Confinement in two-dimensional metals with almost flat Fermi surface

Talk at University of Oxford, March 14, 2007

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work in collaboration with Sascha Ledowski

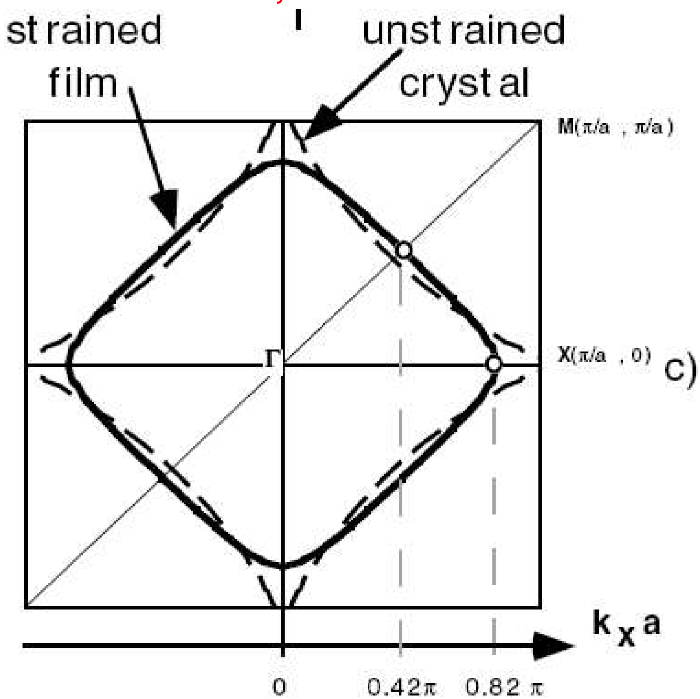
S. Ledowski and P. K., preprint, March 2007
S. Ledowski and P. K., Phys. Rev. B 75, 045134 (2007)
S. Ledowski, P.K. , and A. Ferraz, Phys. Rev. B 71, 235106 (2005)

1. Introduction: Calculating the FS via exact RG
2. Two coupled chains with weak interactions
3. Momentum transfer cutoff scheme
4. Two chains at strong coupling
5. Confinement in 2D

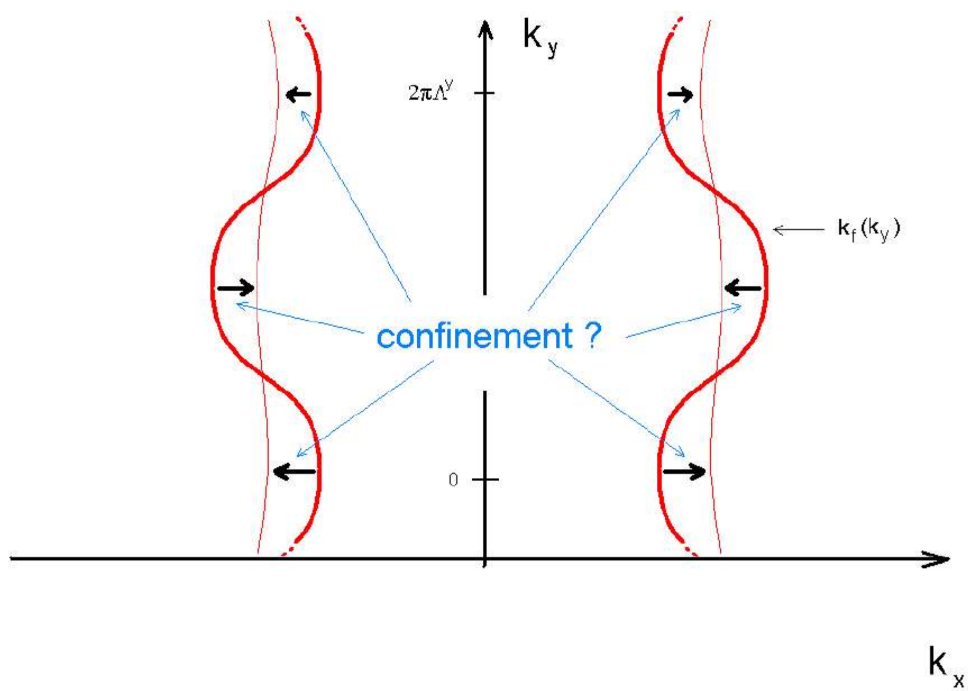
1.Calculating the FS via exact RG

Motivation: flat sectors of a FS lead to non-Fermi liquid behavior in D>1

Doped cuprates under strain:
Abrecht et al., PRL 2003



Quasi 1D metals with open FS:
Clarke,Strong, Anderson, PRL 1994-1997



Problem: Can curved FS become flat due to strong interactions? Confinement!!!

An exact integral equation for the renormalized Fermi surface

S. Ledowski and PK, J. Phys. Cond. Mat. 15, 4779 (2003).

definition of the FS: $\epsilon_{\boldsymbol{k}_F} + \Sigma(\boldsymbol{k}_F, i0) - \mu = 0$

get exact self-energy from RG flow of continuum of relevant couplings:

$$r_l(\boldsymbol{k}_F) = \tilde{\Gamma}_l^{(2)}(\boldsymbol{k}_F, q=0, i\epsilon = i0) = \frac{Z_l(\boldsymbol{k}_F)}{\Lambda_l v_F}\left[\Sigma_l(\boldsymbol{k}_F, i0) - \Sigma_{l=\infty}(\boldsymbol{k}_F, i0)\right]$$

running cutoff $\Lambda_l = \Lambda_0 e^{-l}$

wave-function renormalization $\eta_l(\boldsymbol{k}_F) = -\partial_l \ln Z_l(\boldsymbol{k}_F)$

get flow of $r_l(\boldsymbol{k}_F)$ from exact RG flow equation for (rescaled) two-point vertex:

$$\partial_l \tilde{\Gamma}_l^{(2)}(\boldsymbol{k}_F, Q) = (1 - \eta_l(\boldsymbol{k}_F) - q\partial_q - \epsilon\partial_\epsilon)\tilde{\Gamma}_l^{(2)}(\boldsymbol{k}_F, Q) + \int_{\boldsymbol{k}'_F}\int \frac{dq'd\epsilon'}{(2\pi)^2}\dot{G}_l(\boldsymbol{k}'_F, Q')\tilde{\Gamma}_l^{(4)}(\boldsymbol{k}_F, Q; \boldsymbol{k}'_F, Q'; \boldsymbol{k}'_F, Q', \boldsymbol{k}_F, Q)$$

effective interaction

rescaled variables: $(\boldsymbol{k}, i\omega) \to (\boldsymbol{k}_F, q, i\epsilon)$ $\quad q = \boldsymbol{v}_F \cdot (\boldsymbol{k} - \boldsymbol{k}_F)/\Lambda \quad \epsilon = \omega / v_F \Lambda$

...exact integral equation for the FS...

...follows from requirement that relevant couplings $r_0(\boldsymbol{k}_F)$ flow into RG fixed point

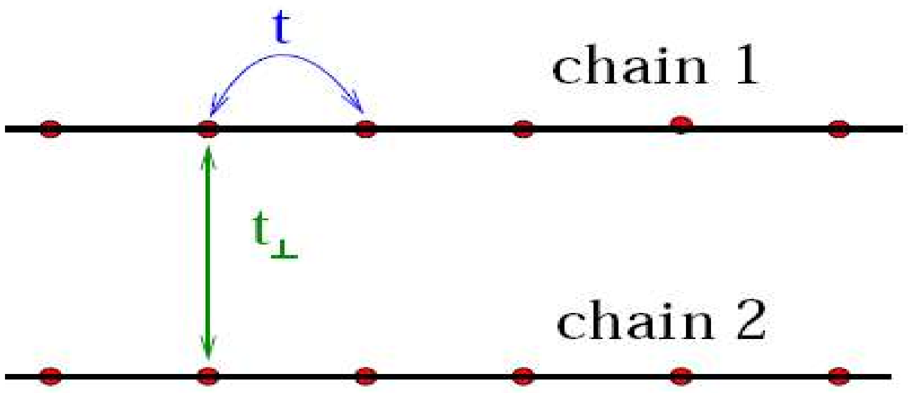
$$r_0(\boldsymbol{k}_F) = \int_0^\infty dl e^{-l+\int_0^l dt \eta_t(\boldsymbol{k}_F)} \int_{\boldsymbol{k}'_F} \int \frac{dq' d\epsilon'}{(2\pi)^2} \dot{G}_l(\boldsymbol{k}'_F, Q') \tilde{\Gamma}_l^{(4)}(\boldsymbol{k}_F, 0; \boldsymbol{k}'_F, Q'; \boldsymbol{k}'_F, Q', \boldsymbol{k}_F, 0)$$

