Magnon-Phonon Interactions: elastic constants & ultrasonic attenuation rate in Cs_2CuCl_4

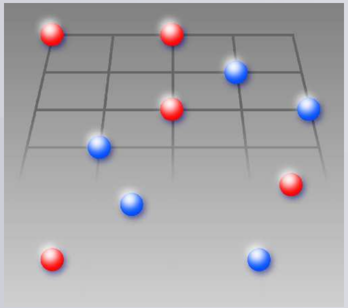
Andreas Kreisel, Peter Kopietz,
Pham Thanh Cong, Bernd Wolf, and Michael Lang

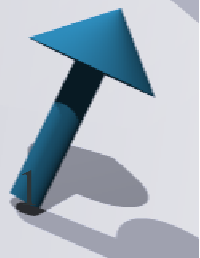
Institut für Theoretische Physik
&
Physikalisches Institut
Goethe Universität Frankfurt
Germany

arXiv: 1102:5342

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

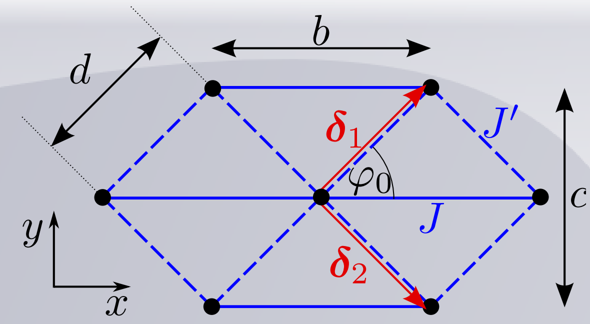
SFB TRR 49





Cs_2CuCl_4 as frustrated antiferromagnet

- model Hamiltonian from high field measurement



spatially anisotropic exchange

Dzyaloshinsky-Moriya anisotropy

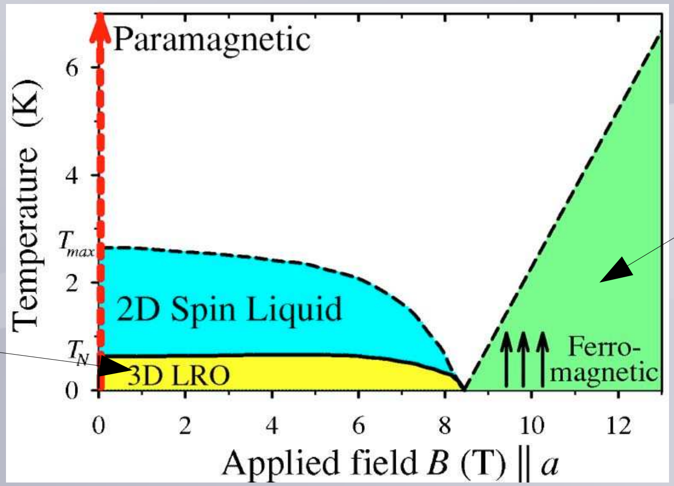
$$\hat{H}_{\rm spin} = \frac{1}{2}\sum_{ij}[J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \mathbf{D}_{ij}\cdot(\mathbf{S}_i\times\mathbf{S}_j)] - \sum_i \mathbf{h}\cdot\mathbf{S}_i\,,$$

$$J_{ij} = J(\mathbf{R}_i - \mathbf{R}_j) = \begin{cases} J & \text{if } \mathbf{R}_i - \mathbf{R}_j = \pm(\delta_1+\delta_2) \\ J' & \text{if } \mathbf{R}_i - \mathbf{R}_j = \pm\delta_1 \text{ or } \pm\delta_2 \end{cases}$$

$$\mathbf{D}_{ij} = \pm D\mathbf{e}_z$$

magnetic field (perpendicular to plane)

- phase diagram



ordered phase at T<0.6 K magnons as excitations

J=0.37 meV
J'=0.13 meV
D=0.02 meV

Coldea *et al.* '03

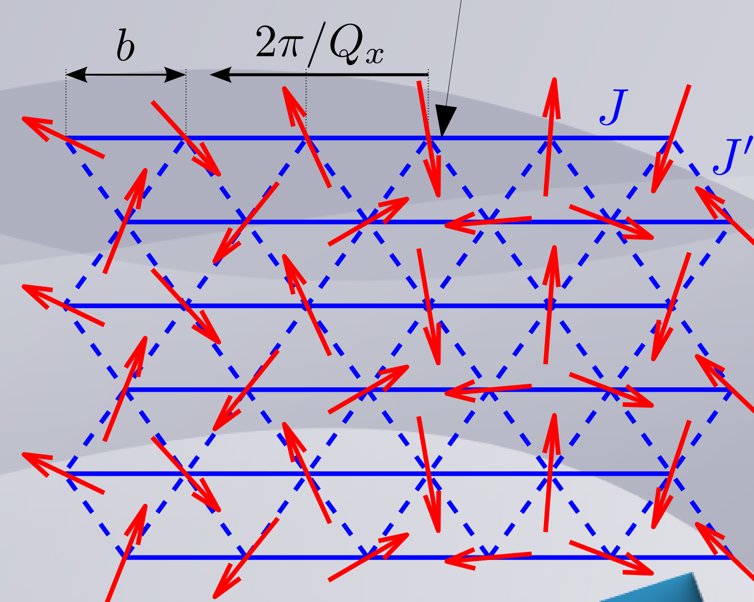
Spin-wave approach

- classical spins: energy

$$E_0^{\mathrm{cl}} = N\frac{S^2}{2}\left[s_\vartheta^2 J_{\mathsf{k}=0} + c_\vartheta^2 J_{\mathsf{Q}}^D\right] - NShs_\vartheta$$

projection of ground state to plane: “Cone state”

