Fermi surface renormalization and confinement in coupled chains

Peter Kopietz and Sascha Ledowski, Frankfurt
ERG 2006, Lefkada, Greece

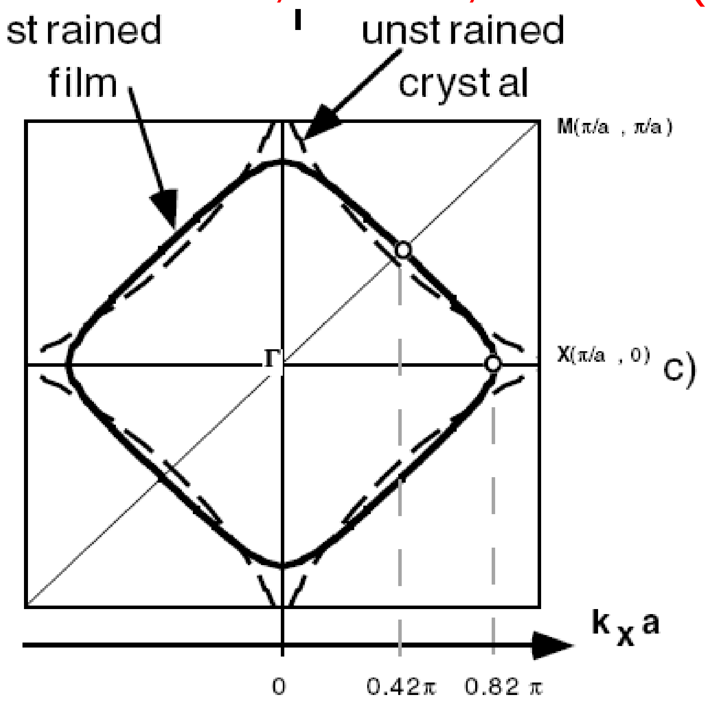
S. Ledowski and P. K., cond-mat/0608119
S. Ledowski, P.K. , and A. Ferraz, Phys. Rev. B 71, 235106 (2005)
F. Schütz, L. Bartosch, and P.K., Phys. Rev. B 72, 035107 (2005).

1. Introduction: Calculating the FS via exact RG
2. Two coupled chains with weak interactions
3. Momentum transfer cutoff scheme
4. Two chains at strong coupling
5. Preliminary results: FS flattening in 2D

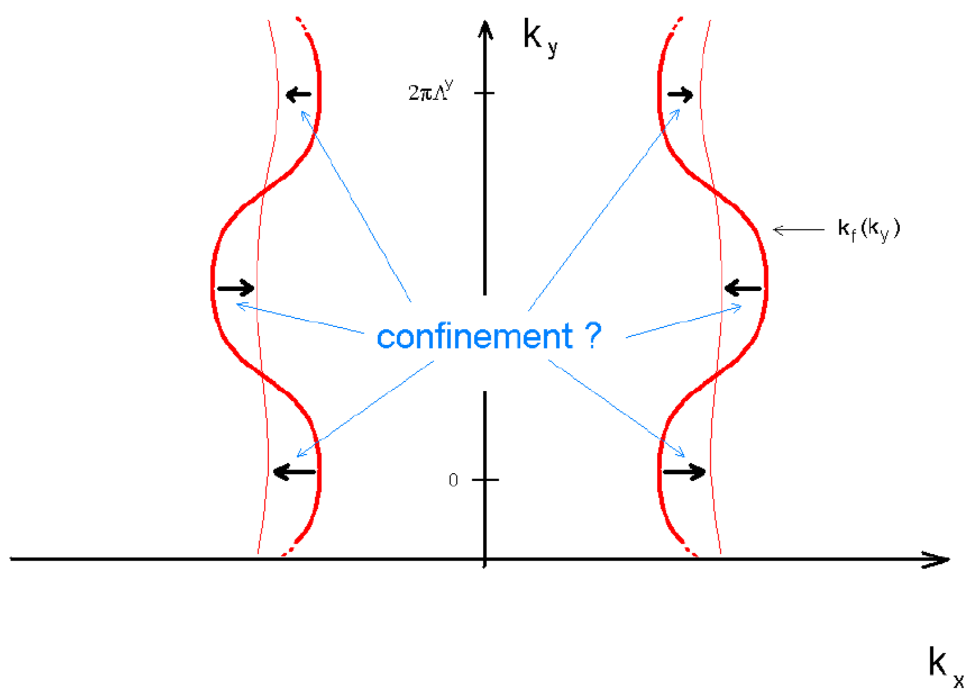
1.Calculating the FS via exact RG

Motivation: flat sectors of a FS lead to non-Fermi liquid behavior in D>1

Doped cuprates under strain:
Abrecht et al., PRL91, 057002 (2003)



Quasi 1D metals with open FS:



Problem: Can curved FS become flat due to strong interactions? Confinement!!!

An exact integral equation for the renormalized Fermi surface

S. Ledowski and PK, J. Phys. Cond. Mat. 15, 4779 (2003).

definition of the FS: $\epsilon_{\boldsymbol{k}_F} + \Sigma(\boldsymbol{k}_F, i0) - \mu = 0$

get exact self-energy from RG flow of continuum of relevant couplings:

$$r_l(\boldsymbol{k}_F) = \tilde{\Gamma}_l^{(2)}(\boldsymbol{k}_F, q=0, i\epsilon = i0) = \frac{Z_l(\boldsymbol{k}_F)}{\Lambda_l v_F}\left[\Sigma_l(\boldsymbol{k}_F, i0) - \Sigma_{l=\infty}(\boldsymbol{k}_F, i0)\right]$$

running cutoff $\Lambda_l = \Lambda_0 e^{-l}$

wave-function renormalization $\eta_l(\boldsymbol{k}_F) = -\partial_l \ln Z_l(\boldsymbol{k}_F)$

get flow of $r_l(\boldsymbol{k}_F)$ from exact RG flow equation for (rescaled) two-point vertex:

$$\partial_l \tilde{\Gamma}_l^{(2)}(\boldsymbol{k}_F, Q) = (1 - \eta_l(\boldsymbol{k}_F) - q\partial_q - \epsilon\partial_\epsilon)\tilde{\Gamma}_l^{(2)}(\boldsymbol{k}_F, Q) + \int_{\boldsymbol{k}'_F}\int \frac{dq' d\epsilon'}{(2\pi)^2}\dot{G}_l(\boldsymbol{k}'_F, Q')\tilde{\Gamma}_l^{(4)}(\boldsymbol{k}_F, Q; \boldsymbol{k}'_F, Q'; \boldsymbol{k}'_F, Q', \boldsymbol{k}_F, Q)$$

effective interaction

rescaled variables: $(\boldsymbol{k}, i\omega) \to (\boldsymbol{k}_F, q, i\epsilon)$ $\quad q = \boldsymbol{v}_F \cdot (\boldsymbol{k} - \boldsymbol{k}_F)/\Lambda \quad \epsilon = \omega / v_F \Lambda$

...exact integral equation for the FS...