- relates counterterm to flow of all couplings
- fine tuning of infinitely many relevant couplings $r_l(\boldsymbol{k}_F)$
- FS can be viewed as multicritical point of infinite order

2.Two spinless chains, weak coupling

kinetic energy:

$$\hat{H}_0 = -t\sum_i \sum_{\sigma=1,2} [\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.] \\ -t_\perp \sum_i [\hat{c}^\dagger_{i,1}\hat{c}_{i,2} + h.c.]$$

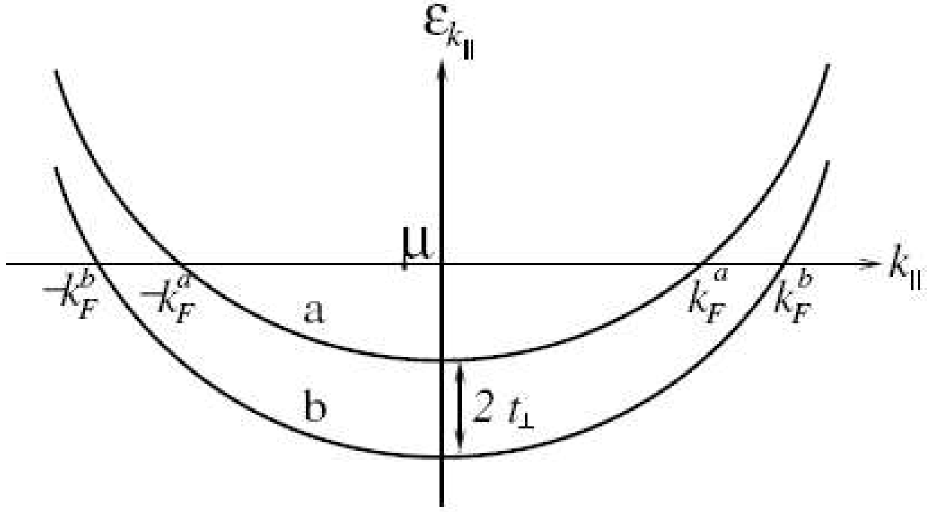


total density fixed:

$$\pi n = k_F^b + k_F^a$$

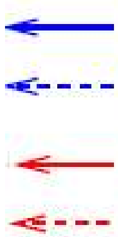
Fermi point distance can be strongly renormalized:

$$\Delta = k_F^b - k_F^a$$



...interactions in 2 spinless chains...

four types of Fermi fields:

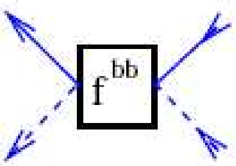


bonding rightmoving

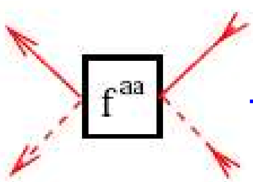
bonding leftmoving

antibonding rightmoving

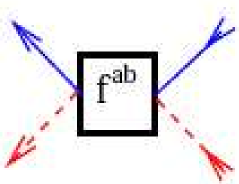
antibonding leftmoving



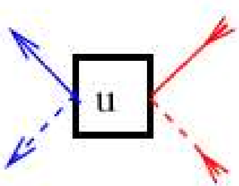
forward scattering bonding



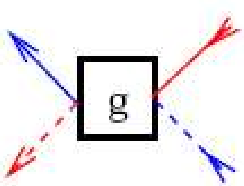
forward scattering antibonding



mixed forward scattering



interchain Umklapp
(pair tunneling)



interchain backscattering

Euclidean action in pseudospin notation:

$$S[\bar{\psi},\psi] = \sum_{\sigma}\int_K (-i\omega + \xi_k^{\sigma})\bar{\psi}_K^{\sigma}\psi_K^{\sigma} + \frac{1}{2}\int_{\bar{K}}\left[f(\bar{k})\bar{\rho}_{\bar{K}}\rho_{\bar{K}} - J^{\|}(\bar{k})\bar{m}_{\bar{K}}m_{\bar{K}}\right] + \int_{\bar{K}}\left[u(\bar{k})\left(\bar{s}_{\bar{K}}\bar{s}_{-\bar{K}} + s_{\bar{K}}s_{-\bar{K}}\right) - 2J^{\perp}(\bar{k})\bar{s}_{\bar{K}}s_{\bar{K}}\right]$$

composite fields:

$$\rho_{\bar{K}} = \sum_{\sigma}\int_K \bar{\psi}_K^{\sigma}\psi_{K+\bar{K}}^{\sigma}\,,$$ density

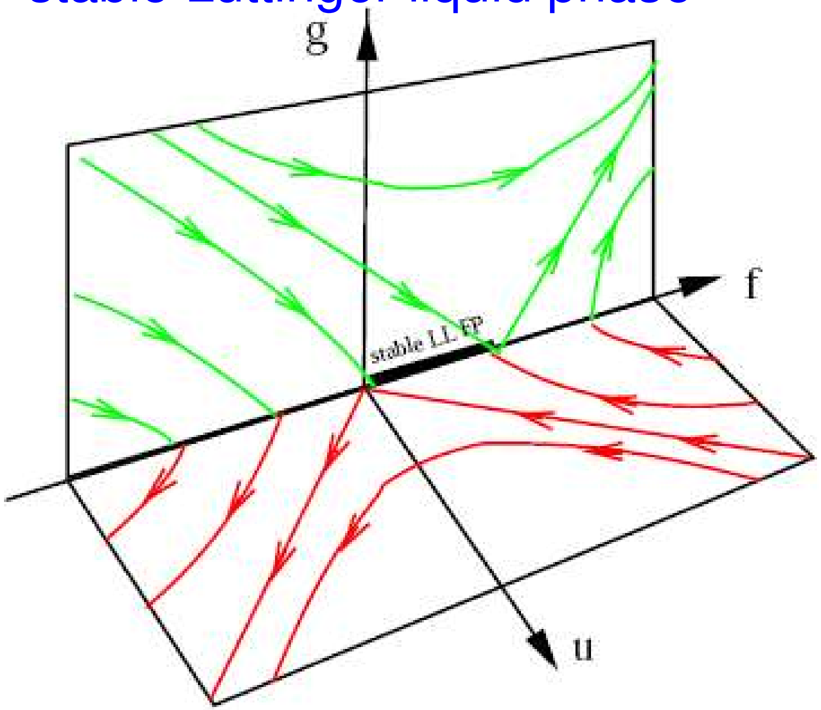
$$m_{\bar{K}} = \sum_{\sigma}\sigma\int_K \bar{\psi}_K^{\sigma}\psi_{K+\bar{K}}^{\sigma}\,,$$ spin density

$$s_{\bar{K}} = \int_K \bar{\psi}_K^{-}\psi_{K+\bar{K}}^{+}\,.$$ spin flip

...weak coupling RG...

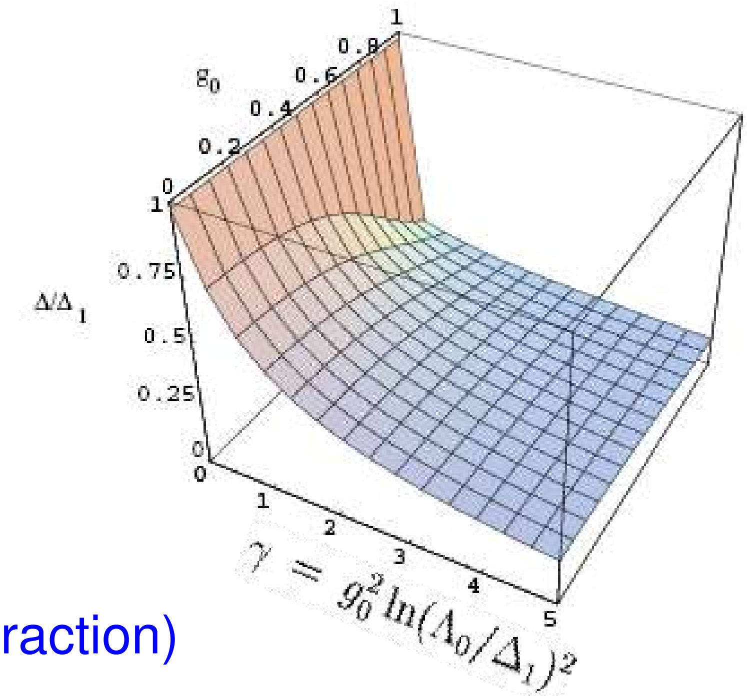
S. Ledowski, PK, A. Ferraz, Phys. Rev. B 71, 057519 (2005).

