Persistent spin currents in mesoscopic Heisenberg rings

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Outline

1. Persistent currents in mesoscopic metal rings: experiments and theory
2. Spin transport: definitions, electrodynamics, linear response
3. Persistent spin currents in spin rings: spin wave calculation, proposal for experiment
4. Conclusions & open problems

1. Persistent currents in mesoscopic metal rings

Experiments

Lévy et al. (Bell Labs), PRL 1990

magnetic moment of 10^7 Cu rings
pierced by magnetic flux ϕ
diffusive: mean free path $\ell \ll L$

measurement of magnetic moment
with SQUID
modulation $\phi(t) = \phi + \phi_{AC} \sin(\Omega t)$

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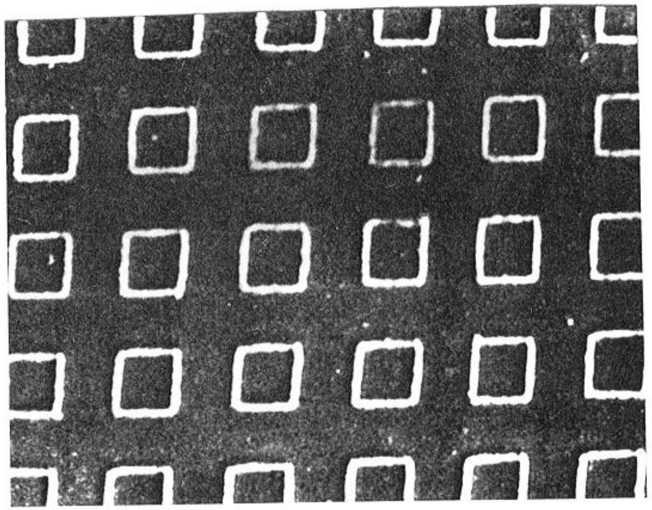


Fig. 1. Scanning electron micrograph of the copper rings used in the experiment. The squares are 0.55 μm on a side. The linewidth is under 500 Å.

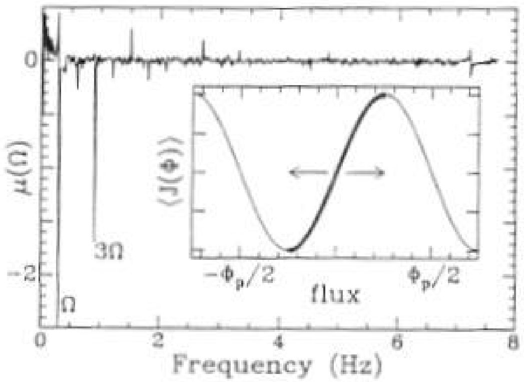


FIG. 1. Fourier transform of the in-phase magnetization signal modulated at 0.3 Hz by a sine wave of 15-G amplitude. The sample temperature is $T=25$ mK, while the magnetometer is held at $T=450$ mK. Inset: Schematic dependence of the average current with flux. A flux modulation of $\pm\phi_p/4$ (arrows and highlighted region of the curve) produces the desired nonlinear response.

average current: $\langle I \rangle \approx 3 \times 10^{-3} \frac{e v_F}{L} \approx 0.4 nA$ per ring

periodic in ϕ with period with half a flux quantum $\phi_0 = hc/e$

Persistent current experiments

Chandrasekhar et al. (IBM), PRL 91

- single Au ring (no self-averaging)
- diffusive regime $\ell \ll L$
- $I_{typ} \approx e v_F / L$
- periodic with ϕ_0

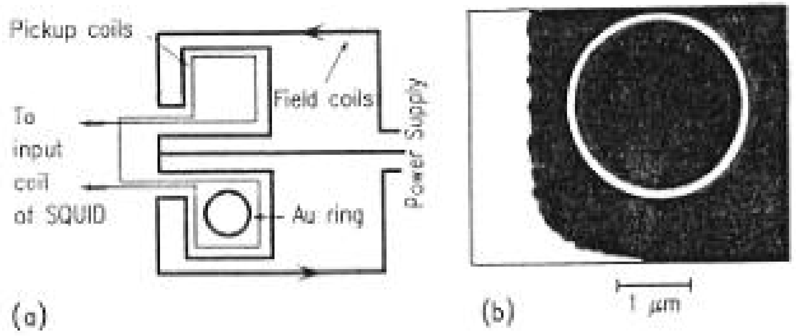


FIG. 1. (a) Schematic diagram of the pickup coil chip, illustrating the counterwound Nb pickup coils and the on-chip magnetic-field coils. (b) Micrograph of the 2.4-μm-diam Au ring in one corner of the 9-μm-inner-diam Nb pickup coil. The loop linewidth is 90 nm and the thickness is 60 nm.

Mailly et al. (Grenoble), PRL 93

- single GaAs ring
- ballistic regime $\ell > L$
- $I \approx e v_F / L$
- periodic with ϕ_0

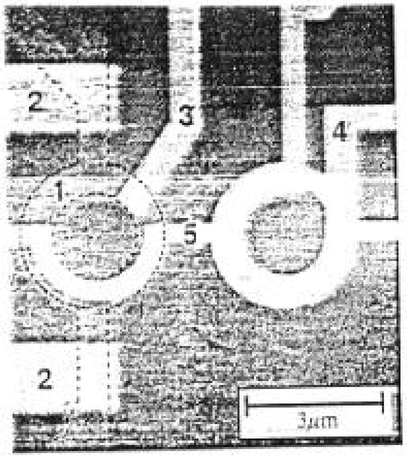
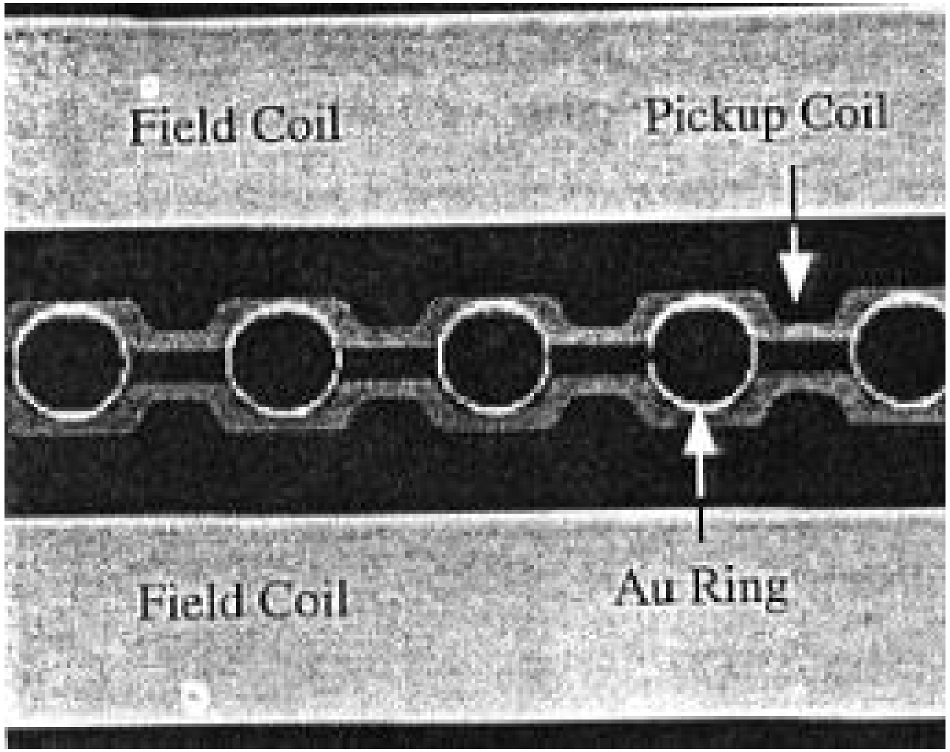


FIG. 1. Electron micrograph of the experimental device. On the left is the ring etched in GaAs 2DEG (labeled 1) (the dashed line has been added because of the poor contrast) with the two gates, (2) and (3). On the right is the calibration coil (4). On the top is the first level of the SQUID fabrication (5) with the two microbridge junctions on the right.