- classical ground state: spiral minimization with respect to ϑ and Q



$$NSc_\vartheta\left[Ss_\vartheta\left(J_0^D - J_{\mathsf{Q}}^D\right) - h\right] = 0$$

$$\nabla_{\mathsf{k}}\left(J_{\mathsf{k}} - iD_{\mathsf{k}}\right)_{\mathsf{k}=\mathsf{Q}} = 0$$

$s_\vartheta = \sin\vartheta = h/h_c \qquad c_\vartheta = \cos\vartheta$

$J_{\mathsf{k}} = 2J\cos(k_x b) + 4J'\cos(k_x b/2)\cos(k_y c/2)$

$J_{\mathsf{k}}^D = J_{\mathsf{k}} - iD_{\mathsf{k}}$

$D_{\mathsf{k}} = -4iD\sin(k_x b/2)\cos(k_y c/2)$

Veillette *et al.* '05

Spin-wave approach

- linear spin-wave theory

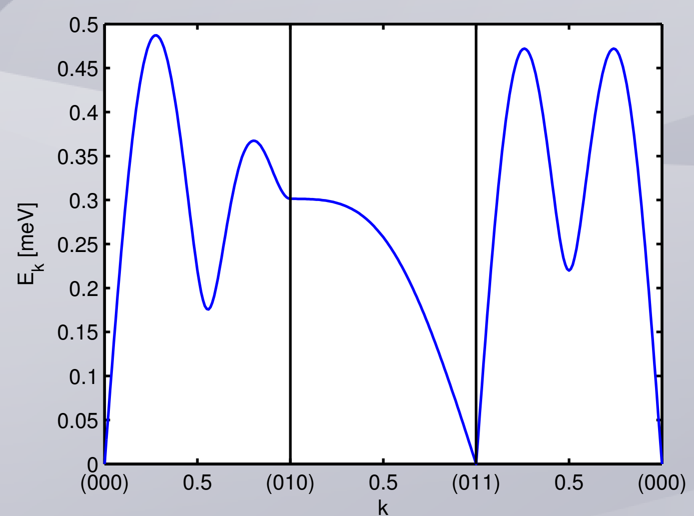
$$H^{2\mathrm{mag}} = \sum_{\mathsf{k}} E_{\mathsf{k}} \beta^\dagger_{\mathsf{k}} \beta_{\mathsf{k}}$$

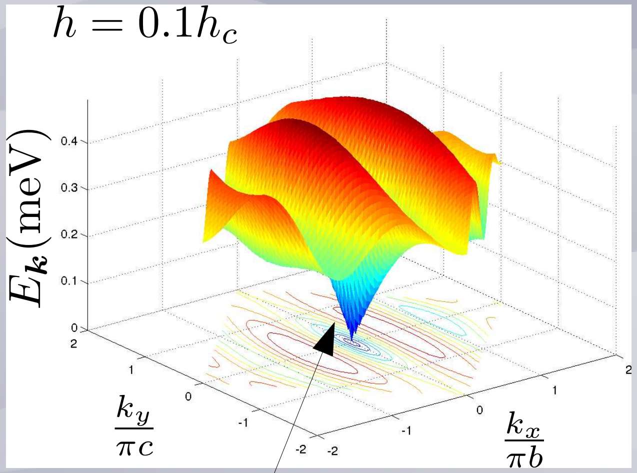
$$E_{\mathsf{k}} = \sqrt{(A^+_{\mathsf{k}})^2 - B^2_{\mathsf{k}}} + A^-_{\mathsf{k}} \neq E_{-\mathsf{k}}$$

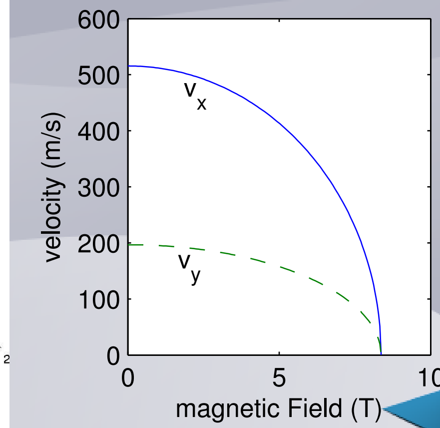
symmetric with respect to k

antisymmetric, but $\propto \mathsf{k}^3$
(Dzyaloshinsky-Moria
anisotropy)

$$E_{\mathsf{k}} = \sqrt{v_x^2 k_x^2 + v_y^2 k_y^2} + \mathcal{O}(\mathsf{k}^3)$$
$$\approx v(\hat{\mathsf{k}})|\mathsf{k}|$$







Goldstone mode: U(1) symmetry

spin wave velocity

$h = 0$ Veillette *et al.* '05 $0 < h < h_c$

Spin-wave approach

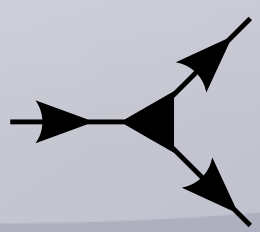
- magnon-magnon interactions
  here: in non-diagonal basis

$$\hat{H}_3 = \frac{1}{\sqrt{N}} \sum_{\mathsf{k}_1\mathsf{k}_2\mathsf{k}_3} \delta_{\mathsf{k}_1+\mathsf{k}_2+\mathsf{k}_3,0} \Big[\frac{1}{2!} \Gamma_3^{b^\dagger b^\dagger b}(\mathsf{k}_1, \mathsf{k}_2; \mathsf{k}_3) b^\dagger_{-\mathsf{k}_1} b^\dagger_{-\mathsf{k}_2} b_{\mathsf{k}_3}$$