stable Luttinger liquid phase



strong reduction of Fermi point distance due to interchain backscattering

$$k_F^b - k_F^a = \quad \Delta = \Delta_1[1 + g_0^2 \ln(\Lambda_0/\Delta)^2]^{-1}$$



effective model for FS renormalization: keep only interchain backscattering (= ferromagnetic XY-interaction)

$$S[\bar{\psi}, \psi] = \sum_{\sigma,\alpha} \int_K (-i\omega + \alpha v_0^\sigma k + \mu_0^\sigma) \bar{\psi}_{K\alpha}^\sigma \psi_{K\alpha}^\sigma - 2 \sum_{\alpha\alpha'} \int_{\bar{K}} J_{\alpha\alpha'}^\perp \bar{s}_{\bar{K}\alpha} s_{\bar{K}\alpha'} ,$$

spin-flip field:

$$s_{\bar{K}} = \int_K \bar{\psi}_K^- \psi_{K+\bar{K}}^+$$

3. Momentum transfer cutoff scheme

F. Schütz, L. Bartosch, PK, Phys. Rev. B 72, 035107 (2005).

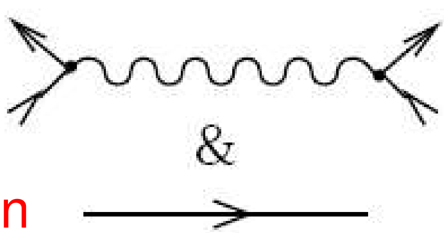
- Question: can Fermi point difference collapse at strong coupling?
- Need RG method for strong coupling regime!
- Idea: partial bosonization (Hubbard-Stratonovich transformation)
- Use bosonic momentum transfer as flow parameter

Example: Tomonaga-Luttinger model:

decouple density-density interaction in zero-sound channel:

original problem:

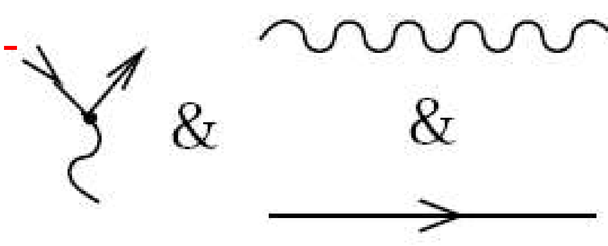
2-body interaction



fermion

after HS transformation:

fermion-
boson-
vertex



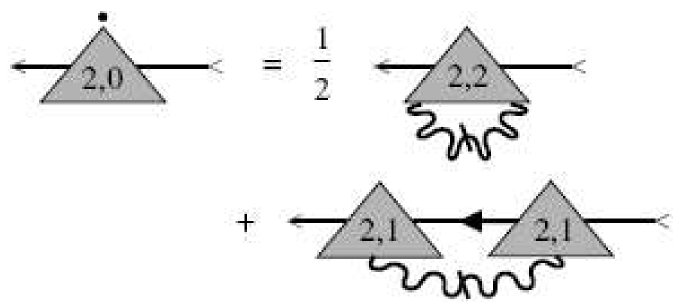
boson

fermion

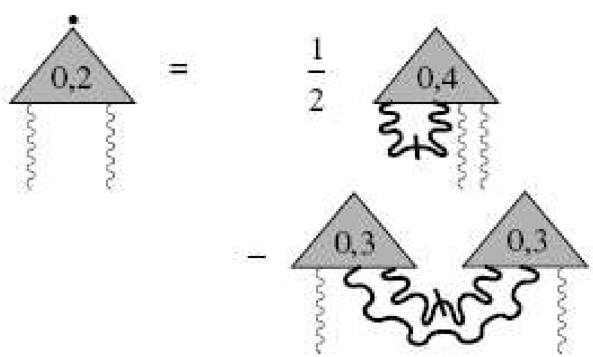
...momentum transfer cutoff scheme...

impose cutoff only in momentum transfered by bosonic field; exact RG equations:

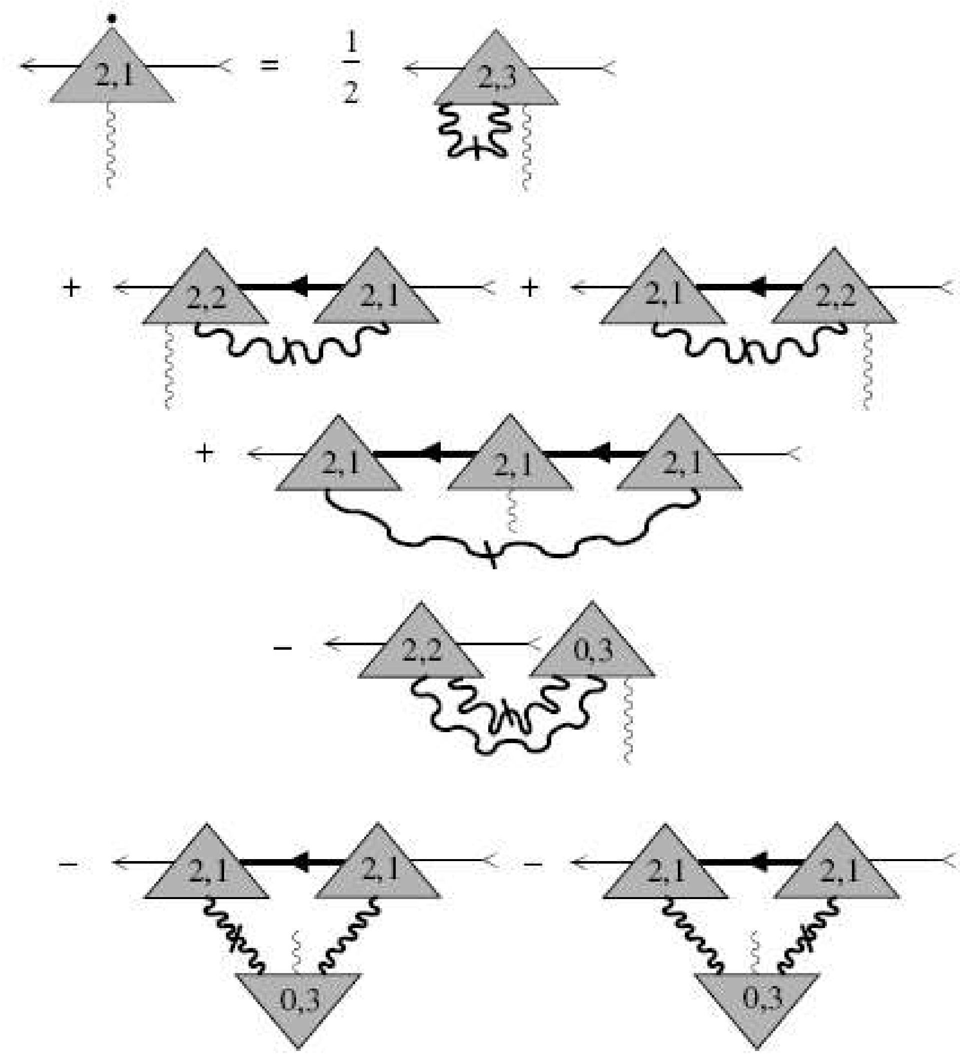
fermionic self-energy:



bosonic self-energy:

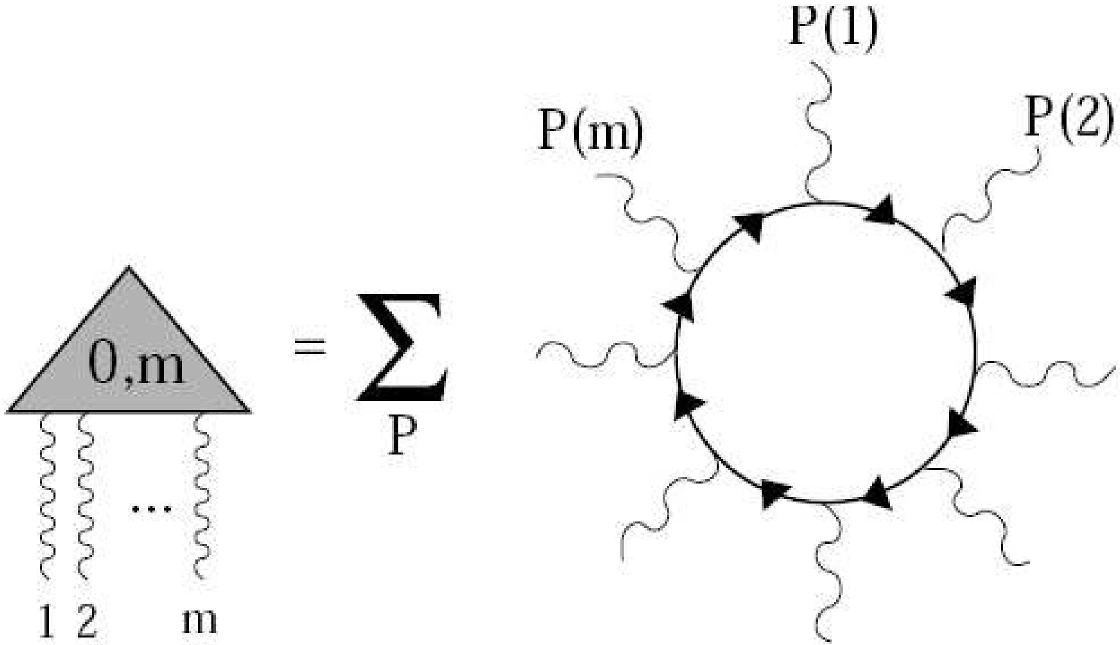


three-legged fermion-boson vertex:



...initial condition in momentum transfer cutoff scheme...

symmetrizied closed fermion loops with arbitrary number of bosonic legs are finite



- cutoff scheme does not violate Ward identities
- exact solution of the Tomonaga-Luttinger model within ERG
- simple truncation gives correct anomalous dimension -even at strong coupling!

