Time-dependent spin-wave theory

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Phys. Rev. **B** 85, 054422 (2012)

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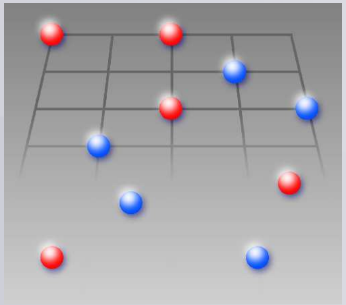


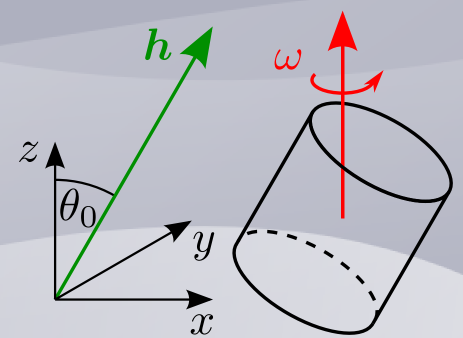
1.1 Model

- Heisenberg ferromagnet

$$\mathcal{H}(t) = -\frac{1}{2}\sum_{ij} J_{ij}\mathsf{S}_i \cdot \mathsf{S}_j - \sum_i \mathsf{h}_i(t)\cdot \mathsf{S}_i,$$

- time-dependent magnetic field (rotating magnet)

$$\mathsf{h}(t) = \begin{pmatrix} h_\perp \cos(\omega t) \\ -h_\perp \sin(\omega t) \\ h_z \end{pmatrix}$$



- Goal: generalization of spin wave-approach to magnet in time-dependent magnetic field

2.1 Spin-wave approach: adiabatic approximation

- static minimization of the classical ground-state energy (as in time-independent case)
- projection of spin operators onto time-dependent basis
- Holstein Primakoff transformation — Holstein, Primakoff (1940)
- local magnetization follows the magnetic field adiabatically

$$\mathrm{M}_{\mathrm{ad}}(t) = \left[S - \frac{1}{N}\sum_{\mathrm{k}} \frac{1}{e^{\beta(\epsilon_{\mathrm{k}}+h)} - 1}\right] \begin{pmatrix} \cos\theta_0 \cos(\omega t) \\ -\cos\theta_0 \sin(\omega t) \\ \sin\theta_0 \end{pmatrix}$$

no dependence on the frequency

spectrum: simple ferromagnet $\epsilon_{\mathrm{k}} = S(J_0 - J_{\mathrm{k}})$

canting angle $\cos\theta_0 = \dfrac{h_z}{\sqrt{h_\perp^2 + h_z^2}}$

2.2 Spin-wave approach: Perturbation theory in $h_\perp$

- split contributions to the magnetic field

Nakata, Tatara, J. Phys. Soc. Jpn. (2011)

$$\mathsf{h}(t) = \begin{pmatrix} h_\perp \cos(\omega t) \\ -h_\perp \sin(\omega t) \\ h_z \end{pmatrix} = \mathsf{h}_0 + \mathsf{h}_1(t)$$

- expand in laboratory frame

$$\mathcal{H}_z \approx -DNJS^2 - Nh_z S + \sum_{\mathsf{k}} (\epsilon_{\mathsf{k}} + h_z) b^\dagger_{\mathsf{k}} b_{\mathsf{k}}$$

perturbation

$$\mathcal{V}(t) \approx -\frac{h_\perp}{2}\sqrt{2S}\sqrt{N}\left[e^{i\omega t} b_{\mathsf{k}=0} + e^{-i\omega t} b^\dagger_{\mathsf{k}=0}\right] \propto h_\perp$$

- calculate magnetization by solving the set of Heisenberg equations of motion

$$i\frac{dA}{dt} = [A, H]$$

- result

unphysical divergenge for $h_z \to \omega$

$$\mathsf{M}_{\text{lab}}(t) = \frac{h_\perp S}{h_z - \omega}[\cos(\omega t)\mathsf{x} - \sin(\omega t)\mathsf{y}] + M_{h_z}\mathsf{z}$$

