Boost-invariant flow from AdS/CFT

Robi Peschanski [a]
(IPhT, Saclay)

Flow and dissipation in ultrarelativistic Heavy Ion Collisions
workshop at ECT Trento September 14-18, 2009

- Boost-invariant dynamics and holography

An Introduction

- Late time flow

Hydrodynamics

- Early time flow

Thermalization

- Conclusions and Prospects

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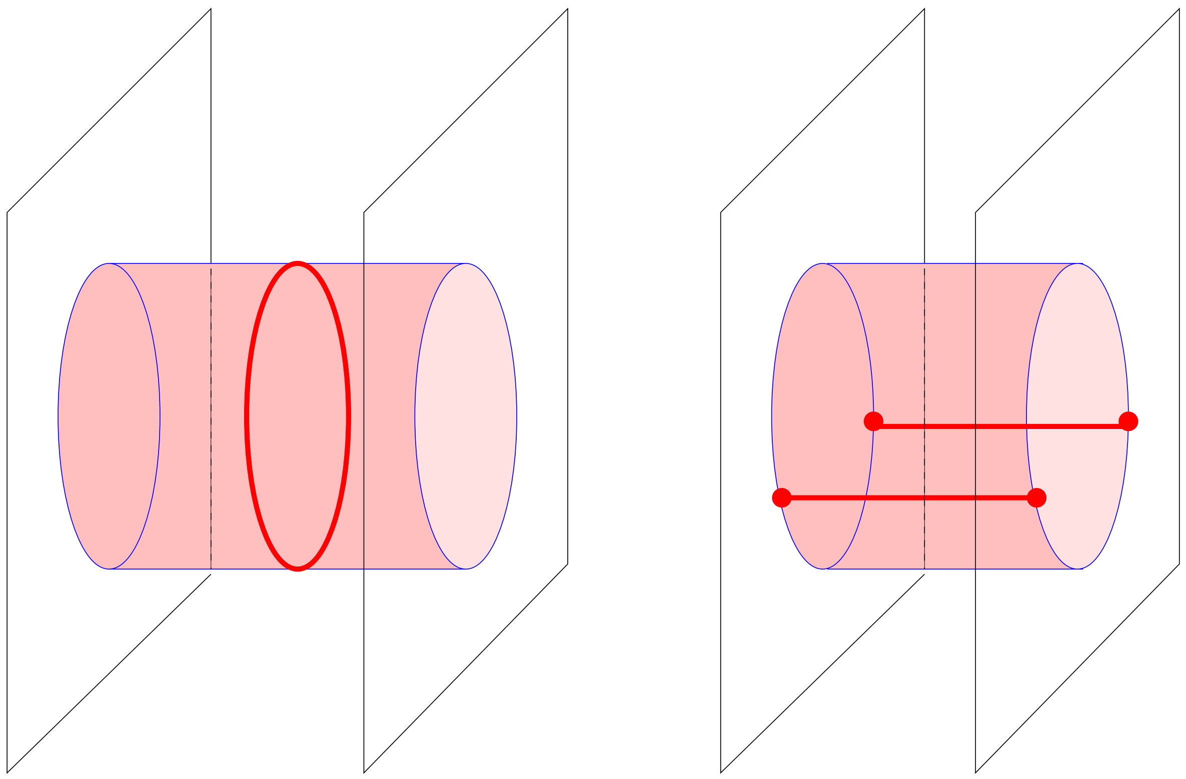
[a] Cracow + Saclay coll. Started and continued with Romuald Janik, then Michal Heller and Guillaume Beuf for the third item, and other contributors.

I. The Gauge-Gravity Duality

Open String $\Leftrightarrow$ Closed String



Schomerus, 2006

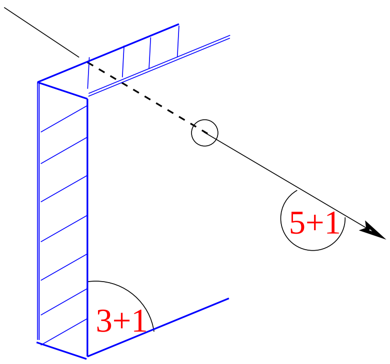
$$\begin{aligned} \textit{Closed String} \quad &\Leftrightarrow \quad 1-\textit{loop Open String} \\ D-\textit{Brane ``Universe''} \quad &\Rightarrow \quad \textit{Open String Ending} \\ \textit{Gravity} \quad &\Leftrightarrow \quad \textit{Gauge} \\ \textit{Large/Small Distance} \quad &\Rightarrow \quad \textit{Gravity/Gauge Correspondence} \end{aligned}$$

