Spin-wave calculations for Heisenberg magnets with reduced symmetry

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1.1 Introduction: Magnetism

- Types of magnetism

$$\chi = \frac{\partial \vec{M}}{\partial \vec{H}}$$

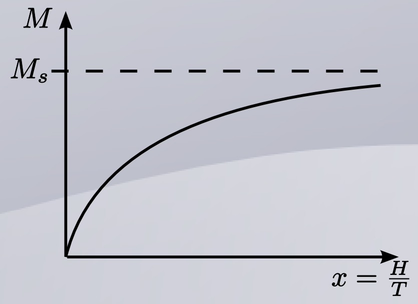
  - Diamagnetism
    response to magnetic field due to Lenz' rule

    $\vec{L}\,|0\rangle = 0 \quad \vec{S}\,|0\rangle = 0$

    $$\chi = -\frac{e^2}{6m}\mu_0 n r_a^2 Z_a \approx -10^{-4} < 0$$

  - Paramagnetism
    alignment of magnetic moment in a magnetic field

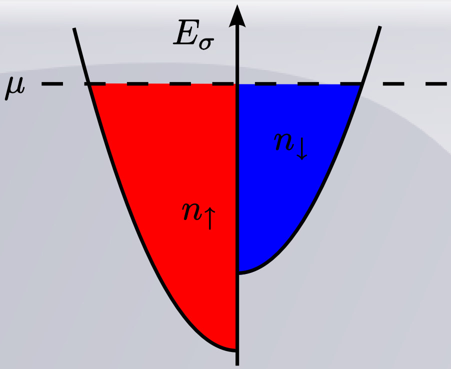
    $$\chi \sim \frac{1}{T} > 0$$



  - **Collective magnetism**
    correlation effects of spin degrees of freedom

    $\chi \to \infty$ (phase transitions)

1.2 Introduction: Collective magnetism

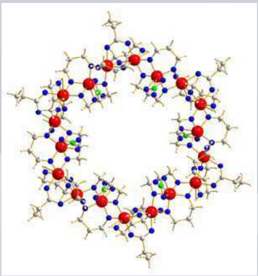


- Itinerant magnetism in metals

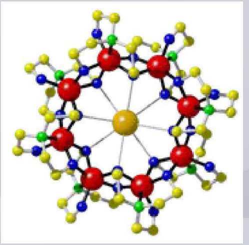
  magnetization from occupation of states

  $M \sim n_\uparrow - n_\downarrow$

- Magnetism on mesoscopic scales

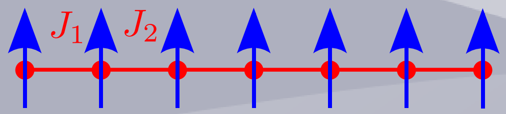


Fe_{18}



$CsFe_8$

finite numbers of spins:
nevertheless collective
behaviour



method: exact diagonalisation
Waldmann *et al.* PRL ('06)

- Magnetism of localized spins in insulators

$$H = J\vec{S}_1 \cdot \vec{S}_2 \quad \rightarrow \quad H = \frac{1}{2}\sum_{ij} J_{ij}\vec{S}_i \cdot \vec{S}_j$$

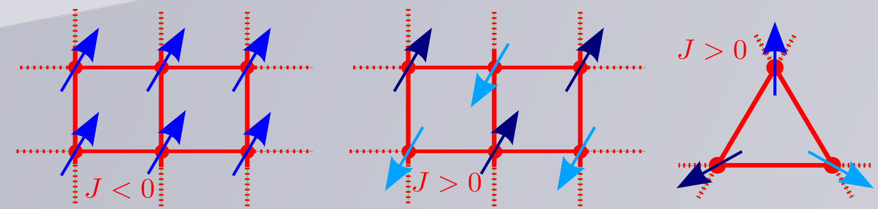
Heisenberg model

W. Heisenberg Z. Phys. **49**, 619 (1928)

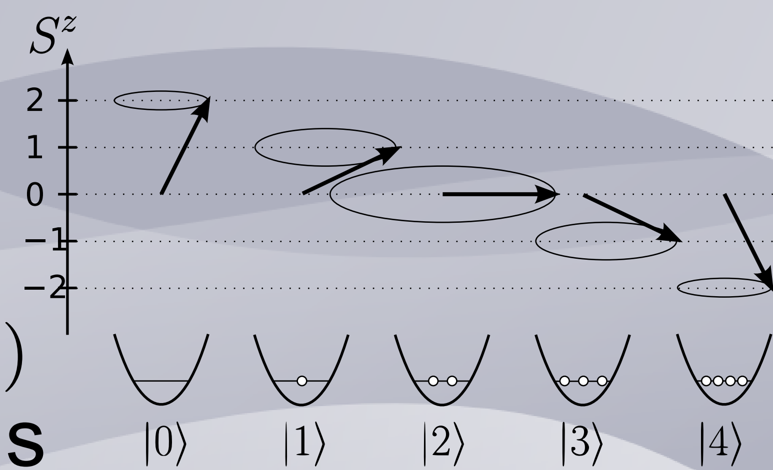
1.3 Introduction: Spin wave theory (method)

- determine classical groundstate

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



- expand in terms of bosons
  - Holstein-Primakoff transformation Holstein, Primakoff Phys. Rev. ('40)
  - Expansion in powers of 1/S exact for $S \to \infty$, but $S = \mathcal{O}(1)$



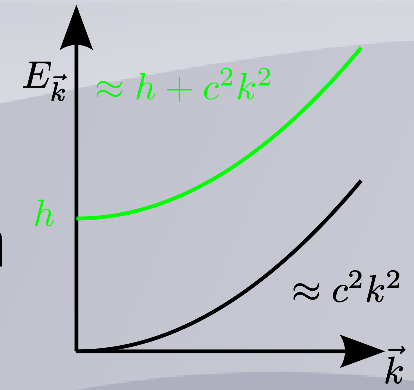
- Interacting theory of bosons
  - Methods of quantum mechanics
  - Theory of many particle systems

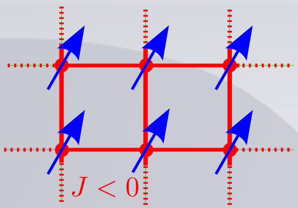
$$H = \sum_{\vec{k}} E_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3} \Gamma^3(\vec{k}_1,\vec{k}_2,\vec{k}_3) b^\dagger_{\vec{k}_1} b_{\vec{k}_2} b_{\vec{k}_3} + \sum_{1,2,3,4} \Gamma^4(1,2;3,4) b^\dagger_1 b^\dagger_2 b_3 b_4 + \ldots$$

1.4 Introduction: Spin waves (general results)

- Ferromagnet
  (exchange interaction only)
  - Quadratic excitation spectrum
  - Vanishing interaction at k=0

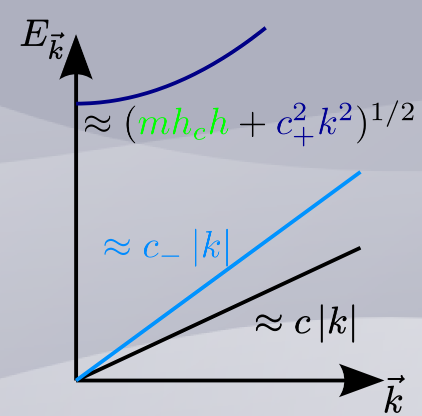
$$\Gamma^4 \sim -(\vec{k}_1 \cdot \vec{k}_2 + \vec{k}_3 \cdot \vec{k}_4)$$

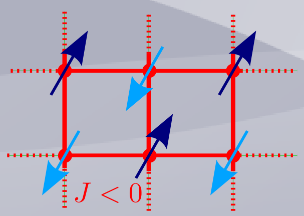


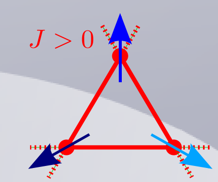


- Antiferromagnet
  - Linear spectrum
    (Goldstone mode)
  - Two modes in magnetic field
    (2 sublattices)
  - Divergent interaction vertices

$$\Gamma^4 \sim \sqrt{\frac{|\vec{k}_1||\vec{k}_2|}{|\vec{k}_3||\vec{k}_4|}} \left(1 \pm \frac{\vec{k}_1 \cdot \vec{k}_2}{|\vec{k}_1||\vec{k}_2|}\right)$$







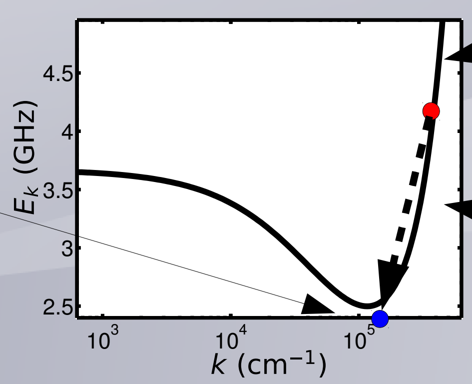
Hasselmann, Kopietz, EPL ('06)
Chernychev, Zhitomirsky, PRB ('09)
Veillette *et al.* PRB ('05)
AK, *et al.* PRB ('08)

