Plasma Astrophysics
Chapter 2: Single Particle Motion

Yosuke Mizuno
Institute of Astronomy
National Tsing-Hua University

Single Particle Motion

- Particle Motion in uniform B-field
  - Gyro-motion
  - E x B drift
- Particle Motion in non-uniform B-field
  - Gradient drift
  - Curvature drift
- Adiabatic invariants

Equation of Motion

- In dense plasma, Coulomb forces couple with particles. So bulk motion is significant
- In rarefied plasma, charge particles does not interact with other particles significantly. So motion of each particles can be treated independently

- In general, equation of motion of particle with mass *m* under influence of Lorentz force is :

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \qquad (2.1)$$

  - ***E***: electric field, ***B***: magnetic field, *q*: particle's charge, ***v***: particle's velocity
  - Which is valid for non-relativistic motion (***v*** << c)
- A wide range of behaviors is possible depending on the nature of ***E*** and ***B*** in space and time

Uniform B-field: Gyration

- If motion is only subject to static and uniform ***B*** field, (***E***=0)

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{v} \times \boldsymbol{B}) \qquad (2.2)$$

- Taking a dot product with ***v***,

$$\boldsymbol{v} \cdot m\frac{d\boldsymbol{v}}{dt} = \boldsymbol{v} \cdot q(\boldsymbol{v} \times \boldsymbol{B})$$

$$m\frac{1}{2}\frac{d(\boldsymbol{v} \cdot \boldsymbol{v})}{dt} = q[\boldsymbol{v} \cdot (\boldsymbol{v} \times \boldsymbol{B})]$$

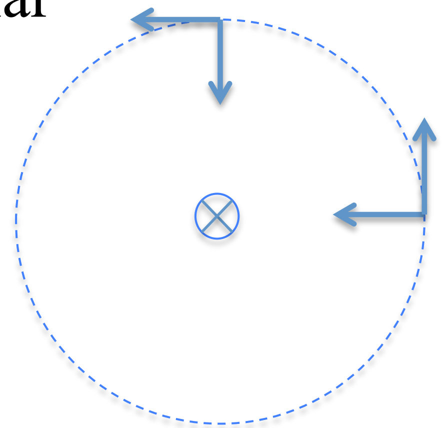
- RHS is zero as $\boldsymbol{v} \perp \boldsymbol{B}$ => $\frac{d}{dt}\left(\frac{mv^2}{2}\right) = 0$

- Therefore, a static magnetic field cannot change the kinetic energy of a particle since force is always perpendicular to direction of motion

Uniform B-field: Gyration (cont.)

- Decompose velocity into parallel and perpendicular components to B:

$$\boldsymbol{v} = \boldsymbol{v}_{\|} + \boldsymbol{v}_{\perp}$$



- Eq (2.2) => $\dfrac{d\boldsymbol{v}_{\|}}{dt} + \dfrac{d\boldsymbol{v}_{\perp}}{dt} = \dfrac{q}{m}(\boldsymbol{v} \times \boldsymbol{B})$

- This equation can be split into two independent equations:

$$\frac{d\boldsymbol{v}_{\|}}{dt} = 0 \quad => \quad \boldsymbol{v}_{\|} = const$$

$$\frac{d\boldsymbol{v}_{\perp}}{dt} = \frac{q}{m}(\boldsymbol{v}_{\perp} \times \boldsymbol{B})$$

- These imply that B field has no effect on the motion along it. Only affects particle velocity perpendicular it

Uniform B-field: Cyclotron Frequency

- To examine the perpendicular motion, we consider ***B*** =(*0*, *0*, B_z)
- Re-write eq (2.2) in each components:

$$m\frac{dv_x}{dt} = qBv_y \qquad (2.3a)$$

$$m\frac{dv_y}{dt} = -qBv_x \qquad (2.3b)$$

$$m\frac{dv_z}{dt} = 0 \qquad (2.3c)$$

- To determine the time variation of v_x and v_y we take derivative eq (2.3a) & eq (2.3b) ,

$$\frac{d^2v_x}{dt^2} + \omega_c^2 v_x = 0 \qquad (2.4a)$$

$$\frac{d^2v_y}{dt^2} + \omega_c^2 v_y = 0 \qquad (2.4b)$$

- Where $\boxed{\omega_c = -\frac{qB}{m}}$ is the *gyrofrequency* or *cyclotron frequency*

Uniform B-field: Cyclotron Frequency (cont.)

- *Gyrofrequency* or *cyclotron frequency*: $$\omega_c = -\frac{qB}{m}$$

- Indicative of the field strength and the charge and mass of particles of plasma