AdS/CFT Correspondence

J.Maldacena, 1998

GRAVITY

AdS



Gravity Source

Macroscopic

$(g_s N)^{-1} \to 0$

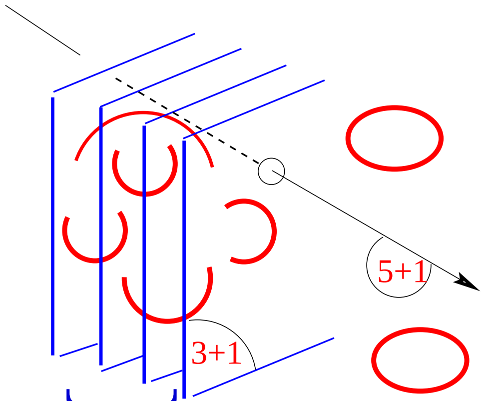
$AdS_5 \times S_5$ Superstring

Weak Gravity

Strong Gravity

GAUGE

CFT



N-Branes

Microscopic

$g_s N \to 0$

SU(N) Gauge Theory on the N-Branes

Strong Coupling

Weak Coupling

Duality

AdS/CFT Background

- D_3-brane Solution of Supergravity: Horowitz,Strominger, 1991

$$\boxed{ds^2 = f^{-1/2}\left(-dt^2 + \sum_1^3 dx_i^2\right) + f^{1/2}\left(dr^2 + r^2 d\Omega_5\right)}$$

"Physical" Brane $\quad + \quad$ Extra-Dimensions

$$f = 1 + \frac{R^4}{r^4}\ ;\ R^4 = 4\pi\alpha'^2 g_{YM}^2 N_c$$

- "Maldacena limit":

$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z\ ,\ R\ fixed\ \Rightarrow g_{YM}^2 N_c \to \infty$$

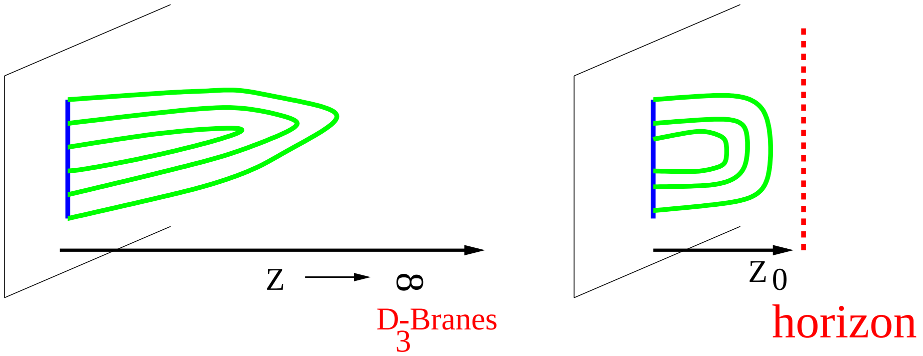
Strong coupling limit

$$\boxed{ds^2 = \frac{1}{R^2 z^2}\left(-dt^2 + \sum_{1-3} dx_i^2 + dz^2\right) + R^2 d\Omega_5}$$

Background Structure: $\mathrm{AdS}_5 \times S_5$ (same R^2)