Persistent current experiments

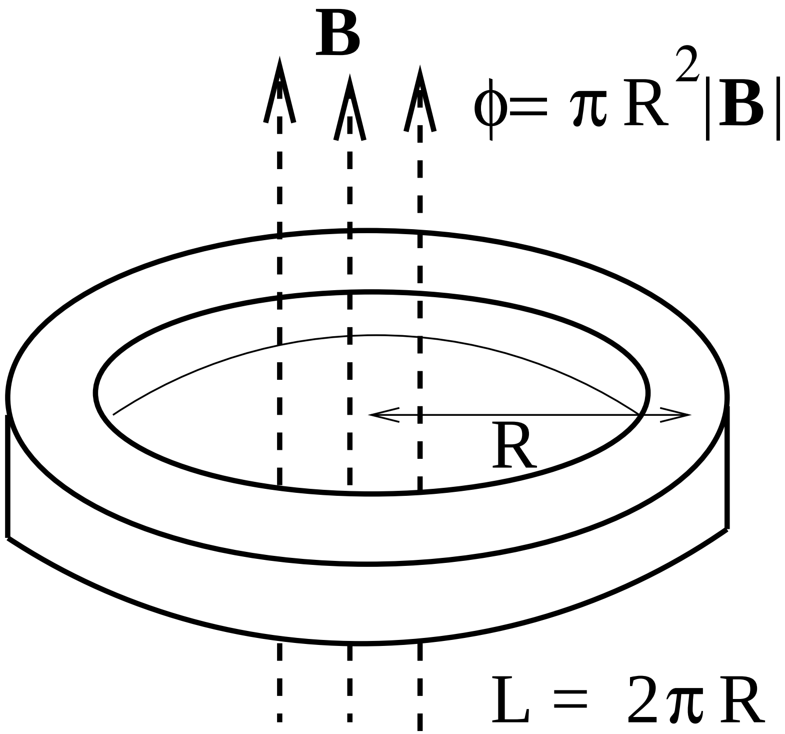
Mohanty, et al. (IBM), 1996

- 30 gold rings
- diffusive regime $\ell \ll L$
- confirmation of Levy-experiment!

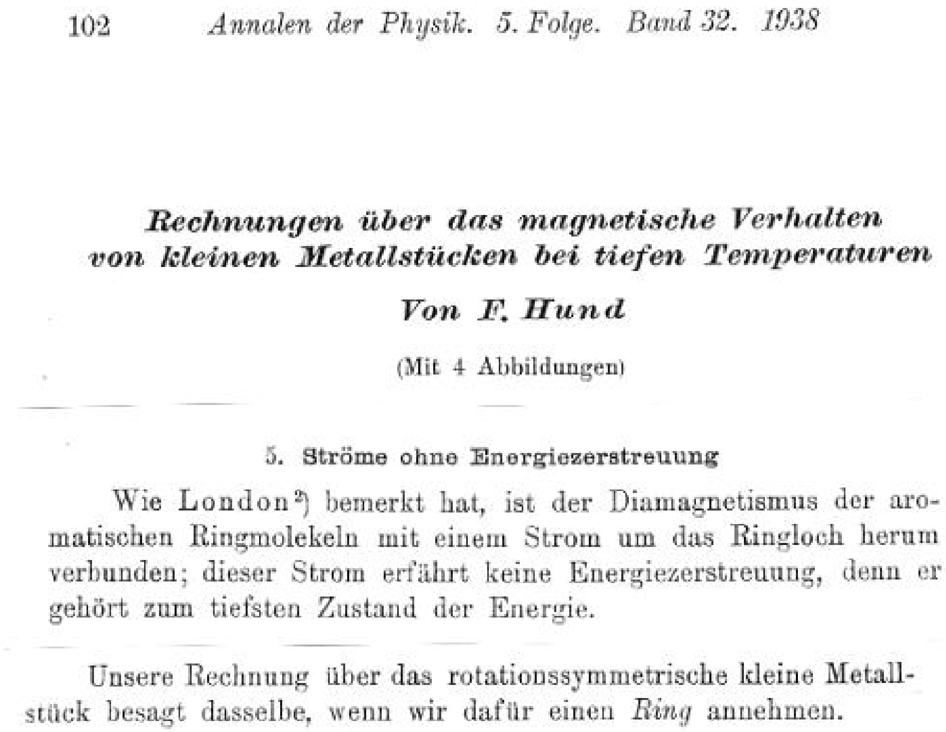


simple theory for persistent currents

mesoscopic ring in Aharonov-Bohm geometry:



early theoretical investigation:
Friedrich Hund, 1938:


102 *Annalen der Physik. 5. Folge. Band 32. 1938*

Rechnungen über das magnetische Verhalten von kleinen Metallstücken bei tiefen Temperaturen

Von F. Hund

(Mit 4 Abbildungen)

5. Ströme ohne Energiezerstreuung

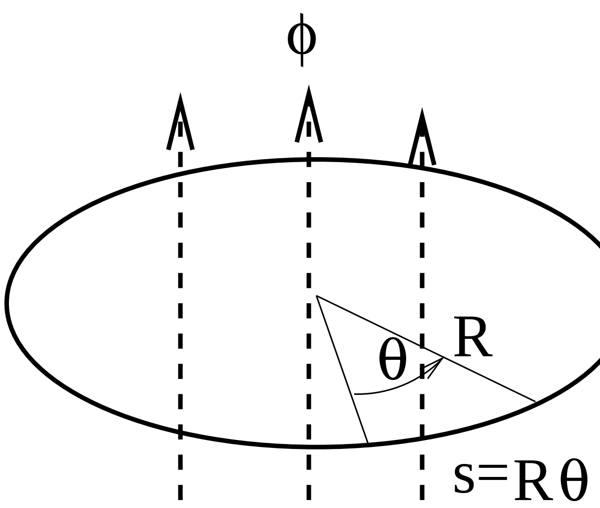
Wie London[2]) bemerkt hat, ist der Diamagnetismus der aromatischen Ringmolekeln mit einem Strom um das Ringloch herum verbunden; dieser Strom erfährt keine Energiezerstreuung, denn er gehört zum tiefsten Zustand der Energie.

Unsere Rechnung über das rotationssymmetrische kleine Metallstück besagt dasselbe, wenn wir dafür einen *Ring* annehmen.