$$\frac{1}{2!} \Gamma_3^{b^\dagger bb}(\mathsf{k}_1; \mathsf{k}_2, \mathsf{k}_3) b^\dagger_{-\mathsf{k}_1} b_{\mathsf{k}_2} b_{\mathsf{k}_3} \Big]$$

$$\Gamma_3^{b^\dagger b^\dagger b}(\mathsf{k}_1, \mathsf{k}_2; \mathsf{k}_3) = -c_\vartheta s_\vartheta \frac{\sqrt{2S}}{i} \frac{h_c}{S} + \mathcal{O}(\mathsf{k}^2)$$

constant term for finite magnetic field
(strong interactions at long wavelengths)



$$s_\vartheta = \sin\vartheta = h/h_c \qquad c_\vartheta = \cos\vartheta$$

Spin-wave approach

- Hermitian parametrization

$$b_{\mathsf{k}} = \sqrt{\frac{\Delta_{\mathsf{k}}}{2}}\hat{\Phi}_{\mathsf{k}} + \frac{i}{\sqrt{2\Delta_{\mathsf{k}}}}\hat{\Pi}_{\mathsf{k}}$$

$$\left[\hat{\Phi}_{\mathsf{k}}, \hat{\Pi}_{\mathsf{k}'}\right] = i\delta_{\mathsf{k},-\mathsf{k}'}$$

$$\Delta_{\mathsf{k}} = A^{+}_{\mathsf{k}} - B_{\mathsf{k}}$$

$$\hat{H}_{2\text{mag}} = \frac{1}{2}\sum_{\mathsf{k}}\Big\{\hat{\Pi}_{-\mathsf{k}}\hat{\Pi}_{\mathsf{k}} + \epsilon^2_{\mathsf{k}}\hat{\Phi}_{-\mathsf{k}}\hat{\Phi}_{\mathsf{k}} + iA^{-}_{\mathsf{k}}(\hat{\Phi}_{-\mathsf{k}}\hat{\Pi}_{\mathsf{k}} + \hat{\Phi}_{\mathsf{k}}\hat{\Pi}_{-\mathsf{k}}) - A^{+}_{\mathsf{k}}\Big\}$$

Hasselmann *et al.* '06
Kreisel *et al.* '08

- sort longitudinal and transversal fluctuations

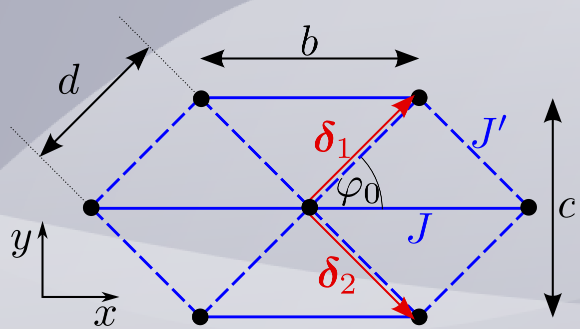
Extension of model

- include lattice vibrations

$$H = \frac{1}{2}\sum_{ij}\left[J_{ij}\,\mathbf{S}_i\cdot\mathbf{S}_j + \mathbf{D}_{ij}\cdot(\mathbf{S}_i\times\mathbf{S}_j)\right] - \sum_i \mathbf{h}\cdot\mathbf{S}_i$$

exchange

DM anisotropy



$$+\sum_{\mathsf{k}\lambda}\left[\frac{P_{-\mathsf{k}\lambda}P_{\mathsf{k}\lambda}}{2M} + \frac{M}{2}\omega^2_{\mathsf{k}\lambda}X_{-\mathsf{k}\lambda}X_{\mathsf{k}\lambda}\right]$$

phonon dispersion (acoustic)

$$\omega_{\mathsf{k}\lambda} = c_\lambda(\hat{\mathsf{k}})|\mathsf{k}|$$

Coldea *et al.* '02
Veillette *et al.* '05

- spin phonon coupling via expansion of exchange integrals

$$J_{ij} = J(\mathsf{R}_{ij}) + (\mathsf{X}_{ij}\cdot\nabla_{\mathsf{R}})\,J(\mathsf{r})|_{\mathsf{r}=\mathsf{r}_{ij}} + \ldots = J(\mathsf{R}_{ij}) + \mathsf{X}_{ij}\cdot\mathsf{J}^{(1)}_{ij} + \ldots$$

bare exchange

magnon phonon coupling

$$D_{ij} \approx D(\mathsf{R}_{ij})$$

Chakraborty *et al.* '87

Magnon-Phonon Interactions

- 1/S expansion of coupling term

$$\hat{H}^{\mathrm{pho}}_{\mathrm{spin}} = \hat{H}_{\mathrm{spin}} + \hat{H}^{\mathrm{1pho}}_{\mathrm{spin}} + \hat{H}^{\mathrm{2pho}}_{\mathrm{spin}} + \dots$$

$$\hat{H}^{n\mathrm{pho}}_{\mathrm{spin}} = \frac{1}{2}\sum_{ij} U^{(n)}_{ij}\, \mathsf{S}_i \cdot \mathsf{S}_j$$

$$U^{(1)}_{ij} = \left(\mathsf{X}_{ij}\cdot\nabla_{\mathsf{r}}\right) J(\mathsf{r})\big|_{\mathsf{r}=\mathsf{R}_{ij}} \equiv \mathsf{X}_{ij}\cdot\mathsf{J}^{(1)}_{ij} \qquad \mathsf{J}^{(1)}_{-\mathsf{k}} = -\mathsf{J}^{(1)}_{\mathsf{k}} = \left(\mathsf{J}^{(1)}_{\mathsf{k}}\right)^*$$

$$U^{(2)}_{ij} = \tfrac{1}{2}\left(\mathsf{X}_{ij}\cdot\nabla_{\mathsf{r}}\right)^2 J(\mathsf{r})\big|_{\mathsf{r}=\mathsf{R}_{ij}} \equiv \tfrac{1}{2}\mathsf{X}^{T}_{ij}\mathsf{J}^{(2)}_{ij}\mathsf{X}_{ij} \qquad \mathsf{J}^{(2)}_{-\mathsf{k}} = \mathsf{J}^{(2)}_{\mathsf{k}}$$