2.1 Model system 1: Thin-film ferromagnet

- Motivation: Experiments on YIG (yttrium iron garnet) films
  - Excitation and detection of spin waves with high energy resolution (parametric pumping, Brillouin light scattering spectroscopy)
  - Low spin-wave damping in YIG
  - Good experimental control

Observation of the occupation number using microwave antennas or Brillouin Light Scattering (BLS)
Sandweg, *et al.*
Rev. Sci. Instrum. ('10)

Condensation of magnons at room temperature!
Demokritov *et al.* Nature ('06)

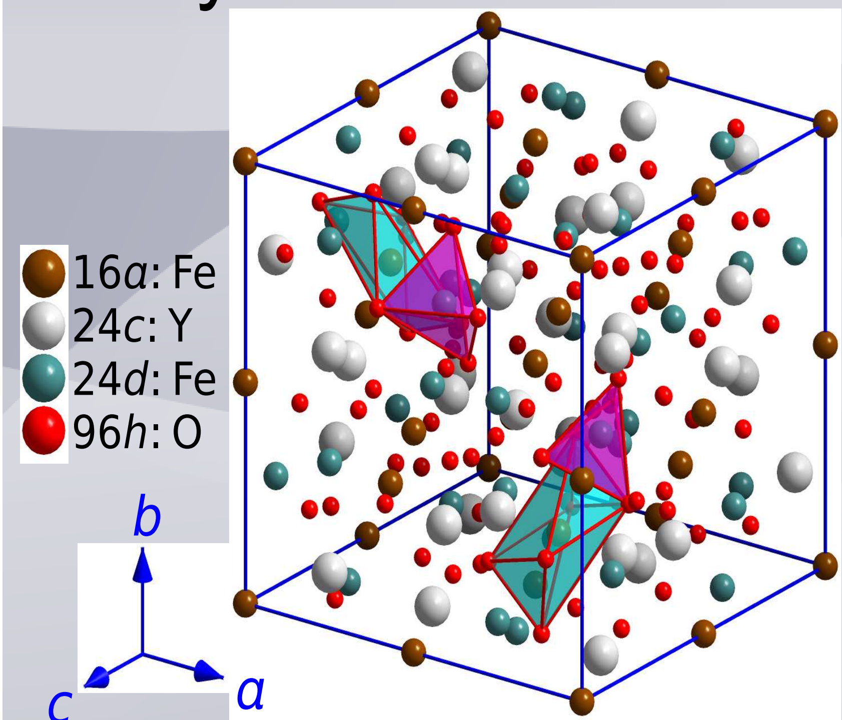


Parametric pumping of magnons at high k-vectors creates magnetic excitations

Open questions:
Time evolution of magnons: Non-equilibrium physics of interacting quasiparticles? Coupling to other degrees of freedom, thermalization?
Kloss, Kopietz PRB ('11)

2.2 Model system 1: Microscopic model

Crystal structure of YIG

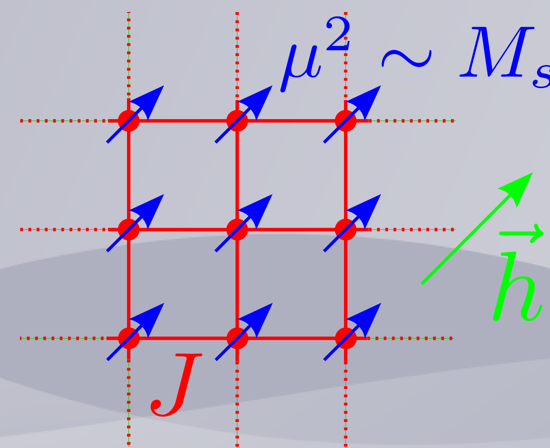


space group: **Ia3d**
Y: 24(c) white
Fe: 24(d) green
Fe: 16(a) brown
O: 96(h) red

Magnetic system:
40 magnetic ions in elementary cell
40 magnetic bands

Gilleo, Geller Phys. Rev. ('58)

Microscopic Hamiltonian



reduced symmetry

quantum spin S ferromagnet

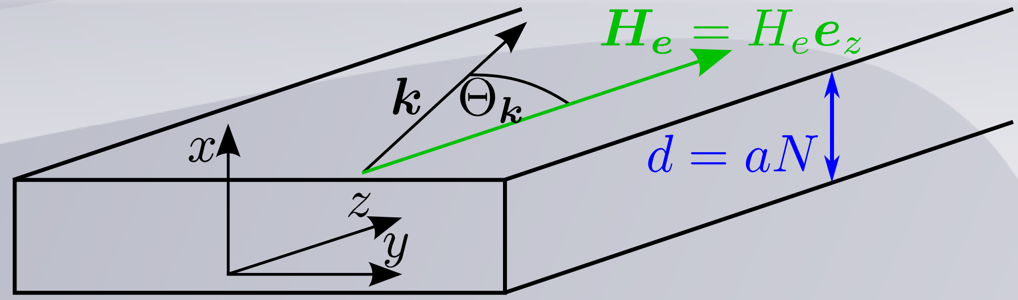
Zeeman term

dipole-dipole interactions

$$\hat{H}_{\mathrm{mag}} = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j - h\sum_i S_i^z - \frac{1}{2}\sum_i \sum_{j\neq i} \frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3(\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j\right]$$

2.3 Model system 1: Linear spin-wave theory

- Geometry (thin film)



- Numerical approach
  - Ewald summation technique
  - Diagonalization of 2N x 2N matrix

$$H_2 = \begin{pmatrix} A_{\vec{k}} & B_{\vec{k}} \\ B^*_{-\vec{k}} & -A^T_{-\vec{k}} \end{pmatrix}$$

- Analytic approaches
  - Approximation for lowest mode
  - Bogoliubov transformation

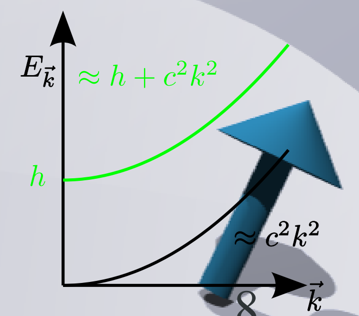
structure factor

$$E_{\vec{k}} = \sqrt{[h + \rho_{\mathrm{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}})\sin^2\Theta_{\vec{k}}][h + \rho_{\mathrm{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

$$\Delta = 4\pi\mu M_S$$

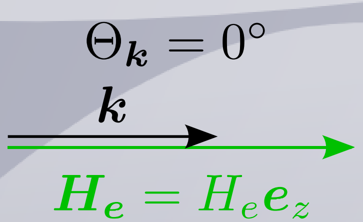
  - No dipolar interaction: known result

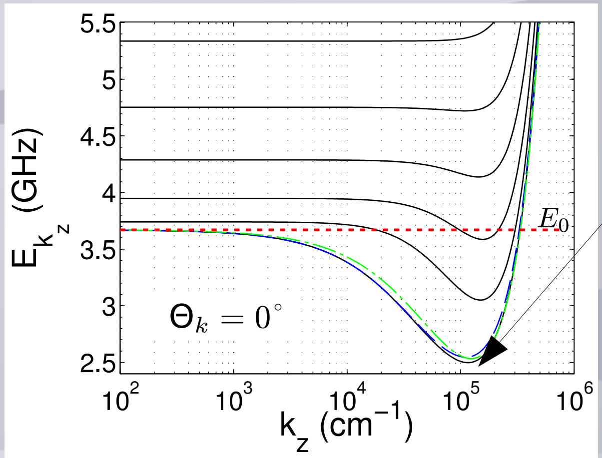
$$E_{\vec{k}} = h + \rho_{\mathrm{ex}}\vec{k}^2 \qquad \Delta = 0$$



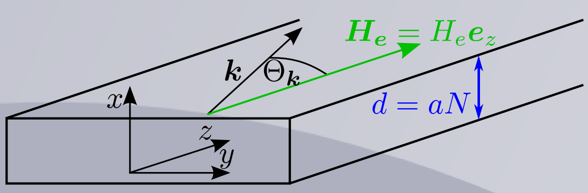
2.4 Model system 1: Results for spectra

- Parallel mode





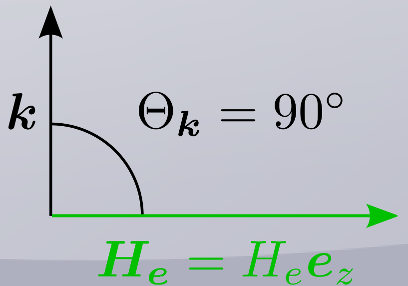
Minimum for BEC
Demokritov *et al.*
Nature ('06)