- mesoscopic systems: wavefunction phase coherent: $L_\varphi \gg L$
- typically: phase coherence length diverges: $L_\varphi \propto 1/T^\alpha$ for $T \to 0$

why persistent current in ground state?

simplest model: free electrons in 1d, no disorder



- vector potential $\mathbf{A} = \frac{\phi}{L}\hat{\mathbf{e}}_\theta$
  magnetic flux $\phi = \oint \mathbf{A} \cdot d\mathbf{r}$
- Schrödinger equation
  $\frac{\hbar^2}{2m^*}[-i\frac{d}{ds} + \frac{2\pi}{L}\frac{\phi}{\phi_0}]^2\psi(s) = E\psi(s)$
- periodic boundary conditions
  $\psi(s+L) = \psi(s)$

- gauge transformation: $\tilde{\psi}(s) = \exp[i\frac{s}{R}\frac{\phi}{\phi_0}]\psi(s)$
  $\Rightarrow$ twisted boundary conditions:

$$\frac{\hbar^2}{2m^*}[-i\frac{d}{ds}]^2\psi(s) = E\psi(s) \quad , \qquad \tilde{\psi}(s+L) = e^{2\pi i\phi/\phi_0}\tilde{\psi}(s)$$

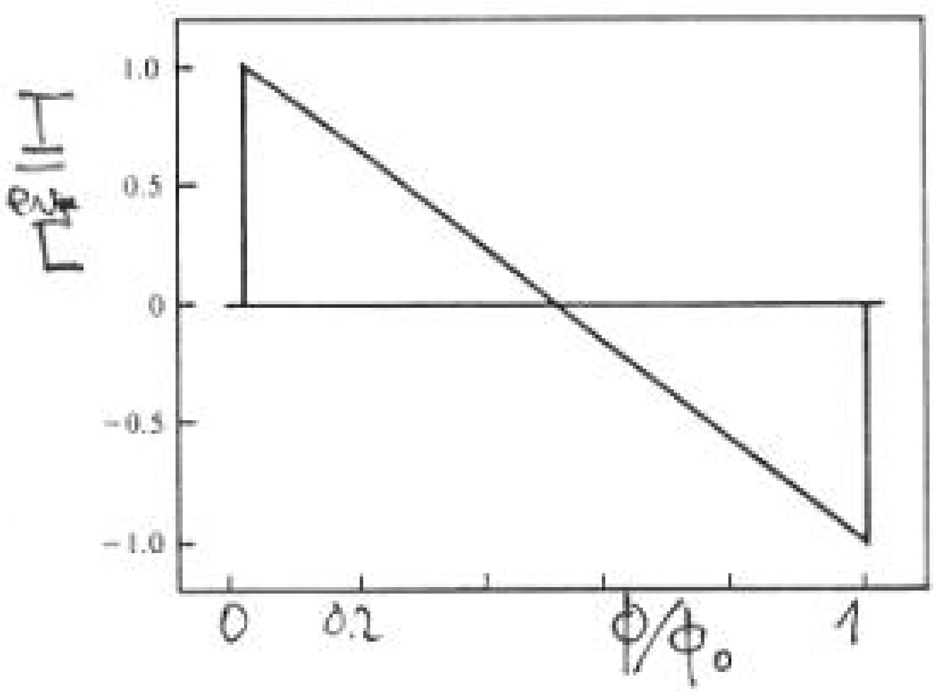
- eigenstates and eigenenergies: $\tilde{\psi}_n(s) = \frac{1}{\sqrt{L}}e^{ik_n s}$, $\epsilon_n = \frac{\hbar^2 k_n^2}{2m^*}$
- quantized wavevectors: $k_n = \frac{2\pi}{L}(n + \frac{\phi}{\phi_0})$, $n = 0, \pm 1, \pm 2 \ldots$

persistent current around 1d ballistic conductor

broken time-reversal invariance: $k_{-n} \neq k_n$ for $\phi \neq m\phi_0$

persistent equilibrium current from thermodynamic potential $\Omega_{gc}(\phi)$:

$$I(\phi) = -c\frac{\partial \Omega_{gc}(\phi)}{\partial \phi} = \frac{-e}{L}\sum_n \frac{v_n}{e^{(\epsilon_n-\mu)/T}+1} \quad , \quad v_n = \frac{1}{\hbar}\frac{\partial \epsilon_n}{\partial k_n} = \frac{\hbar}{m^*}\frac{2\pi}{L}(n+\frac{\phi}{\phi_0})$$



maximum current:
$I_{max} = \frac{ev_F}{L}$

persistent currents: theory + unsolved problems

non-interacting electrons + disorder:

- Landauer, 1957: static disorder ($\Rightarrow$ elastic scattering) preserves phase coherence
- Büttiker, Imry, Landauer, 1983: mesoscopic persistent current exists also in disordered conductor
- several groups, 1990: canonical ensemble, M-channel ring: $I \approx \frac{ev_F}{L}\frac{1}{M}$– too small!

electron-electron interactions + disorder:

- Ambegaokar, Eckern, PRL 1990: 1st order perturbation theory in screened Coulomb interaction:
  $I = \frac{ev_F}{L}\frac{\ell}{L}(g_H + g_F)$– too small!
- P. K., PRL 1993, Völker and P.K., 1997: screening problem in mesoscopic conductors; unscreened Coulomb interactions could explain experiment!
- Kravtsov, Altshuler, PRL 1998: connection with non-equilibrium dephasing time

unsolved problem: experimentally observed current in diffusive regime is larger than all theoretical predictions!
need: non-perturbative method to deal with Coulomb interaction and disorder!

2. Spin transport

Motivation

- Spintronics: information processing based on spin degree of freedom
- many studies (experimental, theoretical) using ferromagnetic metals and semiconductors (Awschalom, Loss, Samarth (Eds): Semiconductor Spintronics and Quantum Computation, Springer, Berlin, 2002)
- simpler problem: spin transport in magnetic insulators,
  for example: systems described by quantum Heisenberg model:

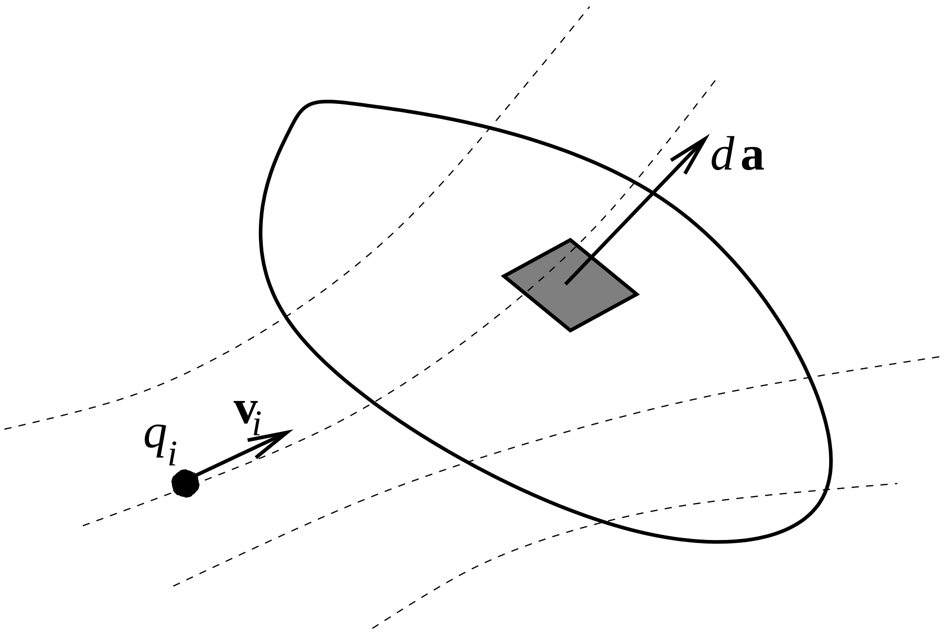
$$\hat{H} = \frac{1}{2} \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B \sum_i \mathbf{B}_i \cdot \mathbf{S}_i$$

quantum spins: $[S_i^x, S_j^y] = i\delta_{ij} S_i^z$, $\mathbf{S}_i^2 = S(S+1)$

- inhomogeneous magnetic field $\mathbf{B}_i$ generates magnetic moments $\mathbf{m}_i(t) = g\mu_B \langle \psi(t) | \mathbf{S}_i | \psi(t) \rangle$
- want: time evolution of magnetization $\mathbf{M}(\mathbf{r}, t) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \mathbf{m}_i(t)$