...follows from requirement that relevant couplings $r_0(\boldsymbol{k}_F)$ flow into RG fixed point

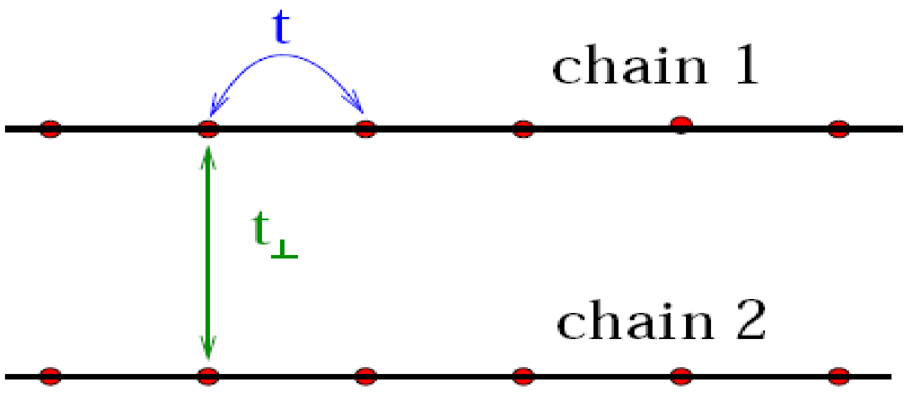
$$r_0(\boldsymbol{k}_F) = \int_0^\infty dl e^{-l+\int_0^l dt \eta_t(\boldsymbol{k}_F)} \int_{\boldsymbol{k}'_F} \int \frac{dq' d\epsilon'}{(2\pi)^2} \dot{G}_l(\boldsymbol{k}'_F, Q') \tilde{\Gamma}_l^{(4)}(\boldsymbol{k}_F, 0; \boldsymbol{k}'_F, Q'; \boldsymbol{k}'_F, Q', \boldsymbol{k}_F, 0)$$

- relates counterterm to flow of all couplings
- fine tuning of infinitely many relevant couplings $r_l(\boldsymbol{k}_F)$
- FS can be viewed as multicritical point of infinite order

2.Two spinless chains, weak coupling

kinetic energy:

$$\hat{H}_0 = -t \sum_i \sum_{\sigma=1,2} [\hat{c}^\dagger_{i,\sigma} \hat{c}_{i+1,\sigma} + h.c.] - t_\perp \sum_i [\hat{c}^\dagger_{i,1} \hat{c}_{i,2} + h.c.]$$

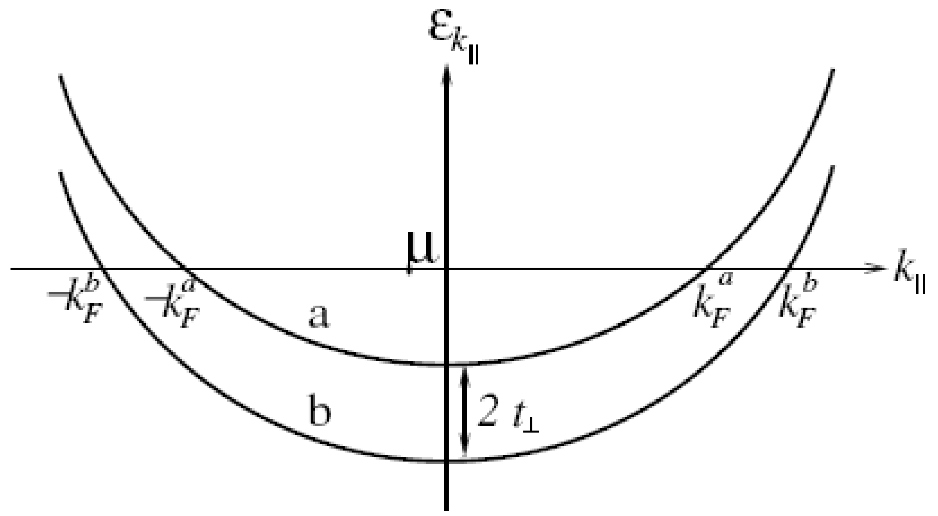


total density fixed:

$$\pi n = k_F^b + k_F^a$$

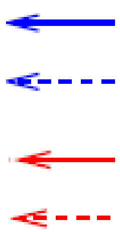
Fermi point distance can be strongly renormalized:

$$\Delta = k_F^b - k_F^a$$



...interactions in 2 spinless chains...

four types of Fermi fields:

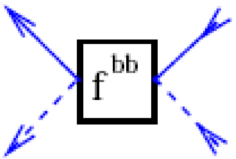


bonding rightmoving

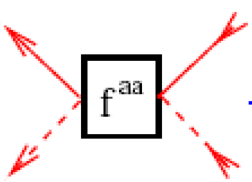
bonding leftmoving

antibonding rightmoving

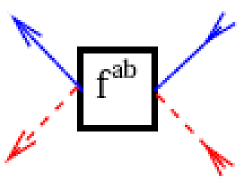
antibonding leftmoving



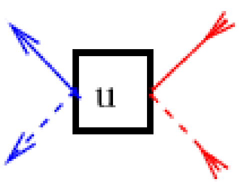
forward scattering bonding



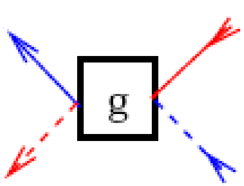
forward scattering antibonding



mixed forward scattering



interchain Umklapp
(pair tunneling)



interchain backscattering

Euclidean action in pseudospin notation:

$$
\begin{aligned}
S[\bar\psi,\psi] &= \sum_\sigma \int_K (-i\omega + \xi_k^\sigma)\bar\psi_K^\sigma \psi_K^\sigma \\
&+ \frac{1}{2}\int_{\bar K} \left[f(\bar k)\bar\rho_{\bar K}\rho_{\bar K} - J^{\|}(\bar k)\bar m_{\bar K} m_{\bar K}\right] \\
&+ \int_{\bar K} \left[u(\bar k)\bar p_{\bar K} p_{\bar K} - 2J^{\perp}(\bar k)\bar s_{\bar K} s_{\bar K}\right] ,
\end{aligned}
$$

composite fields:

$$\rho_{\bar K} = \sum_\sigma \int_K \bar\psi_K^\sigma \psi_{K+\bar K}^\sigma ,$$ density

$$m_{\bar K} = \sum_\sigma \sigma \int_K \bar\psi_K^\sigma \psi_{K+\bar K}^\sigma$$ spin density