4.Two chains at strong coupling

S. Ledowski, PK, Phys. Rev. B 75, 045137 (2007).

Can Fermi point distance collapse in 2-chain system at strong coupling?

Strategy:

a)Start from effective low energy model containing only interchain backscattering: in pseudospin language:ferromagneric XY interaction, magnetic field in z-direction

$t_\perp = h$

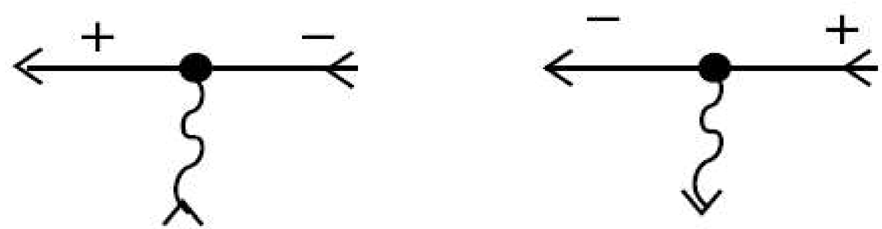
b) Decouple interaction in spin-singlet, particle-hole channel via complex HS field

$$S[\bar{\psi}, \psi] = \sum_{\sigma,\alpha} \int_K (-i\omega + \alpha v_0^\sigma k + \mu_0^\sigma) \bar{\psi}_{K\alpha}^\sigma \psi_{K\alpha}^\sigma - 2 \sum_{\alpha\alpha'} \int_{\bar{K}} J_{\alpha\alpha'}^\perp \bar{s}_{\bar{K}\alpha} s_{\bar{K}\alpha'} \,, \longrightarrow \frac{1}{2} \sum_{\alpha\alpha'} \int_{\bar{K}} [\mathbf{J}^\perp]_{\alpha\alpha'}^{-1} \bar{\chi}_{\bar{K}\alpha} \chi_{\bar{K}\alpha'} + \sum_{\alpha} \int_{\bar{K}} \left[\bar{s}_{\bar{K}\alpha} \chi_{\bar{K}\alpha} + s_{\bar{K}\alpha} \bar{\chi}_{\bar{K}\alpha} \right]$$

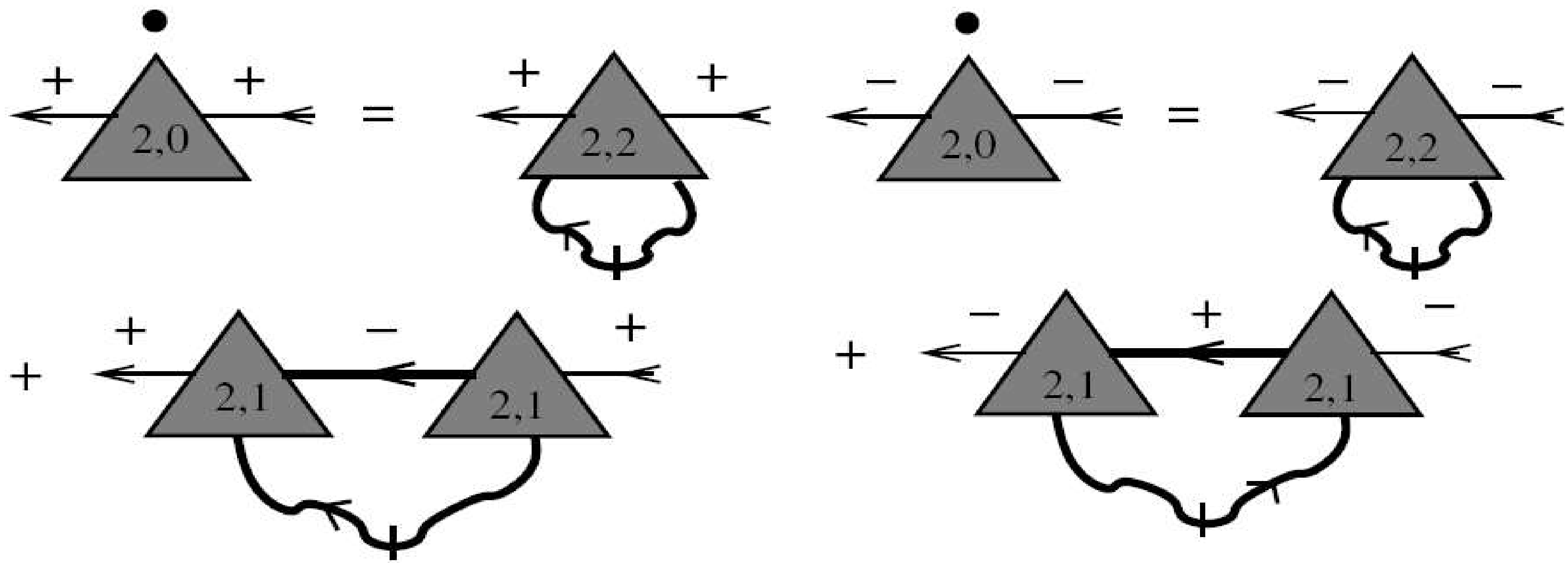
c) Find sensible truncation of resulting mixed Bose-Fermi theory

...ERG flow equations in momentum transfer cutoff scheme...

bare spin-flip vertices:



flow of fermionic self-energy:



$$\partial_\Lambda \Sigma^\sigma_\Lambda(K,\alpha) = \int_{\bar{K}} \dot{F}^{\sigma\bar{\sigma}}_\Lambda(\bar{K},\alpha)\Gamma^{(2,2)}_\Lambda(K\sigma,-K\sigma;\bar{K},-\bar{K},\alpha)$$

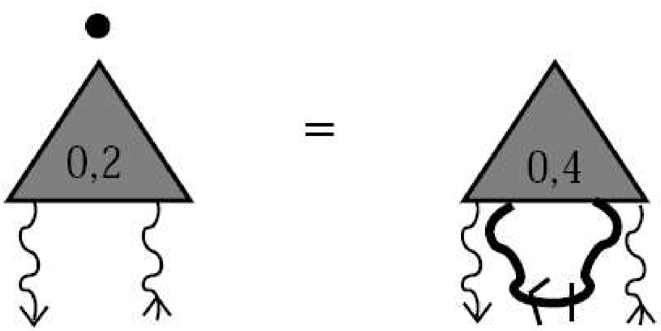
$$+ \int_{\bar{K}} \dot{F}^{\sigma\bar{\sigma}}_\Lambda(\bar{K},\alpha) G^{\bar{\sigma}}_\Lambda(K+\bar{K}+\alpha\sigma\Delta,\alpha)\, \Gamma^{(2,1)}_\Lambda(K\sigma;K+\bar{K},\bar{\sigma};-\bar{K},\alpha)$$

$$\times \Gamma^{(2,1)}_\Lambda(K+\bar{K},\bar{\sigma};K,\sigma;\bar{K},\alpha)$$

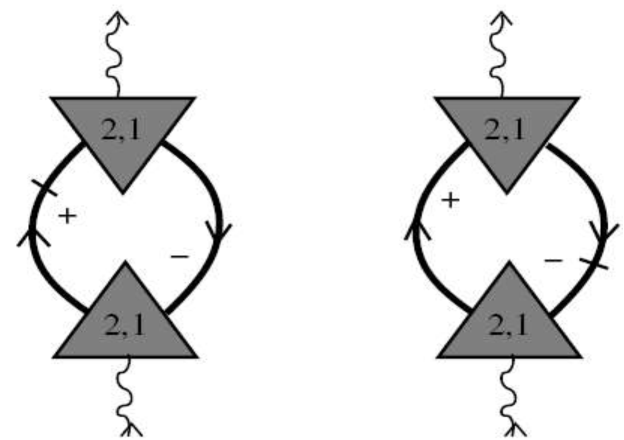
...flow of spin-flip susceptibility and spin-flip vertices...