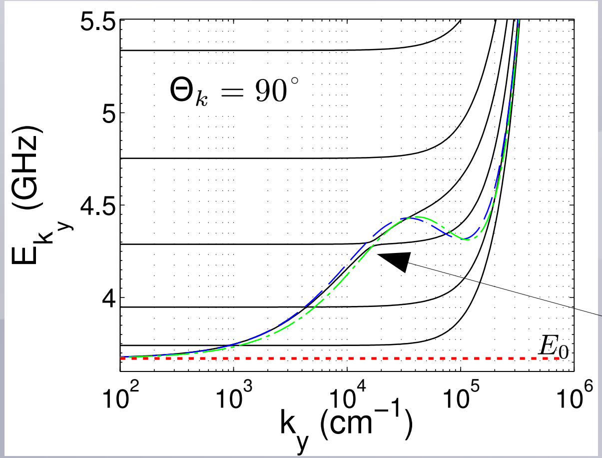


$$d = 400a \approx 0.5 \mu\text{m}$$
$$N = 400$$
$$H_e = 700 \text{ Oe}$$

$$E_0 = \sqrt{h(h + 4\pi\mu M_s)}$$

- Perpendicular mode



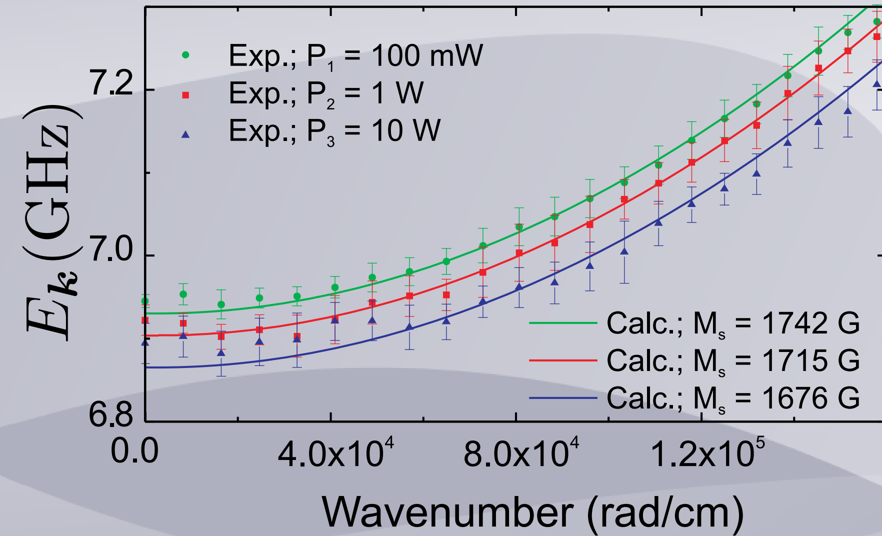


Hybridization:
surface mode

2.5: Model system 1: Comparison to experiments

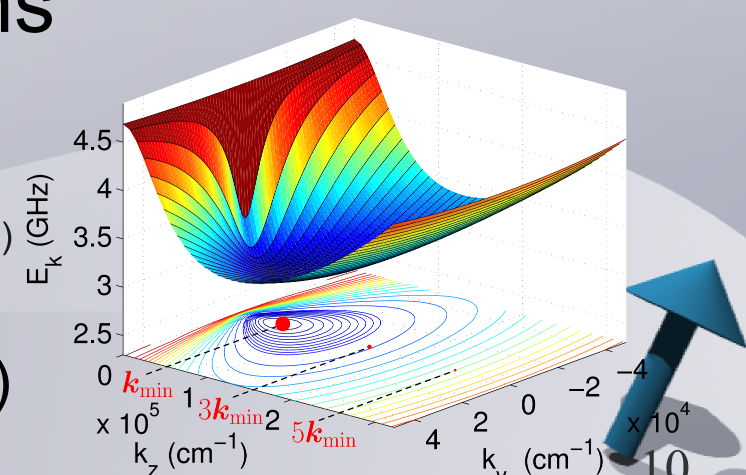
- Excitation and detection of spinwaves using Brillouin light scattering spectroscopy (BLS)

$$\Theta_{\vec{k}} = 90^\circ$$



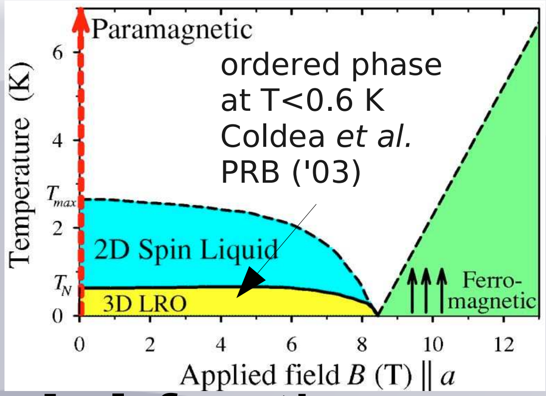
- Outlook, future investigations
  - Condensation of magnons (nonequilibrium, finite momentum)

Hick *et al.* EPJB ('10)



  - Interactions (magnon-magnon, magnon-phonon)

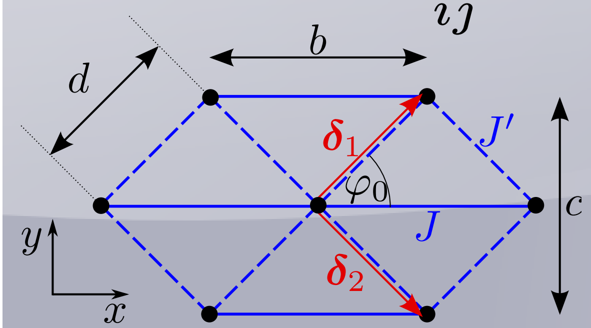
3.1 Model system 2: Triangular antiferromagnet



- Motivation: Frustrated magnets
  - Rich phase diagram
  - Strong quantum effects due to large fluctuations
- Exactly **known microscopic model** for the frustrated antiferromagnet Cs_2CuCl_4

spatially anisotropic exchange — Dzyaloshinsky-Moriya anisotropy — magnetic field (perpendicular to plane)

$$\hat{H}_{\text{spin}} = \frac{1}{2}\sum_{ij}\left[J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \mathbf{D}_{ij}\cdot(\mathbf{S}_i\times\mathbf{S}_j)\right] - \sum_i \mathbf{h}\cdot\mathbf{S}_i\,,$$



$$J_{ij} = J(\mathbf{R}_i - \mathbf{R}_j) = \begin{cases} J & \text{if } \mathbf{R}_i - \mathbf{R}_j = \pm(\delta_1+\delta_2) \\ J' & \text{if } \mathbf{R}_i - \mathbf{R}_j = \pm\delta_1 \text{ or } \pm\delta_2 \end{cases}$$

$$\mathbf{D}_{ij} = \pm D\mathbf{e}_z$$

J=0.37 meV
J'=0.13 meV
D=0.02 meV

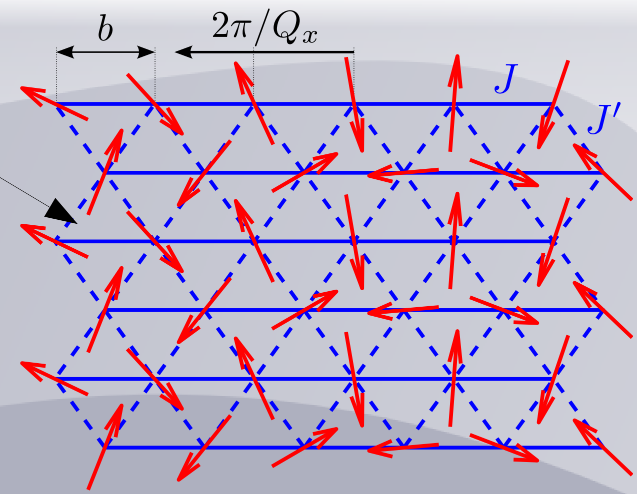
Coldea *et al.* PRL ('02)

3.2 Model system 2: Spin-wave approach

- Classical groundstate: "cone state"

Veillette *et al.* PRB ('05)

Projection on the plane



- Spin-wave spectra

$$E_{\mathsf{k}} = \sqrt{(A^{+}_{\mathsf{k}})^2 - B^2_{\mathsf{k}}} + A^{-}_{\mathsf{k}} \neq E_{-\mathsf{k}}$$

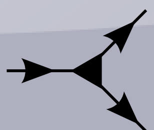
symmetric with respect to k

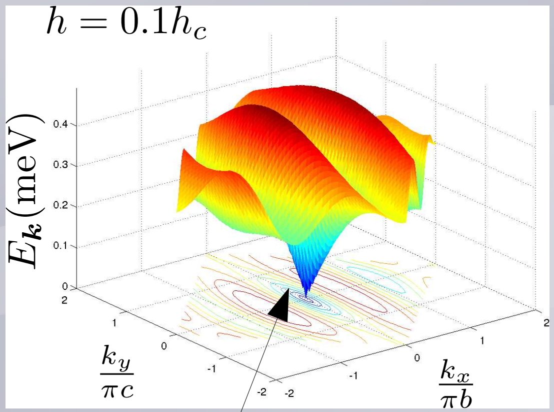
antisymmetric, but $\propto \mathsf{k}^3$ (Dzyaloshinsky-Moria anisotropy)

$$E_{\mathsf{k}} = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2} + \mathcal{O}(\mathsf{k}^3)$$