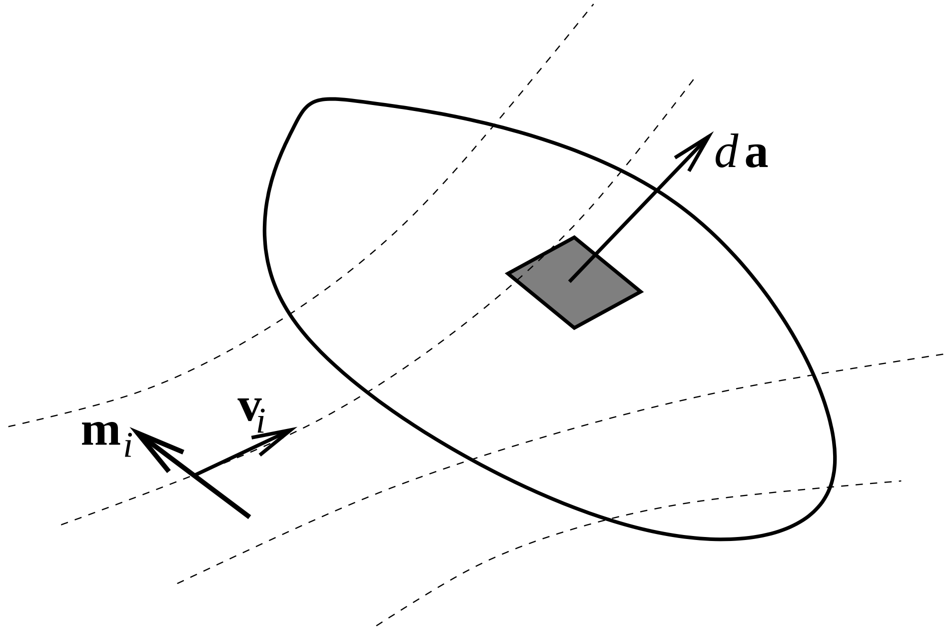
charge currents versus magnetization currents

current of electric monopoles q_i:



- current density: vector
  $j_\mu(\mathbf{r}) = \sum_i q_i (\mathbf{v}_i)_\mu \delta(\mathbf{r} - \mathbf{r}_i)$
- current through surface A:
  $I(A) = \int_A d\mathbf{a} \cdot \mathbf{j}(\mathbf{r})$

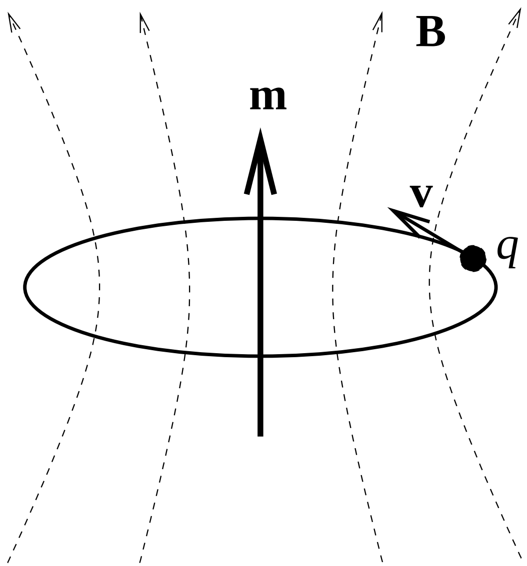
current of magnetic dipoles $\mathbf{m}_i$



- magnetization current density:
  2nd rank tensor
  $j^\alpha_\mu(\mathbf{r}) = \sum_i (\mathbf{m}_i)^\alpha (\mathbf{v}_i)_\mu \delta(\mathbf{r} - \mathbf{r}_i)$
- current through surface A:
  $I^\alpha(A) = \int_A da_\mu j^\alpha_\mu(\mathbf{r})$
  $\alpha = x, y, z$ labels spin components

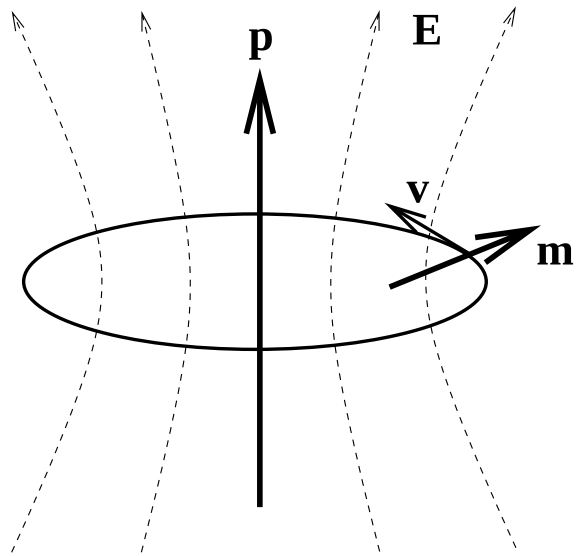
electrodynamics of magnetization currents

stationary charge currents generate static magnetic fields:



- Biot-Savart law:
  $$\mathbf{B}(\mathbf{r}) = \frac{1}{c}\int d^3r'\mathbf{j}(\mathbf{r}') \times \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$
  $$= \frac{I}{c}\oint d\mathbf{r}' \times \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$
- far zone:
  magnetic dipole field:
  $$\mathbf{B} = \nabla \times \mathbf{A} \quad , \quad \mathbf{A} = \frac{\mathbf{m}\times\mathbf{r}}{|\mathbf{r}|^3}$$
  $$\mathbf{m} = \frac{1}{2c}\int d^3r'\mathbf{r}' \times \mathbf{j}(\mathbf{r}')$$

stationary magnetization currents generate static electric fields:



- Biot-Savart-type -law
  $$\phi(\mathbf{r}) = -\frac{1}{c}\int d^3r'[\mathbf{M}(\mathbf{r}') \times \mathbf{v}(\mathbf{r}')] \cdot \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$
  $$= \frac{I_m}{c}\oint [d\mathbf{r}' \times \hat{\mathbf{m}}(\mathbf{r}')] \cdot \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$
- far zone:
  electric dipole field:
  $$\mathbf{E} = -\nabla\phi \quad , \quad \phi = \frac{\mathbf{p}\cdot\mathbf{r}}{|\mathbf{r}|^3}$$
  $$\mathbf{p} = -\frac{1}{c}\int d^3r'\mathbf{M}(\mathbf{r}') \times \mathbf{v}(\mathbf{r}')$$

current conservation

charge current:

$$\frac{\partial\rho(\mathbf{r},t)}{\partial t}+\partial_\mu j_\mu(\mathbf{r},t) = 0$$