3.1 Construction of proper basis

- idea: factorize unitary time-evolution operator

$$i\partial_t \mathcal{U}(t) = \mathcal{H}(t)\mathcal{U}(t) \qquad \mathcal{U}(t) = \mathcal{U}_0(t)\tilde{\mathcal{U}}(t)$$

- choose trivial part to rotate system to the direction of the true magnetization

$$\mathcal{U}_0(t) = e^{-i\sum_i \alpha_i(t)\cdot \mathbf{S}_i} \qquad \alpha_i(t) \propto \mathbf{m}_i(t)$$

- new operator equation determines the dynamics $i\partial_t \tilde{\mathcal{U}}(t) = \tilde{\mathcal{H}}(t)\tilde{\mathcal{U}}(t)$

Lin, Commun. Theor. Phys. ('05)

- effective Hamiltonian

Berry phase contribution

$$\tilde{\mathcal{H}}(t) = \mathcal{U}_0^\dagger(t)\mathcal{H}(t)\mathcal{U}_0(t) + \tilde{\mathcal{H}}_B(t)$$

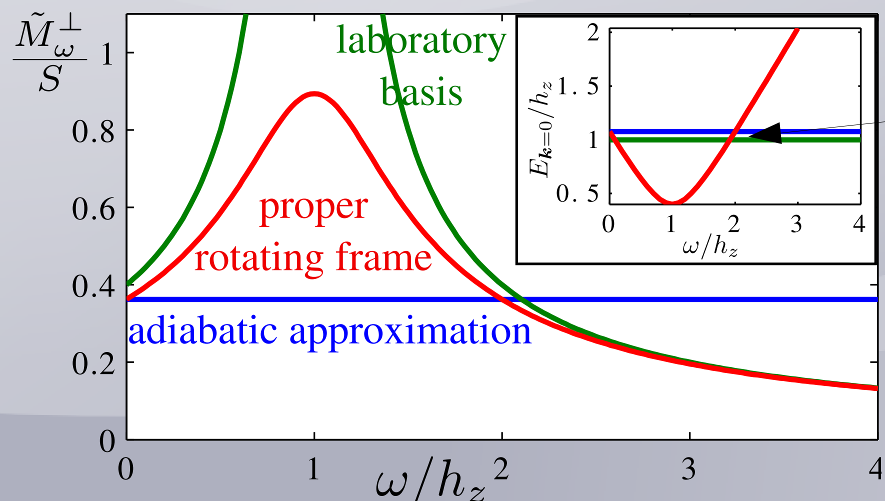
Hamiltonian from adiabatic limit

$$\tilde{\mathcal{H}}_B(t) = -\sum_i \left[\omega_i^{(1)}(t)\tilde{S}_i^{(1)} + \omega_i^{(2)}(t)\tilde{S}_i^{(2)} + \omega_i^{\parallel}(t)\tilde{S}_i^{\parallel}\right] = -i\mathcal{U}_0^\dagger(t)\partial_t\mathcal{U}_0(t)$$

3.2 Results: Spin-wave theory in proper basis

- transform spin operator $\tilde{\mathsf{s}}_i(t) = e^{i\alpha_i(t)\cdot\mathsf{S}_i}\mathsf{s}_i e^{-i\alpha_i(t)\cdot\mathsf{S}_i} = e^{\alpha_i(t)\times}\mathsf{s}_i$
- apply spin-wave theory as expansion around true classical groundstate
- result for magnetization

$$\mathsf{M}\ \ (t) = \left[S - \frac{1}{N}\sum_{\mathsf{k}} \frac{1}{e^{\beta(\epsilon_{\mathsf{k}} + \tilde{h}_\omega)} - 1}\right] \begin{pmatrix} \cos\theta_\omega \cos(\omega t) \\ -\cos\theta_\omega \sin(\omega t) \\ \sin\theta_\omega \end{pmatrix}$$



modified spectrum

frequency dependent canting angle

$$\cos\theta_\omega = \frac{(h_z - \omega)}{\sqrt{h_\perp^2 + (h_z - \omega)^2}}$$