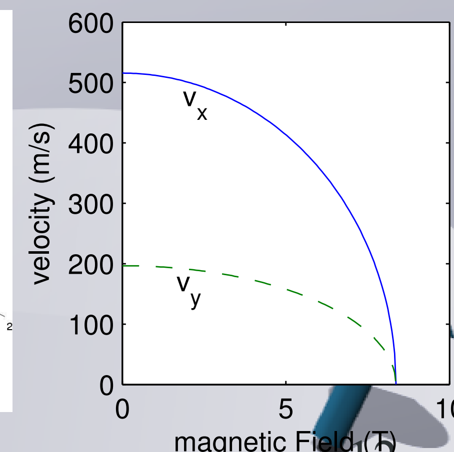
- Interactions
  - Large for finite field

$$\Gamma_3^{b^\dagger b^\dagger b}(\mathsf{k}_1, \mathsf{k}_2; \mathsf{k}_3) \approx -\frac{h}{h_c}\sqrt{1-\left(\frac{h}{h_c}\right)^2}\frac{\sqrt{2S}}{i}\frac{h_c}{S}$$





Goldstone mode: U(1) symmetry



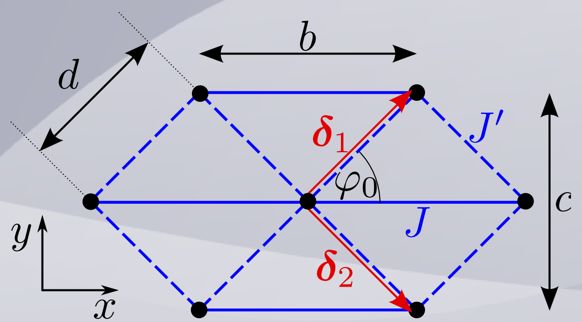
3.3 Model system 2: Coupling to lattice vibrations

- Include **lattice vibrations**

$$H = \frac{1}{2}\sum_{ij}\left[J_{ij}\mathsf{S}_i\cdot\mathsf{S}_j + \mathsf{D}_{ij}\cdot(\mathsf{S}_i\times\mathsf{S}_j)\right] - \sum_i \mathsf{h}\cdot\mathsf{S}_i$$

exchange

DM anisotropy



$$+\sum_{\mathsf{k}\lambda}\left[\frac{P_{-\mathsf{k}\lambda}P_{\mathsf{k}\lambda}}{2M} + \frac{M}{2}\omega^2_{\mathsf{k}\lambda}X_{-\mathsf{k}\lambda}X_{\mathsf{k}\lambda}\right]$$

$$\omega_{\mathsf{k}\lambda} = c_\lambda(\hat{\mathsf{k}})|\mathsf{k}|$$

phonon dispersion (acoustic)

- **Spin-phonon coupling** via expansion of exchange integrals

$$J_{ij} = J(\mathsf{R}_{ij}) + (\mathsf{X}_{ij}\cdot\nabla_{\mathsf{R}})J(\mathsf{r})|_{\mathsf{r}=\mathsf{r}_{ij}} + \ldots = J(\mathsf{R}_{ij}) + \mathsf{X}_{ij}\cdot\mathsf{J}^{(1)}_{ij} + \ldots$$

bare exchange

magnon phonon coupling

$$D_{ij} \approx D(\mathsf{R}_{ij})$$

Chakraborty, Tucker Physica A ('87)

3.4 Model system 2: Calculation of observables

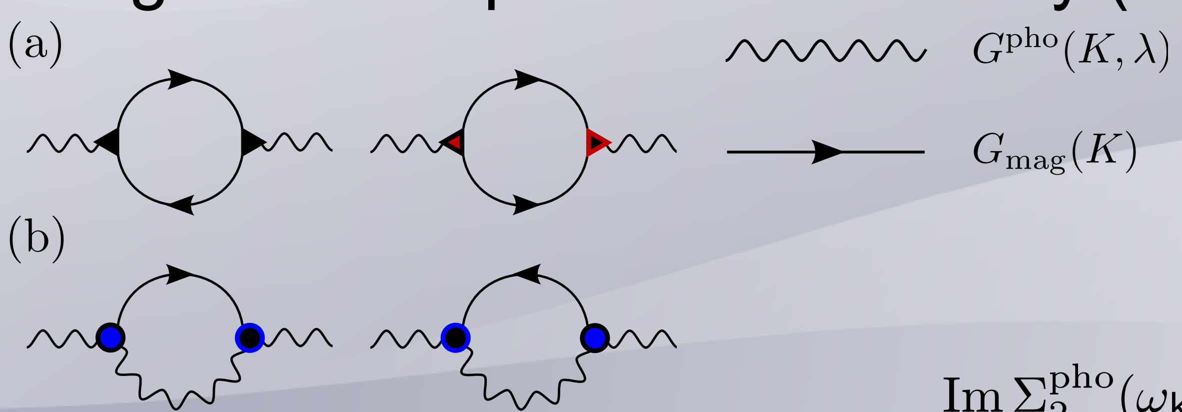
- Shift of the **phonon velocity**
  - Integrate out magnetic degrees of freedom

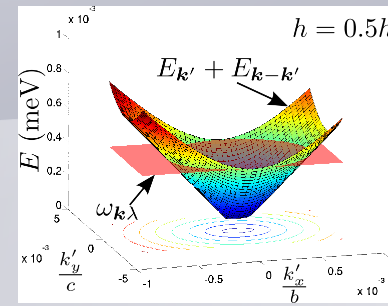
$$e^{-S^{\mathrm{2pho}}_{\mathrm{eff}}[X]} = \int \mathcal{D}[\beta,\bar{\beta}]\, e^{-S^{\mathrm{2pho}}[X]-S_{\mathrm{2mag}}[\bar{\beta},\beta]-S^{\mathrm{1pho}}_{\mathrm{1mag}}[X,\bar{\beta},\beta]}$$

$$\frac{(\Delta c_\lambda)_{\mathrm{tot}}}{c_\lambda} = \lim_{|\mathsf{k}|\to 0}\left(\sqrt{1-\frac{\Sigma_0^{\mathrm{pho}}(\mathsf{k},\lambda)}{\omega^2_{\mathsf{k}\lambda}}} - 1 + \frac{\left|\Gamma^{X\beta}_{\mathsf{k}}\cdot \mathsf{e}_{\mathsf{k}\lambda}\right|^2}{2M\omega^3_{\mathsf{k}\lambda}}\right)$$

Two contributions:
1. Classical spin background
2. Hybridization to magnetoelastic waves

- Ultrasound **attenuation rate**
  - Diagrammatic perturbation theory (1/S expansion)

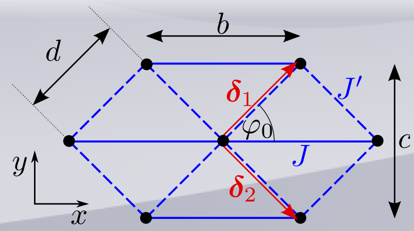




$$\gamma_{\mathsf{k}\lambda} = -\frac{\mathrm{Im}\,\Sigma_2^{\mathrm{pho}}(\omega_{\mathsf{k}\lambda}+i0^+,\mathsf{k},\lambda)}{2\omega_{\mathsf{k}\lambda}}$$

3.5 Model system 2: Comparison to experiments

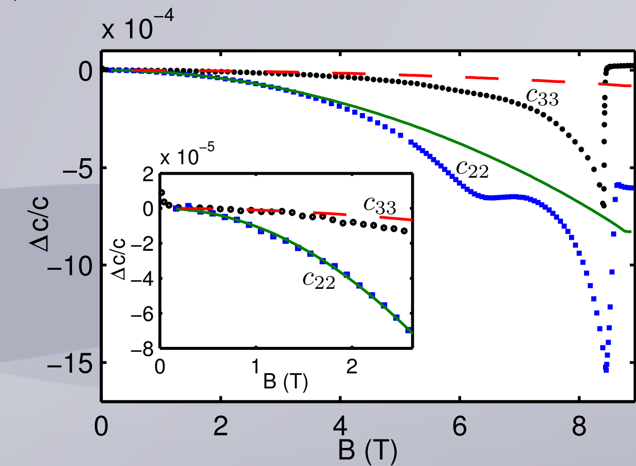
- Model



$$J(x) = J(b)e^{-\kappa(x-b)/b}$$

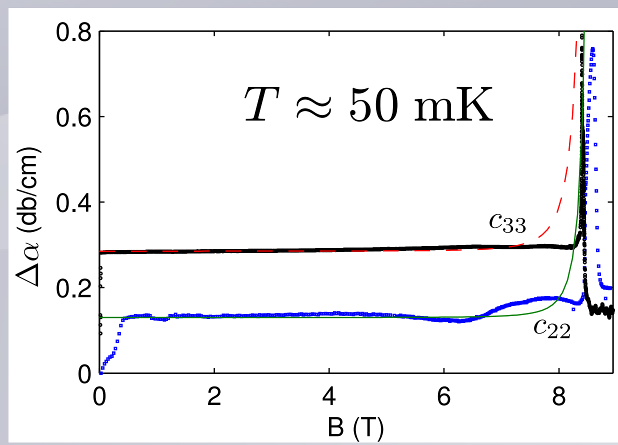
$$J'(r) = J'(d)e^{-\kappa'(r-d)/d}$$

- Shift of ultrasound velocity for c_{22} mode (P. T. Cong):
  - Fix parameters: $\kappa \approx 15 \quad \kappa' \approx 51$
  - Project result for c_{33} mode without adjustable parameters