HOLOGRAPHY

– Holographic Principle: Brane/Bulk correspondence



$\boxed{AdS_5 \otimes S_5}$ $\boxed{Confining\ Geometry}$

– Brane $\rightarrow$ Bulk: Holographic Renormalization

K.Skenderis, 2002

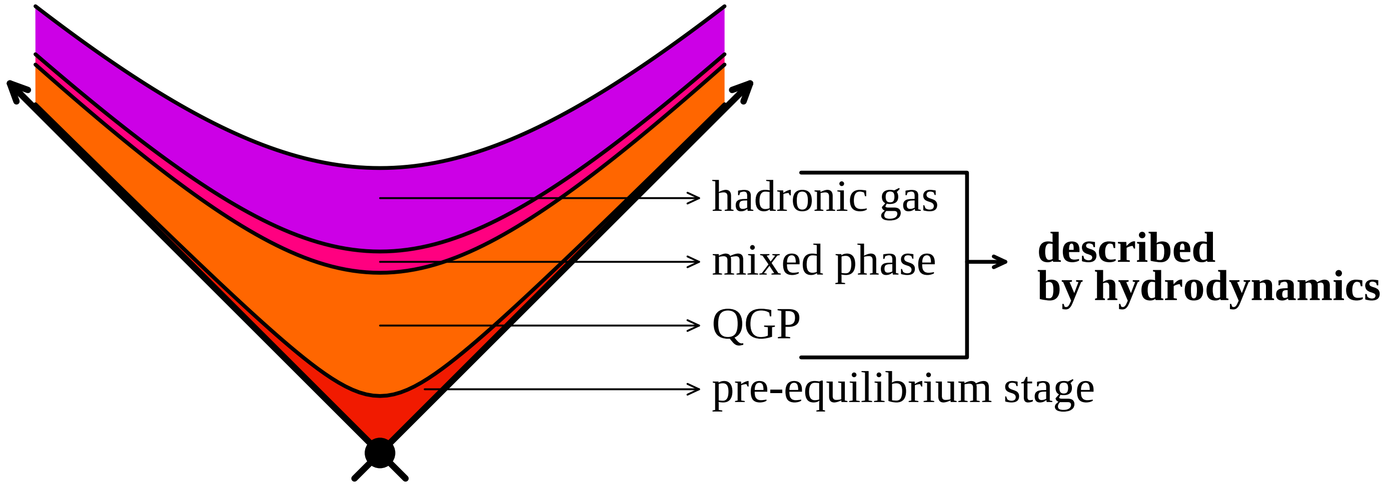
$$\boxed{ds^2 = \frac{g_{\mu\nu}(z)\ dx^\mu dx^\nu + dz^2}{z^2}}$$

$$\boxed{g_{\mu\nu} = g^{(0)}_{\mu\nu}(= \eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(= 0) + z^4 \langle T_{\mu\nu}\rangle \quad + z^6 \ldots +}$$

$+ z^6 \ldots +$: from Einstein Eqs.

Gauge/Gravity and Boost-invariant Dynamics

Janik, R.P., 2005



$$\boxed{\tau = \sqrt{x_0^2 - x_1^2}\ ; y = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1}\ ;\ x_T = x_2, x_3}$$

Questions

– What is the Gravity Dual of a Flow ?

– QGP: (almost) Perfect fluid behaviour ?

– Hydro regime: $\frac{\eta}{s}$, Transport coefficients, Navier-Stokes ?

– Pre-hydrodynamic stage, thermalization ?

II. The late time flow

– Boost-invariant T^{μ}_{ν}

$$T_{\mu\nu}=\begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\frac{d}{d\tau}f(\tau)-\tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau)+\frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

– Proper-time evolution

$f(\tau) \propto \tau^{-S}$: $\mathrm{FamilyIndex} \quad \mathrm{T}_{\mu\nu}\mathrm{t}^\mu\mathrm{t}^\nu \geq 0 \Rightarrow 0 < \mathrm{s} < 4$

$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect Fluid

$f(\tau) \propto \tau^{-1}$: Free streaming

$f(\tau) \propto \tau^{-0}$: Full Anisotropy $\quad \epsilon = p_\perp = -p_L$

– Holographic renormalization:

$\Rightarrow$ Holographic Scaling Variable v at large τ

$$\boxed{v = \frac{z}{\tau^{S/3}}}$$

Calculation of the Gravity Duals

– Boost-Invariant 5-d F-G metric:

$$\boxed{ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}}$$

– Scaling : $\boxed{v = \frac{z}{\tau^{S/4}}}$

$$[a(\tau,z), b(\tau,z), c(\tau,z)] = [a(v), b(v), c(v)] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

$$v(2a'(v)c'(v)+a'(v)b'(v)+2b'(v)c'(v))-6a'(v)-6b'(v)-12c'(v)+vc'(v)^2 = 0$$

$$3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) = 0$$

$$2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 +$$

$$4vsc''(v) + 4sc'(v) - 2vsa'(v)c'(v) + 2vsc'(v)^2 = 0$$

– Asymptotic Solution

$$\begin{aligned} a(v) &= A(v) - 2m(v) \\ b(v) &= A(v) + (2s-2)m(v) \\ c(v) &= A(v) + (2-s)m(v) \end{aligned}$$

$A(v) = \frac{1}{2}\left(\log(1+\Delta(s)\,v^4) + \log(1-\Delta(s)\,v^4)\right)$ $m(v) = \frac{1}{4\Delta(s)}\left(\log(1+\Delta(s)\,v^4) - \log(1-\Delta(s)\,v^4)\right.$ $\Delta(s) = \sqrt{3s^2-8s+8/24}$