- Phonon shift

$$\hat{H}^{\mathrm{2pho}}_{0} = \frac{M}{2}\sum_{\mathsf{k}\lambda}\Sigma^{\mathrm{pho}}_{0}(\mathsf{k},\lambda)\hat{X}_{-\mathsf{k}\lambda}\hat{X}_{\mathsf{k}\lambda}$$

$$\Sigma^{\mathrm{pho}}_{0}(\mathsf{k},\lambda) = \frac{S^2}{M}\mathsf{e}^{\dagger}_{\mathsf{k}\lambda}\left[s^2_{\vartheta}\left(\mathsf{J}^{(2)}_{0} - \mathsf{J}^{(2)}_{\mathsf{k}}\right) + c^2_{\vartheta}\mathsf{J}^{(2+)}_{\mathsf{Q},\mathsf{k}}\right]\mathsf{e}_{\mathsf{k}\lambda}$$

Magnon-Phonon Interactions

- Hybridization (coupled magnoelastic waves)

$$\hat{H}^{\text{1pho}}_{\text{1mag}} = \sum_{\mathsf{k}} \Gamma^{Xb}_{\mathsf{k}} \cdot \left(\mathsf{X}_{-\mathsf{k}} b_{\mathsf{k}} + \mathsf{X}_{\mathsf{k}} b^{\dagger}_{\mathsf{k}} \right)$$

$$\Gamma^{Xb}_{\mathsf{k}} = \frac{i}{4}(2S)^{3/2} c_{\vartheta} \left[\mathbf{J}^{(1+)}_{\mathsf{k},\mathsf{Q}} + s_{\vartheta} \mathbf{J}^{(1-)}_{\mathsf{k},\mathsf{Q}} \right]$$

- Hermitian parametrization (sorts relevant degrees of freedom)

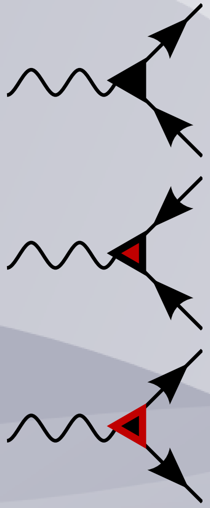
$$\hat{H}^{\text{1pho}}_{\text{1mag}} = \frac{1}{2} \sum_{\mathsf{k}} \Big\{ \Gamma^{X\Phi}_{\mathsf{k}} \cdot (\mathsf{X}_{-\mathsf{k}} \hat{\Phi}_{\mathsf{k}} + \mathsf{X}_{\mathsf{k}} \hat{\Phi}_{-\mathsf{k}})$$

$$+ \Gamma^{X\Pi}_{\mathsf{k}} \cdot (\mathsf{X}_{-\mathsf{k}} \hat{\Pi}_{\mathsf{k}} - \mathsf{X}_{\mathsf{k}} \hat{\Pi}_{-\mathsf{k}}) \Big\}$$

crucial interaction

Magnon-Phonon Interactions

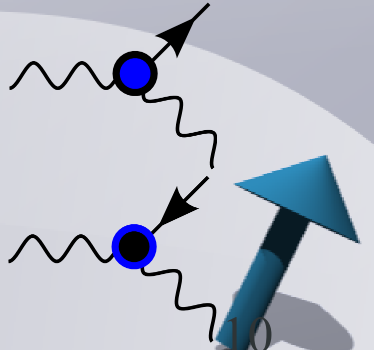
- three particle interactions
  - one phonon two magnon scattering



$$\hat{H}^{\color{red}{1\mathrm{pho}}}_{\color{blue}{2\mathrm{mag}}} = \frac{1}{\sqrt{N}} \sum_{\mathsf{k}\mathsf{k}'} \Big[\Gamma^{b^\dagger b}_{\mathsf{k},\mathsf{k}'} \cdot \color{red}{\mathsf{X}_{\mathsf{k}-\mathsf{k}'}} \color{blue}{b^\dagger_{\mathsf{k}} b_{\mathsf{k}'}}$$

$$+ \frac{1}{2!} \left(\Gamma^{b^\dagger b^\dagger}_{\mathsf{k},\mathsf{k}'} \cdot \color{red}{\mathsf{X}_{\mathsf{k}+\mathsf{k}'}} \color{blue}{b^\dagger_{\mathsf{k}} b^\dagger_{\mathsf{k}'}} + \Gamma^{bb}_{\mathsf{k},\mathsf{k}'} \cdot \color{red}{\mathsf{X}_{-\mathsf{k}-\mathsf{k}'}} \color{blue}{b_{\mathsf{k}} b_{\mathsf{k}'}} \right) \Big]$$