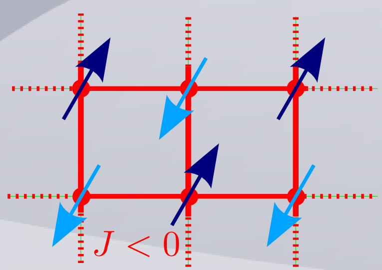
- Attenuation rate calculate from parameters

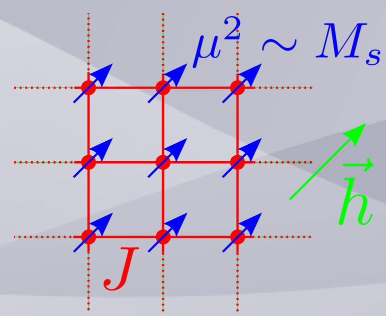
$$\gamma_{\mathsf{k}\lambda} \approx \frac{\pi^2}{64}\left(\frac{\mathsf{k}^2}{2M}\right)\left(\frac{S^2 c_\lambda^2 \mathsf{k}^2}{V_{\mathrm{BZ}} v_x v_y}\right)\frac{\left[\mathsf{f}_1^{X\beta}(\hat{\mathsf{k}})\cdot \mathsf{e}_{\mathsf{k}\lambda}\right]^2}{(1-h/h_c)^2}$$

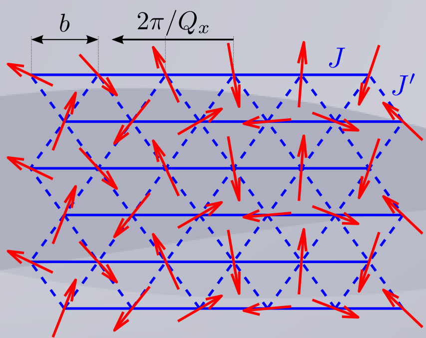


4 Summary

- Theoretical investigations on **Heisenberg magnets with reduced symmetry**
  - 3 different model systems (2 presented in detail)



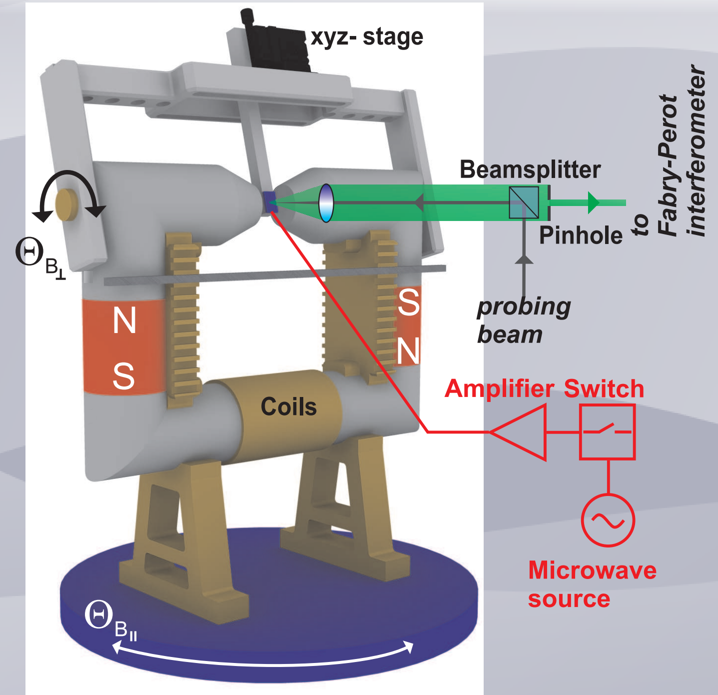


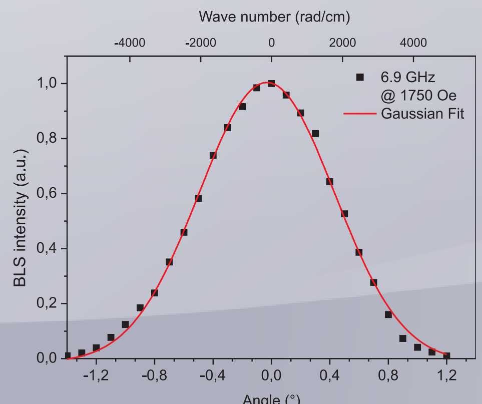


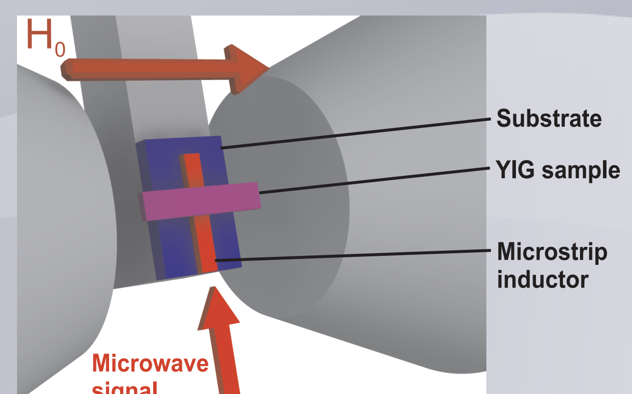
- Application and further **development of concepts** using the spin-wave approach
- Connection to recent experimental research and comparison of corresponding results

5.1 Experimental details BLS spectroscopy

- Wavevector resolved BLS setup (C. Sandweg, TU Kaiserslautern)
- large wave vector range
- high resolution

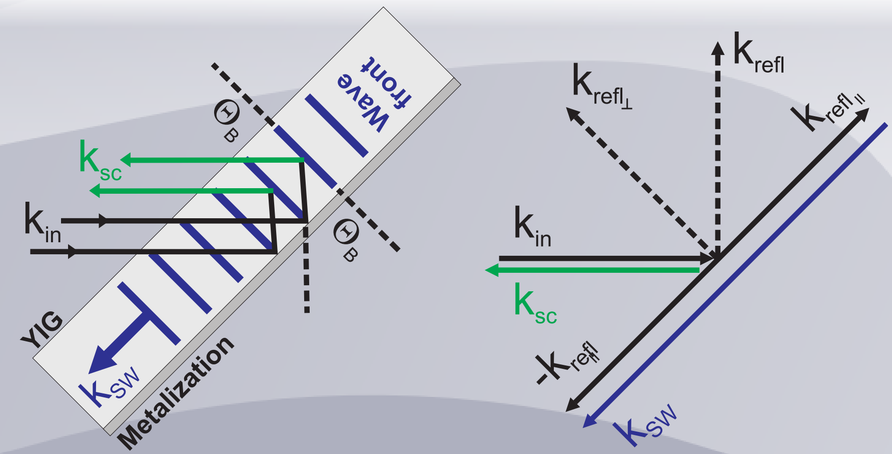


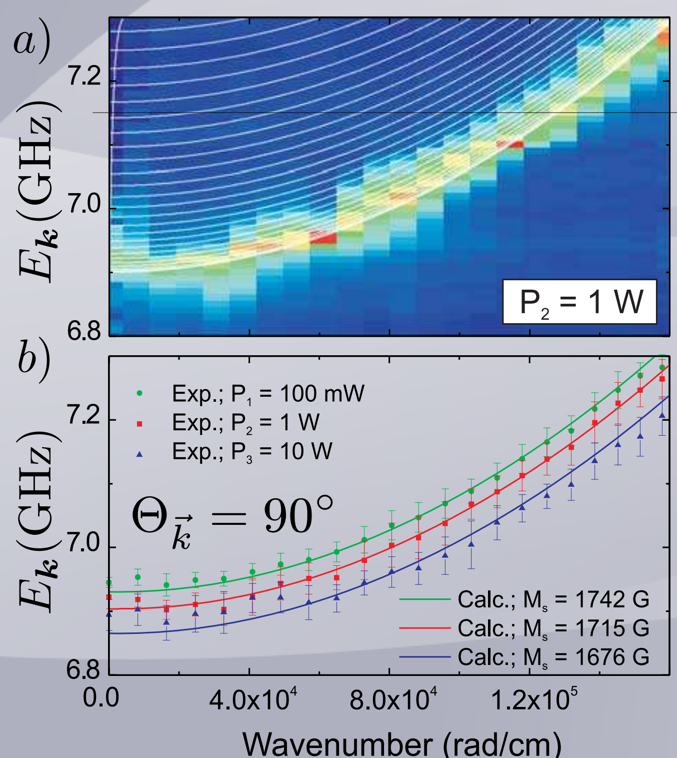




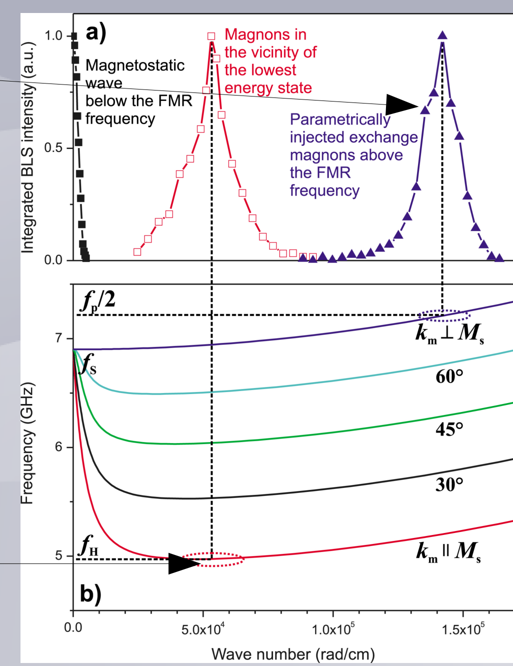
5.2: Experimental Details
BLS spectroscopy