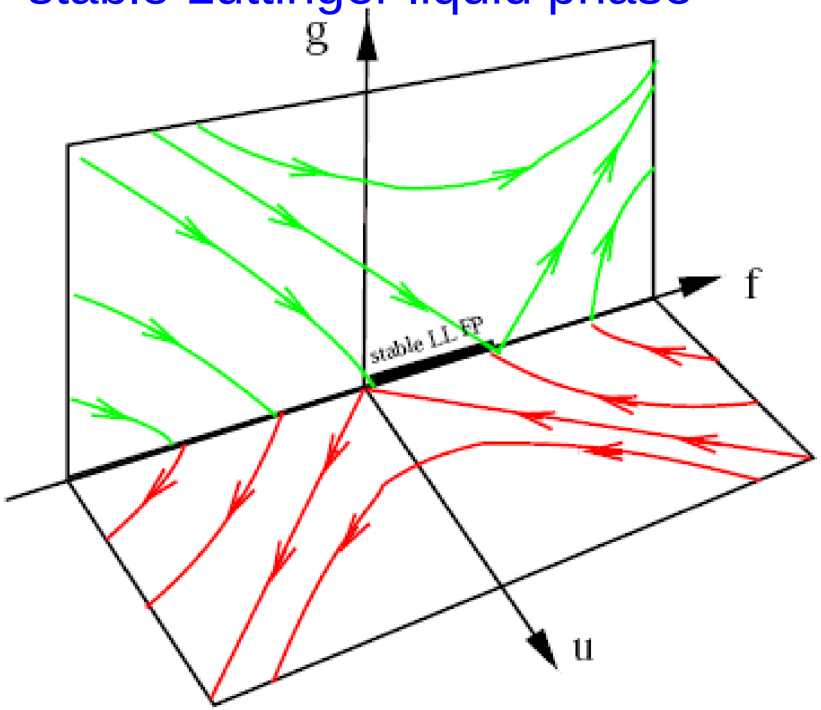
$$p_{\bar K} = \int_K \psi_{-K}^- \psi_{K+\bar K}^+ ,$$ pairing

$$s_{\bar K} = \int_K \bar\psi_K^- \psi_{K+\bar K}^+ .$$ spin flip

...weak coupling RG...

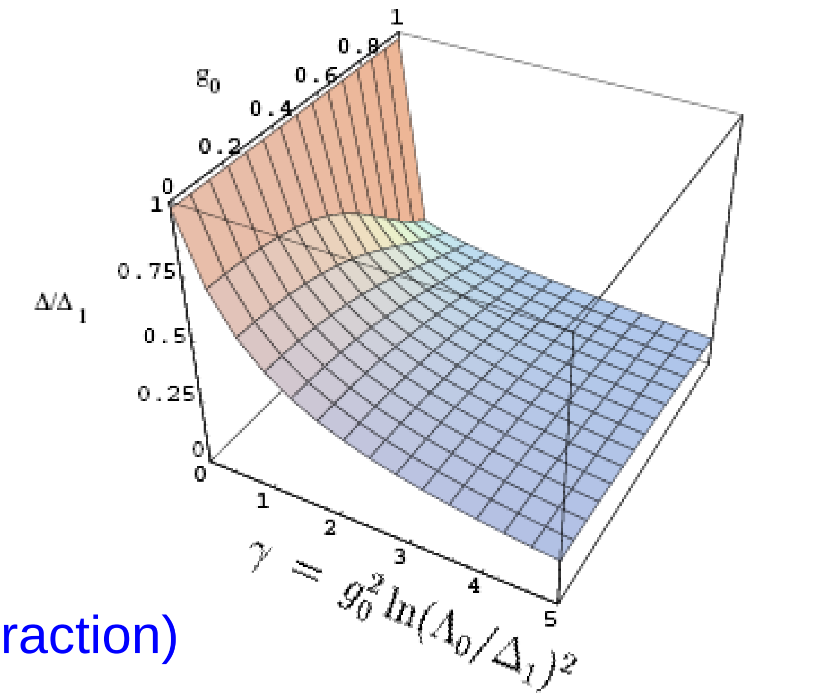
S. Ledowski, PK, A. Ferraz, Phys. Rev. B 71, 057519 (2005).

stable Luttinger liquid phase



strong reduction of Fermi point distance due to interchain backscattering

$$k_F^b - k_F^a = \quad \Delta = \Delta_1 [1 + g_0^2 \ln(\Lambda_0/\Delta)^2]^{-1}$$



effective model for FS renormalization: keep only interchain backscattering (= ferromagnetic XY-interaction)

$$S[\bar{\psi}, \psi] = \sum_{\sigma,\alpha} \int_K (-i\omega + \alpha v_0^\sigma k + \mu_0^\sigma) \bar{\psi}_{K\alpha}^\sigma \psi_{K\alpha}^\sigma - 2 \sum_{\alpha\alpha'} \int_{\bar{K}} J_{\alpha\alpha'}^\perp \bar{s}_{\bar{K}\alpha} s_{\bar{K}\alpha'} \,,$$

spin-flip field:

$$s_{\bar{K}} = \int_K \bar{\psi}_K^- \psi_{K+\bar{K}}^+$$

3. Momentum transfer cutoff scheme

F. Schütz, L. Bartosch, PK, Phys. Rev. B 72, 035107 (2005).

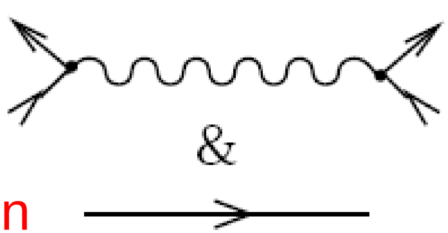
- Question: can Fermi point difference collapse at strong coupling?
- Need RG method for strong coupling regime!
- Idea: partial bosonization (Hubbard-Stratonovich transformation)
- Use bosonic momentum transfer as flow parameter

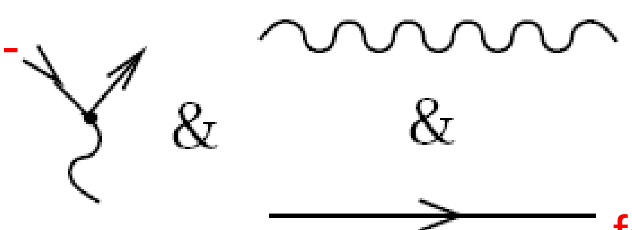
Example: Tomonaga-Luttinger model:

decouple density-density interaction in zero-sound channel:

original problem:

after HS transformation:

