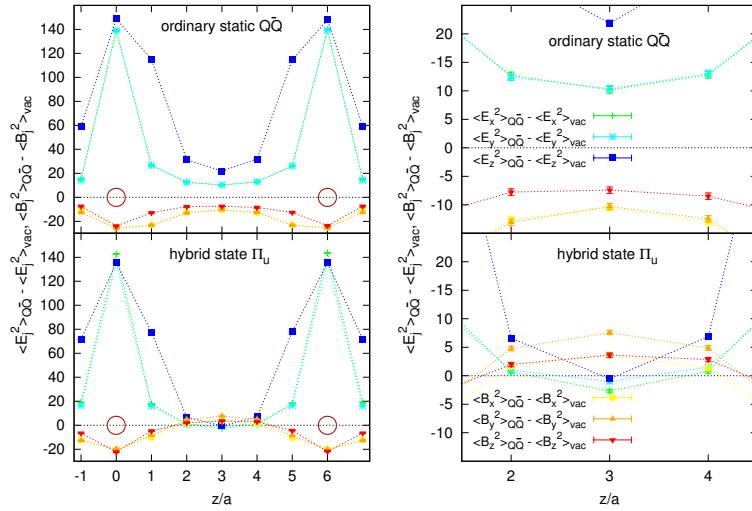


Fig. 2: $\langle E_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle E_j^2 \rangle_{\text{vac}}$ and $\langle B_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle B_j^2 \rangle_{\text{vac}}$ on the $Q\bar{Q}$ separation axis for separation $r = 6a$. Top: ordinary static potential Σ_g^+ . Bottom: hybrid static potential Π_u . Plots on the right are zoomed versions of the plots on the left. Red spheres indicate $Q\bar{Q}$ positions at $z = 0$ and $z = 6a$.

This corresponds to lattice spacing $a \approx 0.073$ fm, when identifying r_0 with 0.46 fm, which is close to the QCD value [8]. For the computation of Wilson loops W APE smeared spatial links have been used, while the links of the plaquettes P_{0j} and P_{kl} are unsmeared.

4. Numerical results

In Fig. 2 we compare the field strengths of the ordinary static potential Σ_g^+ and the hybrid static potential Π_u on the $Q\bar{Q}$ separation axis. In the vicinity of the color charges the field strengths are essentially the same for both potentials. However, a significant difference is observed in the region halfway between the quark and the antiquark. While the chromoelectric field strengths, in particular the z component, i.e. the component parallel to the separation axis, dominate for Σ_g^+ , they almost vanish in the hybrid case Π_u . At the same time, the chromomagnetic field strengths are large and positive for Π_u , while they even yield negative contributions to the energy density for the ordinary static potential Σ_g^+ .

In Fig. 3 we show again the field strengths of the hybrid static potential Π_u , this time along the two axes perpendicular to the separation axis originating in the center between the quark and the antiquark. These axes, which are parallel to the x and the y axes are also referred to as mediator axes. The field strengths along the two axes are not identical, i.e. the hybrid