- Does not depend on kinetic energy

- For electron, ω_c is positive, electron rotates in the right-hand sense

- Plasma can have several cyclotron frequencies

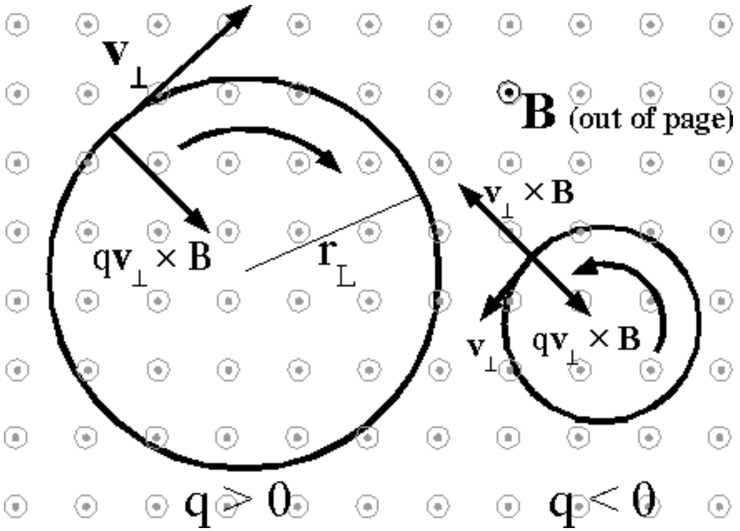
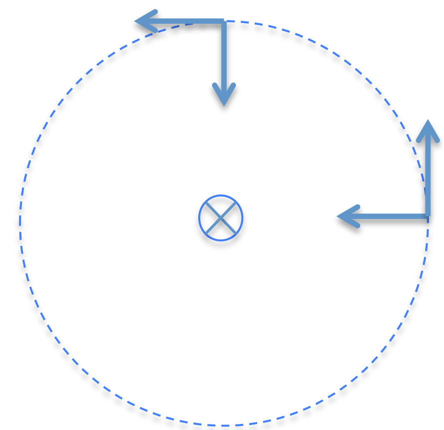


Figure from Schwartz et al., P.23

Uniform B-field: Larmor radius

- The ***v*** x ***B*** force is centripetal, so

$$
\begin{aligned}
-\frac{mv_\perp^2}{r} &= q(\boldsymbol{v} \times \boldsymbol{B}) \\
&= q v_\perp B
\end{aligned}
$$



Larmor radius
or *gyroradius*

$$r_L = \frac{m v_\perp}{|q| B} = \frac{v_\perp}{\omega_c}$$

- Particles with faster velocities or larger mass orbit larger radii

- For electron, the *gyrofrequency* can be written:

$$f_{ce} = \frac{\omega_c}{2\pi} \simeq 2.8 \times 10^6 B$$

where B is in units of Gauss

Uniform B-field: Guiding Center

- What is path of electrons? Solution of eqs (2.4a) & (2.4b) are harmonic:

$$v_x = v_\perp e^{i\omega_c t} = \dot{x} \quad (2.5a)$$

$$v_y = \frac{m}{qB} v_x^2 = \pm \frac{1}{\omega_c} \dot{v_x} = \pm i e^{i\omega_c t} = \dot{y} \quad (2.5b)$$

where $v_\perp = \sqrt{v_x^2 + v_y^2}$ is a constant speed in plane of perpendicular to **B**

- Integrating, we have

$$x - x_0 = -i \frac{v_\perp}{\omega_c} e^{i\omega_c t}$$

$$y - y_0 = \pm \frac{v_\perp}{\omega_c} e^{i\omega_c t}$$

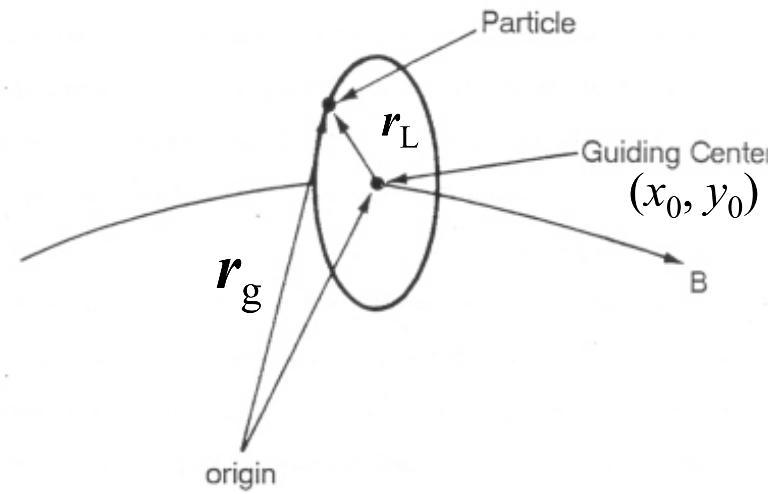
- Using Larmor radius (r_L), and taking a real part of above:

$$\boxed{\begin{aligned} x - x_0 &= r_L \sin(\omega_c t) \\ y - y_0 &= \pm r_L \cos(\omega_c t) \end{aligned}} \quad (2.6)$$

- These describe a circular orbit at a *guiding center* (x_0, y_0)

Uniform B-field: Helical Motion

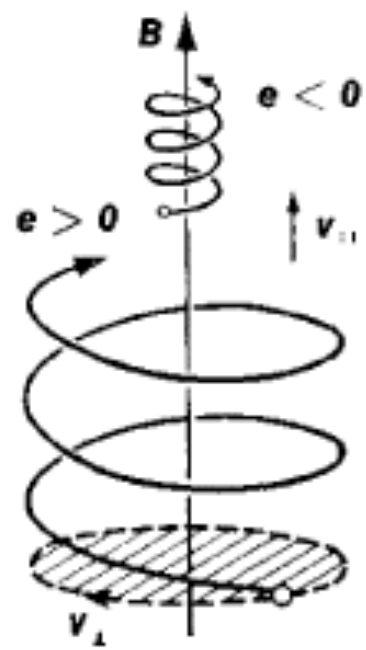
- In addition to this motion, there is a velocity v_z along $\boldsymbol{B}$ which is not effected by $\boldsymbol{B}$
- Combine with eq (2.6), this gives *helical motion* about a guiding center



$$\boldsymbol{r}_g = x_0\hat{\boldsymbol{x}} + y_0\hat{\boldsymbol{y}} + (z_0 + v_{||}t)\hat{\boldsymbol{z}}$$