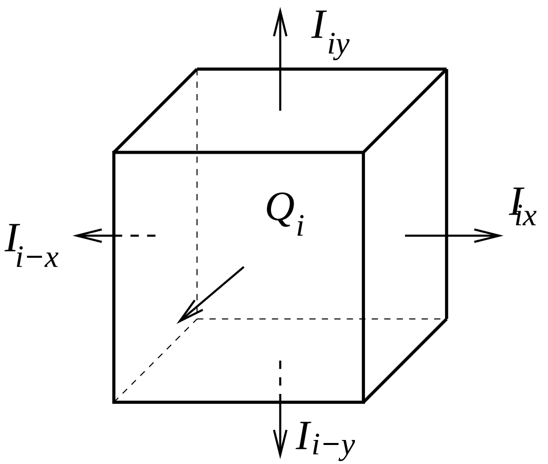
$$\frac{\partial Q_i(t)}{\partial t}+\sum_j I_{i\to j} = 0$$

charge in volume V_i: $Q_i(t)=\int_{V_i} d^3r\,\rho(\mathbf{r},t)$

current flowing out of surface F_{ij}:

$I_{i\to j}(t)=\int_{F_{ij}} d\mathbf{f}\cdot\mathbf{j}(\mathbf{r},t)$

equilibrium: Kirchhoff-law: $\sum_j I_{i\to j}=0$



magnetization current:

$$\frac{\partial M^\alpha(\mathbf{r},t)}{\partial t}+\partial_\mu j^\alpha_\mu(\mathbf{r},t) = -\frac{g\mu_B}{\hbar}[\mathbf{B}(\mathbf{r})\times\mathbf{M}(\mathbf{r},t)]^\alpha$$

integrated version: expectation value of Heisenberg equation of motion for magnetic moments $\mathbf{m}_i(t)=g\mu_B\langle\mathbf{S}_i\rangle=m_i\hat{\mathbf{m}}_i$:

$$\frac{\partial m_i(t)}{\partial t}+\sum_j \hat{\mathbf{m}}_i\cdot\mathbf{I}_{i\to j} = 0$$

magnetization current from i to j:

$\mathbf{I}_{i\to j}(t)=\frac{g\mu_B}{\hbar}J_{ij}\langle\mathbf{S}_i\times\mathbf{S}_j\rangle$

equilibrium: Kirchhoff-law:

$\sum_j \hat{\mathbf{m}}_i\cdot\mathbf{I}_{i\to j}=0$

linear response, Kubo formula

charge conductivity:

$$j_\mu(\mathbf{q},\omega) = \sum_\nu \sigma_{\mu\nu}(\mathbf{q},\omega) E_\nu(\mathbf{q},\omega)$$

Kubo formula:

$$\sigma_{\mu\nu}(\mathbf{q},\omega) = \frac{K_{\mu\nu}(\mathbf{q},\omega)}{i(\omega+i0)}$$

$$K_{\mu\nu}(\mathbf{q},\omega) = -\delta_{\mu\nu}\frac{ne^2}{m} + i\int_0^\infty dt e^{-i(\omega+i0)t}\langle[j_\mu(\mathbf{q},t), j_\nu(-\mathbf{q},0)]\rangle$$

ideal conductor $\Rightarrow$ persistent current

$$\lim_{\omega\to 0} K_{\mu\nu}(0,\omega) = -D_{\mu\nu}$$

$$\mathrm{Re}\sigma_{\mu\nu}(0,\omega) \sim \pi D_{\mu\nu}\delta(\omega)$$

$$\mathrm{Im}\sigma_{\mu\nu}(0,\omega) \sim D_{\mu\nu}/\omega$$

magnetization conductivity:

$$j^\alpha_\mu(\mathbf{q},\omega) = \sum_{\nu,\beta} \sigma^{\alpha\beta}_{\mu\nu}(\mathbf{q},\omega)[\partial_\nu B^\beta](\mathbf{q},\omega)$$

Kubo formula:

$$\sigma^{\alpha\beta}_{\mu\nu}(\mathbf{q},\omega) = \frac{K^{\alpha\beta}_{\mu\nu}(\mathbf{q},\omega)}{i(\omega+i0)}$$

$$K^{\alpha\beta}_{\mu\nu}(\mathbf{q},\omega) = -\delta_{\mu\nu}\delta^{\alpha\beta}D_0 + i\int_0^\infty dt e^{-i(\omega+i0)t}\langle[j^\alpha_\mu(\mathbf{q},t), j^\beta_\nu(-\mathbf{q},0)]\rangle$$

$\Rightarrow$ persistent magnetization current

$$\lim_{\omega\to 0} K^{\alpha\beta}_{\mu\nu}(0,\omega) = -D^{\alpha\beta}_{\mu\nu}$$

$$\mathrm{Re}\sigma^{\alpha\beta}_{\mu\nu}(0,\omega) \sim \pi D^{\alpha\beta}_{\mu\nu}\delta(\omega)$$

$$\mathrm{Im}\sigma^{\alpha\beta}_{\mu\nu}(0,\omega) \sim D^{\alpha\beta}_{\mu\nu}/\omega$$

3. Persistent spin currents in Heisenberg rings

- Starting point: general Heisenberg model in inhomogeneous magnetic field:

$$\hat{H} = \frac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j - \sum_i \mathbf{h}_i \cdot \mathbf{S}_i \quad , \quad \mathbf{h}_i = g\mu_B \mathbf{B}_i$$

- classical ground state: local moments $\mathbf{m}_i = g\mu_B \langle \mathbf{S}_i \rangle$
- low energy excitations above classical ground state: spin-waves
- longitudinal and transverse fluctuations: $\mathbf{S}_i = S_i^{\parallel}\hat{\mathbf{m}}_i + \mathbf{S}_i^{\perp}$, $\mathbf{S}_i^{\perp} \cdot \hat{\mathbf{m}}_i = 0$
- decomposition of Hamiltonian: $\hat{H} = \hat{H}^{\parallel} + \hat{H}^{\perp} + \hat{H}'$
  - longitudinal part: $\hat{H}^{\parallel} = \frac{1}{2}\sum_{i,j} J_{ij}\hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j S_i^{\parallel} S_j^{\parallel} - \sum_i \mathbf{h}_i \cdot \hat{\mathbf{m}}_i S_i^{\parallel}$
  - transverse part: $\hat{H}^{\perp} = \frac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i^{\perp} \cdot \mathbf{S}_j^{\perp}$
  - residual part: $\hat{H}' = -\sum_i \mathbf{S}_i^{\perp} \cdot \left(\mathbf{h}_i - \sum_j J_{ij} S_j^{\parallel}\hat{\mathbf{m}}_j\right)$

classical ground state

Holstein-Primakoff transformation:

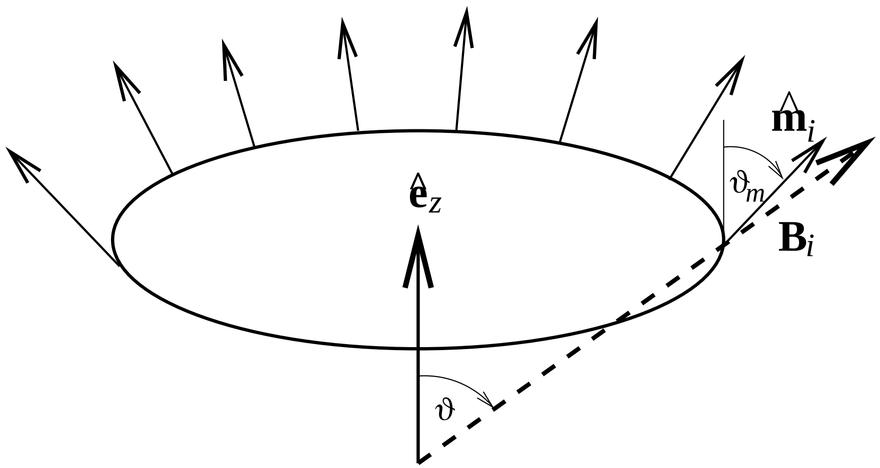
- maps spins ⇒ bosons: $[b_i, b_i^\dagger] = 1$
- good for $b_i^\dagger b_i \ll 2S$!
- square roots can be expanded in $1/S$

