Viscous Evolution of a Quark Gluon Plasma

Derek Teaney

SUNY at Stonybrook and RIKEN Research Fellow

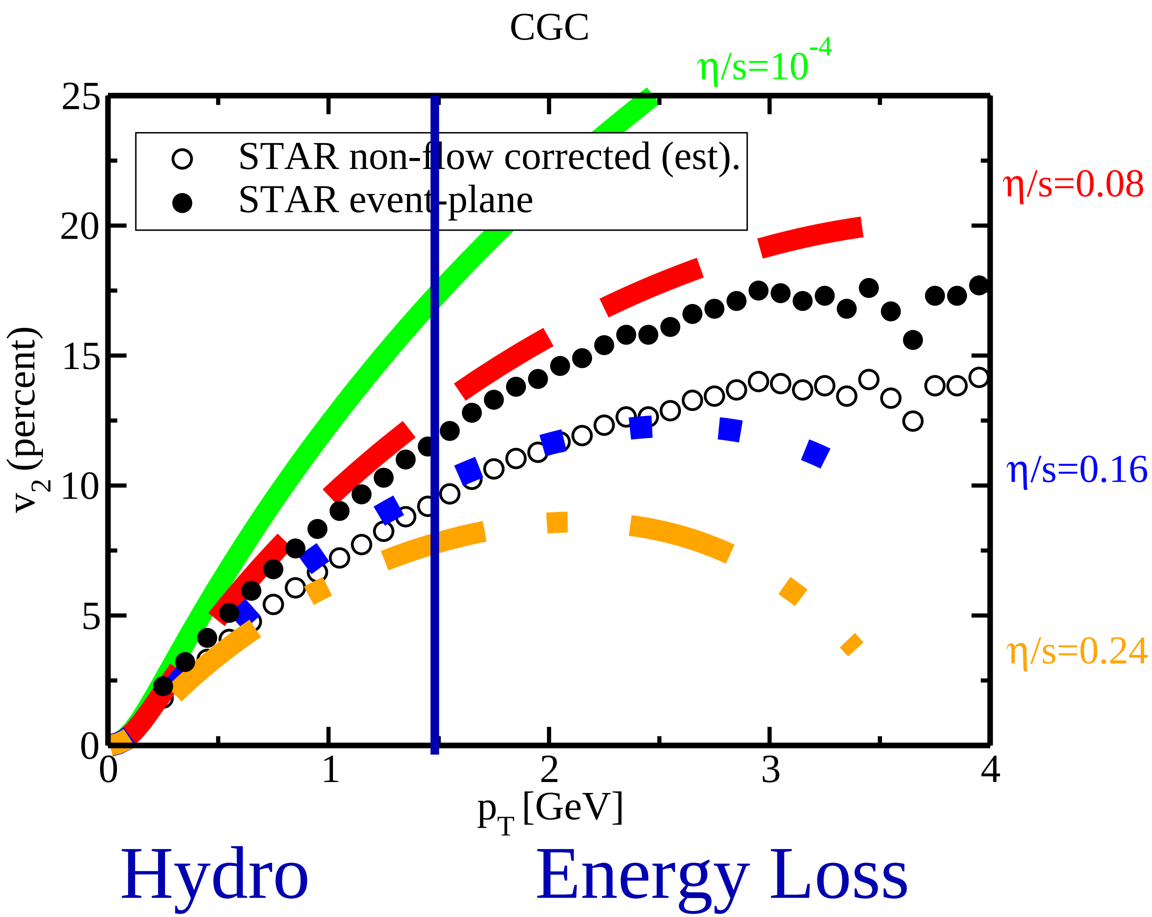
Kevin Dusling, Guy Moore, DT, arXiv:0907.4843

STONY BROOK
STATE UNIVERSITY OF NEW YORK

Talk and paper in two parts

1. Energy loss and $v_2(p_T)$
2. Coalescence hatred

Viscous hydro simulations (Romatschke Luzum)



What are the uncertainties?

How does this become energy loss ? $v_2(p_T)$ or $R_{AA}(\phi)$?

Viscous Corrections

1. Viscous corrections to the equation of motion

$$\partial\, T \quad = 0 \qquad \text{with} \qquad T \quad = \underbrace{(e+p)u\; u \;+ pg}_{\text{ideal}} \; \underbrace{-2\eta\,\langle\partial\;\; u\;\rangle}_{\text{viscous }\pi}$$

2. Viscous corrections to the distribution function

$$f \longrightarrow f_o + \delta f$$

- Must be proportional to strains – must be a scalar
- General form in rest frame and ansatz

$$\delta f \;=\; -\chi(p) \;\times\; f_o(p)\,\hat{p}^i\hat{p}^j\,\langle\partial_i u_j\rangle$$

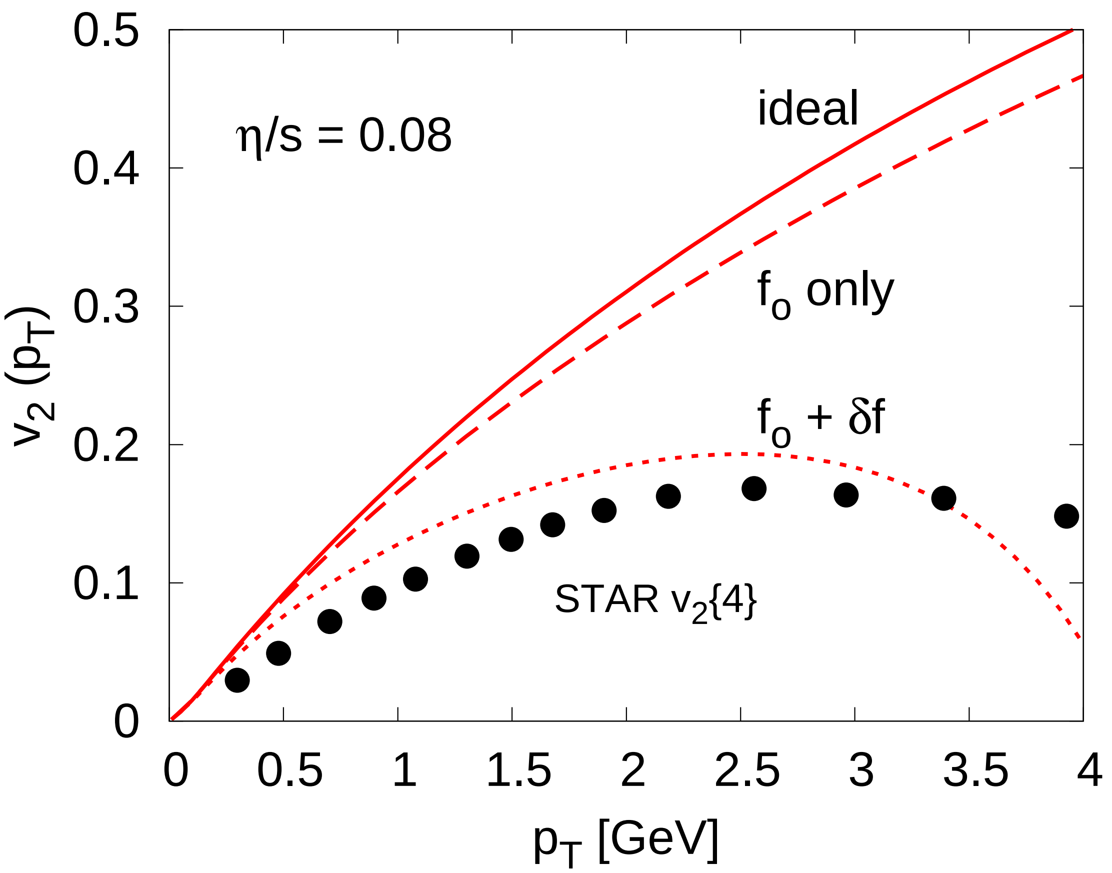
- The Quadratic Ansatz $\chi(p) \propto p^2$

$$\delta f \;=\; -\frac{\eta}{sT^3} \times f_o(p)\,p^i p^j\,\langle\partial_i u_j\rangle$$

All simulations have used the quadratic ansatz!