- Guiding center moves linearly along z with constant velocity, $v_{||}$
- *Pitch angle* of helix is defined as

$$\alpha = \tan^{-1}\left(\frac{v_\perp}{v_{||}}\right)$$



guiding center motion in uniform B-field

Helical Motion: Magnetic Moment

- The charge circulates on the plane perpendicular to B with a uniform angular frequency ω_c
- Charge passes through $\omega_c/2\pi$ times per unit time
- This motion equivalent with a situation that an electric current $I = q\omega_c/2\pi$ is flowing in a circular coil with radius r_L

⇒ It has *Magnetic moment*:

$$\begin{aligned} \mu_m &= IA \\ &= \frac{q\omega_c}{2\pi}\pi r_L^2 \end{aligned}$$

- Therefore, $\boxed{\mu_m = \frac{mv_\perp^2/2}{B}}$

Uniform *E* & *B* field: *E* x *B* drift

- When ***E*** is finite, motion will be sum of two motions: circular Larmor gyration + drift of the guiding center

- Choose ***E*** to lie in x-z plane, so E_y=0. consider B=(0,0,B_z), Equation of motion is

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

- z-component of velocity is: $\frac{dv_z}{dt} = \frac{q}{m}E_z$

$$v_z = \frac{qE_z}{m}t + v_{z,0}$$

- This is a straight acceleration along ***B***. The transverse components are:

$$\begin{aligned} \frac{dv_x}{dt} &= \frac{q}{m}E_x \pm \omega_c v_y \\ \frac{dv_y}{dt} &= 0 \pm \omega_c v_x \end{aligned}$$

Uniform E & B field: E x B drift (cont.)

- Differentiating with constant ***E***,

$$\begin{aligned} \ddot{v}_x &= -\omega_c^2 v_x \\ \ddot{v}_y &= \pm\omega_c \left(\frac{q}{m}E_x \pm \omega_c v_y\right) \\ &= -\omega_c^2 \left(\frac{E_x}{B} + v_y\right) \end{aligned}$$

- Using $v_x = v_y + E_x/B$ we can write this as

$$\frac{d^2}{dt^2}\left(v_y + \frac{E_x}{B}\right) = -\omega_c^2 \left(v_y + \frac{E_x}{B}\right)$$

- In a form similar to Eq (2.5a) & (2.5b):

$$\begin{aligned} v_x &= v_\perp e^{i\omega_c t} \\ v_y &= \pm i e^{i\omega_c t} - \frac{E_x}{B} \end{aligned}$$

- Larmor motion is similar to case when ***E***=0, but now there is super-imposed drift v_g of the guiding center in –y direction

Uniform E & B field: E x B drift (cont.)

- To obtain the general formula for $\boldsymbol{v}_g$, we solve equation of motion

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

- As $md\boldsymbol{v}/dt$ gives a circular motion, already understand this effect, so set to zero

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = 0$$

- Taking cross product with B,

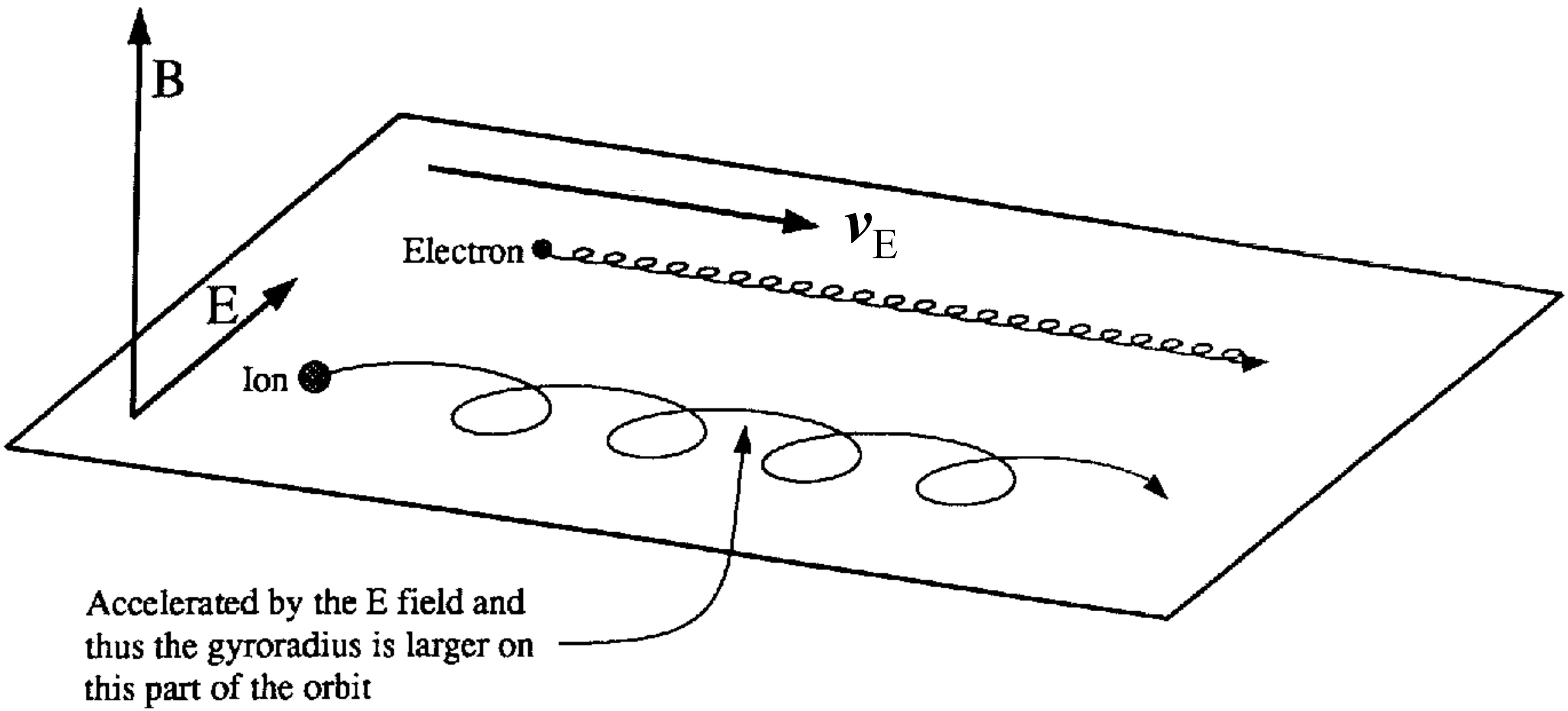
$$\boldsymbol{E} \times \boldsymbol{B} = \boldsymbol{B} \times (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{v}B^2 - \boldsymbol{B}(\boldsymbol{v} \cdot \boldsymbol{B})$$