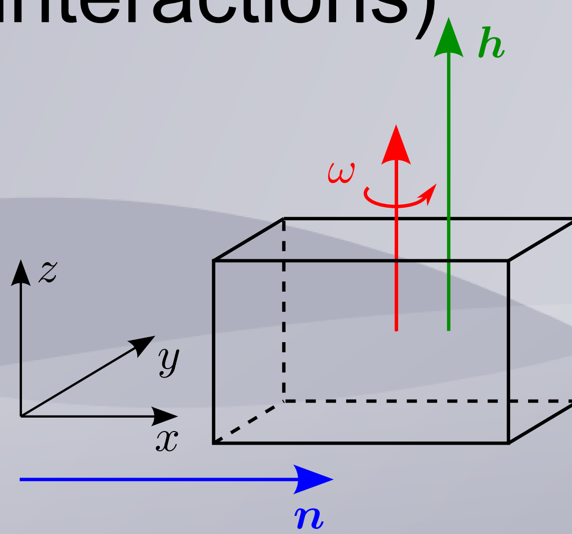
effective magnetic field

$$h_z \to \tilde{h}_\omega = h_z - \omega$$

4.1 Application to pumped ferromagnet

- Model: ferromagnet with rotating single-ion anisotropy (mimics dipole-dipole interactions)

$$\mathcal{H}(t) = -\frac{1}{2}\sum_{ij} J_{ij}\mathsf{S}_i \cdot \mathsf{S}_j - h\sum_i S_i^z$$
$$-\frac{A}{2}\sum_i \left\{[\mathsf{S}_i \cdot \mathsf{n}(t)]^2 - [\mathsf{S}_i \cdot (\mathsf{z} \times \mathsf{n}(t))]^2\right\}$$



- construct proper rotating basis

- use linear spin-wave theory (without interactions between magnons) to calculate the spectrum and magnetization

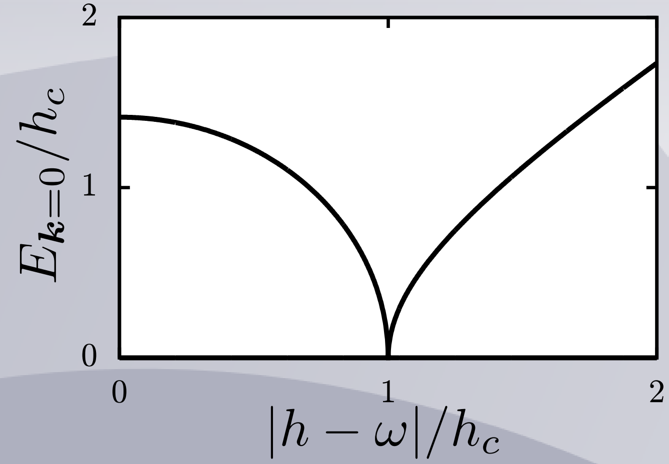
4.2 Results

- spectrum

$$E_{\mathsf{k}}=\sqrt{(\epsilon_{\mathsf{k}}+h-\omega)^2-h_c^2} \qquad |h-\omega|>AS=h_c$$

$$E_{\mathsf{k}}=\sqrt{\left[\epsilon_{\mathsf{k}}+\frac{3h_c}{2}-\frac{(h-\omega)^2}{2h_c}\right]^2-\left[\frac{h_c}{2}+\frac{(h-\omega)^2}{2h_c}\right]^2}$$