The role of δf

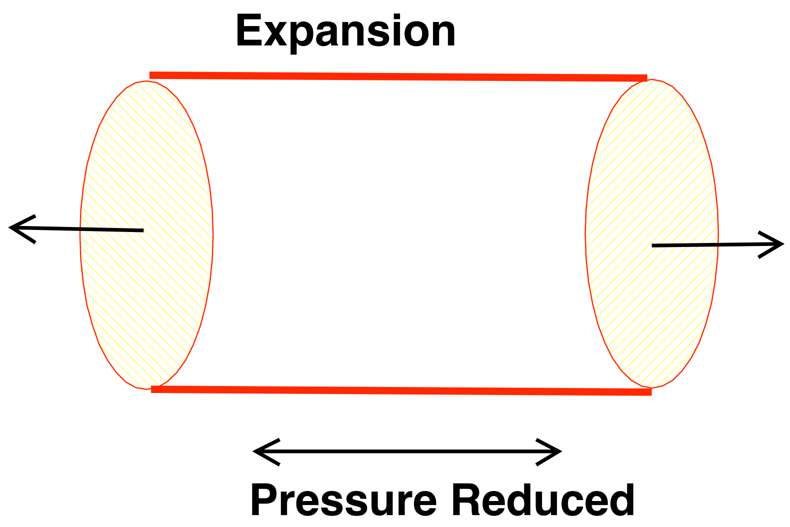
Pure glue, $e_{\rm frz} = 0.6\,{\rm GeV/fm^3}$



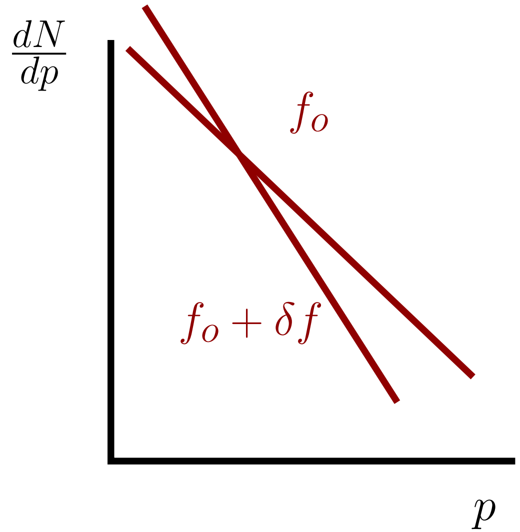
We should understand δf and the Quadratic Ansatz!

Basic Physics of δf

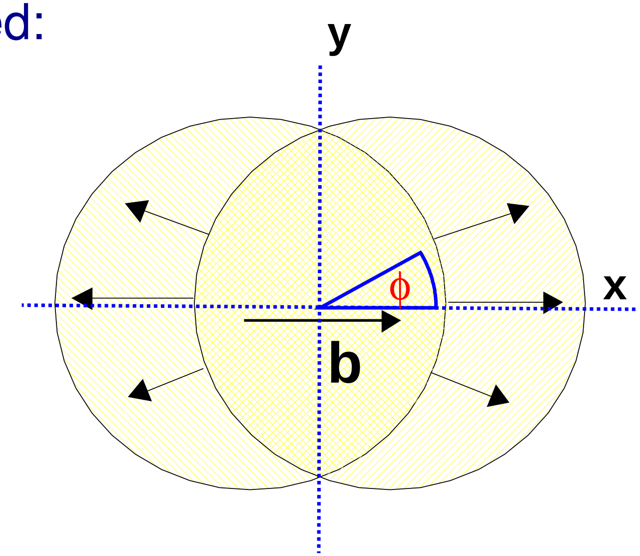
1. When the system is expanding the pressure is reduced:



So



2. Thus elliptic flow is reduced:



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Calculating δf: Relaxation Time Approximation

$$\left[\partial_t + v_{\mathbf{p}} \frac{\partial}{\partial x}\right] f = -\frac{\delta f}{\tau_R(p)}$$

1. Substitute $f = n_p + \delta f$ with

$$\left[\partial_t + v_{\mathbf{p}} \frac{\partial}{\partial x}\right] n_p = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})} \qquad \text{with} \qquad n_p = \frac{1}{e^{-P\cdot u(\mathbf{x};t)} \mp 1}$$

2. With a bit of algebra and classical statistics:

$$n_p \frac{p^i p^j}{T E_{\mathbf{p}}} \langle \partial_i u_j \rangle = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})}$$

3. Find for massless gas

$$\delta f = -\frac{\tau_R(p)}{Tp} n_p \, p^i p^j \langle \partial_i u_j \rangle \qquad \text{or} \qquad \chi(p) = \tau_R(p) \frac{p}{T}$$

Quadratic ansatz corresponds to $\tau_R \propto p$.

What about $\tau_R \propto p$?

Two Extreme Limits: Quadratic and Linear Ansatz

$$\delta f = -\frac{\tau_{\rm R}(p)}{Tp} n_{\rm p}\, p^{\rm i} p^{\rm j} \langle \partial_{\rm i} u_{\rm j} \rangle$$

For the relaxation time take

$$\tau_{\rm R}(p) \sim \frac{p}{\frac{\rm dp}{\rm dt}}$$

1. Relaxation time growing with parton energy – "collisional e-loss"

$$\tau_{\rm R} \propto p \qquad \frac{dp}{dt} \propto \text{const} \qquad \chi(p) \propto p^2$$

2. Relaxation time independent of parton energy – "extreme rad. e-loss"

$$\tau_{\rm R} \propto \text{Const} \qquad \frac{dp}{dt} \propto p \qquad \chi(p) \propto p$$

Reality is probably in-between

Relation between δf and shear viscosity

$$T^{ij} = p\delta^{ij} - \eta \left\langle \partial^{i} u^{j} \right\rangle = \int_{\mathbf{p}} \frac{p^{i}p^{j}}{E}(n_p + \delta f)$$

- First moment of δf determines the shear viscosity

$$\delta f = -\chi(p)\, n_p\, \hat{p}^{i}\hat{p}^{j} \left\langle \partial_i u_j \right\rangle \qquad \text{find} \qquad \eta = \frac{1}{15}\int \frac{d^3p}{(2\pi)^3} n_p\, \chi(p)\, p$$

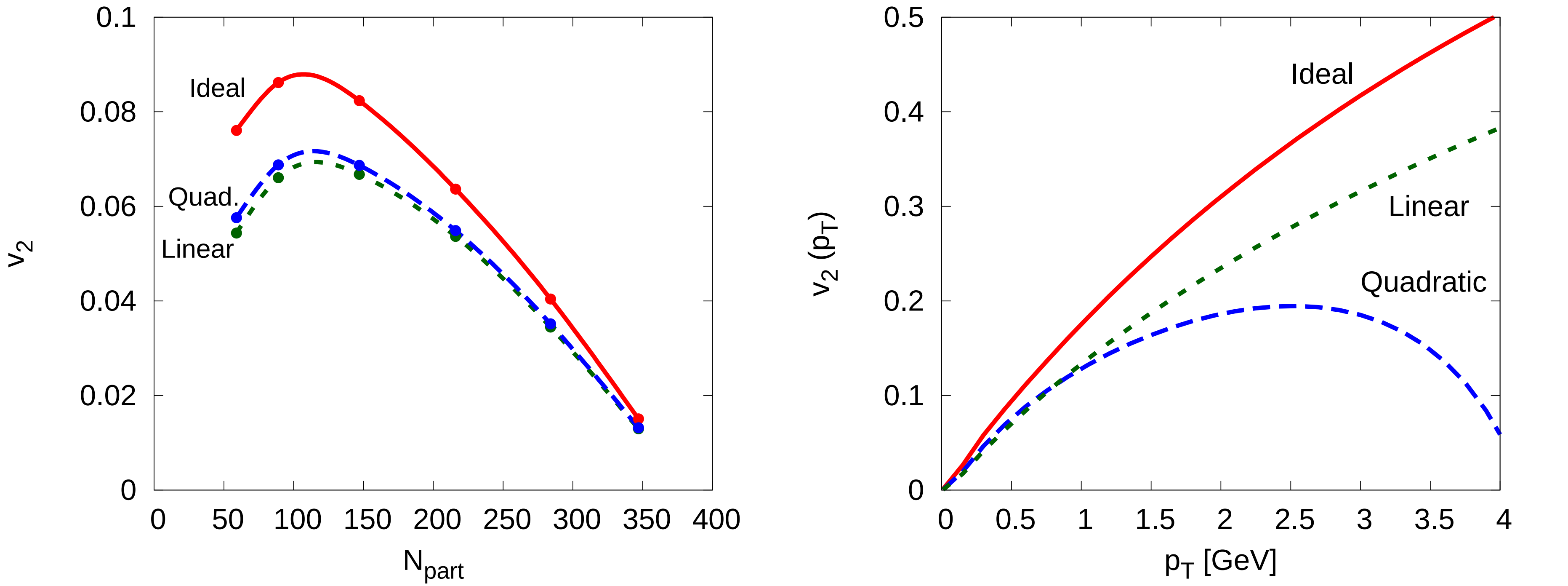
- More General Ansatz – massless gas

$$\chi(p) = Cp^{2-} \qquad \rightarrow \qquad C(\alpha) = \begin{cases} \frac{\;}{sT} & \alpha = 0 \text{ (quadratic)}, \\ 5\frac{\;}{sT} & \alpha = 1 \text{ (linear)}. \end{cases}$$

Ansätze partially constrained by shear viscosity

Two Limits: Quadratic and Linear Ansatz

pure glue, $e_{\rm frz} = 0.6\,{\rm GeV/fm^3}, \eta/s = 0.08$



- The $\overline{v}_2$ independent of δf – see arXiv:0905.2433
  - $\overline{v}_2$ largely determined by $T(e + \mathcal{P})$, u , π

What is reality? Quadratic or Linear?