flow of spin-flip susceptibility (bosonic self-energy)

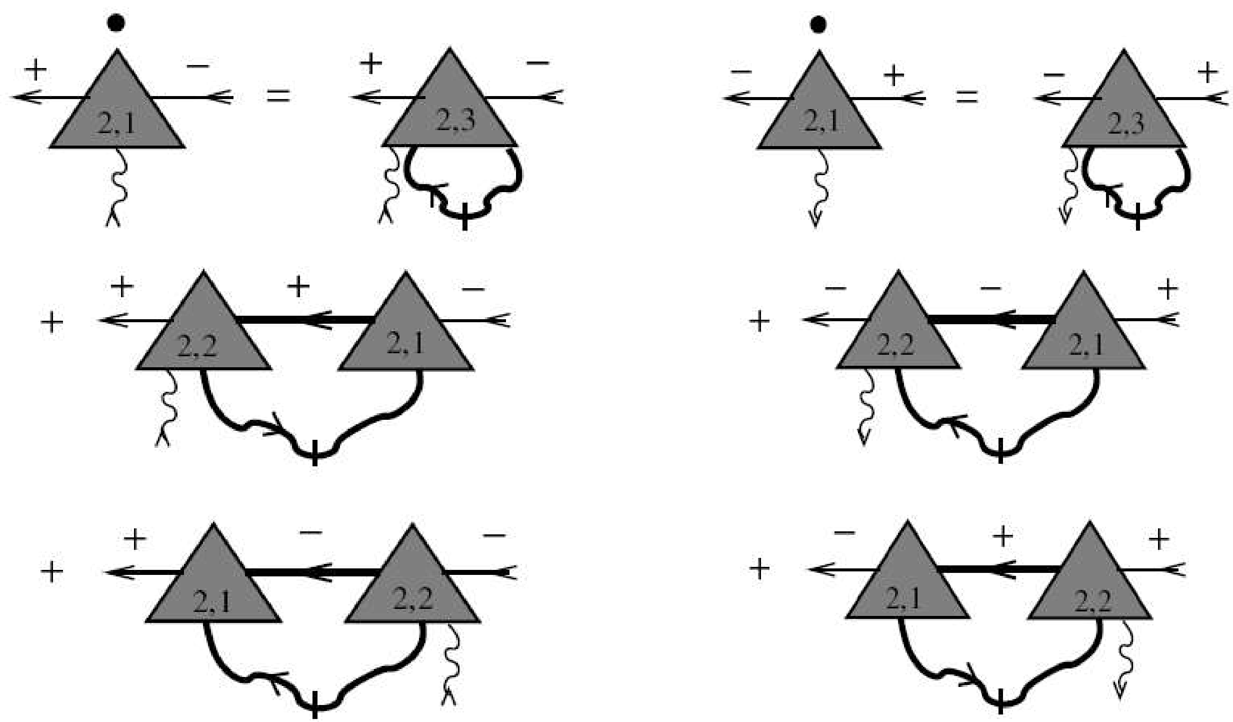
in momentum transfer cutoff scheme:



with fermionic band width cutoff:



flow of spin-flip vertices in momentum transfer cutoff scheme:



...calculating the true Fermi point distance...

From RG flow of momentum-independent part of rescaled self-energy:

$$r_l^\sigma = \tilde{\Sigma}_l^\sigma(0,\alpha) = \frac{Z_l^\sigma}{\Omega_\Lambda}\left[\Sigma_\Lambda^\sigma(0,\alpha) + \mu_0^\sigma\right]$$

Fine tune intitial condition

$$r_0^\sigma = \frac{\mu_0^\sigma}{\Omega_{\Lambda_0}} = -\frac{\Sigma^\sigma(\alpha k^\sigma, i0)}{v_F \Lambda_0}$$

such that relevant coupling r_l^σ flows into a fixed point. From exact flow equation

$$\partial_l r_l^\sigma = (1-\eta_l^\sigma) r_l^\sigma + \dot{\Gamma}_l^\sigma(0,\alpha)$$

we obtain self-consistency equation for true Fermi point distance $\tilde{\Delta} = \frac{k^+ - k^-}{\Lambda_0}$

$$\begin{aligned}\tilde{\Delta} &= \tilde{\Delta}_0 + \left[\frac{r_0^+}{\tilde{v}_0^+} - \frac{r_0^-}{\tilde{v}_0^-}\right] \\ &= \tilde{\Delta}_0 - \sum_\sigma \frac{\sigma}{\tilde{v}_0^\sigma}\int_0^\infty dl e^{-(1-\bar{\eta}_l^\sigma)l}\dot{\Gamma}_l^\sigma(0,\alpha)\end{aligned}$$

...Truncation of hierarchy of flow equations...

Approximation 1: ignore irrelevant vertices which vanish at initial scale

relevant coupling constants:

constant part of self-energy:

$$r_l^\sigma = \tilde{\Sigma}_l^\sigma(0,\alpha) = \frac{Z_l^\sigma}{\Omega_\Lambda}\left[\Sigma_\Lambda^\sigma(0,\alpha) + \mu_0^\sigma\right] \qquad \partial_l r_l^\sigma = (1-\eta_l^\sigma) r_l^\sigma + \dot{\Gamma}_l^\sigma(0,\alpha)$$

marginal coupling constants:

wave-function renormalization:

$$Z_l^\sigma = 1 + \left.\frac{\partial \tilde{\Sigma}_l^\sigma(0,i\epsilon,\alpha)}{\partial(i\epsilon)}\right|_{\epsilon=0} \qquad \partial_l Z_l^\sigma = -\eta_l^\sigma Z_l^\sigma$$

velocity renormalization:

$$\tilde{v}_l^\sigma = Z_l^\sigma + \left.\frac{\partial \tilde{\Sigma}_l^\sigma(q,i0,\alpha)}{\partial(\alpha q)}\right|_{q=0} \qquad \partial_l \tilde{v}_l^\sigma = -\eta_l^\sigma \tilde{v}_l^\sigma + \left.\frac{\partial \dot{\Gamma}_l^\sigma(q,i0,\alpha)}{\partial(\alpha q)}\right|_{q=0}$$

constant part of spin-flip vertex:

$$\gamma_l = \tilde{\Gamma}_l^{(2,1)}(0,\sigma;0,\bar{\sigma};0,\alpha) \qquad \partial_l \gamma_l = -\frac{\bar{\eta}_l + \eta_l^+ + \eta_l^-}{2}\gamma_l + \dot{\Gamma}_l^{(2,1)}$$

...Truncation continued...

Approximation 2: adiabatic approximation for spin-flip susceptibility

vertex correction due to spin-flip vertex

$$\tilde{\Pi}_l^{\sigma\bar{\sigma}}(\bar{Q},\alpha) \approx \frac{\gamma_l^2}{2\pi} \frac{\sigma\tilde{\Delta}_l + \alpha\bar{q}}{\sigma\tilde{\Delta}_l + \alpha\bar{q} - i\epsilon}$$

flowing Fermi point distance at scale l: $\tilde{\Delta}_l = \tilde{\Delta}_l^* - (r_l^+ - r_l^-)$

initial value: bare Fermi point distance $\tilde{\Delta}_{l=0} = \tilde{\Delta}_0 = (k_0^+ - k_0^-)/\Lambda_0$

limit for large l: rescaled true Fermi point distance $\Delta_l^* = e^l(k^+ - k^-)/\Lambda_0 = e^l\tilde{\Delta}$

Justification of adiabatic approximation within two-cutoff RG possible (see Appendix of Ledowski, PK).

...self-consistent one-loop approximation ...