$$\mathbf{e}_i^+ \cdot \mathbf{S}_i^\perp = S_i^+ = \sqrt{2S}\sqrt{1 - \frac{b_i^\dagger b_i}{2S}}\, b_i$$

$$\mathbf{e}_i^- \cdot \mathbf{S}_i^\perp = S_i^- = \sqrt{2S} b_i^\dagger \sqrt{1 - \frac{b_i^\dagger b_i}{2S}}$$

$$\hat{\mathbf{m}}_i \cdot \mathbf{S}_i = S_i^\parallel = S - b_i^\dagger b_i$$

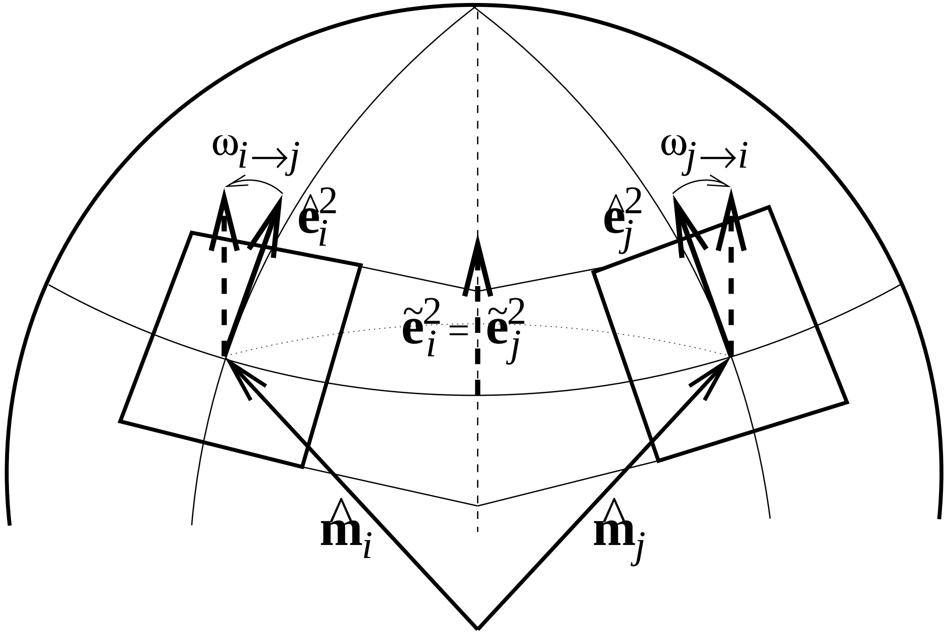
classical ground state: linear fluctuations vanish: $\hat{H}' = 0$ if $S_j^\parallel \approx S$



- general condition:
  $\hat{\mathbf{m}}_i \times (\mathbf{h}_i - \sum_j J_{ij} S \hat{\mathbf{m}}_j) = 0$
- for nearest neighbor ferromagnetic Heisenberg chain $(J_{i,i\pm1} = -J)$ in radial field $(\mathbf{B}(\mathbf{r}) = B\hat{\mathbf{r}})$:
  $\sin(\vartheta_m - \vartheta) =$
  $-(JS/|\mathbf{h}|)\,[1 - \cos(2\pi/N)]\sin(2\vartheta)$

transverse basis: local $U(1)$ gauge freedom

local triad $\{\hat{\mathbf{e}}_i^1, \hat{\mathbf{e}}_i^2, \hat{\mathbf{m}}_i\}$ can be arbitrarily rotated around $\hat{\mathbf{m}}_i$!



transverse Hamiltonian in spherical basis:

$\mathbf{e}_i^p = \hat{\mathbf{e}}_i^1 + ip\hat{\mathbf{e}}_i^2 \;\;,\;\; p = \pm$:

$S_i^p = \mathbf{e}_i^p \cdot \mathbf{S}_i = \mathbf{e}_i^p \cdot \mathbf{S}_i^\perp$

$\hat{H}^\perp = \frac{1}{8} \sum_{i,j} \sum_{p,p'} J_{ij} (\mathbf{e}_i^p \cdot \mathbf{e}_j^{p'}) S_i^{-p} S_j^{-p'}$

general basis: $\mathbf{e}_i^p \cdot \mathbf{e}_j^{p'} = \hat{\mathbf{e}}_i^1 \cdot \hat{\mathbf{e}}_j^1 - pp'\hat{\mathbf{e}}_i^2 \cdot \hat{\mathbf{e}}_j^2 + i[p\hat{\mathbf{e}}_i^2 \cdot \hat{\mathbf{e}}_j^1 + p'\hat{\mathbf{e}}_i^1 \cdot \hat{\mathbf{e}}_j^2]$

parallel transported basis: $\tilde{\mathbf{e}}_i^2 = \tilde{\mathbf{e}}_j^2 \parallel \hat{\mathbf{m}}_i \times \hat{\mathbf{m}}_j \Rightarrow \tilde{\mathbf{e}}_i^p \cdot \tilde{\mathbf{e}}_j^{p'} = \hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j - pp'$

$$\Rightarrow \mathbf{e}_i^p \cdot \mathbf{e}_j^{p'} = [\hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j - pp'] e^{ip\omega_{i\to j} + ip'\omega_{j\to i}}$$

geometric interpretation: $\omega_{i\to j} - \omega_{j\to i}$ = defect angle (anholonomy) for parallel transport from $\hat{\mathbf{m}}_i$ to $\hat{\mathbf{m}}_j$ along geodesic on unit sphere

gauge invariance of $\hat{H}^\perp$: $\omega_{i\to j} \to \omega_{i\to j} + \alpha_i \;\;,\;\; S_i^p \to S_i^p e^{ip\alpha_i}$ with arbitrary α_i

parallel transport on surface of unit sphere

from: article by M. V. Berry, published in book by A. Shapere and F. Wilczek, Geometric Phases in Physics, 1989