  - two phonon one magnon scattering



$$\hat{H}^{\color{red}{2\mathrm{pho}}}_{\color{blue}{1\mathrm{mag}}} = \frac{1}{2!\sqrt{N}} \sum_{\mathsf{k},\mathsf{k}'} \sum_{\lambda\lambda'} \Big[\Gamma^{XXb^\dagger}_{\mathsf{k}\lambda,\mathsf{k}'\lambda'} \color{red}{\hat{X}_{\mathsf{k}\lambda} \hat{X}_{\mathsf{k}'\lambda'}} \color{blue}{b^\dagger_{\mathsf{k}+\mathsf{k}'}}$$

$$+ \Gamma^{XXb}_{\mathsf{k}\lambda,\mathsf{k}'\lambda'} \color{red}{\hat{X}_{-\mathsf{k}\lambda} \hat{X}_{-\mathsf{k}'\lambda'}} \color{blue}{b_{\mathsf{k}+\mathsf{k}'}} \Big]$$

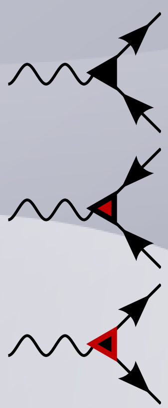
Magnon-Phonon Interactions

- magnon shift performed in Hermitian parametrization

$$\Gamma^{X\Pi}_{\mathsf{k}} \to 0 \qquad\qquad \lambda_{\mathsf{k}} = \frac{i}{\sqrt{2\Delta_{\mathsf{k}}}}\Gamma^{X\Pi}_{\mathsf{k}}$$

$$b_{\mathsf{k}} = \tilde{b}_{\mathsf{k}} + \lambda_{\mathsf{k}} \cdot X_{\mathsf{k}}$$

- renormalization of one-phonon two magnon interaction



$$\Gamma^{b^\dagger b^\dagger}_{\mathsf{k},\mathsf{k}'} \to \tilde{\Gamma}^{b^\dagger b^\dagger}_{\mathsf{k},\mathsf{k}'} = \Gamma^{b^\dagger bb}_{3}(-\mathsf{k}-\mathsf{k}';\mathsf{k},\mathsf{k}')\lambda_{\mathsf{k}+\mathsf{k}'}$$

$$\tilde{H}^{1\mathrm{pho}}_{2\mathrm{mag}} = \frac{1}{\sqrt{N}}\sum_{\mathsf{k}\mathsf{k}'}\Big[\tilde{\Gamma}^{b^\dagger b}_{\mathsf{k},\mathsf{k}'} \cdot X_{\mathsf{k}-\mathsf{k}'}\tilde{b}^\dagger_{\mathsf{k}}\tilde{b}_{\mathsf{k}'}$$

$$+\frac{1}{2!}\left(\tilde{\Gamma}^{b^\dagger b^\dagger}_{\mathsf{k},\mathsf{k}'} \cdot X_{\mathsf{k}+\mathsf{k}'}\tilde{b}^\dagger_{\mathsf{k}}\tilde{b}^\dagger_{\mathsf{k}'} + \tilde{\Gamma}^{bb}_{\mathsf{k},\mathsf{k}'} \cdot X_{-\mathsf{k}-\mathsf{k}'}\tilde{b}_{\mathsf{k}}\tilde{b}_{\mathsf{k}'}\right)\Big]$$

Lagrangian functional integral

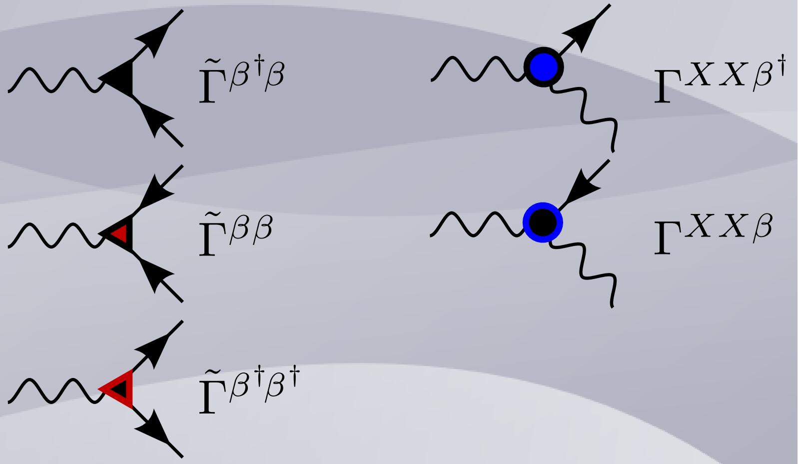
- Transform to Bogoliubov basis

$$\begin{pmatrix} b_{\mathsf{k}} \\ b^{\dagger}_{-\mathsf{k}} \end{pmatrix} = \begin{pmatrix} u_{\mathsf{k}} & -v_{\mathsf{k}} \\ -v_{\mathsf{k}} & u_{\mathsf{k}} \end{pmatrix} \begin{pmatrix} \beta_{\mathsf{k}} \\ \beta^{\dagger}_{-\mathsf{k}} \end{pmatrix}$$

- integrate out canonical momentum fields

$$\mathcal{Z} = \int \mathcal{D}[\mathsf{P}, \mathsf{X}, \beta, \bar{\beta}] e^{-S'[\mathsf{P}, \mathsf{X}, \beta, \bar{\beta}]}$$

$$= \int \mathcal{D}[\mathsf{X}, \beta, \bar{\beta}] e^{-S[\mathsf{X}, \beta, \bar{\beta}]}$$



$$S[\mathsf{X}, \bar{\beta}, \beta] = S^{\mathrm{2pho}}[\mathsf{X}] + S_{\mathrm{2mag}}[\bar{\beta}, \beta] + S^{\mathrm{1pho}}_{\mathrm{1mag}}[\mathsf{X}, \bar{\beta}, \beta]$$
$$+ S^{\mathrm{1pho}}_{\mathrm{2mag}}[\mathsf{X}, \bar{\beta}, \beta] + S_{\mathrm{3mag}}[\bar{\beta}, \beta] + \ldots$$