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Solving for δf with the Boltzmann Equation:

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_x f = C \circ f$$

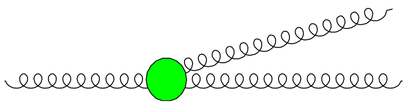
- Substitute $f = n_p + \delta f$

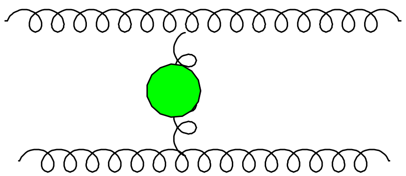
$$\partial_t n_p + v_{\mathbf{p}} \cdot \partial_x n_p = C \circ \delta f$$

- with a bit of algebra

$$n_p \frac{p^i p^j}{T E_{\mathbf{p}}} \langle \partial_i u_j \rangle = C \circ \delta f$$

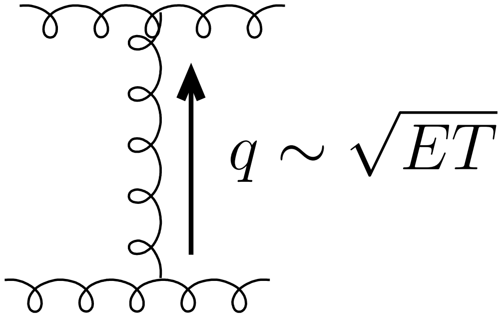
- The collisions and bremsstrahlung is all in C.





Can go invert this matrix and determine δf

Simple Scattering



- **Transition Rate**

$$ {}_{12\to 34} = \frac{|\mathcal{M}|^2}{(2E_1)(2E_2)(2E_3)(2E_4)} (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4) $$

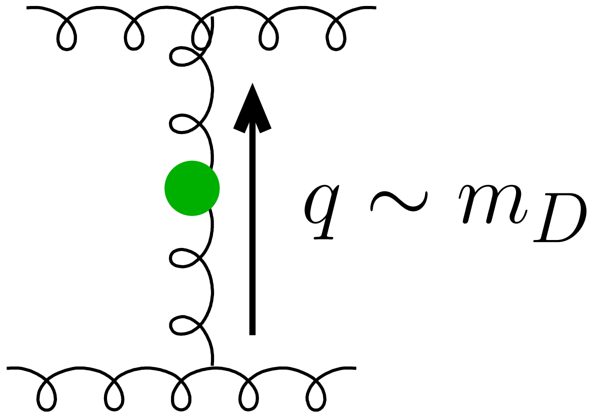
- **Linearized equation**

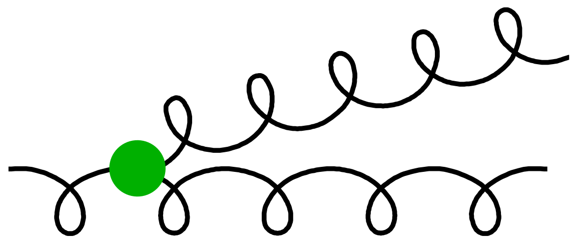
$$ n^0_{\mathbf{p}} \frac{p^i p^j}{T E_{\mathbf{p}}} \langle \partial_i u_j \rangle \quad = \quad - \int_{234} \quad {}_{12\to 34}\, n^0_{\mathbf{p}} n^0_2 \left[\frac{\delta f(\mathbf{p})}{n^0_{\mathbf{p}}} + \frac{\delta f_2}{n^0_2} - \frac{\delta f_3}{n^0_3} - \frac{\delta f_4}{n^0_4} \right] $$

Matrix Equation for δf

$$ b_p = [\quad]_{pp'} \; \delta f_{p'} $$

QCD Boltzmann equation is rich





1. Scattering of soft classical field

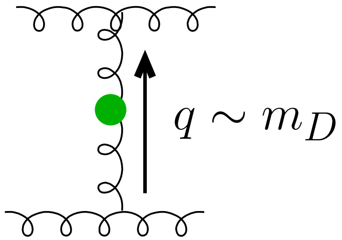
2. Collinear Brem with interference

All these processes influence δf

Three Models of Energy Loss

1. Soft Scattering

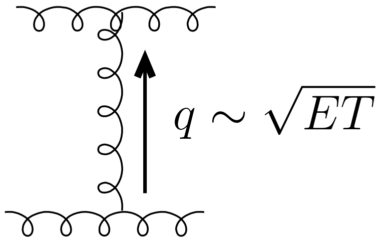
$$\frac{dp}{dt} \propto C_R\, \alpha_s^2 T^2 \log\left(\frac{T}{m_D}\right)$$



find $\chi(p) \propto p^2$

2. Collisional Energy Loss

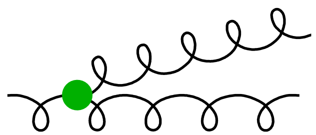
$$\frac{dp}{dt} \sim C_R\, \alpha_s^2 T^2 \log\left(\frac{p}{m_D}\right)$$



find $\chi(p) \propto \frac{p^2}{\log(p\ )}$

3. Radiative + Collisional Energy Loss in Infinite medium

$$\frac{\Delta p}{\Delta t} \sim \alpha_s \sqrt{\hat{q} E_{\mathbf{p}}}$$

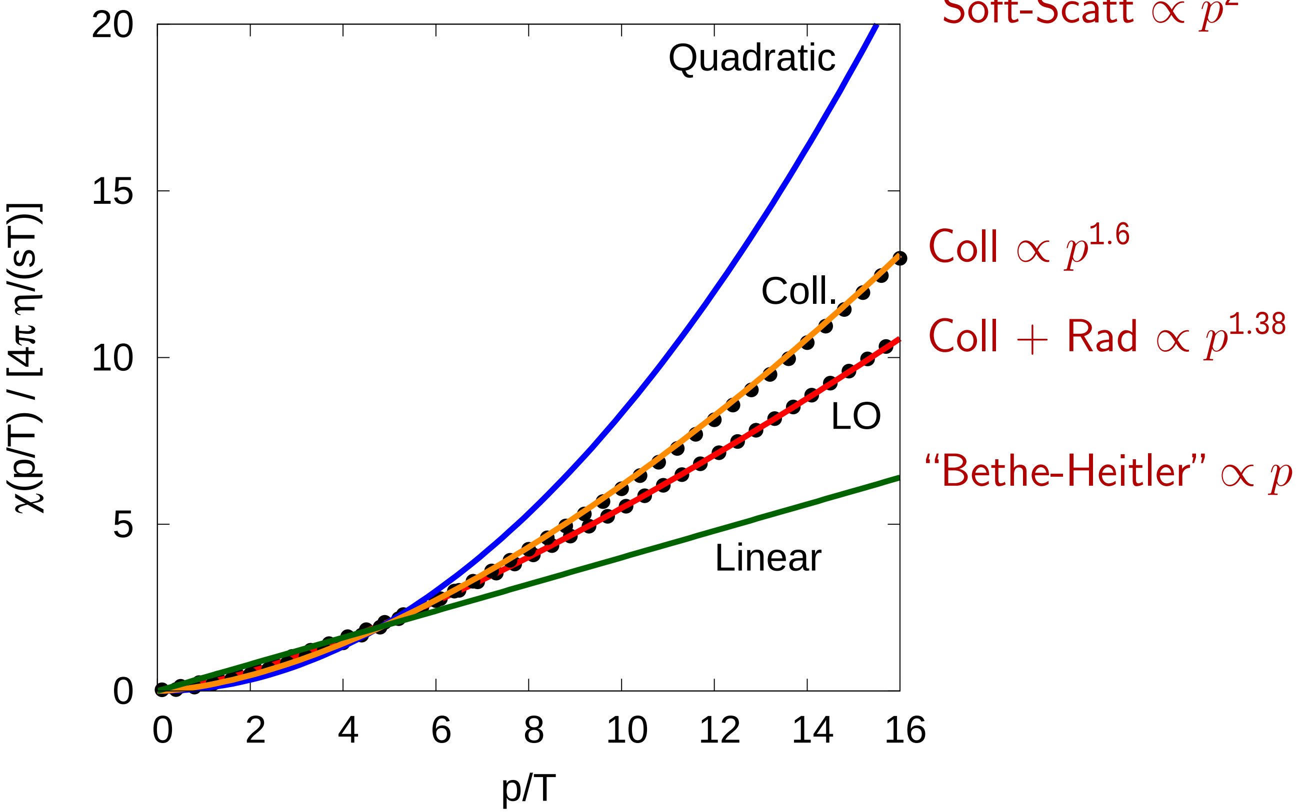


find $\chi \propto p^{3=2}$

These estimates are borne out by our numerical work

Summary – Energy Loss and δf

$$\delta f = -\chi(p)\, n_p\, \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle \qquad \text{fit with} \qquad C p^{2-}$$