$$|h-\omega|<AS$$



- magnetization

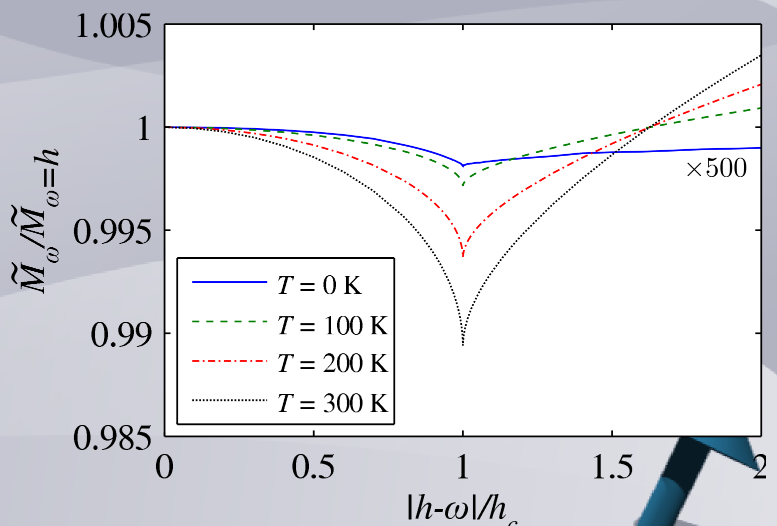
$$\mathsf{M}(t)=\tilde{M}_\omega \mathsf{m}_\omega(t)$$

$$\mathsf{m}_\omega(t)=\sin\theta[\cos(\omega t)\mathsf{x}-\sin(\omega t)\mathsf{y}]+\cos\theta\mathsf{z}$$

frequency dependent canting angle $\cos\theta=\frac{h-\omega}{h_c}$

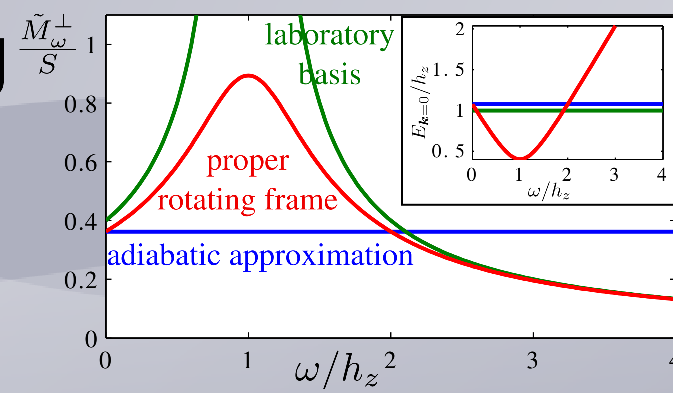
$$\tilde{M}_\omega=S+\frac{1}{2}-\frac{1}{N}\sum_{\mathsf{k}}\frac{1}{E_{\mathsf{k}}}\left[\epsilon_{\mathsf{k}}+\frac{3h_c}{2}-\frac{(h-\omega)^2}{2h_c}\right]$$

$$\times\left[\frac{1}{e^{\beta E_{\mathsf{k}}}-1}+\frac{1}{2}\right]$$

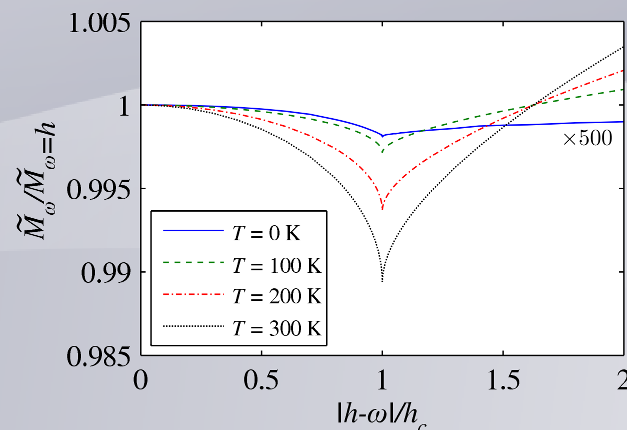


5.1 Summary

- Conventional approaches fail:
  - adiabatic approximation for nonzero frequencies
  - perturbation theory for large effective field
- Construction of proper rotating basis by factorization of time-evolution operator allows to use linear spin-wave theory.



- Testable predictions for pumped ferromagnet for example on thin films of YIG



Demokritov, *et al.*, Nature, 06