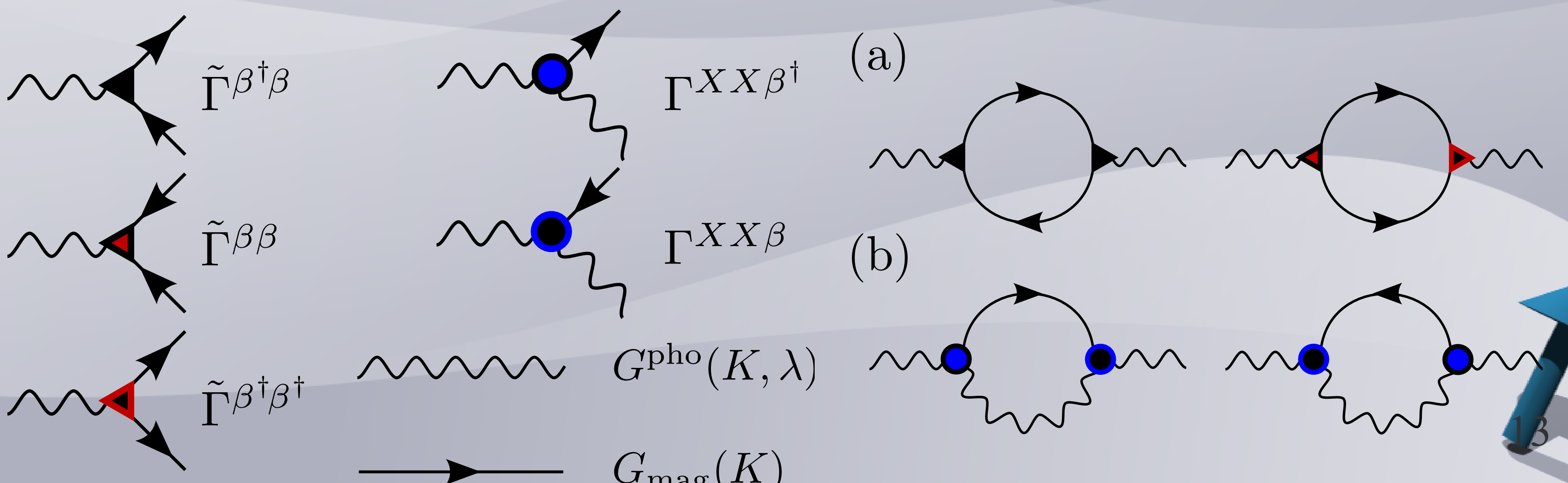
Many particle methods: Phonon renormalization

- full phonon propagator (diagrammatics)

$$G^{\mathrm{pho}}(K,\lambda) = \frac{M}{T}\langle X_{-K\lambda}X_{K\lambda}\rangle = \frac{1}{\omega^2 + \omega_{\mathbf{k}\lambda}^2 + \Sigma^{\mathrm{pho}}(K,\lambda)}$$

self-energy

$$\Sigma^{\mathrm{pho}}(K,\lambda) \approx \Sigma_0^{\mathrm{pho}}(\mathbf{k},\lambda) + \Sigma_1^{\mathrm{pho}}(K,\lambda) + \Sigma_2^{\mathrm{pho}}(K,\lambda) + \mathcal{O}(1/S)$$



Shift of elastic constants

- classical spin background

$$\frac{(\Delta c_\lambda)_0}{c_\lambda} = \sqrt{1 - \lim_{|\mathsf{k}|\to 0} \frac{\Sigma_0^{\mathrm{pho}}(\mathsf{k},\lambda)}{\omega^2_{\mathsf{k}\lambda}}} - 1$$

- magnon-phonon Hybridization (equivalent to diagonalization in Hamilton formulation)

$$S_1[\mathsf{X},\bar\beta,\beta] = S^{2\mathrm{pho}}[\mathsf{X}] + S_{2\mathrm{mag}}[\bar\beta,\beta] + S^{1\mathrm{pho}}_{1\mathrm{mag}}[\mathsf{X},\bar\beta,\beta]$$

$$e^{-S^{2\mathrm{pho}}_{\mathrm{eff}}[\mathsf{X}]} = \int \mathcal{D}[\beta,\bar\beta]\, e^{-S_1[\mathsf{X},\bar\beta,\beta]}$$

$$\frac{(\Delta c_\lambda)_1}{c_\lambda} = \lim_{|\mathsf{k}|\to 0} \frac{\left|\Gamma^{X\beta}_{\mathsf{k}} \cdot \mathsf{e}_{\mathsf{k}\lambda}\right|^2}{2M\omega^3_{\mathsf{k}\lambda}}$$