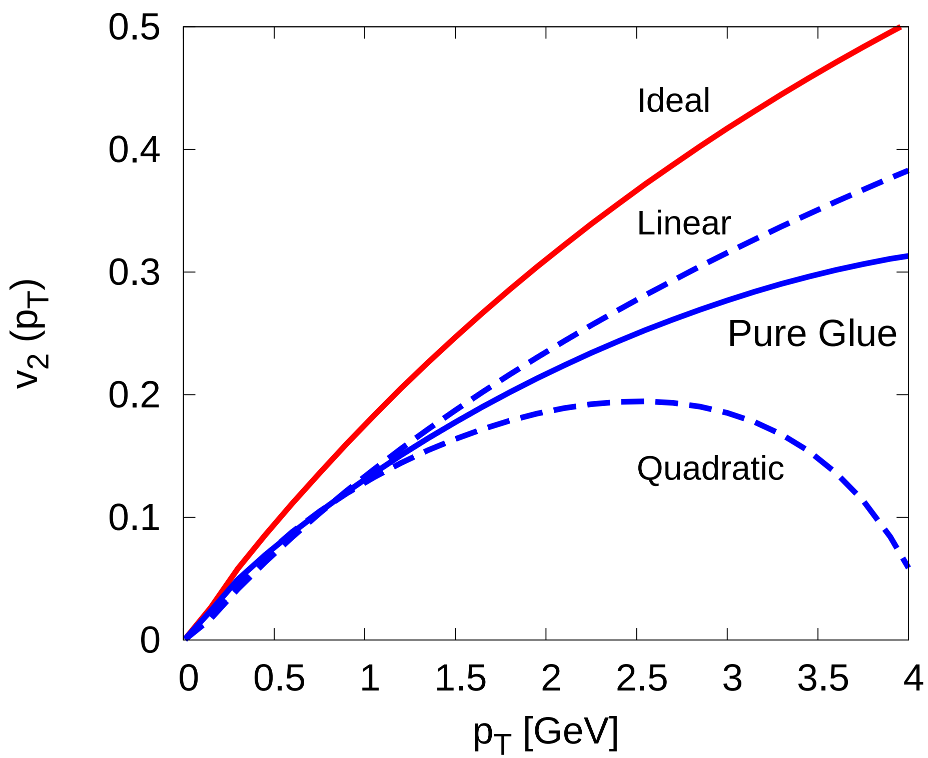


Energy loss determines $\chi(p)$

QCD kinetic theory expectation $\chi(p) \propto p^{1:38}$ in relevant range

Phenomenological Summary

pure glue, $\eta/s = 0.08$, $e_{\rm frz} = 0.6\,{\rm GeV/fm^3}$



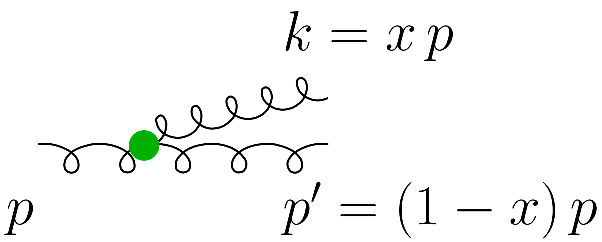
pQCD is closer to a linear (τ_R = const) rather than a quadratic ansatz

Connection to energy loss

1. At large momentum brem dominates the Boltzmann collision term

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{1\leftrightarrow 2}[f].$$

2. The collision kernel is



$$\mathcal{C}^{1\to 2} \propto \underbrace{\int_0^1 dx}_{\text{Phase Space}} \times \underbrace{\gamma^{g}_{gg}(p; p', k)}_{|\mathcal{M}|^2} \times \underbrace{\left[f_p(1+f_{p'})(1+f_k)\right]}_{\text{Stimulation Factors}}$$

3. The QCD splitting function is medium modified see P.Arnold, C.Dogan, BDMPS

$$\gamma^{g}_{gg} \propto \alpha_s C_A d_A \sqrt{p\hat{q}}\,\frac{[1-x(1-x)]^{5=2}}{[x(1-x)]^{3=2}}$$

Linearizing the Boltzmann equation

$$\delta f = -\chi(p)\, n_p(1 + n_p)\, \hat{p}^i \hat{p}^j \, \langle \partial_i u_j \rangle$$

1. The linearized Boltzmann equation becomes in high momentum limit

$$\frac{p}{T} = -\frac{(2\pi)^3}{32p} \int_0^\infty dx \, \underbrace{\gamma(p; xp, (1-x)p)}_{\propto \;_s\sqrt{\hat{q}p}} \left[\chi_p - \chi_{xp} - \chi_{(1-x)p}\right] .$$

2. Guess a solution $\chi = Cp^{3=2}$

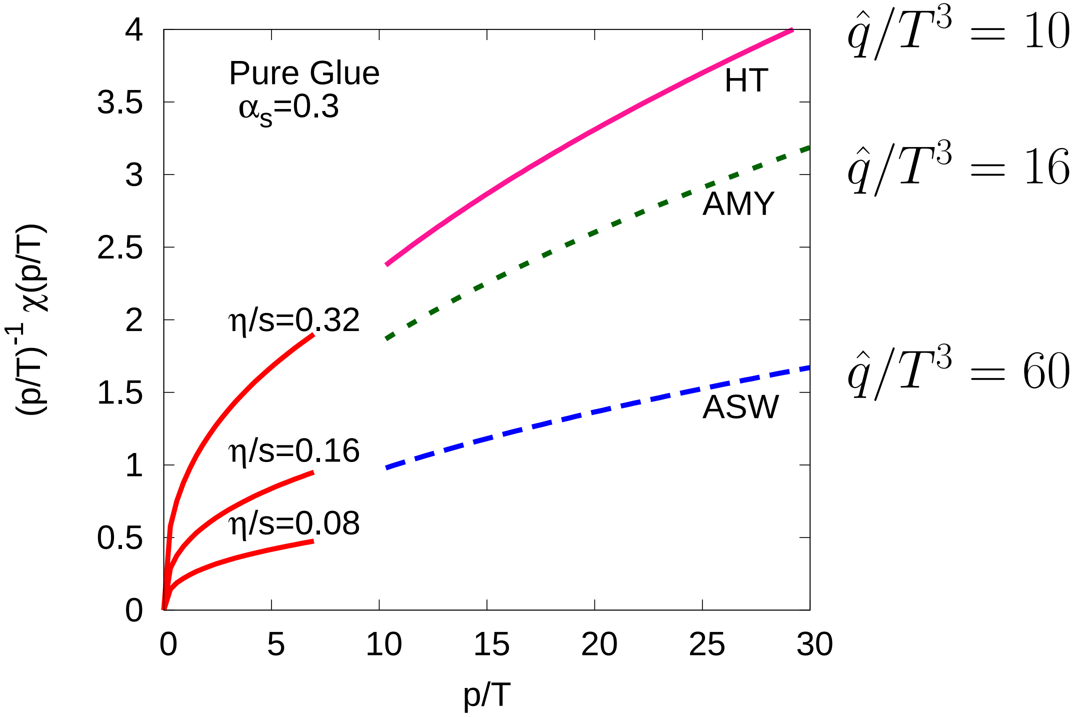
3. Find:

$$\chi(p) = \underbrace{0.7}_{\text{an } x \text{ integ. over split. fcn}} \times \frac{p^{3=2}}{\alpha_s T \sqrt{\hat{q}}}$$

$\hat{q}$ and viscous corrections at high momenta: A nifty formula

$$\underbrace{\chi(p)}_{\text{Viscous Correction}} = \underbrace{0.7 \times \frac{p^{3/2}}{\alpha_s T \sqrt{\hat{q}}}}_{\text{radiative loss}}$$

1. At low momentum $\chi(p)$ is determined by the shear viscosity η/s
2. At high momentum $\chi(p)$ is determined by $\hat{q}$



So far only single component (gluon) plasmas

Next: multi-component plasmas (coalesence hatred)

Quarks and Gluons (simple model)