Simplest approximation: ignore flow of marginal couplings, amounts to:
ladder approximation with self-consistency condition for $\tilde{\Delta} = \dfrac{k^+ - k^-}{\Lambda_0}$

$$\tilde{\Delta} = \frac{\tilde{\Delta}_0}{1 + R(\tilde{\Delta})} \qquad R(\tilde{\Delta}) = -2g_{c,0} + \frac{2g_{n,0}^2}{\sqrt{(1-g_{c,0})^2 - g_{n,0}^2}} \ln \left[\frac{1 + \sqrt{1 + \frac{\tilde{\Delta}^2 g_{n,0}^2}{(1-g_{c,0})^2 - g_{n,0}^2}}}{\tilde{\Delta}\left(1 + \sqrt{\frac{(1-g_{c,0})^2}{(1-g_{c,0})^2 - g_{n,0}^2}}\right)} \right]$$

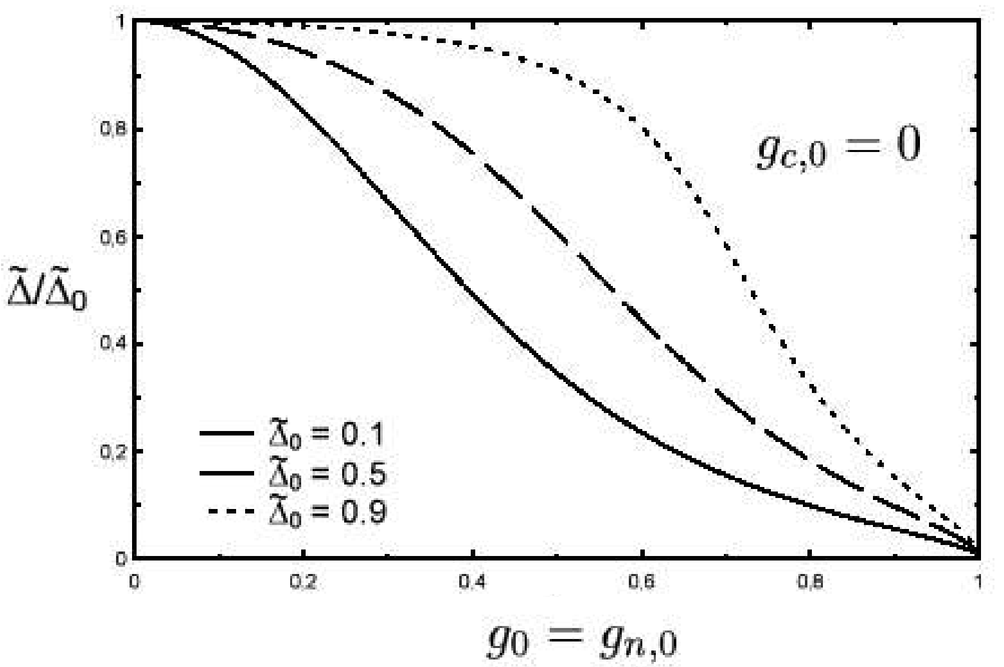
2 types of interchain backscattering:

chiral: $2\nu_0 J^{\perp}_{\alpha,-\alpha} = 2\pi g_{n,0}$

non-chiral: $2\nu_0 J^{\perp}_{\alpha\alpha} = 2\pi g_{c,0}$

weak coupling expansion:

$$R(\tilde{\Delta}) = -2g_{c,0} + 2g_{n,0}^2 \ln(1/\tilde{\Delta}) + O(g_{i,0}^3)$$



- strong confinement for $g_{c,0} + g_{n,0} \to 1$
- confinement is driven by non-chiral part of interchain backscattering

...including wave-function and vertex corrections...

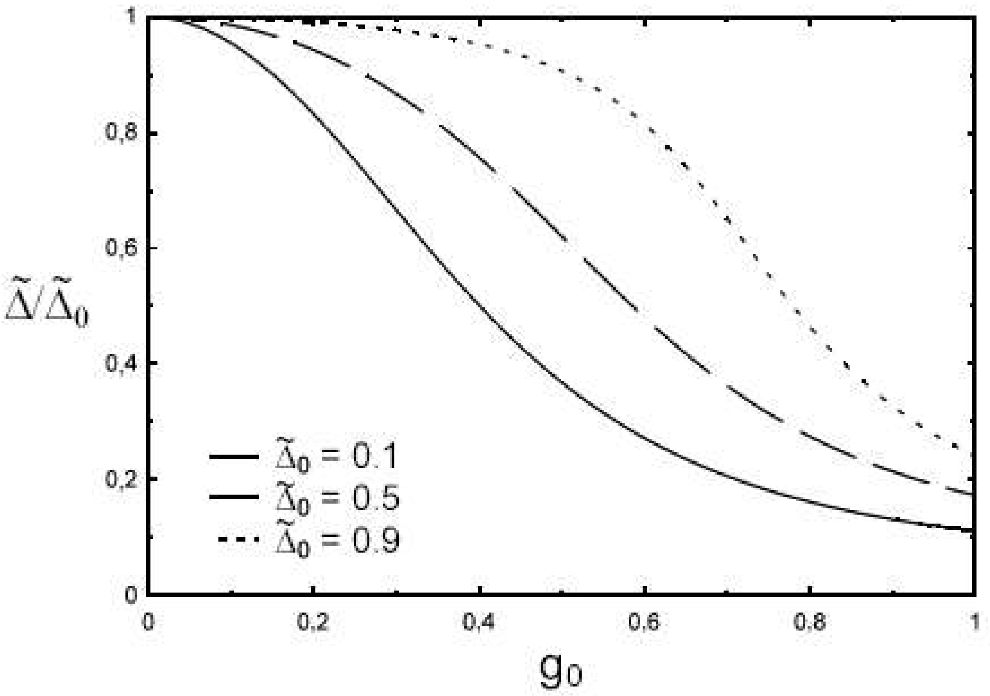
$$\tilde{\Delta} = \tilde{\Delta}_0 - \int_0^{\infty} dl e^{-l} \frac{2\Theta(1-\tilde{\Delta}_l)\tilde{\Delta}_l g_l^2}{\sqrt{1-g_l^2(1-\tilde{\Delta}_l)^2}}$$

$$\tilde{\Delta}_l = \tilde{\Delta}_l^* - 2r_l = \tilde{\Delta} e^l - 2r_l$$

flow of constant part of self-energy:

$$\partial_l r_l = r_l + A(g_l, \tilde{\Delta}_l)$$

$$A(g_l, \tilde{\Delta}_l) = \frac{\Theta(1-\tilde{\Delta}_l)\tilde{\Delta}_l g_l^2}{\sqrt{1-g_l^2(1-\tilde{\Delta}_l^2)}}$$



flow of non-chiral part of interchain backscattering:

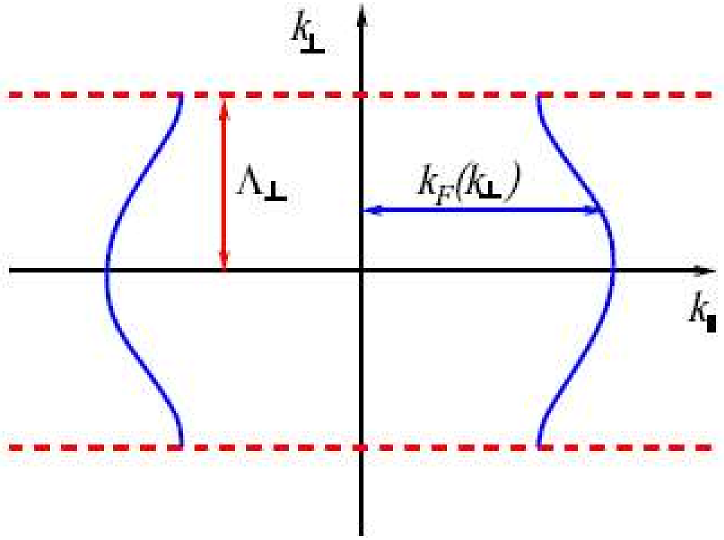
$$\partial_l g_l = B(g_l, \tilde{\Delta}_l)$$

$$B(g_l, \tilde{\Delta}_l) = \frac{-2\Theta(1-\tilde{\Delta}_l)g_l^3}{\sqrt{1-g_l^2(1-\tilde{\Delta}_l^2)}\left[1+\sqrt{1-g_l^2(1-\tilde{\Delta}_l^2)}\right]}$$

No confinement, even at strong coupling!

5. Confinement in two dimensions

S. Ledowski, PK, preprint in preparation (March 2007)



$$S[\bar\psi,\psi] = \sum_\alpha \int_K [-i\omega + \alpha v_F \delta k_\parallel + \mu_0(k_\perp)] \bar\psi_{K\alpha}\psi_{K\alpha} + \frac{1}{2}\sum_{\alpha\alpha'}\int_{\bar K} f_{\alpha\alpha'} \bar\rho_{\bar K\alpha}\rho_{\bar K\alpha'}$$

$$\delta k_\parallel = k_\parallel - \alpha k_F(k_\perp)$$

$$\mu_0(k_\perp) = -\Sigma(\boldsymbol{k}_F, i0)$$

$$\int_K = \int_{k_\perp}\int_{-\Lambda_\parallel}^{\Lambda_\parallel} \frac{d\delta k_\parallel}{2\pi}\int \frac{d\omega}{2\pi}$$

Fermi surface: $k_\parallel = \alpha k_F(k_\perp)$

density-density interaction:
no momentum transfer between sheets:

$$2\pi g_2 = \nu_0 f_{+-} = \nu_0 f_{-+}$$

$$2\pi g_4 = \nu_0 f_{++} = \nu_0 f_{--}$$

$$\nu_0 = \Lambda_\perp(\pi v_F)^{-1} = (a_\perp v_F)^{-1}$$

...second order self-consistent perturbation theory ...