- The transverse components of this equation are

$$\boxed{\boldsymbol{v}_{\perp gc} = (\boldsymbol{E} \times \boldsymbol{B})/B^2 = \boldsymbol{v}_E}$$

- Where $\boldsymbol{v}_{\mathrm{E}}$ is the *E x B drift velocity* of the guiding center

Uniform E & B field: E x B drift (cont.)



External force drift

- E x B drift of guiding center is : $\boldsymbol{v}_E = \dfrac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}$

  which can be extended to a form for a general force *F*:

$$\boldsymbol{v}_f = \frac{1}{q} \frac{\boldsymbol{F} \times \boldsymbol{B}}{B^2}$$

External-force drift

- Example: In a gravitational field,

$$\boldsymbol{v}_g = \frac{m}{q} \frac{\boldsymbol{g} \times \boldsymbol{B}}{B^2}$$

- Similar to the drift $\boldsymbol{v}_{\mathrm{E}}$, in that drift is perpendicular to both forces, but in this case particles with opposite charge drift in opposite direction.

Drift in non-uniform field

- Uniform fields provide poor descriptions for many phenomena, such as planetary fields, coronal loops in the Sun, Tokamaks, which have spatially and temporally varying fields.

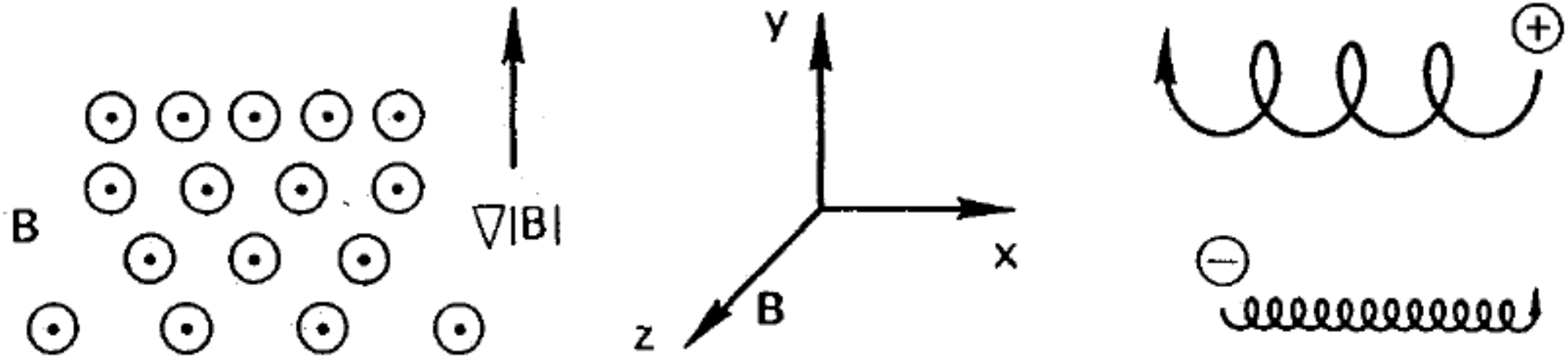
- Particle drifts in inhomogeneous fields are classified several way
- In this lecture, consider two drifts associated with spatially non-uniform ***B***: *gradient drift* and *curvature drift*. (There are many other)

- In general, introducing inhomogeneity is too complicated to obtain exact solutions for guiding center drifts

- Therefore use *orbit theory* approximation:
  - Within one Larmor orbit, B is approximately uniform, i.e., typical length-scale L over which B varies is $L >> \boldsymbol{r}_L$ => gyro-orbit is nearly circle

Grad-B Drift

- Assumes lines of forces are straight, but their strength is increases in *y*-direction



- Gradient in $|B|$ causes the Larmor radius ($r_A = mv/qB$) to be larger at the bottom of the orbit than at the top, which leads to a drift
- Drift should be perpendicular to ∇B and $\boldsymbol{B}$
- Ions and electrons drift in opposite directions

Grad-B Drift (cont.)