- Quarks and Gluons have different relaxation times and δfj

$$\begin{aligned} \delta f^{Q} &= -C_{q}\, n_{p} p^{i} p^{j} \, \langle \partial_{i} u_{j} \rangle \\ \delta f^{G} &= -C_{g}\, n_{p} p^{i} p^{j} \, \langle \partial_{i} u_{j} \rangle \end{aligned}$$

- Casimir Scaling

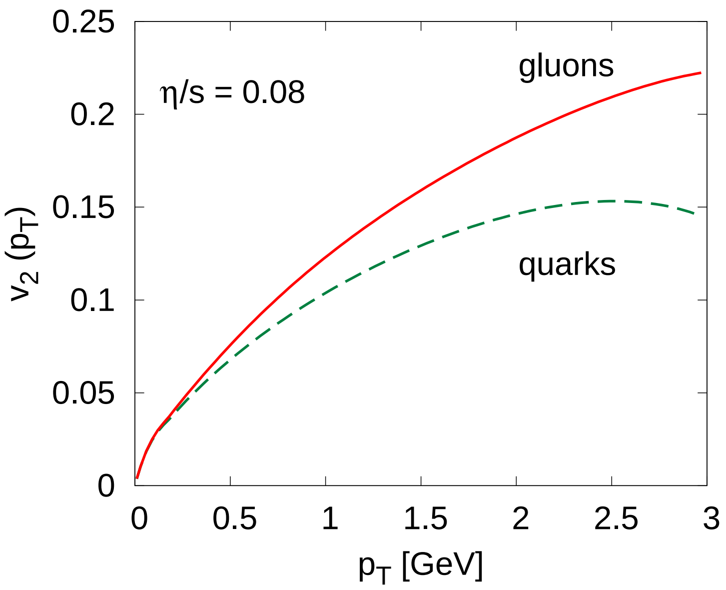
$$\frac{C_q}{C_g} = \frac{\tau_R^Q}{\tau_R^G} = \frac{C_A}{C_F} = \frac{9}{4}$$

- One constraint is provided by the shear viscosity

$$\eta = \frac{1}{15} \sum_{s=q;g} \nu_s C_s \int \frac{d^3p}{(2\pi)^3} p^3 n_p \,(1 \pm n_p) \, .$$

Can now solve for C_q and C_g

Simple Casimir Scaling – Quadratic Ansatz

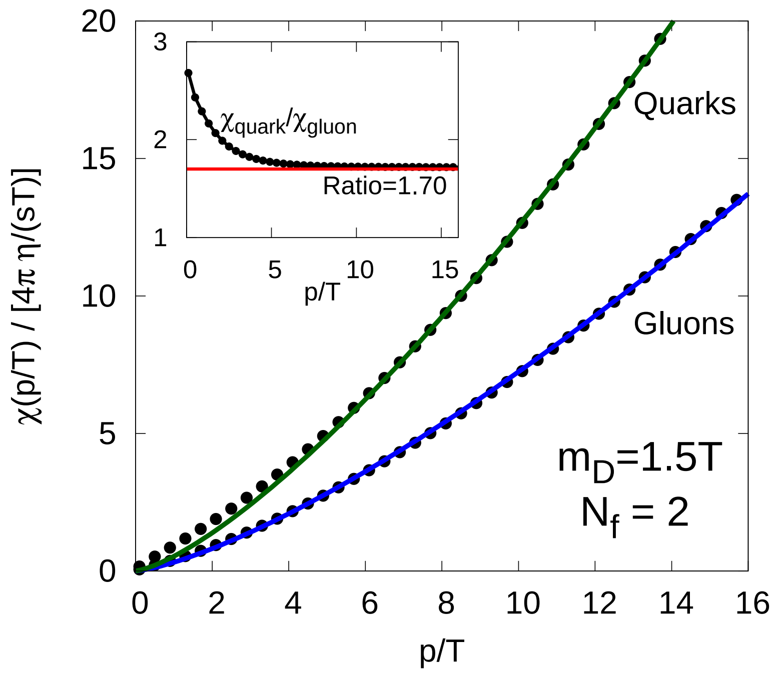


Now we can do a real calculation

- All kinds of processes: $g \longrightarrow qq,\, gq \longrightarrow gq$
- As before we can linearize the Boltzmann equation and write a matrix equation

$$\begin{bmatrix} b_{\rm p}^{\rm g} \\ b_{\rm p}^{\rm q} \end{bmatrix} = \begin{bmatrix} \rm gg & \rm gq \\ \rm qg & \rm qq \end{bmatrix} \begin{bmatrix} \delta f_{\rm g} \\ \delta f_{\rm q} \end{bmatrix}$$

Quark and gluons:



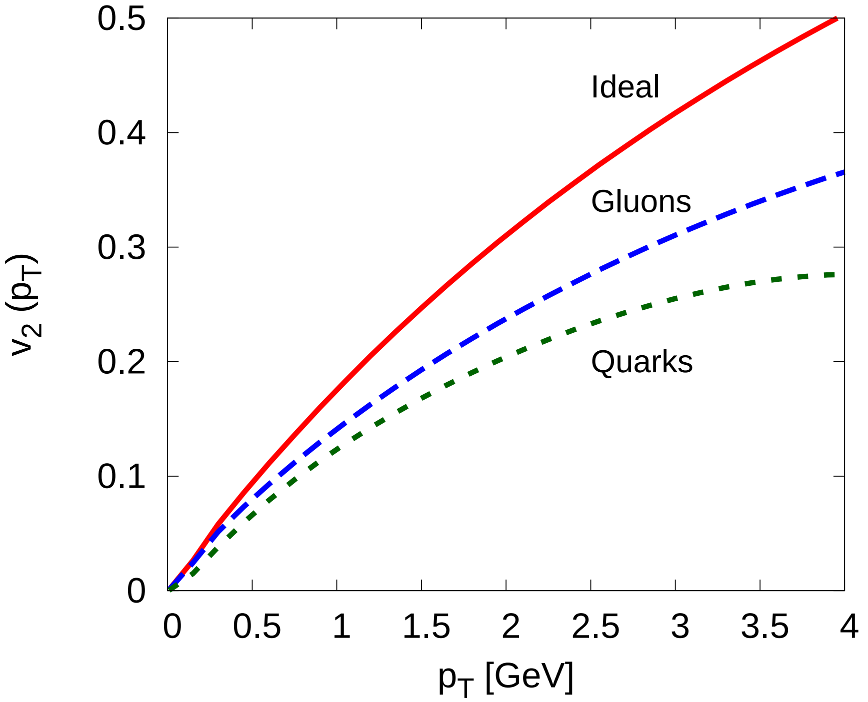
$$\frac{\chi_{\rm quark}}{\chi_{\rm glue}} \sim \frac{\tau_R^Q}{\tau_R^G} \sim 1.7$$

High momentum behavior – not just Casimirs

- Other splitting processes $g \to qq$ and spin dependence in splitting fcn.

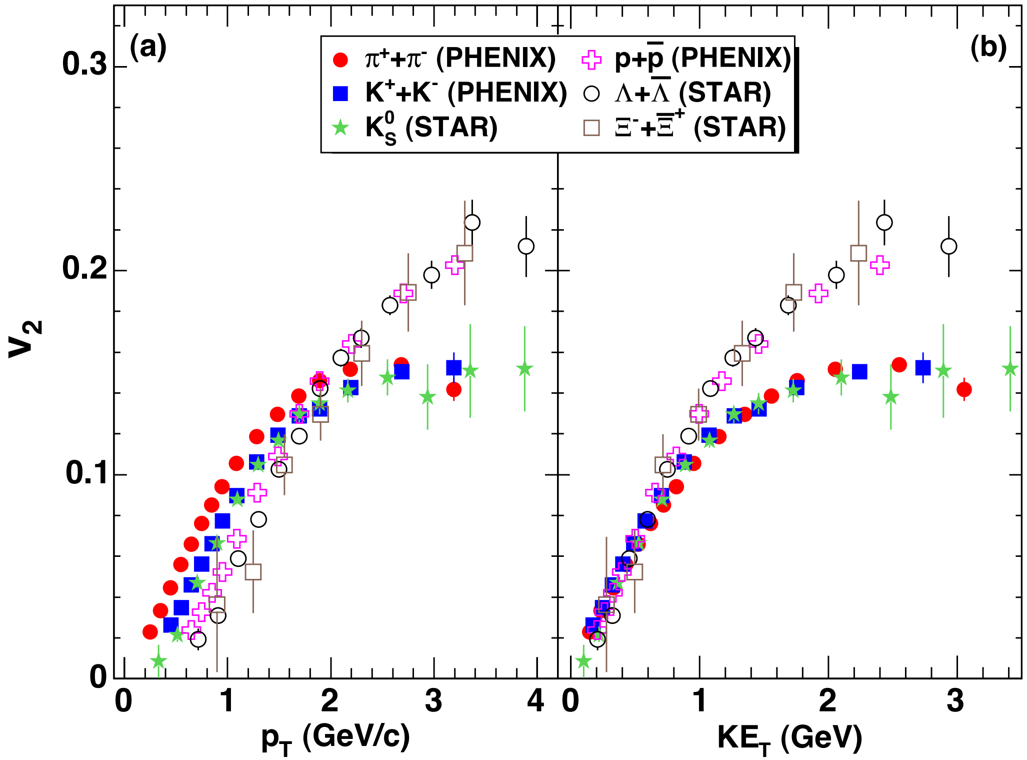
Derive the ratio $\frac{\rm quark}{\rm glue}$ = 1.7 analytically by analyzing collinear emission.