AdS/CFT: Selection of the Perfect Fluid

– Kreschtmann Scalar: $\boxed{\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}}$

$$\mathfrak{R}^2 = \frac{4}{\left(1-\Delta(s)^2 v^8\right)^4} \cdot \Big[10\,\Delta(s)^8 v^{32} - 88\,\Delta(s)^6 v^{24} + 42\,v^{24} s^2 \Delta(s)^4 +$$
$$+112\,v^{24}\Delta(s)^4 - 112\,v^{24}\Delta(s)^4 s + 36\,v^{20} s^3 \Delta(s)^2 - 72\,v^{20} s^2 \Delta(s)^2 +$$
$$+828\,\Delta(s)^4 v^{16} + 288\,v^{16}\Delta(s)^2 s - 288\,v^{16}\Delta(s)^2 - 108\,v^{16} s^2 \Delta(s)^2 +$$
$$-136\,v^{16} s^3 + 27\,v^{16} s^4 - 320\,v^{16} s + 160\,v^{16} + 296\,v^{16} s^2 + 36\,v^{12} s^3 +$$
$$-72\,v^{12} s^2 - 88\,\Delta(s)^2 v^8 + 42\,v^8 s^2 + 112\,v^8 - 112\,v^8 s + 10 \Big] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

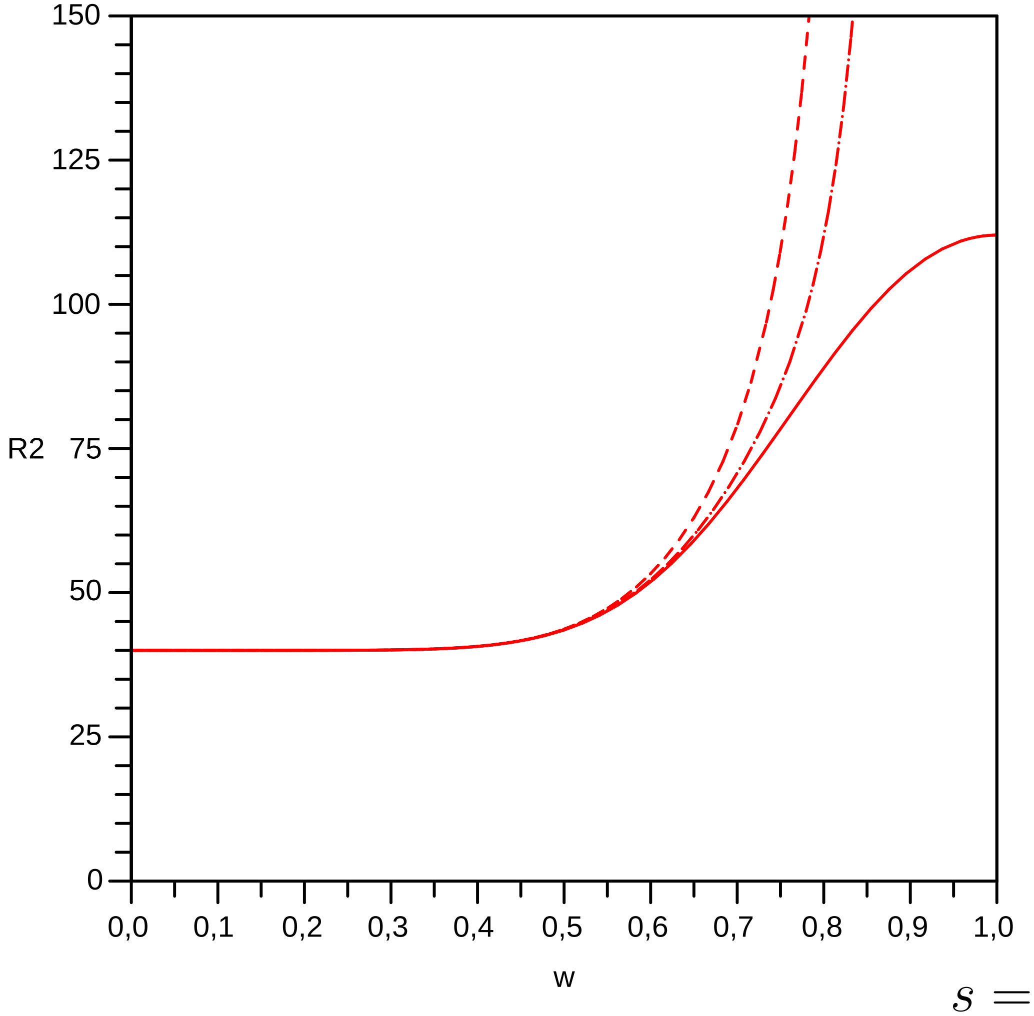
– $\mathfrak{R}^2$ for $s = \frac{4}{3}$:

$$\boxed{\mathfrak{R}^2{}_{\text{perfect fluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1+w^4)^4}}$$

where $w = v/\Delta(\frac{4}{3})^{\frac{1}{4}} \equiv \sqrt[4]{3}\,v$.

AdS/CFT $\Rightarrow$ Perfect Fluid at large τ

Kreschtmann Scalar: $\mathfrak{R}^2 = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$



$s = \frac{4}{3} \pm .1$

A nonsingular background selects a moving Black Hole geometry corresponding to the perfect fluid at large proper-times

The Perfect fluid is Dual to a Fifth-d Moving Black Brane

$$\boxed{v = \frac{z}{\tau^{1/3}}}$$

– Asymptotic metric

$$\boxed{ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\ \frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2+\left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)(\tau^2 dy^2+dx_\perp^2)\right]+\frac{dz^2}{z^2}}$$

– BH off in the 5th dimension $\Leftrightarrow$ Hwa-Bjorken flow

$$Horizon: \ z_h = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}} \ .$$

$$Temperature: \ T(\tau) \sim \frac{1}{z_h} \sim \tau^{-\frac{1}{3}}$$

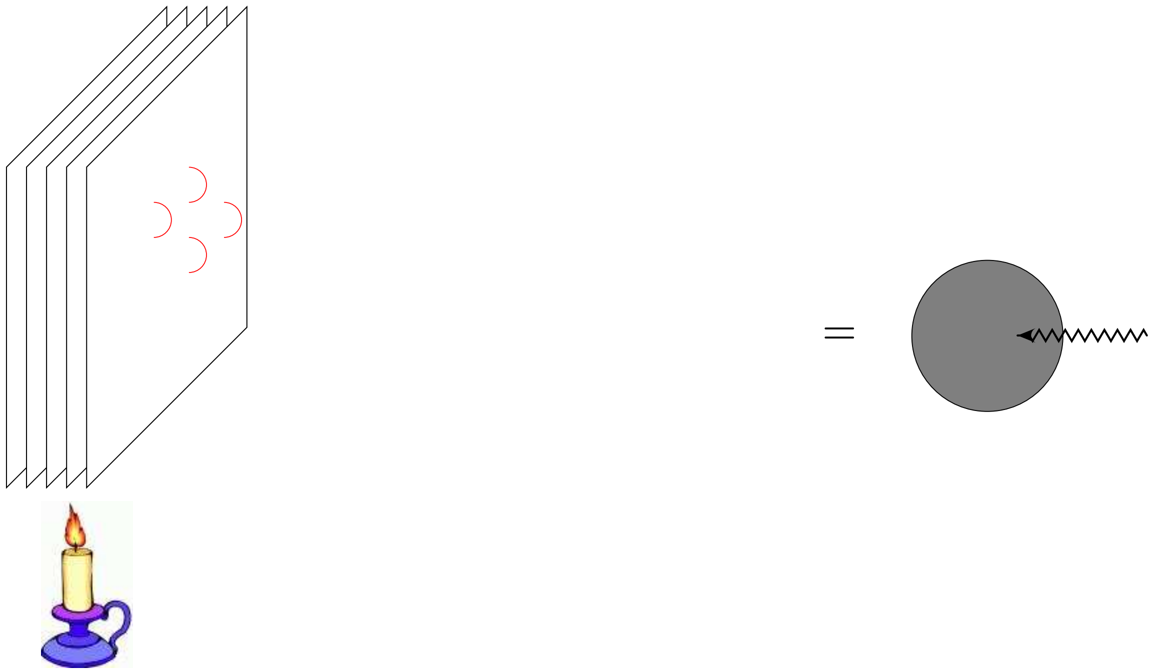
$$Entropy: \ S(\tau) \sim Area \sim \tau \cdot \frac{1}{z_h^3} \sim const$$

Hydro beyond the Perfect fluid
Static Case

Kovtun, Policastro, Son, Starinets (2001)

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

$$\boxed{\sigma_{abs}(\omega) \propto \int d^4x \, \frac{e^{i\omega t}}{\omega} \, \langle [T_{x_2x_3}(x), T_{x_2x_3}(0)] \rangle \Rightarrow \boxed{\frac{\eta}{s} \equiv \frac{\sigma_{abs}(0)/(16\pi \; G)}{A/(4 \; G)} = \frac{1}{4\pi}}}$$

Hydro beyond the Perfect fluid
Dynamic Case

– Going beyond perfect fluid

In-flow Viscosity, Relaxation time, Transport Coeff., etc...