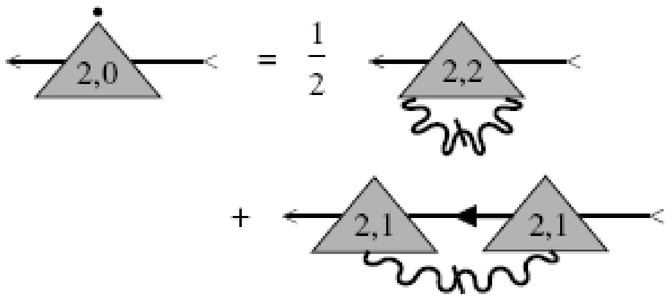




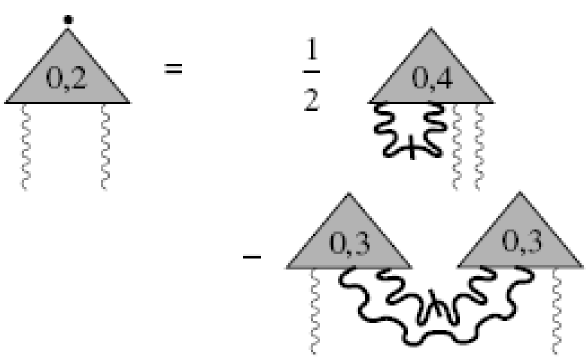
...momentum transfer cutoff scheme...

impose cutoff only in momentum transfered by bosonic field; exact RG equations:

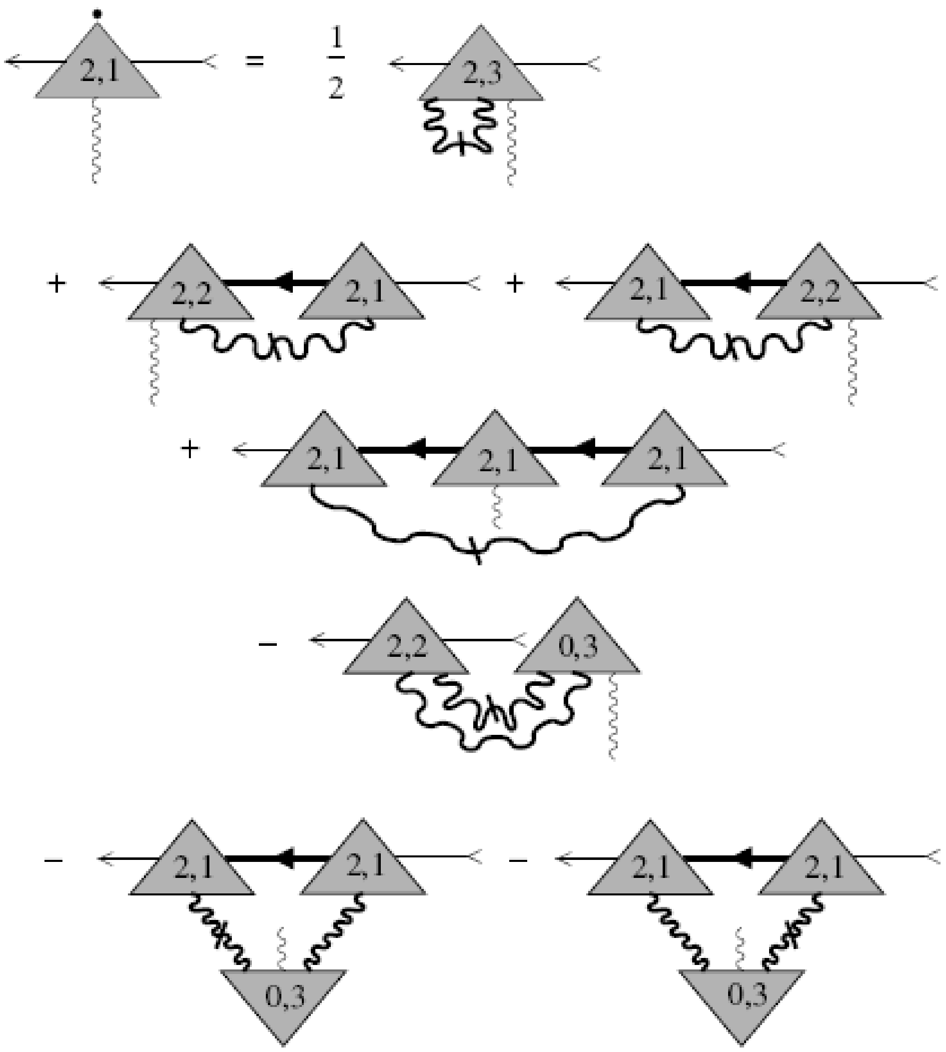
fermionic self-energy:



bosonic self-energy:

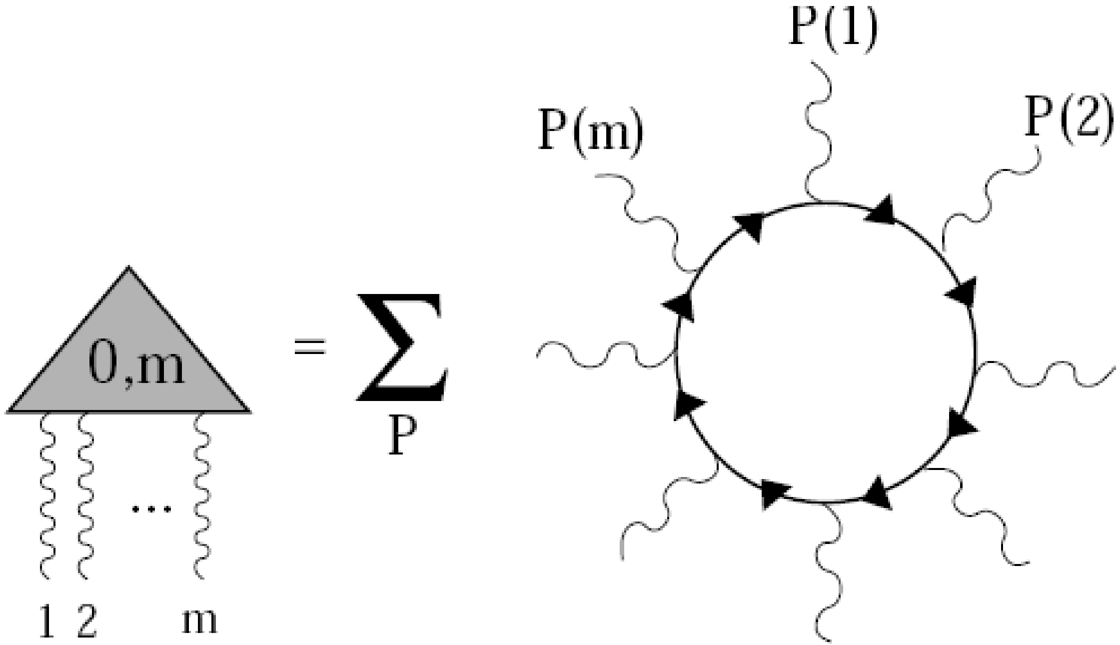


three-legged fermion-boson vertex:



...initial condition in momentum transfer cutoff scheme...

symmetrizied closed fermion loops with arbitrary number of bosonic legs are finite



- cutoff scheme does not violate Ward identities
- exact solution of the Tomonaga-Luttinger model within ERG
- simple truncation gives correct anomalous dimension -even at strong coupling!