Quarks and Gluons – Real Calculation



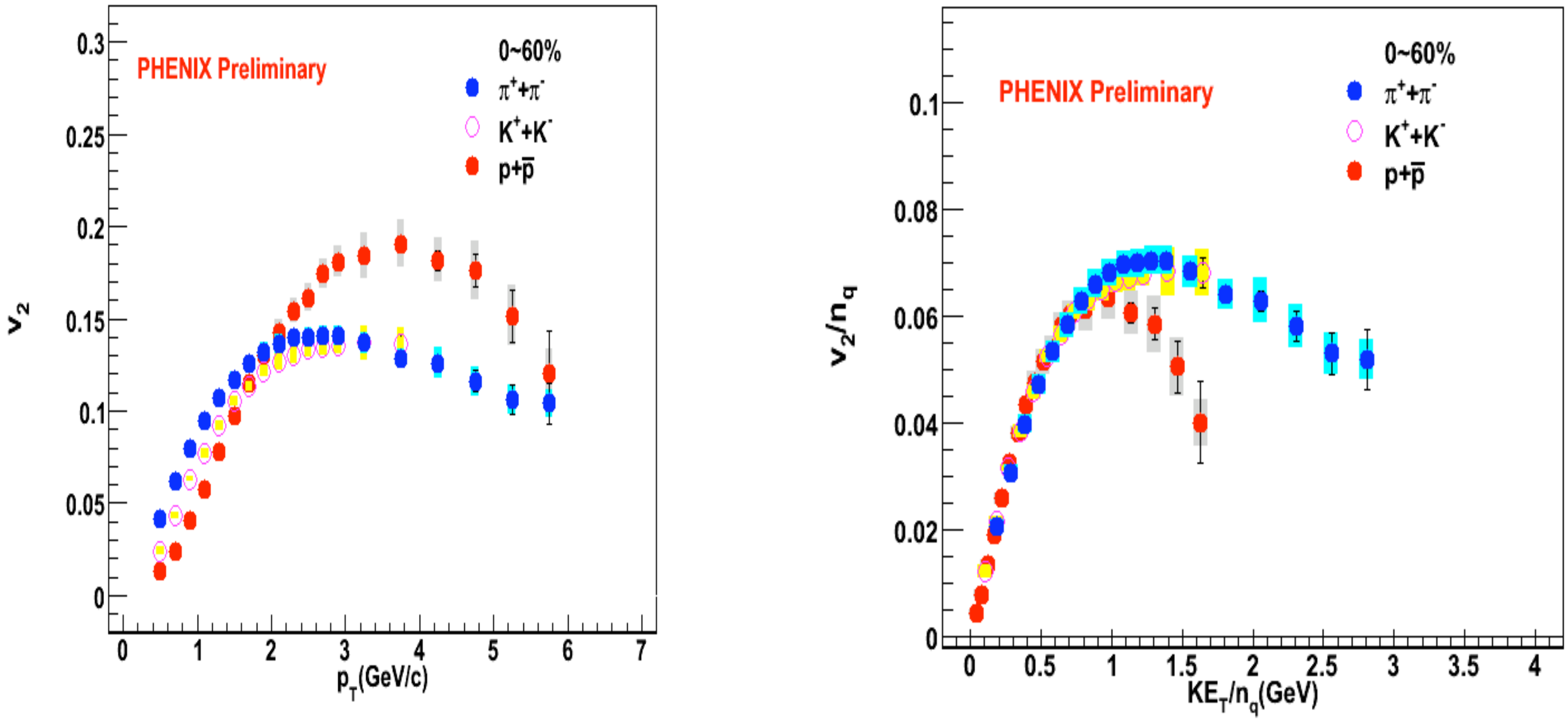
Different species with different relaxation times have different flows

Mesons and Baryons have different flows

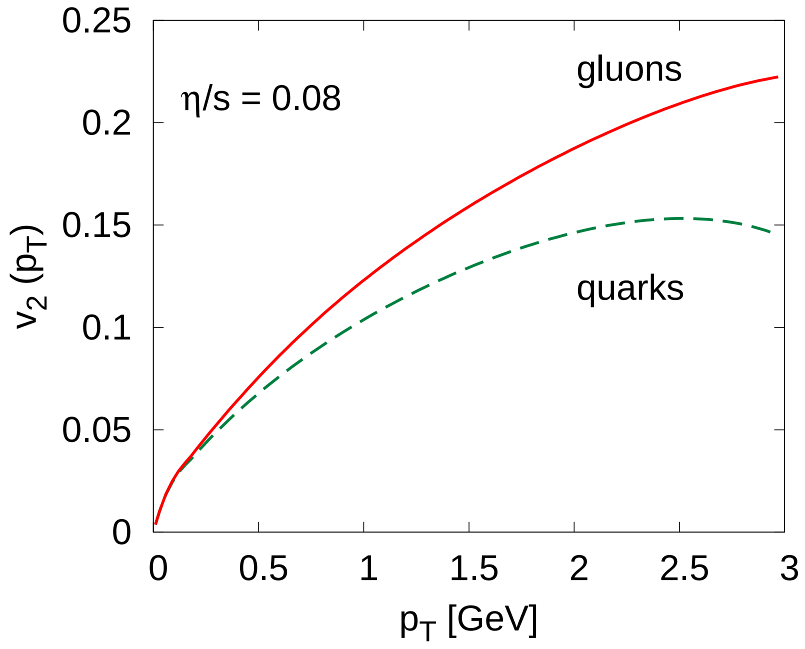


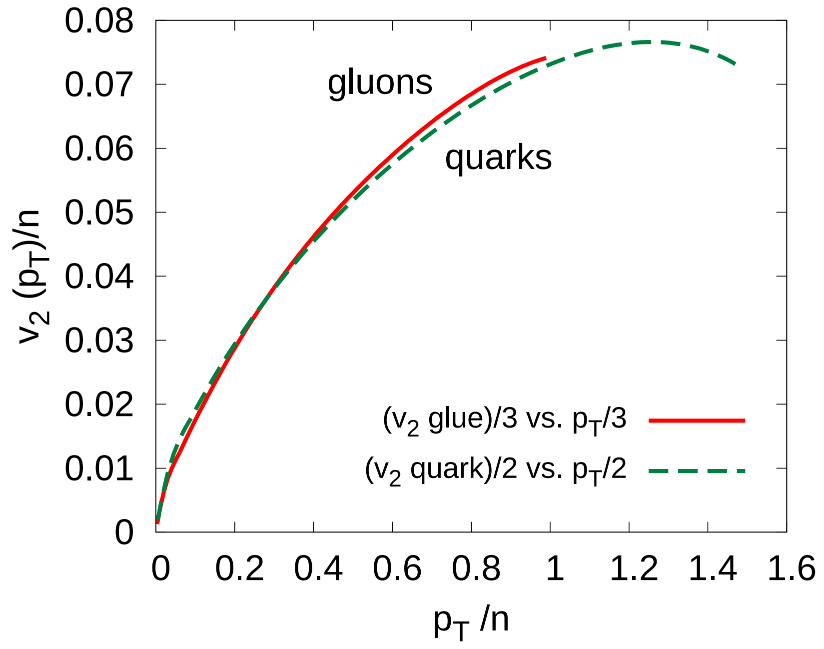
Perhaps they have different relaxation times

Two components interpreted with Coalescence



Simple quark and gluon model





"Scaling" can be an artifact of two different relaxation times

Try two different relaxation times for mesons and baryons

Two component meson/baryon gas – relaxation time

$$\delta f_m(p) = -n_p(1 + n_p)\chi_m(p)\hat{p}^i\hat{p}^j \langle\partial_i u_j\rangle$$
$$\delta f_b(p) = -n_p(1 - n_p)\chi_b(p)\hat{p}^i\hat{p}^j \langle\partial_i u_j\rangle$$