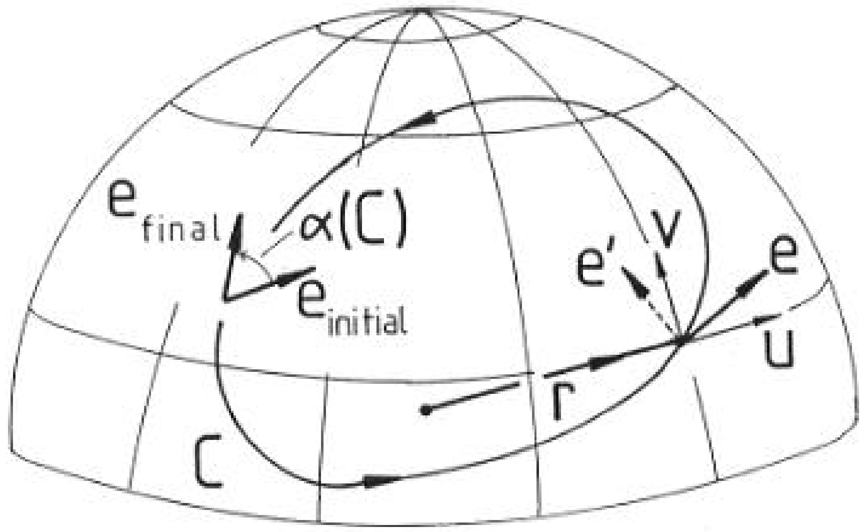
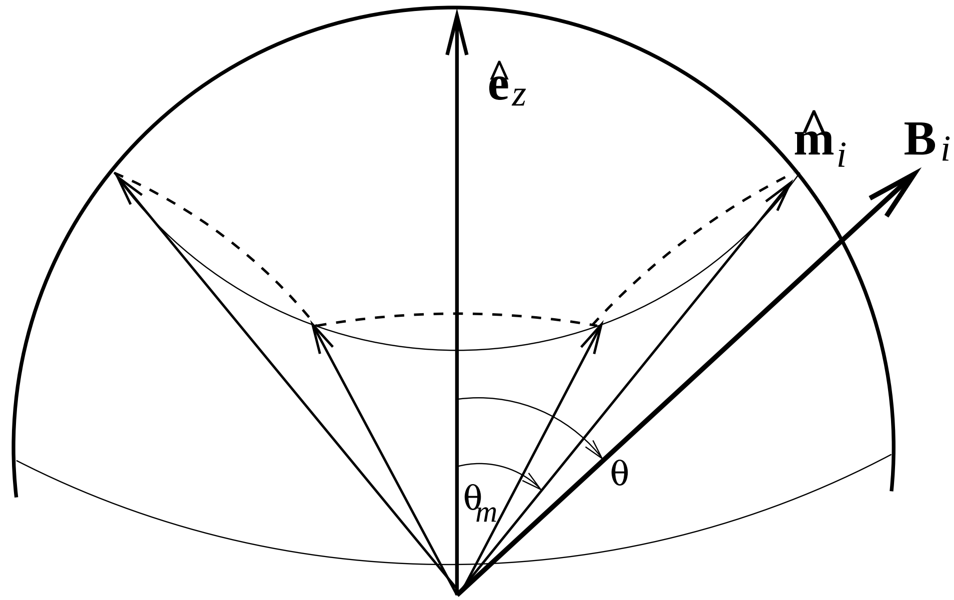


Figure 1. Rotation by $\alpha(C)$ after parallel transport of vector **e** round circuit C on a sphere.



discrete parallel transport along geodesics connecting $\hat{\mathbf{m}}_i$ and $\hat{\mathbf{m}}_j$.

- given: curve on unit sphere $\hat{\mathbf{m}}(s)$
- parallel transported transverse basis is determined by $\frac{d\hat{\mathbf{e}}(s)}{ds} = \boldsymbol{\omega}(s) \times \hat{\mathbf{e}}(s)$ with angular velocity $\boldsymbol{\omega}(s) = \hat{\mathbf{m}}(s) \times \frac{d\hat{\mathbf{m}}(s)}{ds}$

gauge invariance and current conservation

transverse part of spin Hamiltonian:

$$\hat{H}^{\perp} = \frac{1}{8}\sum_{i,j} J_{ij}\Big[(1+\hat{\mathbf{m}}_i\cdot\hat{\mathbf{m}}_j)e^{i(\omega_{i\to j}-\omega_{j\to i})}S_i^- S_j^+ - (1-\hat{\mathbf{m}}_i\cdot\hat{\mathbf{m}}_j)e^{i(\omega_{i\to j}+\omega_{j\to i})}S_i^- S_j^- + \mathrm{h.c.}\Big]$$

invariant under gauge transformation: $\omega_{i\to j}\to\omega_{i\to j}+\alpha_i \quad , \quad S_i^p \to S_i^p e^{ip\alpha_i}$
Noether-theorem: conserved spin current:

$$0 = \langle\frac{\partial \hat{H}^{\perp}}{\partial \alpha_i}\rangle = \langle\sum_j \frac{\partial \hat{H}^{\perp}}{\partial \omega_{i\to j}}\rangle = -\langle\sum_j J_{ij}\hat{\mathbf{m}}_i\cdot\left(\mathbf{S}_i^{\perp}\times\mathbf{S}_j^{\perp}\right)\rangle$$

follows from Heisenberg equation of motion:

$$\hbar\frac{\partial S_i^{\parallel}}{\partial t} = -\sum_j J_{ij}\hat{\mathbf{m}}_i\cdot\left(\mathbf{S}_i^{\perp}\times\mathbf{S}_j^{\perp}\right) - \mathbf{S}_i^{\perp}\cdot[\hat{\mathbf{m}}_i\times(\mathbf{h}_i-\sum_j J_{ij}S_j^{\parallel}\hat{\mathbf{m}}_j)]$$

gauge invariant persistent current

charge current:

current operator:

$$j_\mu(\mathbf{r}) = -c\frac{\delta \hat{H}}{\delta A_\mu(\mathbf{r})}$$

gauge invariant magnetic flux through surface S:

$$\phi(S) = \int_S d\mathbf{s} \cdot \mathbf{B} = \oint_{\partial S} d\mathbf{r} \cdot \mathbf{A}$$

persistent charge current through surface S:

$$I(\phi) = -c\frac{\partial F(\phi)}{\partial \phi}$$

where $F(\phi)$ = free energy

magnetization (spin) current:

current operator:

$$\hat{\mathbf{m}}_i \cdot \mathbf{I}_{i\to j} = \hat{\mathbf{m}}_i \cdot (J_{ij}\mathbf{S}_i^\perp \times \mathbf{S}_j^\perp) = -\frac{\partial \hat{H}^\perp}{\partial \omega_{i\to j}}$$

gauge invariant geometric flux:

$$\Omega = \sum_{i=1}^{N}(\omega_{i\to i+1} - \omega_{i\to i-1})$$

total defect angle (anholonomy) for parallel transport in spin space along classical ground state configuration

persistent spin current along chain:

$$I_s(\Omega) = -\frac{\partial F(\Omega)}{\partial \Omega}$$

magnetization current:

$I_m(\Omega) = \frac{g\mu_B}{\hbar} I_s(\Omega)$

persistent spin current due to spin-waves

spin-wave Hamiltonian for ferromagnet (long-wavelength approximation):

$$\hat{H} \approx -JS^2 \sum_{<ij>} \hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j - g\mu_B S \sum_i \mathbf{B}_i \cdot \hat{\mathbf{m}}_i + \sum_n \left[JSa^2k_n^2 + g\mu_B B\right] b_n^\dagger b_n$$

quantized wavevectors: $k_n = \frac{2\pi}{L}(n - \frac{\Omega}{2\pi})$ $, n = 0, \pm 1, \pm 2 \ldots$

energies: $\epsilon_n = JSa^2k_n^2 + g\mu_B B$

velocities: $v_n = \hbar^{-1}\partial\epsilon_n/\partial k_n = 2JSa^2k_n$

$\Rightarrow$ free energy $F(\Omega) = E_{cl} + T\sum_n \ln\{1 - \exp[-(\epsilon_n + h)/T]\}$

$$I_m(\Omega) = \frac{g\mu_B}{L}\sum_n \frac{v_n}{e^{(\epsilon_n + h)/T} - 1}$$

$$\approx \frac{g\mu_B T}{\hbar}\frac{\sin\Omega}{\cosh(h/\Delta) - \cos\Omega}$$