- Scattering of light on grating created from spin waves





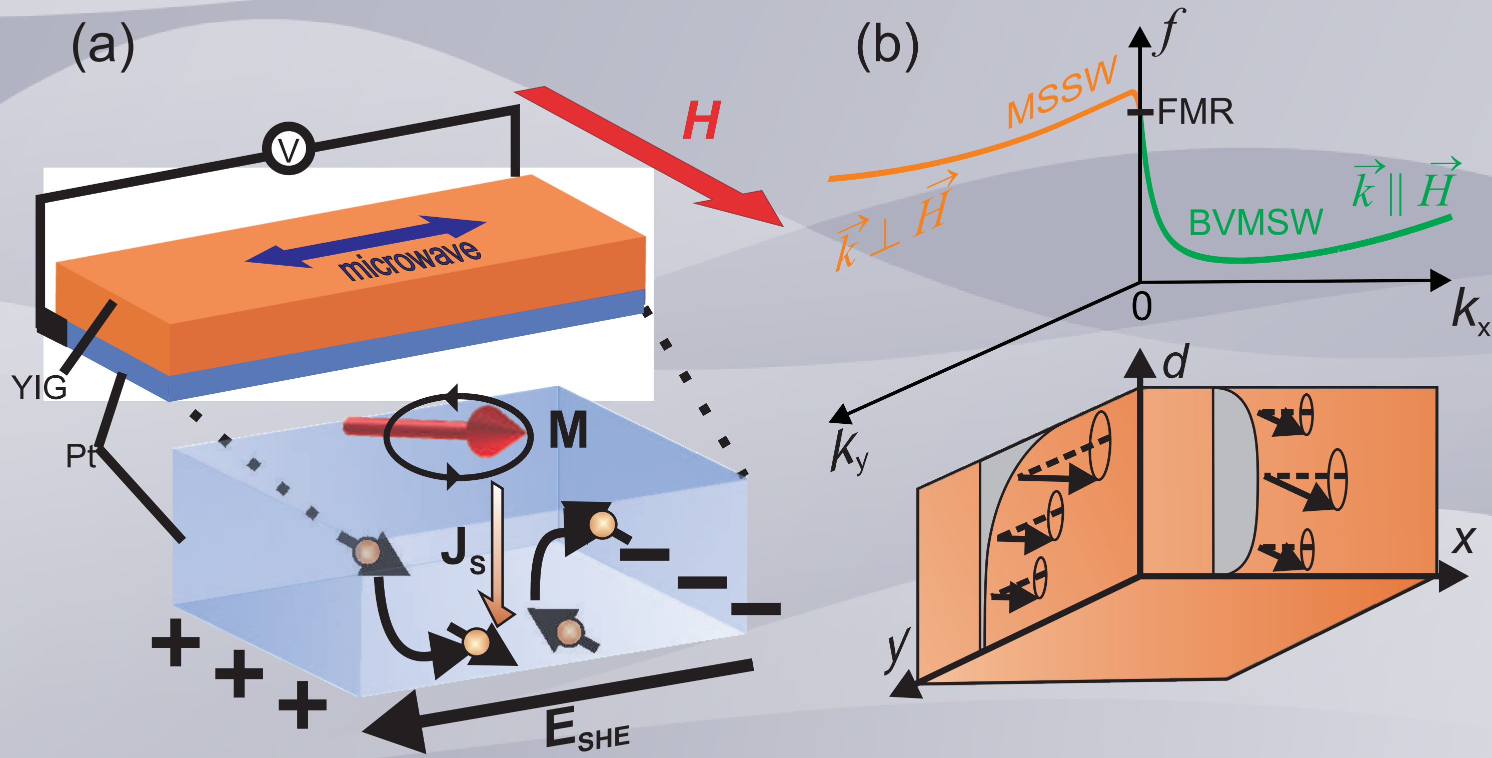
cut along line



wavenumber from geometry
energy from interferometer

5.3 Experimental details
Techniques: Thin-film magnets

- Inverse spin-hall effect (ISHE):
  spin polarized current

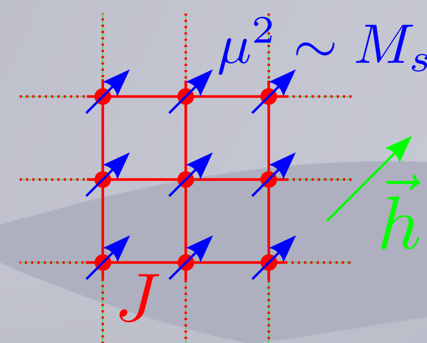


C. Sandweg, et al., Appl. Phys. Lett, '10

5.4 Experimental details
Material parameters for YIG

- Heisenberg magnet with dipole-dipole interactions

$$\hat{H}_{\text{mag}} = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j - h\sum_i S_i^z$$
$$-\frac{1}{2}\sum_i\sum_{j\neq i}\frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3(\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j\right]$$



material parameters

$a = 12.376$ Å

Gilleo *et al.* '58

$4\pi M_s = 1750$ G

Tittmann '73

$\frac{\rho_{\text{ex}}}{\mu} = 5.17\cdot 10^{-13}\text{Oe m}^2$

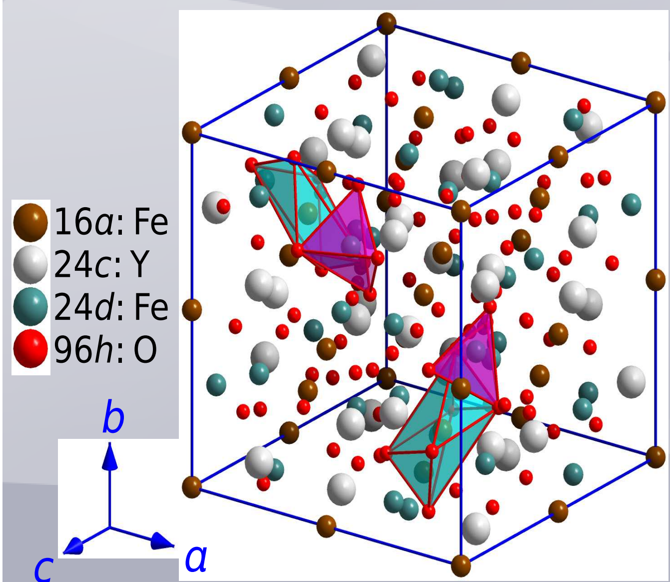
Cherepanov *et al.* '93

alternatively

$J = 1.29$ K

$S = 14.2 \quad \mu = 2\mu_B$

Tupitsyn *et al.* '08



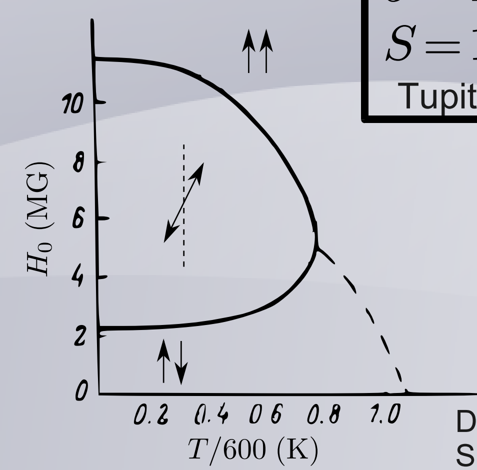
Crystal structure:

space group: **Ia3d**
Y: 24(c) white
Fe: 24(d) green
Fe: 16(a) brown
O: 96(h) red