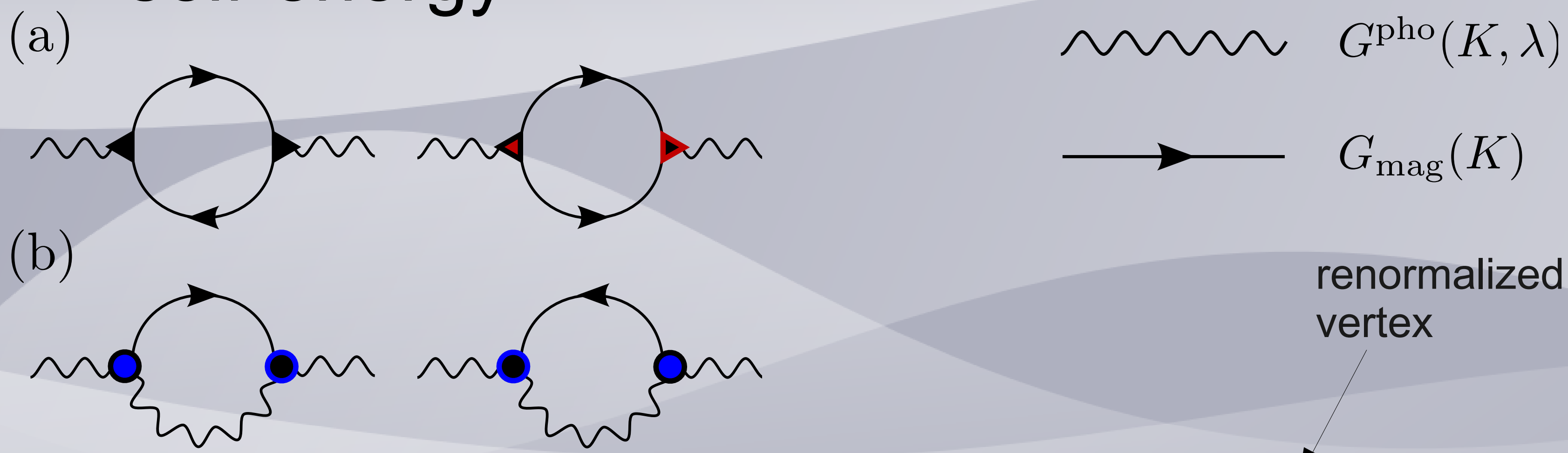
$s_\vartheta = \sin\vartheta = h/h_c \qquad c_\vartheta = \cos\vartheta$

magnetic field dependence

$$\frac{(\Delta c_\lambda)_1}{c_\lambda} = \frac{S^3}{4}\left(\frac{v(\hat{\mathsf{k}})}{c_\lambda}\right)\left(\frac{h_c}{Mc_\lambda^2}\right)\left|s_\vartheta \mathsf{f}_1^{X\beta}(\hat{\mathsf{k}})\cdot \mathsf{e}_{\mathsf{k}\lambda} - c_\vartheta^2 \mathsf{f}_2^{X\beta}(\hat{\mathsf{k}},\hat{\mathsf{k}})\cdot\mathsf{e}_{\mathsf{k}\lambda}\right|^2$$

Ultrasound Attenuation Rate

- self-energy



renormalized vertex

$$S_2[\mathsf{X}\ ,\bar\beta,\beta] = S^{2\mathrm{pho}}[\mathsf{X}\ ] + S_{2\mathrm{mag}}[\bar\beta,\beta] + \tilde S^{1\mathrm{pho}}_{2\mathrm{mag}}[\mathsf{X}\ ,\bar\beta,\beta] + S^{2\mathrm{pho}}_{1\mathrm{mag}}[\mathsf{X}\ ,\bar\beta,\beta]$$

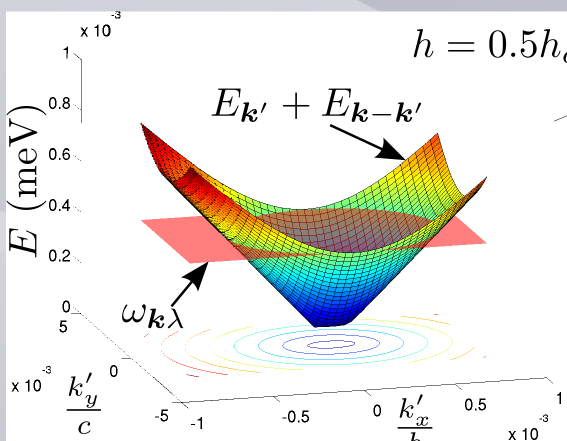
- attenuation rate

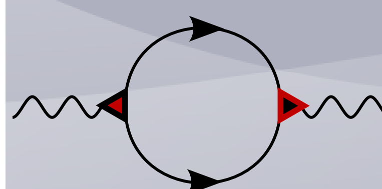
$$\gamma_{\mathsf{k}\lambda} = -\frac{\mathrm{Im}\Sigma_2^{\mathrm{pho}}(\omega_{\mathsf{k}\lambda}+i0^+,\mathsf{k},\lambda)}{2\omega_{\mathsf{k}\lambda}}$$

Ultrasound attenuation rate

- process (a): scattering surface

$$\gamma^{(a)}_{\mathsf{k}\lambda} = \frac{\pi}{2\omega_{\mathsf{k}\lambda}} \frac{1}{N} \sum_{\mathsf{k}'} \frac{|\tilde{\Gamma}^{\beta^\dagger \beta^\dagger}_{\mathsf{k}',\mathsf{k}-\mathsf{k}'} \cdot \mathsf{e}_{\mathsf{k}\lambda}|^2}{2M} \delta(\omega_{\mathsf{k}\lambda} - E_{\mathsf{k}'} - E_{\mathsf{k}-\mathsf{k}'})$$

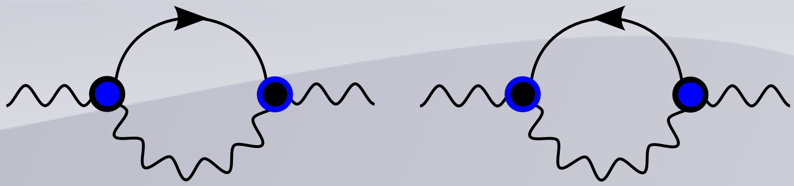




$$\gamma^{(a)}_{\mathsf{k}\lambda} = \frac{\pi^2}{64} \left(\frac{\mathsf{k}^2}{2M}\right) \left(\frac{S^2 c_\lambda^2 \mathsf{k}^2}{V_{\mathrm{BZ}} v_x v_y}\right) \frac{I_\lambda(\hat{\mathsf{k}})}{\sqrt{1 - r^2_{\mathsf{k}\lambda}}} \propto \mathsf{k}^4$$