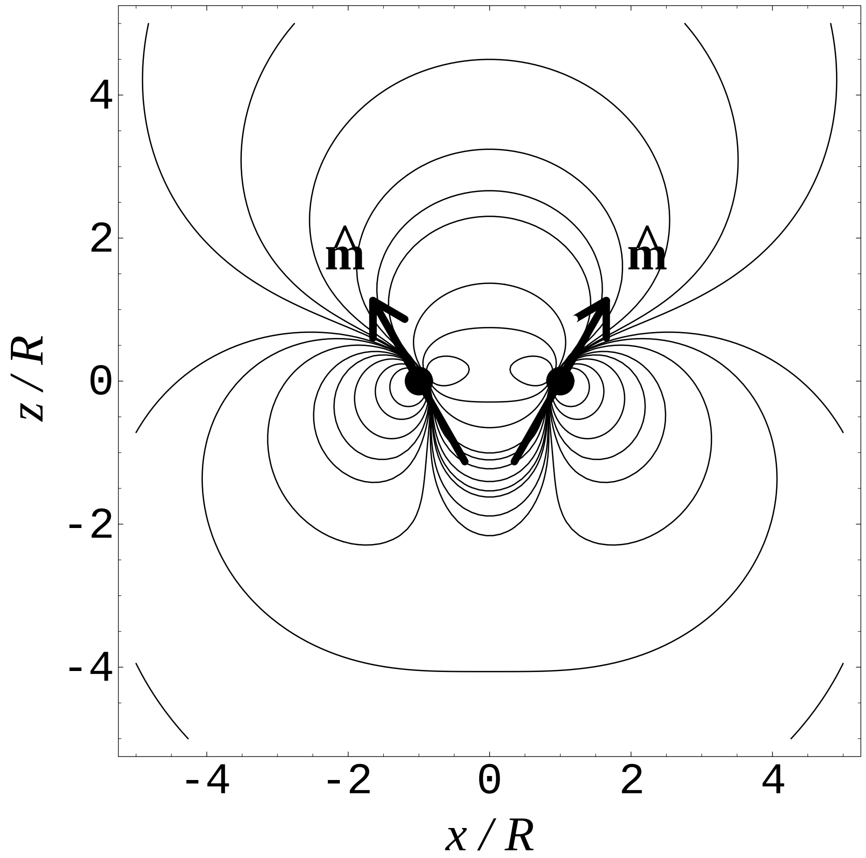
compare with charge current:

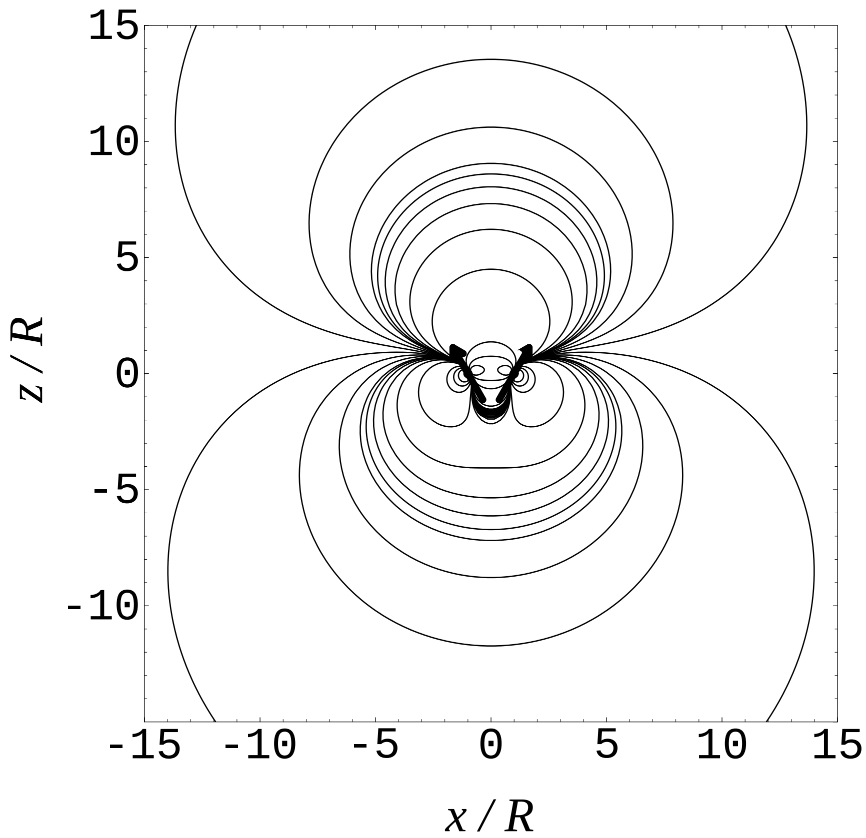
$$\boxed{I(\phi) = \frac{-e}{L}\sum_n \frac{v_n}{e^{(\epsilon_n - \mu)/T} + 1}}$$

for $\Delta = JS(\frac{2\pi}{N})^2 \ll T \ll J$

electric field around spin current loop

lines of constant electric potential, $\vartheta_m = 30^o$





generalized Biot-Savart-law:

$$\phi(\mathbf{r}) = \frac{I_m}{c} \oint [d\mathbf{r}' \times \hat{\mathbf{m}}(\mathbf{r}')] \cdot \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

electric field: $\mathbf{E}(\mathbf{r}) = -\nabla\phi(\mathbf{r})$

for zone: $\phi(\mathbf{r}) = \frac{\mathbf{p}\cdot\mathbf{r}}{|\mathbf{r}|^3}$

electric dipole moment:

$$\mathbf{p} = -\hat{\mathbf{e}}_z \frac{I_m}{c} \sin\vartheta_m$$

possible experiment

- measure voltage difference ΔU a distance L above and below Heisenberg ring along z-axis.
- optimal parameters for N site ferromagnetic Heisenberg chain
  - solid angle: $\Omega = \pi/2$; with $\Omega = 2\pi(1 - \cos\vartheta_m) \Rightarrow \vartheta_m \approx 41^o$
  - $g\mu_B B \approx \Delta = JS(2\pi/N)^2$
  - $\Delta \ll T \ll J$.
- estimate for optimal parameters $\frac{\Delta U}{\mathrm{nV}} = 0.24 \times g \times \frac{(T/\mathrm{Kelvin})}{(L/\mathrm{nm})}$
  for $T = 50K$, $L = 100nm$, $g = 2 \Rightarrow \Delta U \approx 0.2nV$
  for $N = 100$, $J = 100K$ and $S = 1/2$ optimal magnetic field is $B \approx 0.1T$.
- experimental requirements:
  - material: well-characterized Heisenberg-ring with large J in submicron-range
  - magnetic field: inhomogeneity in submicron-range
  - measurement of electric field: nanovolt-sensitivity

4. Summary and Conclusions

- persistent magnetization currents in mesoscopic Heisenberg rings: new mesoscopic quantum interference effect
(spin analogue of mesoscopic persistent charge current in normal metals)
- magnetization current generates electric field which (for optimal parameters) corresponds to potential drops in the nano-volt range $\Rightarrow$ experimentally measurable
- experimental challenge: nanovoltages, controlled submicron-inhomogeneities of magnetic fields, material fabrication of small well-defined Heisenberg rings. ($\longrightarrow$ new “Frankfurter Forschergruppe” (chemistry+physics): “Spin- und Ladungskorrelationen in niedrigdimensionalen metallorganischen Festkörpern”)
- many open theoretical problems:
  - antiferromagnetic, ferrimagnets, multi-channel rings (spin-ladders)
  - collective effects: spin diffusion, disorder, weak localization in spin chains
  - dephasing in spin systems