Janik, Heller, Bak, Benincasa, Buchel, Nakamura, Sin,......
Kinoshita, Mukoyama, Nakamura, Oda,Natsuume, Okamura,...

$$\boxed{\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\eta}{\tau^2} + \cdots \Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}}$$

– Going beyond boost-invariance

Fluid/Gravity Duality

Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$
$$+ \underbrace{(\pi T^2)\left(\log 2T^{\mu\nu}_{2a} + 2T^{\mu\nu}_{2b} + (2-\log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

Going beyond hydrodynamics? Out-of-Equilibrium?
Boost-Invariant Early-time dynamics: part III

III. Early-time Boost-Invariant Flow

– General Boost-Invariant Fefferman-Graham metric:

$$\boxed{ds^2 = \frac{-e^{\,a(\tau,z)}\ d\tau^2 + \tau^2 e^{\,b(\tau,z)}\ dy^2 + e^{\,c(\tau,z)}\ dx_\perp^2}{z^2} + \frac{dz^2}{z^2}}$$

– Einstein Equation:

$$\boxed{R_{AB} + 4G_{AB} = 0}$$

– To be solved: $(\dot a = \partial_\tau a; a' = \partial_z a; \cdots)$

$$(\tau\tau): \ddot b + 2\ddot c - \frac{\dot a}{2}(\dot b + 2\dot c) + \frac{1}{2}(\dot b^2 + 2\dot c^2) - \frac{1}{\tau}(\dot a - 2\dot b) = e^a\left\{a'' - \frac{3a'}{z} + \left(\frac{a'}{2} - \frac{1}{z}\right)(a' + b' + 2c')\right\}$$

$$(yy): \ddot b - \dot a\dot b + \frac{1}{\tau}(\dot b - 2\dot a) + \frac{1}{2}(\dot a + \dot b + 2\dot c)\left(\dot b + \frac{2}{\tau}\right) = e^a\left\{b'' - \frac{3b'}{z} + \left(\frac{b'}{2} - \frac{1}{z}\right)(a' + b' + 2c')\right\}$$

$$(\perp\perp): \ddot c - \dot a\dot c + \frac{\dot c}{2}\left(\dot a + \dot b + 2\dot c + \frac{2}{\tau}\right) = e^a\left\{c'' - \frac{3c'}{z} + \left(\frac{c'}{2} - \frac{1}{z}\right)(a' + b' + 2c')\right\}$$

$$(\tau z): 2\dot b' + 4\dot c' + b'\left(\dot b + \frac{2}{\tau}\right) + 2\dot c c' - a'\left(\dot b - 2\dot c + \frac{2}{\tau}\right) = 0$$

$$(zz): a'' + b'' + 2c\text{""} - \frac{1}{z}(a' + b' + 2c') + \frac{1}{2}(a'^2 + b'^2 + 2c'^2) = 0$$

Preliminaries (1): Quasi-Normal Modes

R.Janik,R.P., 2006

– Scalar Excitation of a Moving Black Hole

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_n\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

– Scalar "Quasi-Normal Modes"

$$\phi(\tau, v \equiv z/\tau^{1/3}) = f(\tau) \times \phi(v)$$

$$\boxed{f(\tau) = \sqrt{\tau} J_{\pm\frac{3}{4}}\left(\frac{3}{2}\omega\tau^{\frac{2}{3}}\right) \ \sim\ \tau^{\frac{1}{6}}\ e^{\frac{3}{2}i\omega\tau^{2/3}}}$$

– Short Excitation Decay

$$\boxed{\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\, i \Rightarrow \tau \sim \frac{1}{8.3\ T}}$$

– e-folding conjecture

JJ Friess, SS Gubser, G. Michalogiorgakis,SS Pufu, 2007

$$\boxed{\tau_{therm} \sim\ 4\tau_{e-fold} = 4 \times \frac{1}{8.3\ T_{peak}} \sim 4 \times .1 fermi}$$