- Consider spatially-varying magnetic field, $\boldsymbol{B}$=(0,0,$B_z(y)$), i,.e., B only has z-component and the strength of it varies with y.
- Assume that $\boldsymbol{E}$=0, so equation of motion is $\boldsymbol{F}=q(\boldsymbol{v} \times \boldsymbol{B})$
- Separating into components,

$$F_x = q(v_y B_z) \quad (2.7a)$$

$$F_y = -q(v_x B_z) \quad (2.7b)$$

$$F_z = 0 \quad (2.7c)$$

- The gradient of B_z is

$$\frac{dB_z}{dy} \approx \frac{B_z}{L} \ll \frac{B_z}{r_L}$$

$$\Rightarrow \quad r_L \frac{dB_z}{dy} \ll B_z$$

- This means that the magnetic field strength can be expanded in a Taylor expansion for distances $y < r_L$,

$$B_z(y) = B_0 + y\frac{dB_z}{dy} + \ldots$$

Grad-B Drift (cont.)

- Expanding B_z to first order in Eqs (2.7a) & (2.7b) :

$$F_x = qv_y \left(B_0 + y \frac{dB_z}{dy} \right) \quad (2.8a)$$

$$F_y = -qv_x \left(B_0 + y \frac{dB_z}{dy} \right) \quad (2.8b)$$

- Particles in B-field traveling around a guiding center (0,0) with helical trajectory:

$$x = r_L \sin(\omega_c t)$$
$$y = \pm r_L \cos(\omega_c t)$$

- The velocities can be written in a similar form:

$$v_x = v_\perp \cos(\omega_c t)$$
$$v_y = \pm v_\perp \sin(\omega_c t)$$

- Substituting these into eq(2.8) gives:

$$F_x = -qv_\perp \sin(\omega_c t) \left[B_0 \pm r_L \cos(\omega_c t) \frac{dB_z}{dy} \right]$$

$$F_y = -qv_\perp \cos(\omega_c t) \left[B_0 \pm r_L \cos(\omega_c t) \frac{dB_z}{dy} \right]$$

Grad-B Drift (cont.)

- Since we are only interested in the guiding center motion, we average force over a gyro-period. Therefore in *x*-direction:

$$\langle F_x \rangle = -qv_\perp \left[B_0 \langle \sin(\omega_c t) \rangle \pm r_L \langle \sin(\omega_c t) \cos(\omega_c t) \rangle \frac{dB_z}{dy} \right] \quad (2.9a)$$

- But $\langle \sin(\omega_c t) \rangle = 0$ and $\langle \sin(\omega_c t) \cos(\omega_c t) \rangle = 0$ => $\langle F_x \rangle = 0$

- In the y-direction:

$$\begin{aligned} \langle F_y \rangle &= -qv_\perp \left[B_0 \langle \cos(\omega_c t) \rangle \pm r_L \langle \cos^2(\omega_c t) \rangle \frac{dB_z}{dy} \right] \\ &= \mp \frac{qv_\perp r_L}{2} \frac{dB_z}{dy} \quad (2.9b) \end{aligned}$$

- Where $\langle \cos(\omega_c t) \rangle = 0$ but $\langle \cos^2(\omega_c t) \rangle = 1/2$

Grad-B Drift (cont.)

- In general, drift of guiding center is $\boldsymbol{v}_f = \frac{1}{q}\frac{\boldsymbol{F}\times\boldsymbol{B}}{B^2}$
- Using Eq(2.9b): $$v_{\nabla B} = \frac{1}{q}\frac{\langle F_y\rangle\hat{y}\times B_z\hat{z}}{B_z^2} = \mp\frac{v_\perp r_L}{2B_z}\frac{dB_z}{dy}\hat{x}$$
- So positive charged particles drift in –*x* direction and negative charged particles drift +*x* direction
- In 3D, the result can be generalized to:

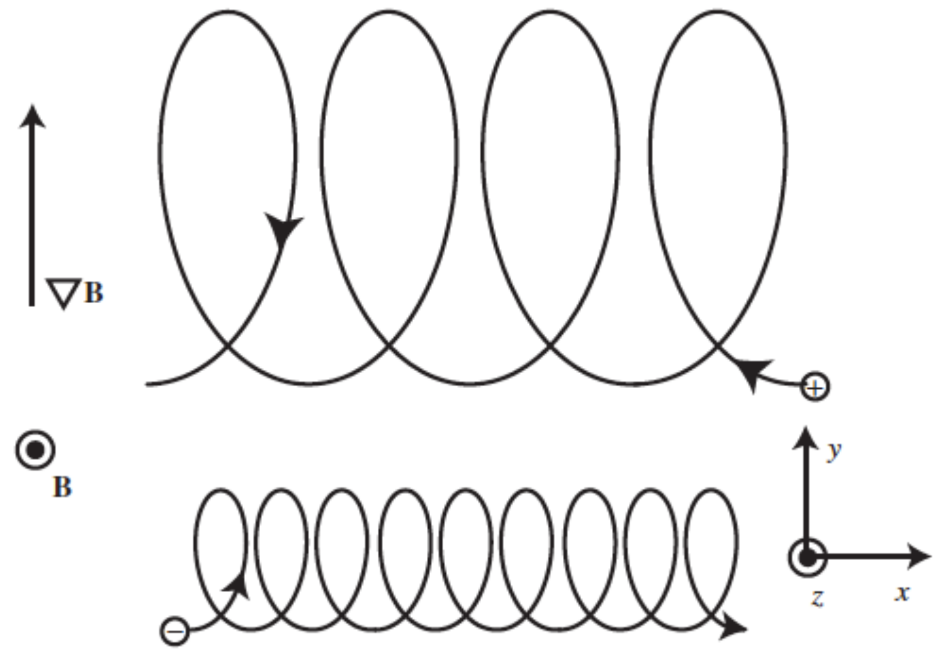
$$v_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\boldsymbol{B}\times\nabla B}{B^2}$$

