Hydrodynamics and Magnetohydrodynamics: Solutions of the exercises in Lecture XII

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Lecture XII, Exercise 1.

The gyro frequencies and Larmor radii are given by

$$\omega_c := \frac{qB}{m}, \quad (1)$$

$$r_L := \frac{mv_\perp}{qB}, \quad (2)$$

where $v_\perp$ is the velocity perpendicular to the magnetic field. We assume the plasma is in thermal equilibrium and the particles have the thermal velocity. Therefore the velocity is given by

$$v_\perp \approx v_{th} = \sqrt{\frac{k_B T}{m}}. \quad (3)$$

Here we compare the gyro frequencies with plasma frequencies which can be obtained

$$\omega_P = \sqrt{\frac{4\pi n q^2}{m}}. \quad (4)$$

We list the results in different physical conditions (1 rad/s $= 1/2\pi$ Hz).

		ω_c [Hz]	r_L (cm)	ω_p [Hz]
fusion machine	electron	8.4×10^{20}	2.3×10^{-13}	2.3×10^{12}
	proton	4.6×10^{17}	1.0×10^{-11}	5.3×10^{10}
Earth's magnetosphere	electron	8.4×10^{14}	2.3×10^{-9}	2.3×10^{6}
	proton	4.6×10^{11}	1.0×10^{-7}	5.3×10^{4}
center of the Sun	electron	8.4×10^{22}	3.0×10^{-15}	2.3×10^{17}
	proton	4.6×10^{19}	1.3×10^{-13}	5.3×10^{15}
solar corona	electron	8.4×10^{16}	7.4×10^{-10}	2.3×10^{8}
	proton	4.6×10^{13}	3.2×10^{-8}	5.3×10^{6}
solar wind	electron	8.4×10^{11}	2.3×10^{-5}	7.1×10^{4}
	proton	4.6×10^{8}	1.0×10^{-3}	1.7×10^{3}
neutron star's atmosphere	electron	8.4×10^{28}	2.3×10^{-21}	2.3×10^{10}
	proton	4.6×10^{25}	1.0×10^{-19}	5.3×10^{-8}

Lecture XII, Exercise 2.

Consider the non relativistic motion of particles moving under the combined influence of uniform magnetic and gravity fields. The equation of motion can be given as

$$\frac{d\vec{\boldsymbol{v}}}{dt} = \vec{\boldsymbol{g}} + \frac{q}{m}(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}), \tag{5}$$

where $m\vec{\boldsymbol{g}}$ is the gravitational force on a particle of mass, m. This equation is identical to that for motion in an "effective" electric field and the same magnetic field. Thus in this case, the effective electric field is written as

$$\vec{\boldsymbol{E}}_{\text{eff}} = \frac{m}{q}\vec{\boldsymbol{g}}. \tag{6}$$

From the derivation of the $\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$ drift we know that $\vec{\boldsymbol{v}}_d = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}/B^2$, the drift component of the motion is given by

$$\vec{\boldsymbol{v}}_d = \frac{m}{q}\frac{\vec{\boldsymbol{g}} \times \vec{\boldsymbol{B}}}{B^2}. \tag{7}$$

The drift velocity depends on both charge and mass. Electrons and ions are drift in opposite direction, producing a current in the system. For the simple case of uniform plasma consisting only of protons and electrons, the current density is given by

$$\vec{\boldsymbol{j}} = -ne\vec{\boldsymbol{v}}_{d,e} + ne\vec{\boldsymbol{v}}_{d,p}, \tag{8}$$

where $\vec{\boldsymbol{v}}_{d,e}$ and $\vec{\boldsymbol{v}}_{d,p}$ are the drift velocities of the electrons and of the protons, respectively. From eq (7), the current density can then be expressed as

$$\vec{\boldsymbol{j}} = \rho\frac{\vec{\boldsymbol{g}} \times \vec{\boldsymbol{B}}}{B^2}, \tag{9}$$

where $\rho := nm_e + nm_p$.

Lecture XII, Exercise 3.

See Lecture XIII, Exercise 1. for the solution of this problem.