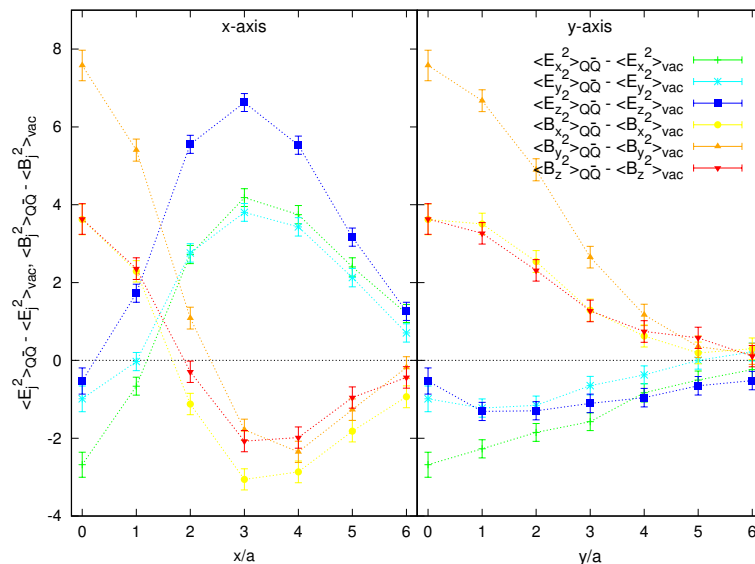


Fig. 3: $\langle E_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle E_j^2 \rangle_{\text{vac}}$ and $\langle B_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle B_j^2 \rangle_{\text{vac}}$ on the mediator axes for the hybrid static potential Π_u for separation $r = 6a$.

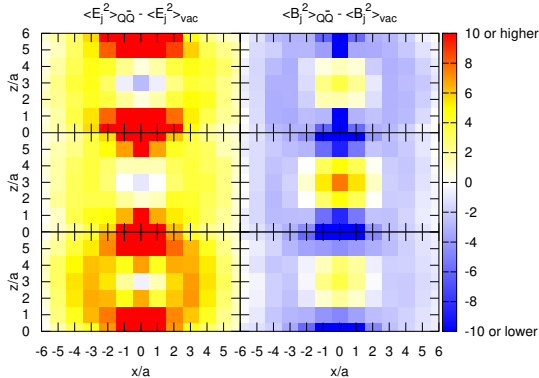


Fig. 4: $\langle E_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle E_j^2 \rangle_{\text{vac}}$ and $\langle B_j(\mathbf{x})^2 \rangle_{Q\bar{Q}} - \langle B_j^2 \rangle_{\text{vac}}$ in the x - z plane for the hybrid static potential Π_u for separation $r = 6a$.

Π_u state we excite with the spatial links shown in Fig. 1 has a rotationally non-invariant gluon distribution. The general picture is the same as before: In the hybrid case Π_u the chromomagnetic field strengths dominate, while the chromoelectric field strengths contribute in a negative way. Such a behavior is not surprising, since it is also possible and common to excite the Π_u sector by locally inserting $\mathbf{r} \times \mathbf{B}$ halfway in both spatial parts of an ordinary Wilson loop (cf. e.g. the pNRQCD investigation [9] or the recent lattice field theory study [10]).

Finally, in Fig. 4 we show the field strengths of the hybrid static potential Π_u in the whole x - z plane. Again, one can clearly see that chromomagnetic flux is localized halfway between the quark and the antiquark, while in the same region chromoelectric flux is strongly suppressed. We interpret this localized bump of chromomagnetic flux as gluons generating the hybrid quantum numbers of the meson.

It is also very interesting to compare our results to the results of the previously mentioned very recent and quite similar lattice field theory study [6], where the computation of flux tubes of hybrid static potentials has been carried out for the first time. The spatial parts of the Wilson loops used in [6] to excite the Π_u sector are somewhat different than in our work. While our two mass-degenerate Π_u hybrid states have defined ϵ , the two mass-degenerate Π_u hybrid states in [6] can be distinguished by the sign of the angular momentum, i.e. by $L_z = +1$ and $L_z = -1$. It is, however, easy to relate the two sets of states as well as their gluon distribution analytically: adding our field strengths in x , y and z direction corresponds to the rotationally invariant quantities computed in [6]. Comparing e.g. our Fig. 2 and 3 to Fig. 3, 4, 5 and 6 in [6] shows, that there is a severe qualitative discrepancy between the results of [6] and our results. While we observe a strong

dominance of chromomagnetic flux near the center and of chromoelectric flux close to the color charges, no similar change from chromomagnetic to chromoelectric dominance has been found in [6]. From a technical point of view the main difference between our work and [6] is that we study gauge group $SU(2)$, not $SU(3)$, but it seems rather unlikely that the different gauge groups lead to flux tubes, which are drastically different. It will be very interesting to explore the reason for the observed discrepancy.

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