Grad-B drift

- The \pm stands for sign of charge. The grad-B drift is in opposite direction for electrons and ions and causes a current transverse to ***B***

Grad-B Drift (cont.)

- Below are shown particle drifts due to a magnetic field gradient, where $\boldsymbol{B}(y) = \mathbf{z}B_z(y)$

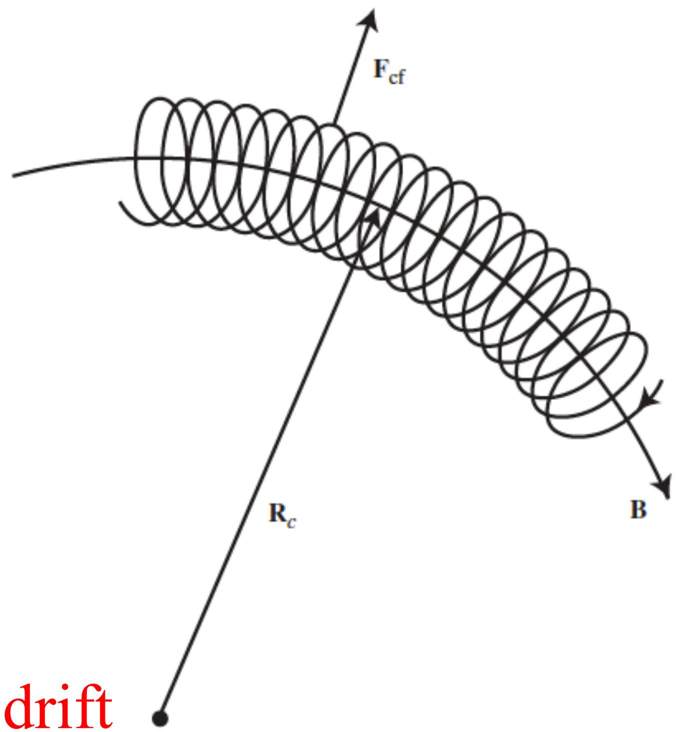


- Consider $r_L = mv/q\boldsymbol{B}$, local gyroradius is large where $\boldsymbol{B}$ is small and is small where $\boldsymbol{B}$ is large which gives to a drift

Curvature Drift

- When charged particles move along curved magnetic field lines, experience centrifugal force perpendicular to magnetic field lines
- Assume radius of curvature (R_c) is >> r_L.
- The outward centrifugal force is

$$F_{cf} = \frac{mv_{\parallel}^2}{R_c}\hat{r}$$



- This can be directly inserted into general form for guiding-center drift

$$\boldsymbol{v}_f = \frac{1}{q}\frac{\boldsymbol{F}\times\boldsymbol{B}}{B^2}$$

=> $$v_R = \frac{mv_{\parallel}^2}{qR_c^2}\frac{\boldsymbol{R_c}\times\boldsymbol{B}}{B^2}$$ Curvature drift

- Drift is into or out of page depending on sign of q

Adiabatic invariance of magnetic moment

- Gyrating particle constitutes an electric current loop with a dipole moment:

$$\mu = \frac{1/2mv_\perp^2}{B}$$

- The dipole moment is conserved, i.e., is invariant. Called the *first adiabatic invariant*.

- $\mu = const$ even if $\boldsymbol{B}$ varies spatially or temporally. If B varies, then v_{perp} varies to keep $\mu = const$ =>$v_{||}$ also changes.

- It gives rise to *magnetic mirrorng*. Seen in planetary magnetospheres, coronal loops etc.

Magnetic mirroring

- Consider ***B***-field is in *z*-direction and whose magnitude varies in *z*-direction. If ***B***-field is axisymmetric, $B_\theta=0$ and $d/d\theta=0$ (r, θ, z)
- This has cylindrical symmetry, so write $\boldsymbol{B} = B_r\hat{r} + B_z\hat{z}$
- How does this configuration gives a force that can trap a charged particle?
- Can obtain B_r from $\nabla \cdot \boldsymbol{B} = 0$. In cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial}{\partial r}(rB_r) = -r\frac{\partial B_z}{\partial z}$$

- If $\partial B_z/\partial z$ is given at $r=0$ and does not change much with r, then

$$rB_r = -\int_0^r r\frac{\partial B_z}{\partial z}dr \approx -\frac{1}{2}r^2\left[\frac{\partial B_z}{\partial z}\right]_{r=0}$$

$$B_r = -\frac{1}{2}r\left[\frac{\partial B_z}{\partial z}\right]_{r=0} \qquad (2.10)$$

Magnetic mirroring (cont.)