4.Two chains at strong coupling

S. Ledowski, PK, cond-mat/0608119.

Can Fermi point distance collapse in 2-chain system at strong coupling?

Strategy:

a)Start from effective low energy model containing only interchain backscattering:
in pseudospin language:ferromagnetic XY interaction, magnetic field in z-direction

$$t_\perp = h$$

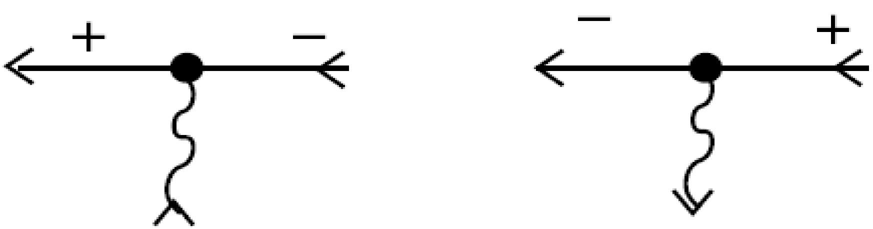
b) Decouple interaction in spin-singlet, particle-hole channel via complex HS field

$$S[\bar{\psi},\psi] = \sum_{\sigma,\alpha}\int_K (-i\omega + \alpha v_0^\sigma k + \mu_0^\sigma)\bar{\psi}^\sigma_{K\alpha}\psi^\sigma_{K\alpha} - 2\sum_{\alpha\alpha'}\int_{\bar{K}} J^\perp_{\alpha\alpha'}\bar{s}_{\bar{K}\alpha}s_{\bar{K}\alpha'}\,, \longrightarrow \frac{1}{2}\sum_{\alpha\alpha'}\int_{\bar{K}}[\mathbf{J}^\perp]^{-1}_{\alpha\alpha'}\bar{\chi}_{\bar{K}\alpha}\chi_{\bar{K}\alpha'} + \sum_\alpha\int_{\bar{K}}\left[\bar{s}_{\bar{K}\alpha}\chi_{\bar{K}\alpha} + s_{\bar{K}\alpha}\bar{\chi}_{\bar{K}\alpha}\right]$$

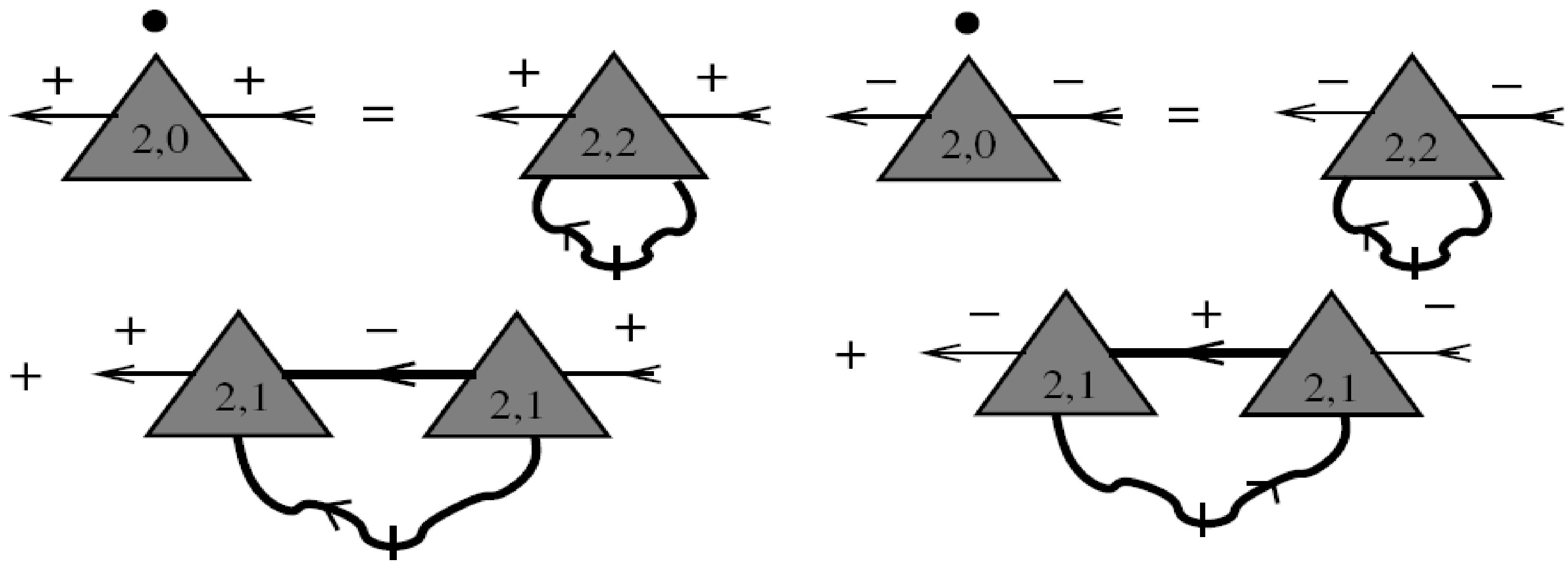
c) Find sensible truncation of resulting mixed Bose-Fermi theory

...ERG flow equations in momentum transfer cutoff scheme...

bare spin-flip vertices:



flow of fermionic self-energy:



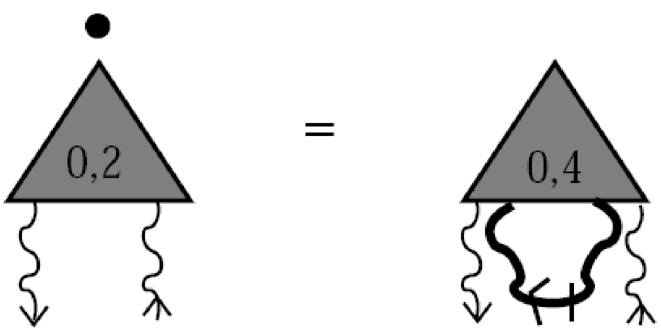
$$\partial_\Lambda \Sigma^\sigma_\Lambda(K,\alpha) = \int_{\bar{K}} \dot{F}^{\sigma\bar{\sigma}}_\Lambda(\bar{K},\alpha)\Gamma^{(2,2)}_\Lambda(K\sigma,-K\sigma;\bar{K},-\bar{K},\alpha)$$

$$+\int_{\bar{K}} \dot{F}^{\sigma\bar{\sigma}}_\Lambda(\bar{K},\alpha)G^{\bar{\sigma}}_\Lambda(K+\bar{K}+\alpha\sigma\Delta,\alpha)\;\Gamma^{(2,1)}_\Lambda(K\sigma;K+\bar{K},\bar{\sigma};-\bar{K},\alpha)$$
$$\times\Gamma^{(2,1)}_\Lambda(K+\bar{K},\bar{\sigma};K,\sigma;\bar{K},\alpha)$$

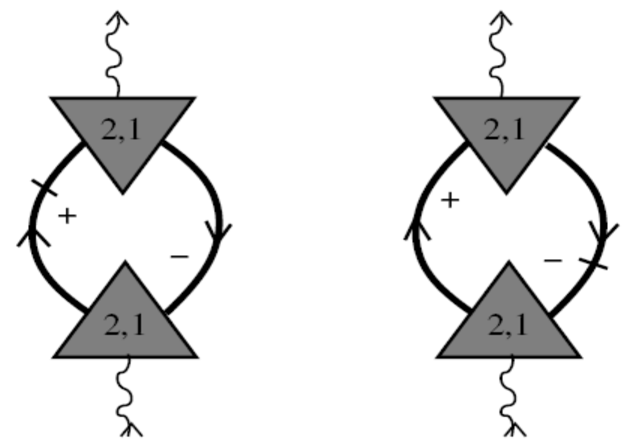
...flow of spin-flip susceptibility and spin-flip vertices...

flow of spin-flip susceptibility (bosonic self-energy)

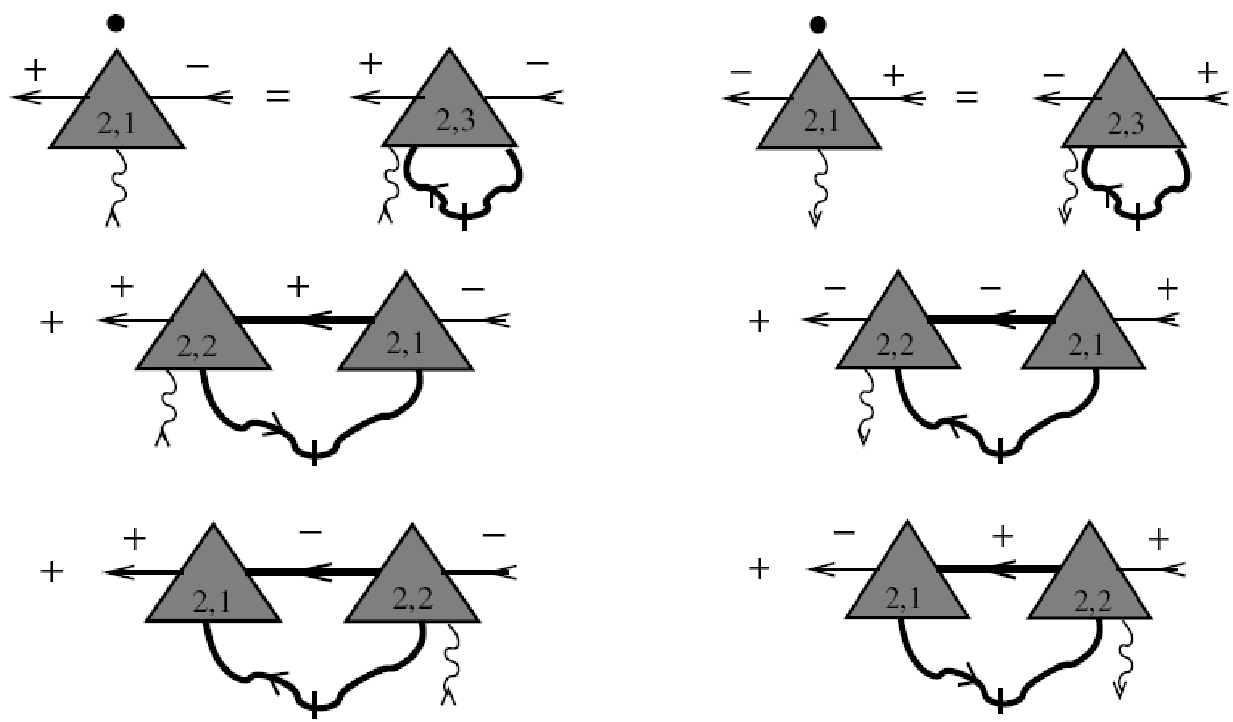
in momentum transfer cutoff scheme:



with fermionic band width cutoff:



flow of spin-flip vertices in momentum transfer cutoff scheme:



...calculating the true Fermi point distance...

From RG flow of momentum-independent part of rescaled self-energy:

$$r_l^\sigma = \tilde{\Sigma}_l^\sigma(0,\alpha) = \frac{Z_l^\sigma}{\Omega_\Lambda}\left[\Sigma_\Lambda^\sigma(0,\alpha) + \mu_0^\sigma\right]$$

Fine tune intitial condition

$$r_0^\sigma = \frac{\mu_0^\sigma}{\Omega_{\Lambda_0}} = -\frac{\Sigma^\sigma(\alpha k^\sigma, i0)}{v_F \Lambda_0}$$

such that relevant coupling r_l^σ flows into a fixed point. From exact flow equation

$$\partial_l r_l^\sigma = (1-\eta_l^\sigma) r_l^\sigma + \dot{\Gamma}_l^\sigma(0,\alpha)$$

we obtain self-consistency equation for true Fermi point distance $\tilde{\Delta} = \frac{k^+ - k^-}{\Lambda_0}$

$$\begin{aligned}\tilde{\Delta} &= \tilde{\Delta}_0 + \left[\frac{r_0^+}{\tilde{v}_0^+} - \frac{r_0^-}{\tilde{v}_0^-}\right] \\ &= \tilde{\Delta}_0 - \sum_\sigma \frac{\sigma}{\tilde{v}_0^\sigma}\int_0^\infty dl e^{-(1-\bar{\eta}_l^\sigma)l}\dot{\Gamma}_l^\sigma(0,\alpha)\end{aligned}$$

...Truncation of hierarchy of flow equations...