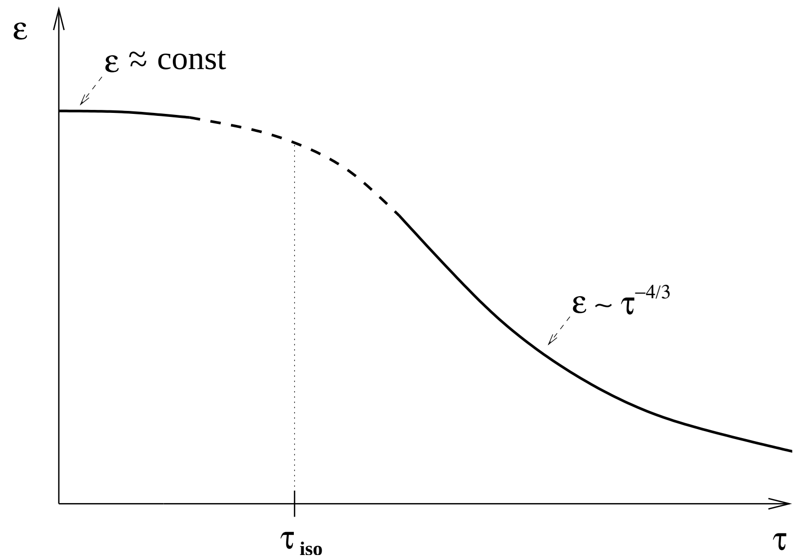
The Black Hole as an "Attractor"

Preliminaries (2): Scaling Solution

Kovchegov, Taliotis, 2007

– Evolution at small ($S = 0$) *vs.* large ($S = 4/3$) proper-time

Assuming Monodromy $\in$ Regularity



– Evaluation of The Isotropization/Thermalization time

$$Matching: \; z_h^{late}(\tau) = (3/e_0)^{\frac{1}{4}} \equiv z_h^{early}(\tau) = \tau$$

$$Isotropization: \; \tau_{iso} = \left(3N_c^2/2\pi^2 e_0\right)^{3/8}$$

$$Typical\ Scale: \epsilon(\tau) \; = \; e_0 \; \tau^{4/3}\big|_{\tau=.6} \sim 15 \; GeV fermi^{-3}$$

$$\Rightarrow \boxed{\tau_{iso} \sim .3 \; fermi}$$

Early-Time; General Features

G.Beuf, M.Heller, R.Janik, R.P., 2009

– Problems with Scaling at $S = 0$: Initial Conditions matter

$$\boxed{a(\tau, z) = \ldots + z^8 \ \{-1/16\,\tau^{-2\,s} s^2 - 1/6\,\tau^{-2\,s} + 1/6\,\tau^{-2\,s} s\} + \frac{z^4}{\tau^s}\left\{\frac{1}{96}\frac{z^4}{\tau^4}s^2 - \frac{1}{384}\frac{z^4}{\tau^4}s^4\right\} + \ldots}$$

scaling canceled when s=0

– The metric is singular at all times (including $\tau = 0$!):

$$\text{Set}: \quad u(z^2) = \frac{1}{4z}a_0'(z) \qquad v(z^2) = \frac{1}{4z}b_0'(z) \qquad w(z^2) = \frac{1}{4z}c_0'(z)$$

$$\boxed{[u + v + w]_0^\infty \equiv \int_0^\infty (u' + v' + w')dz^2 = -2\int_0^\infty (u^2 + v^2 + w^2) z dz^2}$$

– The geometry should stay regular:

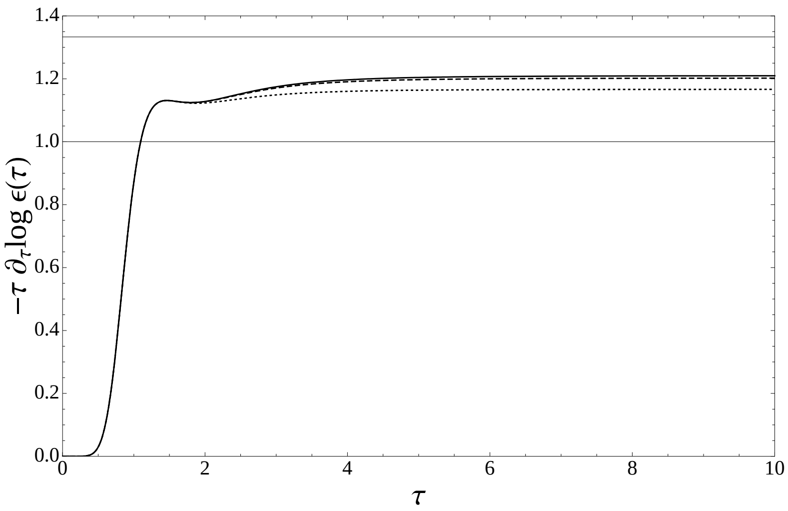
Holographic constraint at $\tau = 0, z \sim z_{sing}$