shift of FS due to interactions: $\delta k_F(k_\perp) = k_F(k_\perp) - k_{F,0}(k_\perp)$

satisfies non-linear integral equation:

$$\frac{\delta k_F(k_\perp)}{\Lambda_0} = \left[-g_4 + \frac{g_4^2 + g_2^2}{2}\right] \int_{\bar{k}_\perp} \tilde{\Delta}(k_\perp, \bar{k}_\perp) - g_2^2 \int_{\bar{k}_\perp} \int_{k'_\perp} Y(\tilde{\Delta}(k_\perp, \bar{k}_\perp); \tilde{\Delta}(k'_\perp, \bar{k}_\perp))$$

$$\int_{k_\perp} = \int_{-\Lambda_\perp}^{\Lambda_\perp} \frac{dk_\perp}{2\Lambda_\perp}$$

$$\bar{\Lambda}_{||} = \Lambda_0$$

kernel: $$Y(\tilde{\Delta}; \tilde{\Delta}') = \frac{\tilde{\Delta} + \tilde{\Delta}'}{4} \ln \left[\frac{4 - (\tilde{\Delta} - \tilde{\Delta}')^2}{(\tilde{\Delta} + \tilde{\Delta}')^2}\right]$$

$$\tilde{\Delta}(k_\perp, \bar{k}_\perp) = [k_F(k_\perp) - k_F(k_\perp + \bar{k}_\perp)]/\Lambda_0$$

numerical solution possible; more instructive: expansion in harmonics.

...approximate solution of perturbative integral equation ...

bare FS: $k_{F,0}(k_\perp) = \bar{k}_F \pm t_0 \cos(k_\perp a_\perp)$

nearest neighbor hopping:

$$t_0 = 2t_\perp / v_F \ll \Lambda_0$$

renormalized FS: $k_F(k_\perp) \overset{\sim}{=} \bar{k}_F + t \cos(k_\perp a_\perp) + \ldots$

self-consistency condition:

$$t/t_0 = [1 + R(t)]^{-1}$$

$$R(t) \approx \frac{g_4}{2} - \frac{g_4^2}{4} + \frac{g_2^2}{2} \ln\left(\frac{\Lambda_0}{|t|}\right)$$

logarithmic correction dominates for small interchain hopping

Question: can renormalzed hopping vanish at strong coupling?

...functional RG for the two-dimensional FS ...

S. Ledowski and PK, J. Phys. Cond. Mat. (2003), PRB 2005, 2007.

FS can be defined as RG fixed point by fine tuning initial conditions of relevant couplings: $r_l(k_\perp) = Z_l(k_\perp)[\Sigma_\Lambda(\boldsymbol{k}_F, i0, \alpha) + \mu_0(k_\perp)]/v_F\Lambda$

exact RG flow equation: $\partial_l r_l(k_\perp) = [1 - \eta_l(k_\perp)] r_l(k_\perp) + \dot{\Gamma}_l(k_\perp)$

flowing anomalous dimension: $\eta_l(k_\perp) = -\partial_l \ln Z_l(k_\perp)$

exact integral equation for FS shift:

$$\frac{\delta k_F(k_\perp)}{\Lambda_0} = r_0(k_\perp) = -\int_0^\infty dl e^{-l+\int_0^l dt \eta_t(k_\perp)} \dot{\Gamma}_l(k_\perp)$$

approximation for inhomogeneity on rhs:

vertex correction

$$\dot{\Gamma}_l(k_\perp) = -\int_{\bar{k}_\perp} \int \frac{d\bar{q} d\bar{\epsilon}}{(2\pi)^2} \frac{\delta(|\bar{q}| - 1)[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp)]_{\alpha\alpha} e^{i\bar{\epsilon}0}}{i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp, \bar{k}_\perp)} \gamma_l(k_\perp, \bar{k}_\perp) \gamma_l(k_\perp + \bar{k}_\perp, -\bar{k}_\perp)$$

$$\tilde{\Delta}_l(k_\perp, \bar{k}_\perp) = \tilde{k}_{F,l}(k_\perp) - \tilde{k}_{F,l}(k_\perp + \bar{k}_\perp) \qquad \tilde{k}_{F,l}(k_\perp) = k_F(k_\perp)/\Lambda - r_l(k_\perp)$$

...functional RG for the two-dimensional FS, continued....

adiabatic approximation for propagator of density fluctuations:

$$[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp)]^{-1}_{\alpha\alpha'} = [\nu_0 \mathbf{f}]^{-1}_{\alpha\alpha'} + \delta_{\alpha\alpha'} \tilde{\Pi}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp, \alpha)$$

$$\tilde{\Pi}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp, \alpha) = \frac{1}{2\pi} \int_{k_\perp} \frac{\tilde{\Delta}_l(k_\perp, \bar{k}_\perp) + \alpha\bar{q}}{\tilde{\Delta}_l(k_\perp, \bar{k}_\perp) + \alpha\bar{q} - i\bar{\epsilon}} \gamma_l(k_\perp, \bar{k}_\perp)\gamma_l(k_\perp + \bar{k}_\perp, -\bar{k}_\perp)$$

flowing anomalous dimension:

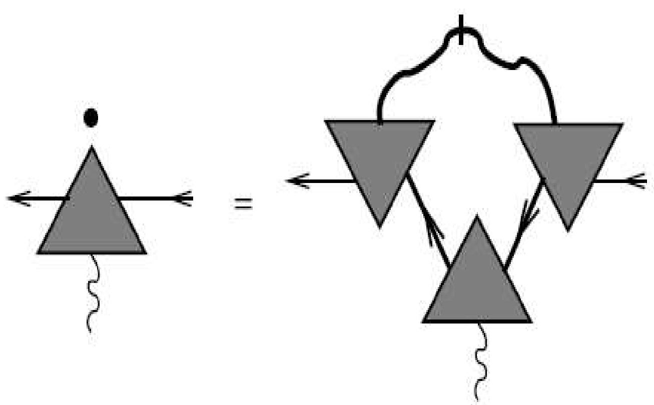
$$\eta_l(k_\perp) = -\int_{\bar{k}_\perp} \int \frac{d\bar{q}d\bar{\epsilon}}{(2\pi)^2} \frac{\delta(|\bar{q}| - 1)[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}_\perp)]_{\alpha\alpha}}{[i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp, \bar{k}_\perp)]^2} \gamma_l(k_\perp, \bar{k}_\perp)\gamma_l(k_\perp + \bar{k}_\perp, -\bar{k}_\perp)$$

flowing vertex correction:

$$\partial_l \gamma_l(k_\perp, \bar{k}_\perp) = -\frac{1}{2}[\eta_l(k_\perp) + \eta_l(k_\perp + \bar{k}_\perp)]\gamma_l(k_\perp, \bar{k}_\perp)$$

$$-\int_{\bar{k}'_\perp} \int \frac{d\bar{q}d\bar{\epsilon}}{(2\pi)^2} \delta(|\bar{q}| - 1)[\mathbf{F}_l(\bar{q}, i\bar{\epsilon}, \bar{k}'_\perp)]_{\alpha\alpha}$$

$$\times \frac{\gamma_l(k_\perp + \bar{k}'_\perp, \bar{k}_\perp)\gamma_l(k_\perp, \bar{k}'_\perp)\gamma_l(k_\perp + \bar{k}_\perp + \bar{k}'_\perp, -\bar{k}'_\perp)}{[i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp, \bar{k}'_\perp)][i\bar{\epsilon} - \alpha\bar{q} - \tilde{\Delta}_l(k_\perp + \bar{k}_\perp, \bar{k}'_\perp)]}$$



...simplest truncation: self-consistent ladder approximation....

ignore vertex and wave-function renormalization:

$$\gamma_l(k_\perp, \bar{k}_\perp) \approx 1$$

$$\eta_l(k_\perp) \approx 0$$

self-consistency equation:

$$\frac{\delta k_F(k_\perp)}{\Lambda_0} = \frac{1-\sqrt{1-g_2^2}}{\sqrt{1-g_2^2}} \int_{\bar{k}_\perp} \tilde{\Delta}(k_\perp, \bar{k}_\perp) \ln |\tilde{\Delta}(k_\perp, \bar{k}_\perp)|$$

solution for first harmonic: $t/t_0 = [1 + R(t)]^{-1}$

$$R(t) \approx \frac{1-\sqrt{1-g_2^2}}{\sqrt{1-g_2^2}} \ln\Big(\frac{\Lambda_0}{|t|}\Big)$$

Confinement transition for $g_2 \to 1$

...RG flow of vertex correction...