Gilleo *et al.* '58

Magnetic system:
40 magnetic ions in elementary cell
40 magnetic bands

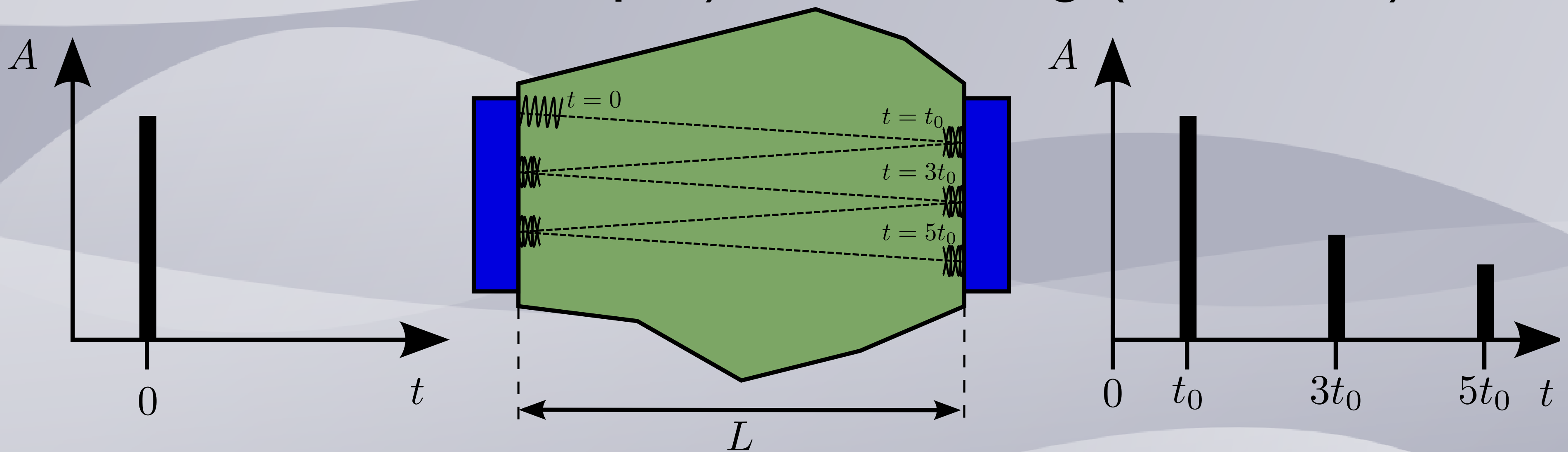
Elastic system:
160 atoms in elementary cell
3x160 phonon bands



Druzinin et. al
Sov. Phys. Solid. ('81)

5.5 Experimental details Ultrasonic technique

- pulse echo method (phase-sensitive detection technique), P. T. Cong (Uni FFM)

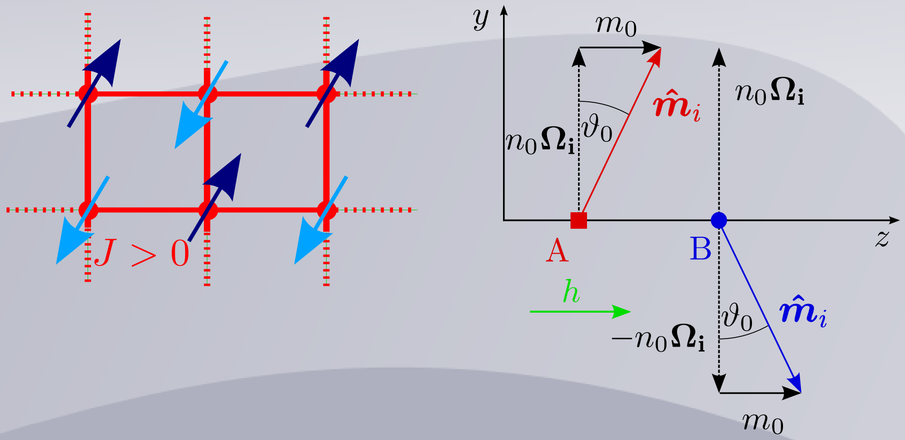


- sound velocity $c_\lambda = L/t_0$
- change of sound velocity $\frac{df}{f} = \frac{dc_\lambda}{c_\lambda}$
- (relative) attenuation rate $\Delta\alpha \propto \frac{1}{L}\ln(A_1/A_0)$

6.1 Theoretical details
QAF in a magnetic field

- model Hamiltonian

$$\hat{H} = \frac{1}{2}\sum_{ij} J_{ij}\mathsf{S}_i \cdot \mathsf{S}_j - h\sum_i S_i^z$$



- spin-wave modes

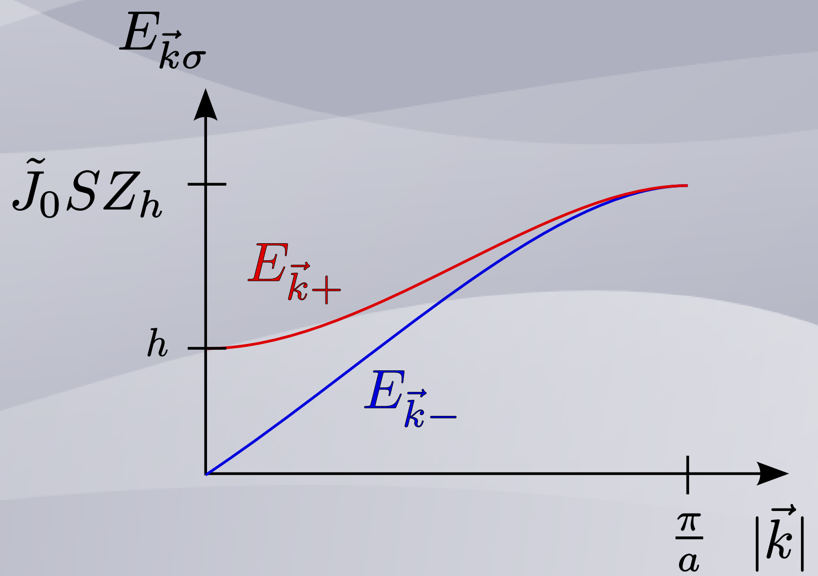
$$E^2_{\vec{k}+} = h^2 + c_+^2\vec{k}^2,$$

$$E^2_{\vec{k}-} = c_-^2\vec{k}^2,$$

- spin-wave velocities

$$c_+^2 = c_0^2(1 - \frac{3}{\Delta_0^2}h^2)$$

$$c_-^2 = c_0^2(1 - \frac{1}{\Delta_0^2}h^2)$$



6.2 Theoretical details QAF in a magnetic field

- Hermitian parametrization (compare harmonic oscillator)

$$[\hat{X}_{\vec{k}\sigma}, \hat{P}_{\vec{k}'\sigma'}] = i\delta_{\vec{k},-\vec{k}'}\delta_{\sigma,\sigma'}$$

$$\hat{\Psi}_{\vec{k}\sigma} = p_\sigma \left[\sqrt{\frac{\nu_{\vec{k}\sigma}}{2}} \hat{X}_{\vec{k}\sigma} + \frac{i}{\sqrt{2\nu_{\vec{k}\sigma}}} \hat{P}_{\vec{k}\sigma} \right]$$

- physical meaning for $k \ll 1$

$\hat{P}_{\vec{k}\sigma}$: uniform spin fluctuations

$\hat{X}_{\vec{k}\sigma}$: staggered spin fluctuations

- effective action

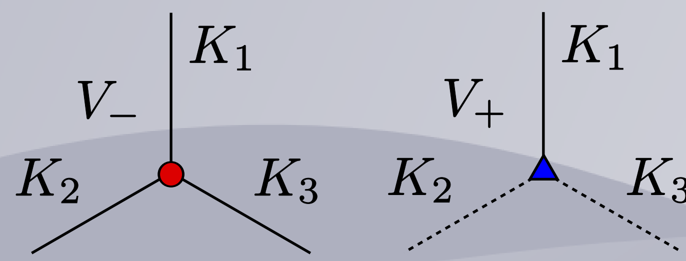
$$\begin{aligned} S_{\text{eff}}[X_\sigma] &= S_0 + \frac{\beta}{2} \sum_{K\sigma} \frac{E^2_{\mathsf{k}\sigma} + \omega^2}{\Delta_{\mathsf{k}\sigma}} X_{-K\sigma} X_{K\sigma} \\ &+ \beta \sqrt{\frac{2}{N}} \sum_{K_1 K_2 K_3} \delta_{K_1+K_2+K_3,0} \Big[\frac{1}{3!} \Gamma^{(3)}_{---}(K_1, K_2, K_3) X_{K_1-} X_{K_2-} X_{K_3-} \\ &+ \frac{1}{2!} \Gamma^{(3)}_{-++}(K_1; K_2, K_3) X_{K_1-} X_{K_2+} X_{K_3+} \Big] \end{aligned}$$

6.3 Theoretical details
QAF in a magnetic field

- diagrammatic perturbation theory

$$S_{\text{eff}}^{\text{int}}[X_\sigma] \quad = \beta\sqrt{\frac{2}{N}}\sum\left[\frac{1}{3!}V_-^{(3)}X_-X_-X_- + \frac{1}{2!}V_+^{(3)}X_-X_+X_+\right]$$