Ultrasound attenuation rate

- process (b)



$$\gamma^{(\mathrm{b})}_{\mathbf{k}\lambda} = \frac{\pi S^3}{4}\left(\frac{\mathsf{k}^2}{2M}\right)\left(\frac{\mathsf{k}^2}{V_{\mathrm{BZ}}}\right)\sum_{\lambda'}\left(\frac{h_c}{Mc^2_{\lambda'}}\right)\left(\frac{c_\lambda}{c_{\lambda'}}\right)^2\int_0^{2\pi} d\varphi' \frac{u(\hat{\mathsf{k}},\varphi')}{c_{\lambda'}}|\mathsf{e}^\dagger_{\mathsf{k}\lambda}\mathsf{F}^{XX\beta}(\hat{\mathsf{k}},\varphi')\mathsf{e}_{\mathsf{k}'\lambda'}|^2\,,$$

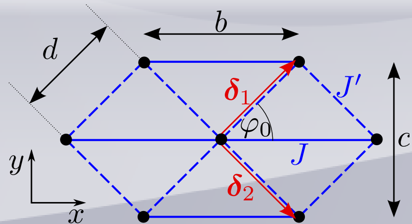
higher power in v/c<<1

- both contributions

$$\gamma_{\mathsf{k}\lambda} = \gamma^{(\mathrm{a})}_{\mathsf{k}\lambda} + \gamma^{(\mathrm{b})}_{\mathsf{k}\lambda} \propto \mathsf{k}^4$$

Comparison to experiments

- model

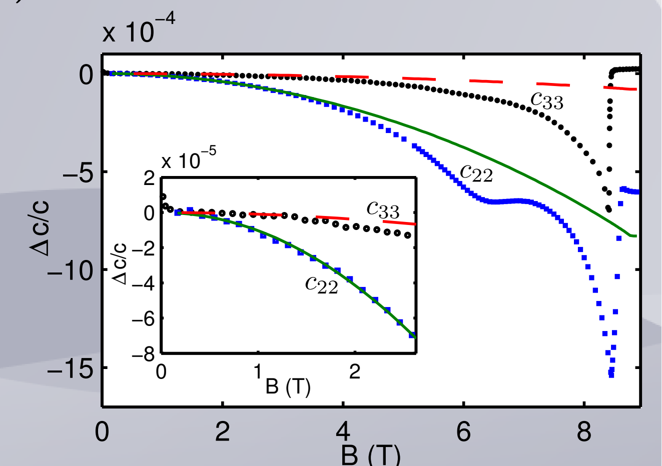


$$J(x) = J(b)e^{-\kappa(x-b)/b}$$

$$J'(r) = J'(d)e^{-\kappa'(r-d)/d}$$

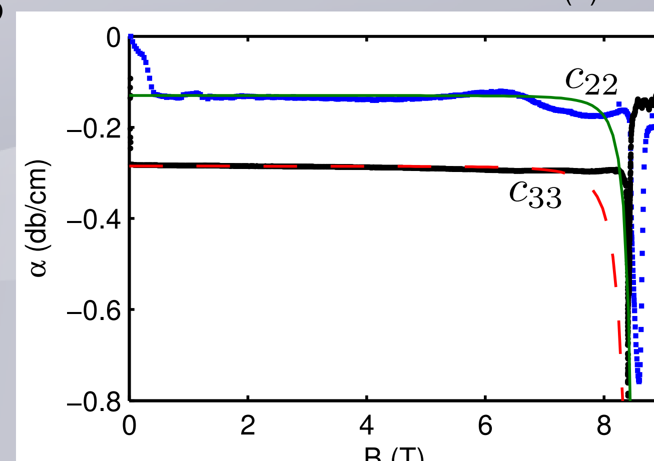
- shift of ultrasound velocity for c_{22} mode: fix parameters

$$|\kappa| \approx 15 \quad |\kappa'| \approx 51$$



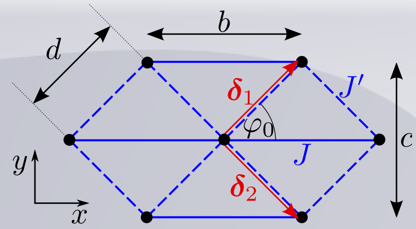
- attenuation rate calculate from parameters

$$\gamma_{\mathrm{k}\lambda} \approx \frac{\pi^2}{64}\left(\frac{\mathrm{k}^2}{2M}\right)\left(\frac{S^2 c_\lambda^2 \mathrm{k}^2}{V_{\mathrm{BZ}} v_x v_y}\right)\frac{\left[\mathrm{f}_1^{X\beta}(\hat{\mathrm{k}}) \cdot \mathrm{e}_{\mathrm{k}\lambda}\right]^2}{(1-h/h_c)^2}$$

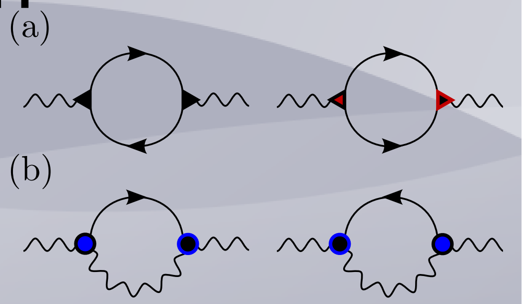


P. T. Cong *et. al* (in preparation)

Summary



- ultrasonic technique: probe magnetic properties
- combine spin-wave approach for ordered “cone-state” with expansion in terms of lattice vibrations



- calculate renormalization of phonons using an effective action
- good description of phonon properties (sound velocity, damping) away from critical point