- Now have B_r in terms of B_z, which use to find Lorentz force on particle.
- The components of Lorentz force are:

$$F_r = q(v_\theta B_z - \cancel{v_z B_\theta})$$
(1)

$$F_\theta = q(-v_r B_z + v_z B_r)$$
(2) (3)

$$F_z = q(\cancel{v_r B_\theta} - v_\theta B_r)$$
(4)

- As B_θ=0, two terms vanish and terms (1) & (2) give Larmor gyration. Term (3) vanishes on the axis and cause a drift in radial direction. Term (4) is therefore the one of interest
- Substitute from eq (2. 10):

$$\begin{aligned} F_z &= -q v_\theta B_r \\ &= \frac{q v_\theta r}{2} \frac{\partial B_z}{\partial z} \end{aligned}$$

Magnetic mirroring (cont.)

- Averaging over one gyro-orbit, and putting $v_\theta = \mp v_\perp$ & $r = r_\mathrm{L}$

$$F_z = \mp \frac{q v_\perp r_L}{2} \frac{\partial B_z}{\partial z}$$

- This is called the *mirror force*, where -/+ arises because particles of opposite charge orbit the field in opposite directions.
- Above is normally written:

$$\begin{aligned} F_z &= \mp \frac{1}{2} q \frac{v_\perp^2}{\omega_c} \frac{\partial B_z}{\partial z} \\ &= -\frac{1}{2} \frac{m v_\perp^2}{B} \frac{\partial B_z}{\partial z} \end{aligned}$$

- or $F_z = -\mu \frac{\partial B_z}{\partial z}$ where $\mu = \frac{1/2 m v_\perp^2}{B}$
- In 3D, this can be generalized to: $F_\| = -\mu \frac{dB}{ds} = -\mu \nabla_\| B$

  where $F_\|$ is the mirror force parallel to ***B*** and *ds* is a line element along ***B***

Adiabatic invariants

- Symmetry principles:
  - Periodic motion ⇔conserved quantity
  - symmetry ⇔ conserved law

- What if the motion is almost periodic?
- Hamiltonian formulation:
- q and p is canonical variables and motion almost periodical =>

  $I = \oint p dq$ is constant, called *adiabatic invariant*

- Adiabatic invariant is a property of physical system that stays constant when changes occur slowly

First adiabatic invariant

- As particle moves into regions of stronger or weaker ***B***, Larmor radius changes, but μ remains invariant
- To prove this, consider component of equation of motion along ***B***:

$$m\frac{dv_\parallel}{dt} = -\mu\frac{dB}{ds}$$

- Multiplying by $v_\parallel$: $mv_\parallel\frac{dv_\parallel}{dt} = -\mu v_\parallel\frac{dB}{ds}$
- Then,

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{ds}{dt}\frac{dB}{ds}$$

$$= -\mu\frac{dB}{dt} \qquad (2.11)$$

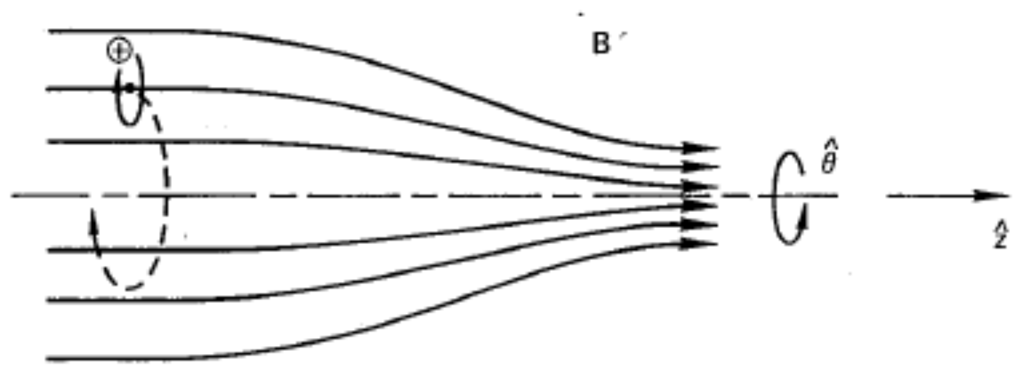
- The particle energy must be conserved: $\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right) = 0$
- Using $\mu = \frac{1/2mv_\perp^2}{B}$ ⇒ $\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \mu B\right) = 0$

First adiabatic invariant (cont.)