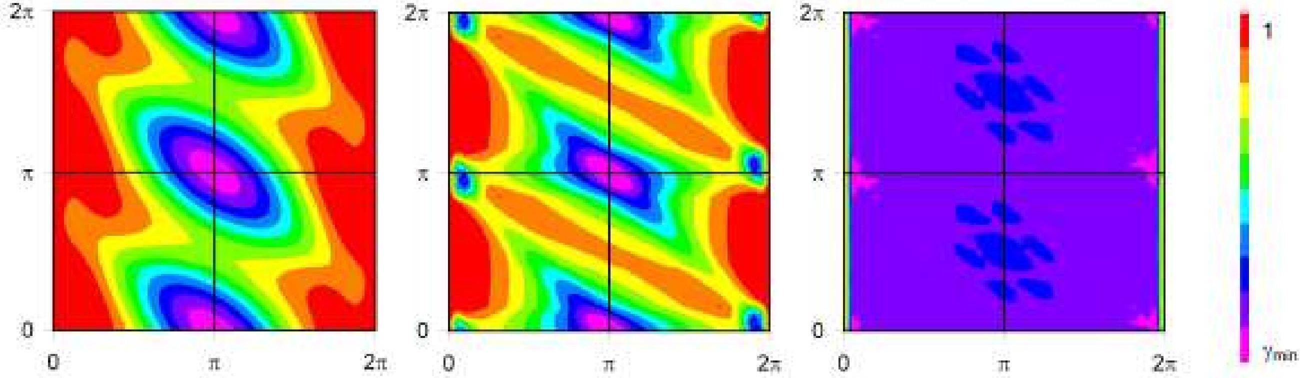
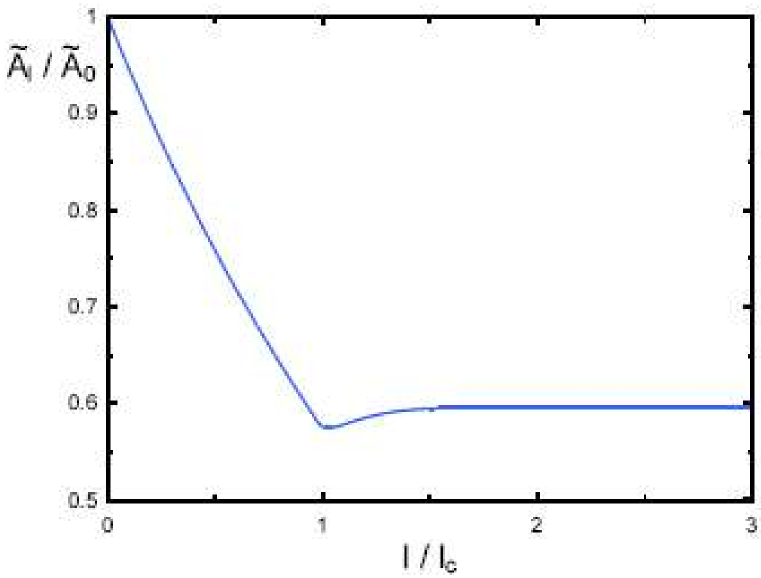


FIG. 3: (Color online) Evolution of the vertex $\gamma_l(k_\perp, \bar{k}_\perp)$ for a harmonic bare FS with amplitude $t_0/\Lambda_0 = 10^{-3}$ and bare coupling $g_2 = 0.4$ for different values of the flow parameter l. The abscissa indicates $\bar{k}_\perp$ and the ordinate $k_\perp$. From left to right $l = \frac{1}{2}l_c, l_c, 3l_c$ and $\gamma_{\min} = 0.999989, 0.981, 0.85$. To evaluate the flow we have expanded in Eqs. (7) and (11) up to g_2^2.

- vertex becomes approximately independent of fermionic momentum
- effective momentum-dependent interaction $g_l(\bar{k}_\perp) = g_2\gamma_l^2(\bar{k}_\perp)$

...RG flow of effective interchain hopping...



$g_2 = 0.4$

$t_0/\Lambda_0 = 10^{-3}$

- at scale $l_c \approx -\ln(2t_0/\Lambda_0)$ effective cutoff comparable with bare hopping:

$$\Lambda_c = \Lambda_0 e^{-l_c} = 2t_0$$

- vertex becomes independent of fermionic momentum

...numerical solution: confinement transition:

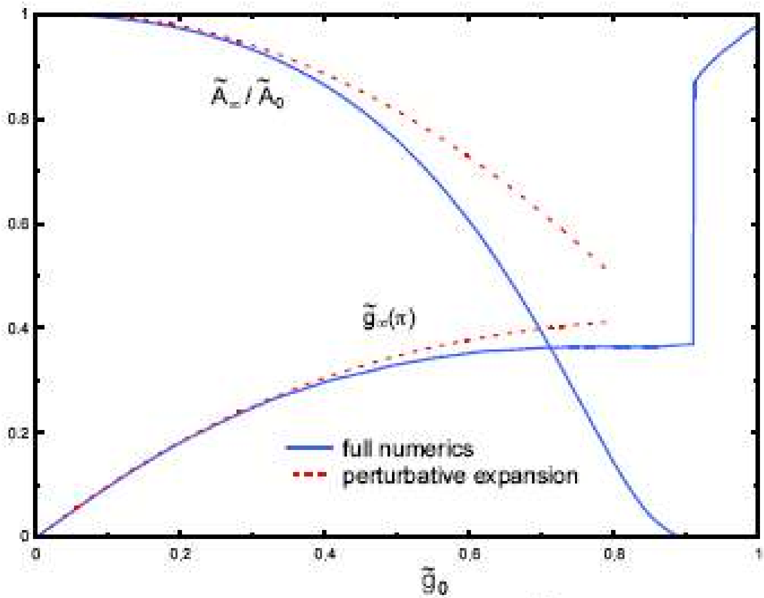


FIG. 6: (Color online) RG flow of $\tilde{t}_l$ and u_l as a function of the bare interaction g_2 for $\tilde{t}_0 = 0.1$ as obtained from the numerical solution of Eqs. (7) and (11).

...qualitative understanding from simplified flow equations:

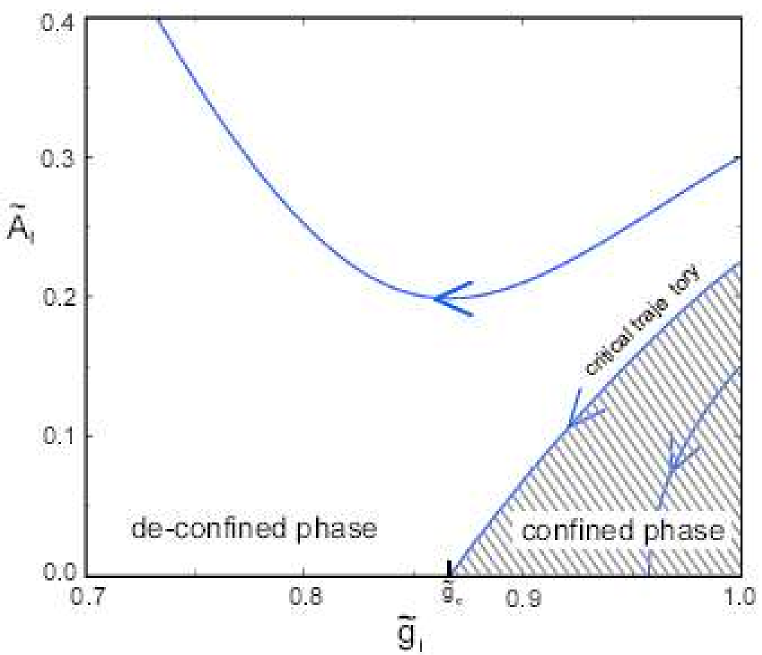
effective interchain hopping:

$$\partial_l \tilde{t}_l = \left[1 - 2 \int_{\bar{k}_\perp} \sin^2(\bar{k}_\perp a_\perp/2) \frac{1 - \sqrt{1 - g_l^2(\bar{k}_\perp)}}{\sqrt{1 - g_l^2(\bar{k}_\perp)}} \right] \tilde{t}_l$$

effective interaction:

$$\partial_l g_l(\bar{k}_\perp) = -\frac{4 \sin^2(\bar{k}_\perp a_\perp/2) g_l(\bar{k}_\perp) u_l^2 \tilde{t}_l^2}{\sqrt{1 - u_l^2}[1 + \sqrt{1 - u_l^2}]^3}$$

$$u_l = g_l(\pi/a_\perp)$$



Summary, Conclusions

- Understand confinement in quasi 1D metals:
  - functional RG with partial bosonization
  - for 2 chains: fluctuations beyond one loop destroy confinement
  - two-dimensional array of chains: confinement transtion possible!
- Method is very general:
  - applicable to all problems where dominant scattering channel is known
  - reasonable extrapolation to strong coupling
- Extension: problems where several scattering channels compete,
  for example Anderson-Impurity model in Kondo regime