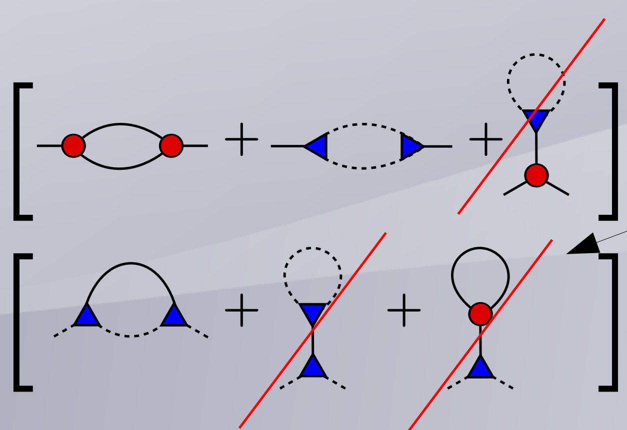
$$G_\sigma(K) = \frac{\Delta_{\mathsf{k}\sigma}}{E_{\mathsf{k}\sigma}^2 + \omega^2}$$



- perturbation theory: 1/S corrections to self energy

$\Sigma_- = -\frac{1}{2}$

$\Sigma_+ = -\frac{1}{2}$



no frequency dependence, negligible

6.4 Theoretical details
QAF in a magnetic field

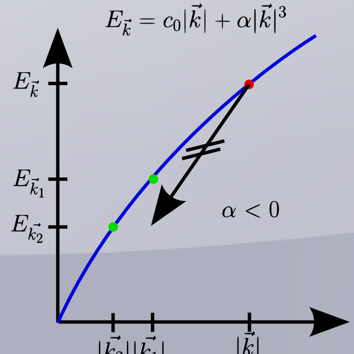
- results
  - spin-wave velocity

nonanalytic in h²

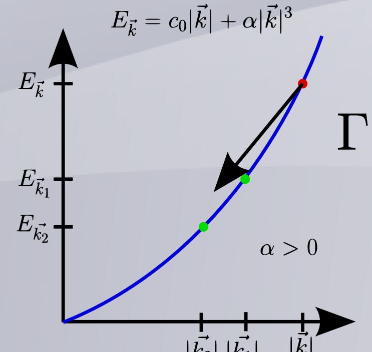
$$\frac{c_-^2}{c_0^2} \approx 1 - \frac{6\sqrt{3}\,\tilde{h}^2}{\pi^2 S} \ln\left(\frac{2}{\tilde{h}}\right) \qquad D = 3$$

$$\frac{c_-^2}{c_0^2} \approx 1 - \frac{2\tilde{h}}{\pi S} \qquad D = 2 \qquad\qquad \tilde{h} = \frac{h}{\Delta_0}$$

  - magnon damping $\Gamma_{\vec{k}-} \propto \frac{1}{S}\left(\frac{h}{h_c}\right)^2 \left(\sqrt{6\bar{A}_-}\right)^{D-3} a^{D+1}|\vec{k}|^{2D-1}$



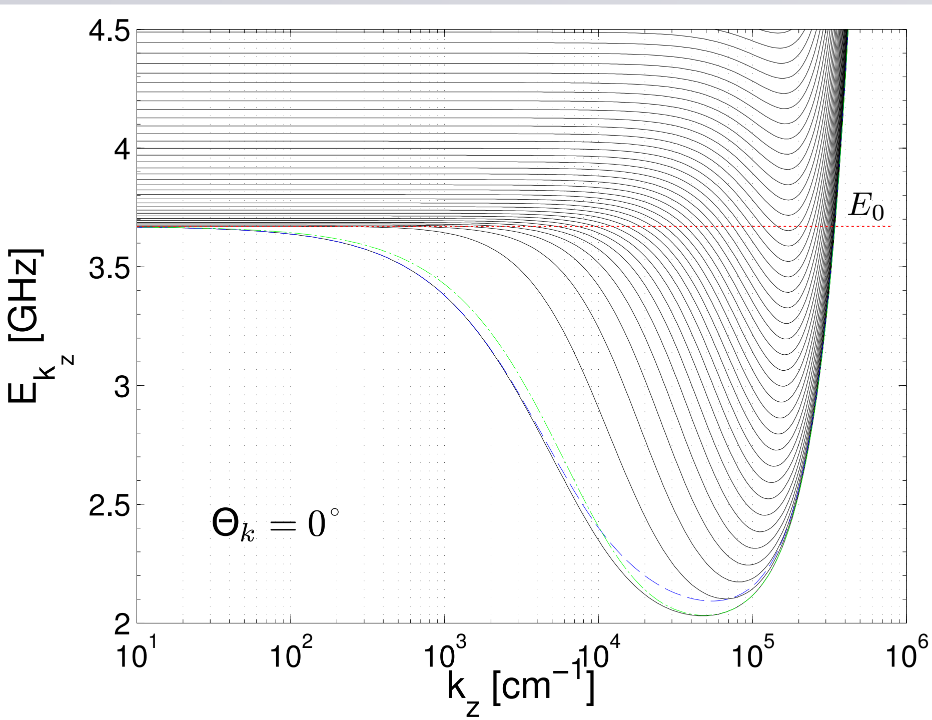
$\Gamma_{\vec{k}} \approx 0$

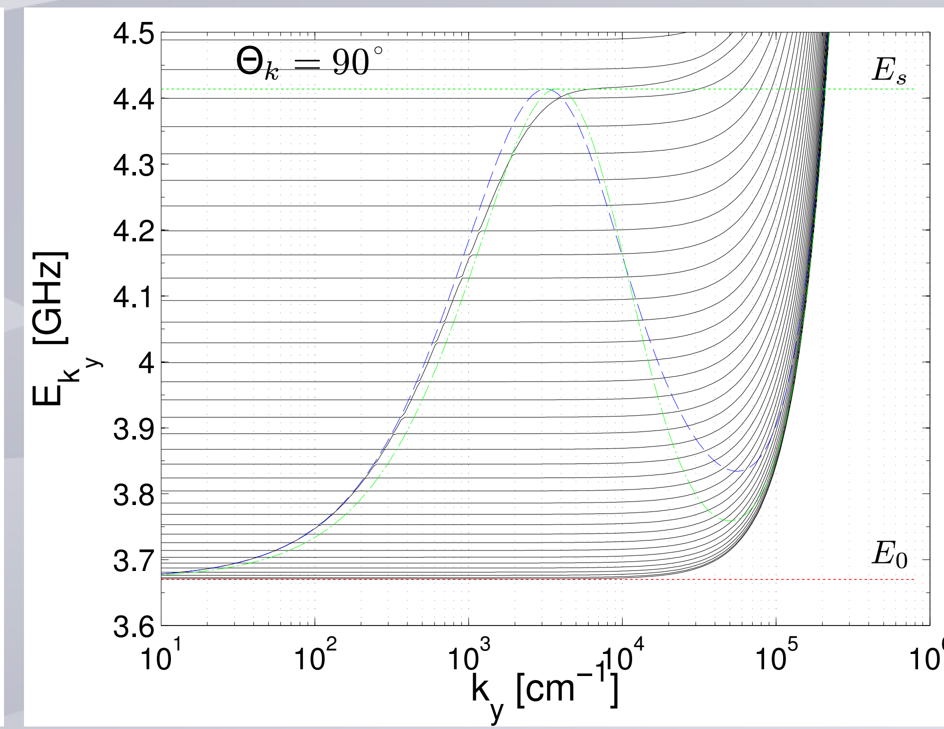


$\Gamma_{\vec{k}} \propto k^{2D-1}$

6.5 Theoretical details
Spin-wave theory for thin films

- spectrum for realistic samples



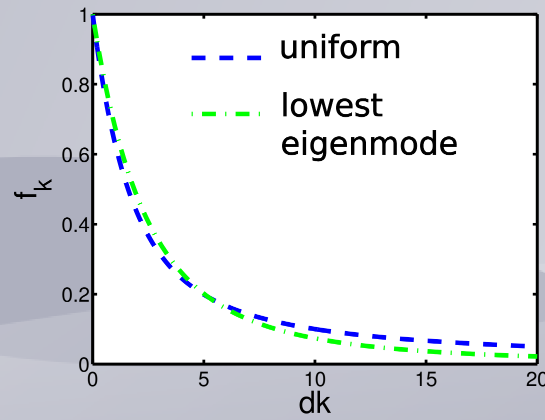


$d = 4040a \qquad H_e = 700 \text{ Oe}$

6.6: Theoretical details
Spin-wave theory for thin films

- Comparison $E_{\vec{k}} = \sqrt{[h + \rho_{\mathrm{ex}} \vec{k}^2 + \Delta(1 - f_{\vec{k}}) \sin^2 \Theta_{\vec{k}}][h + \rho_{\mathrm{ex}} \vec{k}^2 + \Delta f_{\vec{k}}]}$
  - analytical result
    - uniform mode approximation

$$f_{\vec{k}} = \frac{1 - e^{-|\vec{k}|d}}{|\vec{k}|d} \qquad \Delta = 4\pi\mu M_S$$



    - eigenmode approximation

$$f_{\vec{k}} = 1 - |\vec{k}d| \frac{|\vec{k}d|^3 + |\vec{k}d|\pi^2 + 2\pi^2(1 + e^{-|\vec{k}d|})}{(\vec{k}^2 d^2 + \pi^2)^2}$$

  - numerical result