- Parameterize the viscous corrections as

$$\chi_m(p) = C_m p^2$$
$$\chi_b(p) = C_b p^2$$

- Fit

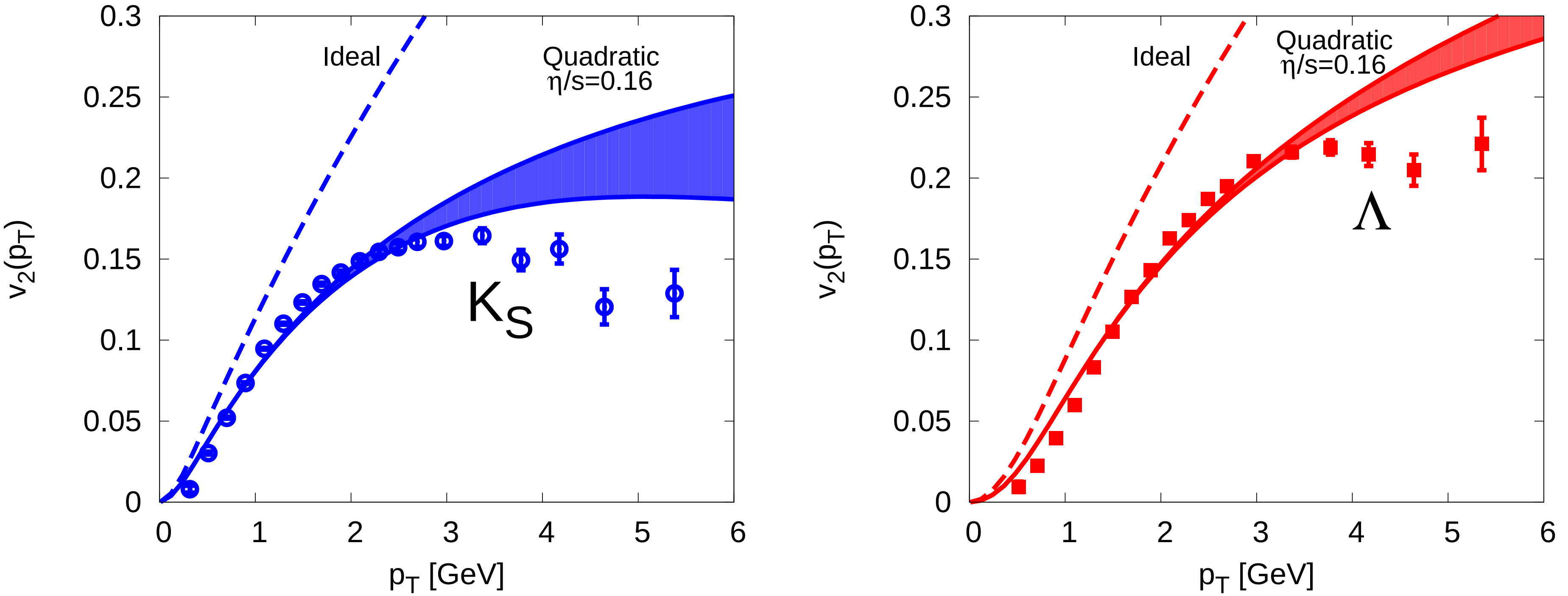
$$\frac{C_m}{C_b} = 1.6$$

- Constrained by shear viscosity

$$\eta = \frac{1}{15} \sum_{a=\pi,K,p,\ldots} \nu_a C_{m=b} \int \frac{d^3p}{(2\pi)^3 E_a} p^4 n(E_a)\left[1 \pm n(E_a)\right],$$

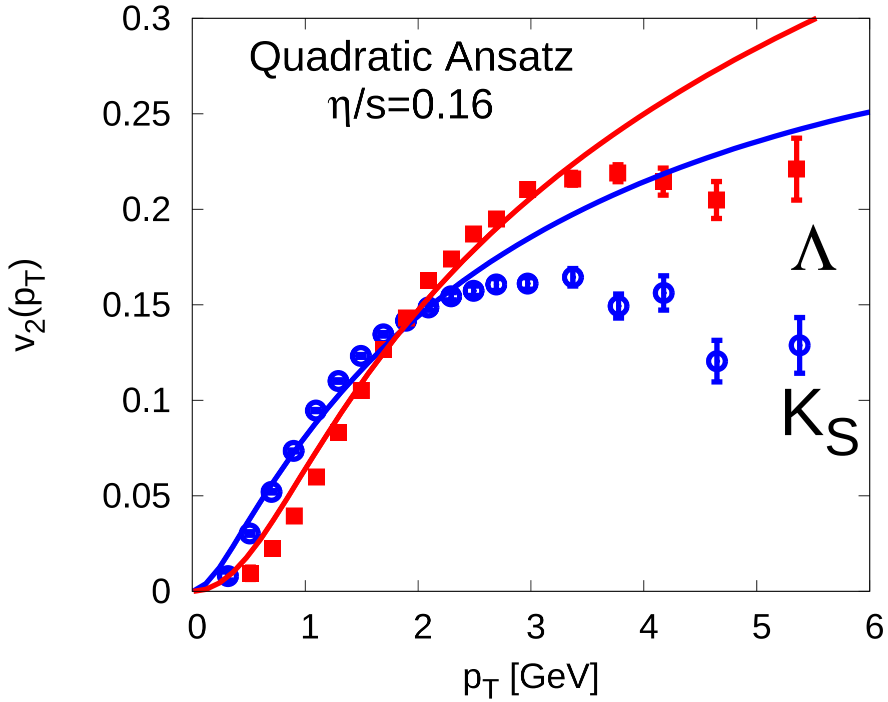
No reason to think the relaxation times of baryons are the same as mesons

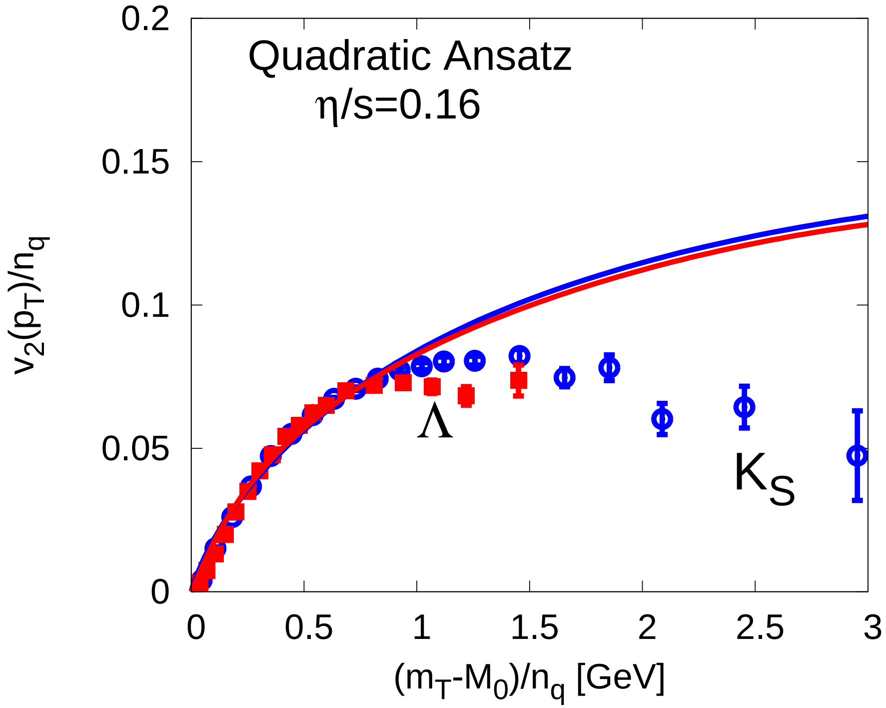
Results



0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
$v_2(p_T)$
p_T [GeV]
Ideal
Quadratic
η/s=0.16
K_S
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
$v_2(p_T)$
p_T [GeV]
Ideal
Quadratic
η/s=0.16
Λ