- Use eq(2.11), $$-\mu\frac{dB}{dt}+\frac{d}{dt}(\mu B)=0$$
$$-\mu\frac{dB}{dt}+\mu\frac{dB}{dt}+B\frac{d\mu}{dt}=0$$
$$\Rightarrow \quad B\frac{d\mu}{dt}=0$$
- As *B* is not equal to 0, this implies that: $\frac{d\mu}{dt}=0$
- That is $\mu = const$ in time (invariant)
- μ is known as the *first adiabatic invariant* of the particle orbit.
- As a particle moves from a weak-field to a strong-field region, it sees ***B*** increasing and therefore v_{perp} must increase in order to keep μ constant. Since total energy must remain constant, $v_{||}$ must decrease.
- If ***B*** is high enough, $v_{||}$ essentially => 0 and particle is reflected back to weak-field

Consequence of invariant μ

- Consider $\boldsymbol{B}_0$ & $\boldsymbol{B}_1$ in the weak- and strong-field regions. Associated speeds are v_0 & v_1

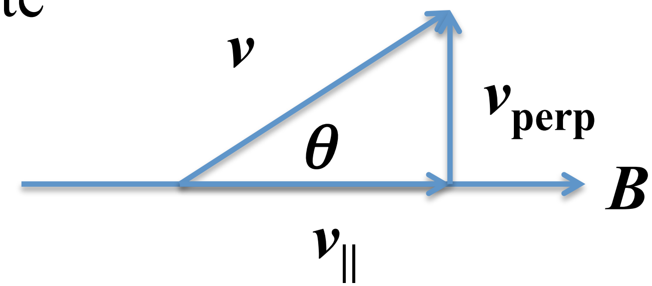


- The conservation of μ implies that $\mu = \dfrac{mv_0^2}{2B_0} = \dfrac{mv_1^2}{2B_1}$
- So as $\boldsymbol{B}$ increases, the perpendicular component of particle velocity increases => particles move more and more perpendicular to $\boldsymbol{B}$
- However, since $\boldsymbol{E}$=0, the total particle energy cannot increase. Thus as v_{perp} increases, $v_{||}$ must decrease. The particle slow down in its motion along the filed
- If field is strong enough, at some point the particle may have $v_{||}$=0

Consequence of invariant μ *(cont.)*

- At $\boldsymbol{B}_1$, $v_{1,\|}=0$, From conservation of energy: $v_1^2 = v_{1,\perp}^2 = v_0^2$
- Using $\mu = \frac{mv_0^2}{2B_0} = \frac{mv_1^2}{2B_1}$ we can write

$$\frac{B_0}{B_1} = \frac{v_{0,\perp}^2}{v_{1,\perp}^2} = \frac{v_{0,\perp}^2}{v_0^2}$$

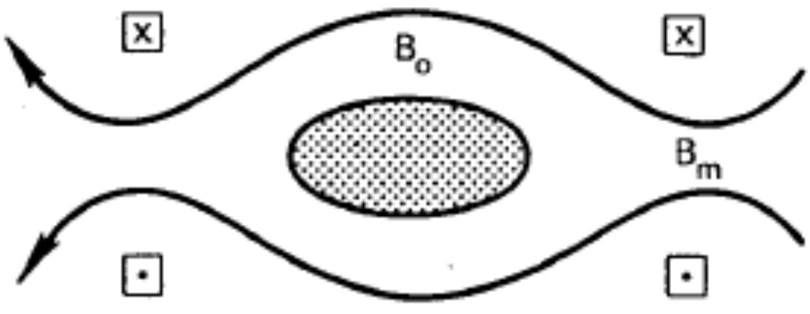


- But $\sin(\theta) = v_\perp / v_0$, where θ is the *pitch angle*
- Therefore $\frac{B_0}{B_1} = \sin^2(\theta)$
- Particle with smaller θ will mirror in regions of higher $\boldsymbol{B}$. If θ is too small, $\boldsymbol{B}_1 >> \boldsymbol{B}_0$ and particle does not mirror

Consequence of invariant μ *(cont.2)*

- Mirror ratio is defined as

$$R_m = \frac{B_m}{B_0}$$



A plasma trapped between magnetic mirrors.

- The smallest θ of a confined particle is

$$\sin^2(\theta) = \frac{B_0}{B_m} = \frac{1}{R_m}$$

- This defines region in velocity space in the shape of a cone, called the *loss cone*.
- If a particle is in a region between two high field. The particle may be reflected at one, travel towards the second, and also reflect there. Thus the particle motion is confined to a certain region of space, this process is known as *magnetic trapping*.

Other adiabatic invariants

- Second adiabatic invariant: longitudinal invariant of a trapped particle in the magnetic mirror

$$J = \oint p_{||} ds$$

- This property is used in Fermi acceleration.

- Third adiabatic invariant: flux invariant, which means the magnetic flux through the guiding center orbit is conserved

$$\Phi = \oint A \cdot d\boldsymbol{x}$$

Application for Astrophysics

- Particle trapping: due to magnetic mirroring in the Earth's van Allen radiation belt and energetic electrons confined in solar coronal magnetic loops
- Particle transport of energetic particles (galactic cosmic-rays, solar energetic particles, ultra-relativistic particles from extragalactic sources): governed by variety of particle drifts
- Particle acceleration: due to electric fields in pulsar magnetosphere and solar flares. Magnetic mirroring due to either stochastic motion of high field regions, or systematic motion between converging mirrors in the vicinity of shock waves, leads to Fermi acceleration of particles
- Radiation: given by relativistic gyrating particles (synchrotron radiation) or accelerating particles (bremsstrahlung)

Summary of single particle motion

- Charged particle motion in ***B*** & ***E*** fields is very unique and has interesting physical properties

- Particle Motion in uniform ***B***-field (no ***E***-field)
  - Gyration: gyro (cyclotron)-frequency, Larmor radius, guiding center
- Particle Motion in uniform *E* & *B*-fields
  - E x B drift
- Particle Motion in non-uniform *B*-field
  - Grad-B drift, Curvature drift
  - magnetic mirroring
- Adiabatic invariants
  - Magnetic moment