Approximation 1: ignore irrelevant vertices which vanish initially

relevant coupling constants:

constant part of self-energy:

$$r_l^\sigma = \tilde{\Sigma}_l^\sigma(0,\alpha) = \frac{Z_l^\sigma}{\Omega_\Lambda}[\Sigma_\Lambda^\sigma(0,\alpha) + \mu_0^\sigma] \qquad \partial_l r_l^\sigma = (1-\eta_l^\sigma)r_l^\sigma + \dot{\Gamma}_l^\sigma(0,\alpha)$$

marginal coupling constants:

wave-function renormalization:

$$Z_l^\sigma = 1 + \left.\frac{\partial\tilde{\Sigma}_l^\sigma(0,i\epsilon,\alpha)}{\partial(i\epsilon)}\right|_{\epsilon=0} \qquad \partial_l Z_l^\sigma = -\eta_l^\sigma Z_l^\sigma$$

velocity renormalization:

$$\tilde{v}_l^\sigma = Z_l^\sigma + \left.\frac{\partial\tilde{\Sigma}_l^\sigma(q,i0,\alpha)}{\partial(\alpha q)}\right|_{q=0} \qquad \partial_l\tilde{v}_l^\sigma = -\eta_l^\sigma\tilde{v}_l^\sigma + \left.\frac{\partial\dot{\Gamma}_l^\sigma(q,i0,\alpha)}{\partial(\alpha q)}\right|_{q=0}$$

constant part of spin-flip vertex:

$$\gamma_l = \tilde{\Gamma}_l^{(2,1)}(0,\sigma;0,\bar{\sigma};0,\alpha) \qquad \partial_l\gamma_l = -\frac{\bar{\eta}_l + \eta_l^+ + \eta_l^-}{2}\gamma_l + \dot{\Gamma}_l^{(2,1)}$$

...Truncation continued...

Approximation 2: adiabatic approximation for spin-flip susceptibility

vertex correction due to spin-flip vertex

$$\tilde{\Pi}_l^{\sigma\bar{\sigma}}(\bar{Q},\alpha) \approx \frac{\gamma_l^2}{2\pi}\frac{\sigma\tilde{\Delta}_l + \alpha\bar{q}}{\sigma\tilde{\Delta}_l + \alpha\bar{q} - i\epsilon}$$

flowing Fermi point distance at scale l: $\tilde{\Delta}_l = \tilde{\Delta}_l^* - (r_l^+ - r_l^-)$

initial value: bare Fermi point distance $\tilde{\Delta}_{l=0} = \tilde{\Delta}_0 = (k_0^+ - k_0^-)/\Lambda_0$

limit for large l: rescaled true Fermi point distance $\Delta_l^* = e^l(k^+ - k^-)/\Lambda_0 = e^l\tilde{\Delta}$

Justification of adiabatic approximation within two-cutoff RG possible (see Appendix of cond-mat/0608119).

...self-consistent one-loop approximation ...

Simplest approximation: ignore flow of marginal couplings, amounts to:
ladder approximation with self-consistency condition for $\tilde{\Delta} = \frac{k^+ - k^-}{\Lambda_0}$

$$\tilde{\Delta} = \frac{\tilde{\Delta}_0}{1 + R(\tilde{\Delta})} \qquad R(\tilde{\Delta}) = -2g_{c,0} + \frac{2g_{n,0}^2}{\sqrt{(1-g_{c,0})^2 - g_{n,0}^2}} \ln\left[\frac{1 + \sqrt{1 + \frac{\tilde{\Delta}^2 g_{n,0}^2}{(1-g_{c,0})^2 - g_{n,0}^2}}}{\tilde{\Delta}\left(1 + \sqrt{\frac{(1-g_{c,0})^2}{(1-g_{c,0})^2 - g_{n,0}^2}}\right)}\right]$$

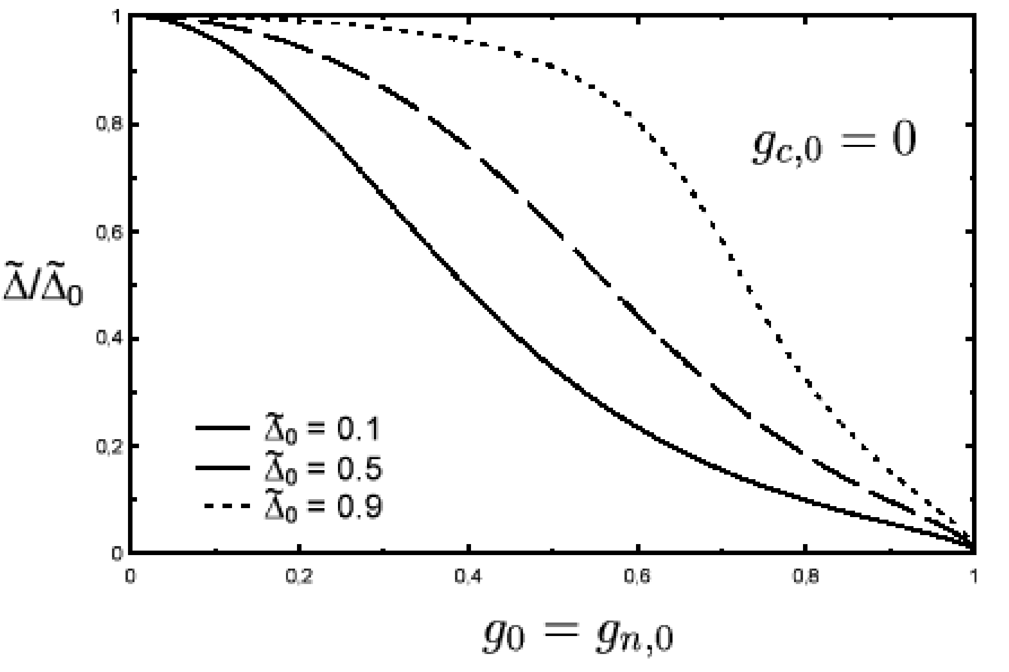
2 types of interchain backscattering:

chiral: $2\nu_0 J^{\perp}_{\alpha,-\alpha} = 2\pi g_{n,0}$

non-chiral: $2\nu_0 J^{\perp}_{\alpha\alpha} = 2\pi g_{c,0}$

weak coupling expansion:

$$R(\tilde{\Delta}) = -2g_{c,0} + 2g_{n,0}^2 \ln(1/\tilde{\Delta}) + O(g_{i,0}^3)$$



- strong confinement for $g_{c,0} + g_{n,0} \to 1$
- confinement is driven by non-chiral part of interchain backscattering

...including wave-function and vertex corrections...