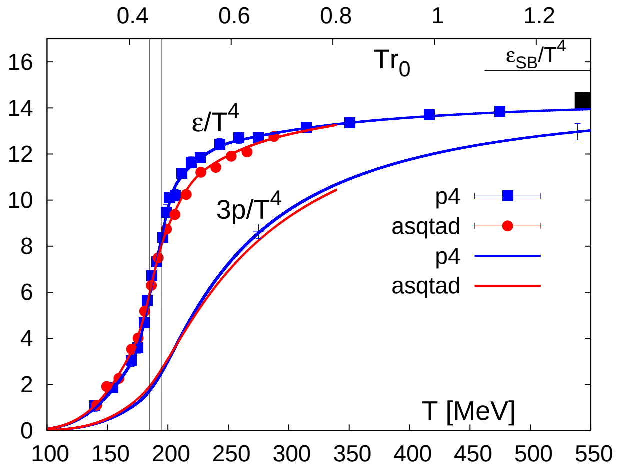
Scaling

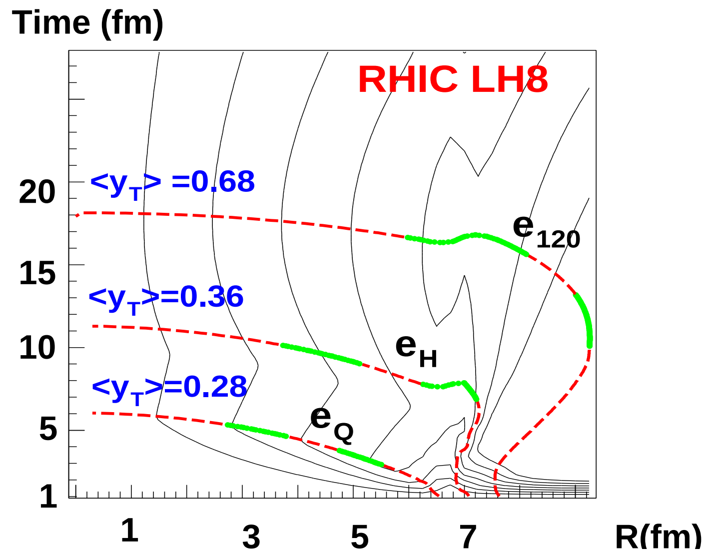




Perhaps quark number scaling is simply *Relaxation Time Scaling* (RTS)

Transition Region – long lived, not hadronic or partonic



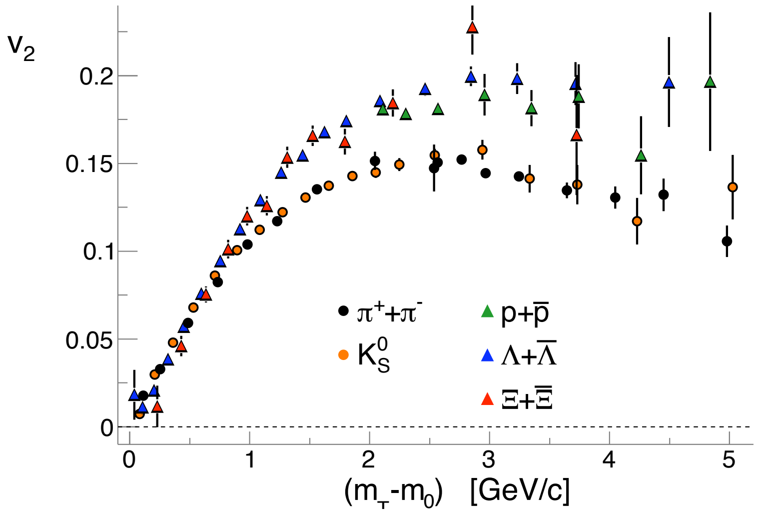


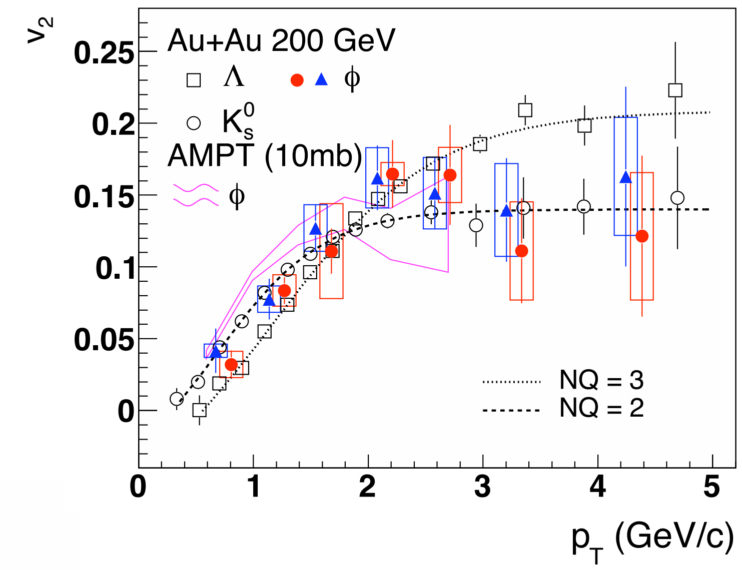
1. The transition region is long lived $\sim 3\,\mathrm{fm}$
2. The interactions are very inelastic in this momentum range
3. Results suggest the additive quark model (Bleicher et al)

$$\frac{C_m}{C_b} = \frac{\sigma_B}{\sigma_M} = 1.5$$

Transition Region – approximately $SU(3)$ symmetric

- In the high temperature range expect $SU(3)$ symmetric to be better
  - In $SU(3)$ symmetric world differences Baryon-Meson and spin diffs
  - $\underbrace{\pi, K}_{R1}$ and $\underbrace{p, \quad , \quad ,}_{R2}$ and $\underbrace{\phi}_{R3}$ and $\underbrace{\bar{\quad}}_{R4}$





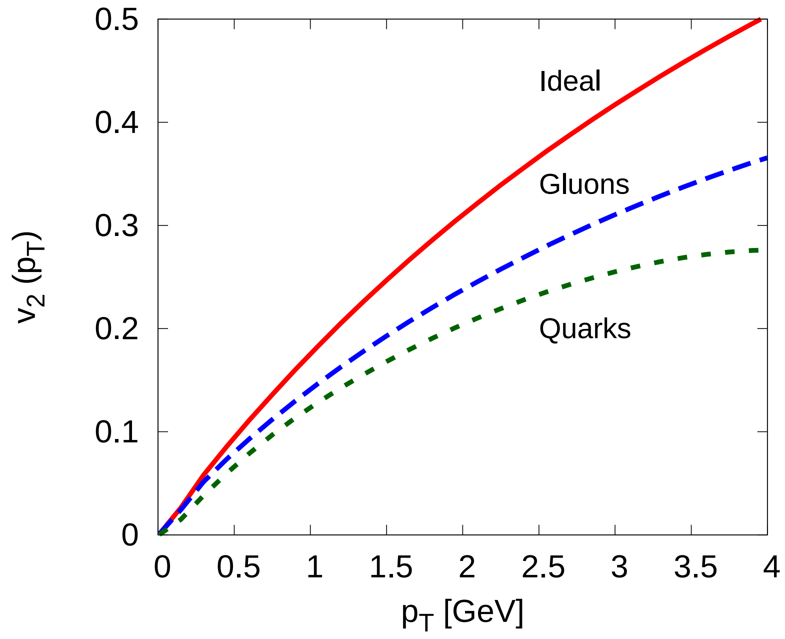
Conclusions

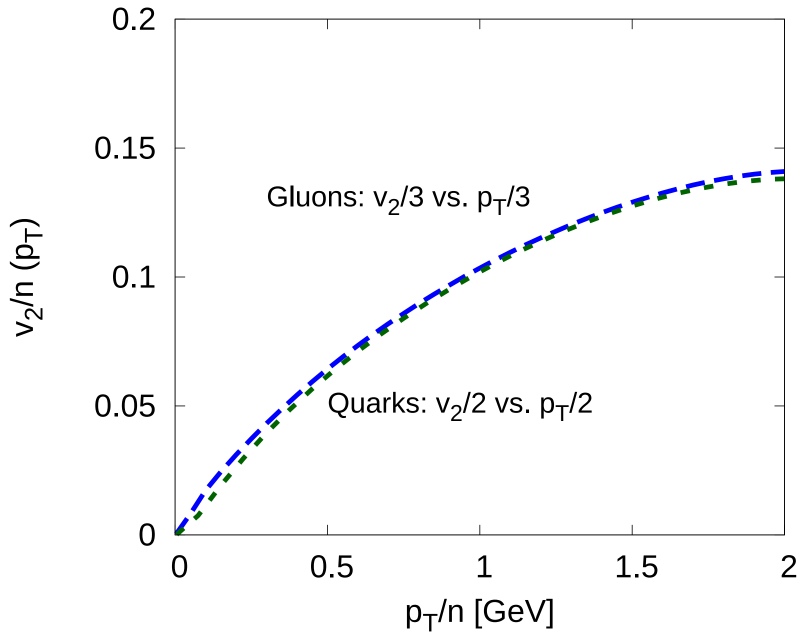
- Studied the kinetics of Quarks and Gluons and the imprints on elliptic flow.
- Radiative energy loss increases the elliptic flow in a certain range

$$p_T \simeq 1.5 \leftrightarrow 2.5\,\mathrm{GeV}$$

- Makes precise the connection between energy loss and viscosity.
- Observed *Relaxation Time Scaling (RTS)* in measured elliptic flow
  - I believe that such relaxation time fits will do as well as coalescence.

Backup I: perturbative quark and gluon model





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