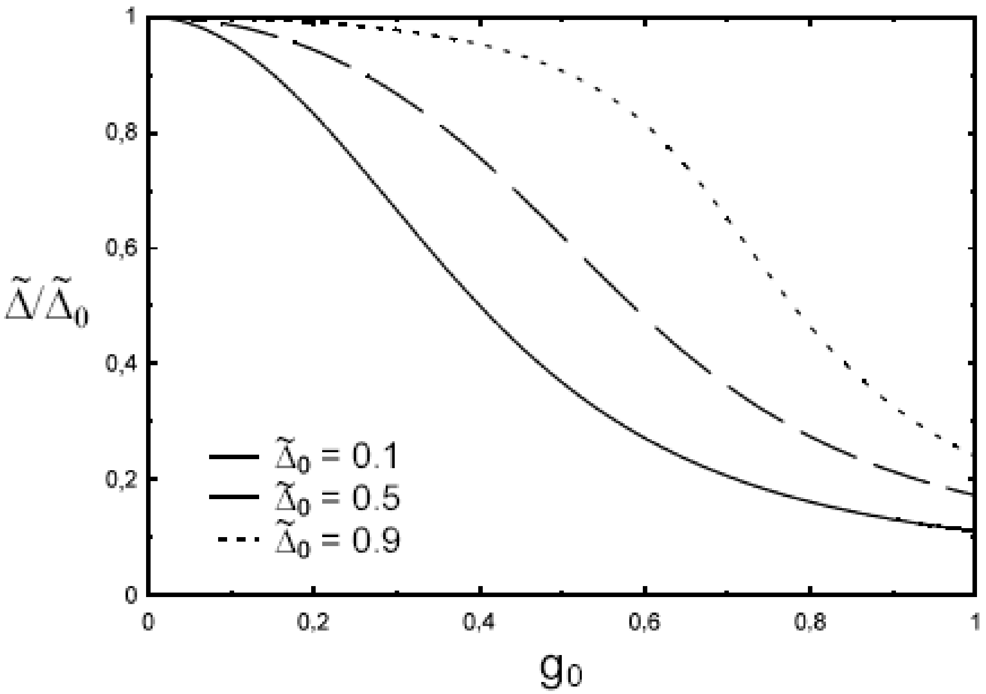
$$\tilde{\Delta} = \tilde{\Delta}_0 - \int_0^\infty dl e^{-l} \frac{2\Theta(1-\tilde{\Delta}_l)\tilde{\Delta}_l g_l^2}{\sqrt{1-g_l^2(1-\tilde{\Delta}_l)^2}}$$

$$\tilde{\Delta}_l = \tilde{\Delta}_l^* - 2r_l = \tilde{\Delta} e^l - 2r_l$$

flow of constant part of self-energy:

$$\partial_l r_l = r_l + A(g_l, \tilde{\Delta}_l)$$

$$A(g_l, \tilde{\Delta}_l) = \frac{\Theta(1-\tilde{\Delta}_l)\tilde{\Delta}_l g_l^2}{\sqrt{1-g_l^2(1-\tilde{\Delta}_l^2)}}$$



flow of non-chiral part of interchain backscattering:

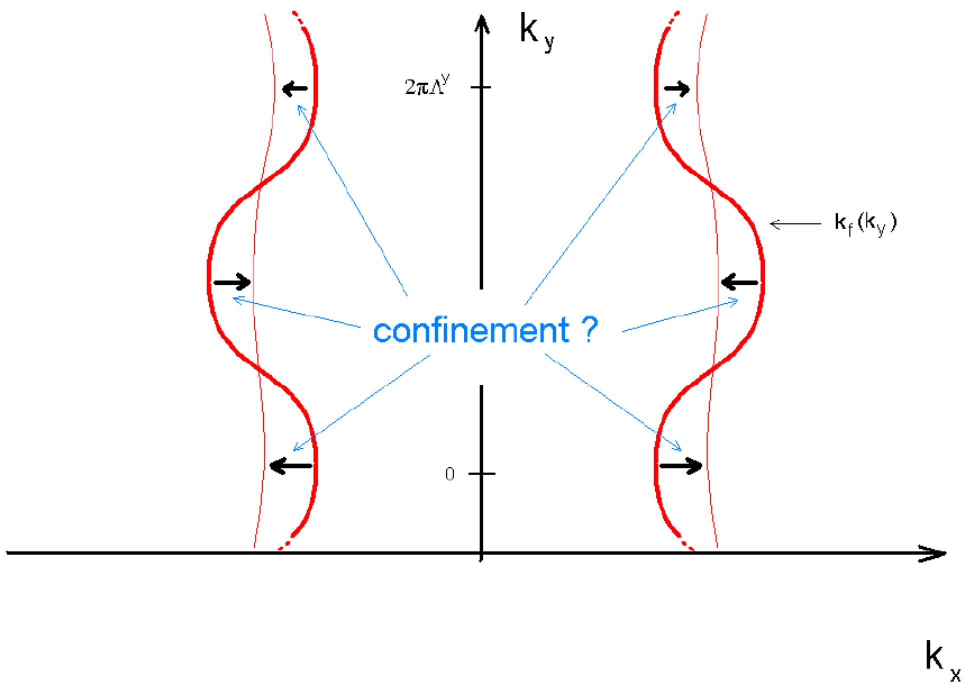
$$\partial_l g_l = B(g_l, \tilde{\Delta}_l)$$

$$B(g_l, \tilde{\Delta}_l) = \frac{-2\Theta(1-\tilde{\Delta}_l)g_l^3}{\sqrt{1-g_l^2(1-\tilde{\Delta}_l^2)}\left[1+\sqrt{1-g_l^2(1-\tilde{\Delta}_l^2)}\right]}$$

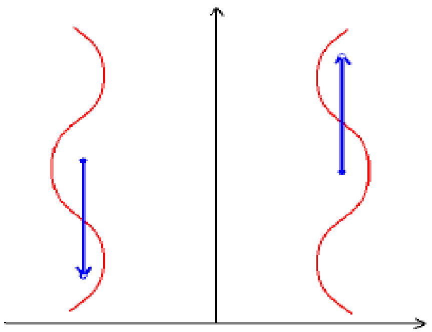
No confinement, even at strong coupling!

5. Confinement in two dimensions

S. Ledowski, PK, A. Ferraz, work in progress --cond-mat/0611XXX



- method can be generalized to 2D
- integral equation for the true FS
- strong chiral density-density interaction can generate flat FS



- self-consistent one loop qualitatively ok

Summary, Conclusions

- Understand confinement in 1D toy model: two chains:
  - identify scattering channel driving FS renormalization
  - treat this channel non-perturbatively via partial bosonization
  - in 1D: fluctuations beyond one loop destroy confinement
- Method is very general:
  - confinement in 2D
  - symmetry breaking
  - all problems where the dominant scattering channel is known
- Extension: problems where several scattering channels compete, for example Anderson-Impurity model in Kondo regime