$$\boxed{ds^2(z \sim z_{sing}) \sim \frac{1}{z^2}\left(1 - \frac{z}{z_{sing}}\right)^2 d\tau^2 + \ldots + \frac{1}{z^2}dz^2}$$

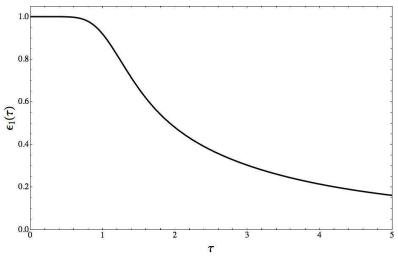
– To be satisfied: Initial Conditions + Constraints

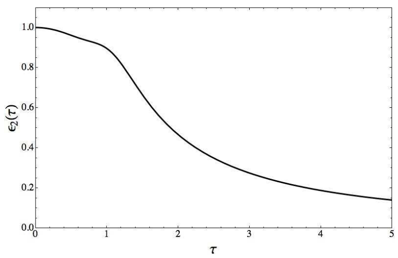
Investigations on Thermalization: Energy Density

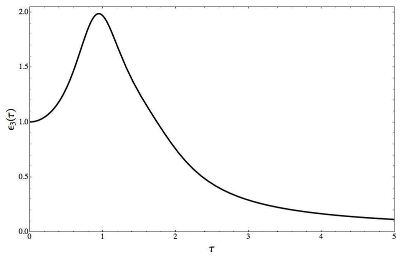
– "Family Index": $\mathbf{S} = -\tau\partial_\tau\epsilon(\tau)$



– I.C. Dependence







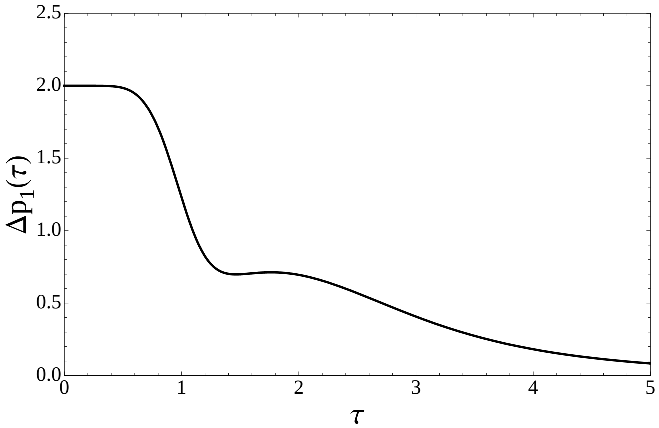
$$v+w = A) : \tanh(z^2)-\tan(z^2) \quad B) : \tanh(z^2+z^8/6)-\tan(z^2) \quad C) : 2/3z^6(1+z^2/2)/(z^2-1)$$

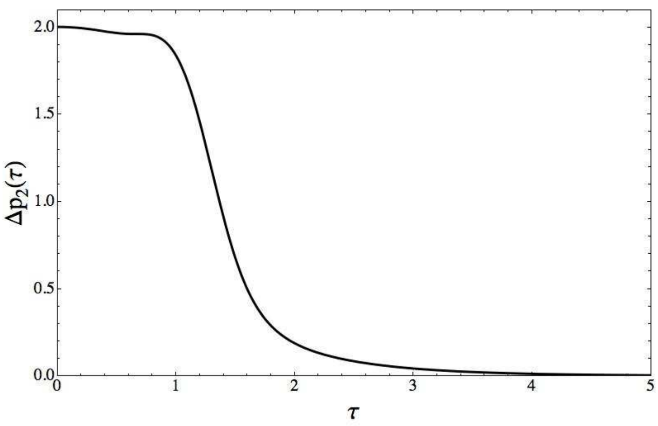
B): Temporary violation of positivity on $T_{\mu\nu}$

$$\frac{4\epsilon\,(\tau)}{\tau} \;\leq\; \epsilon'\,(\tau) \;\leq\; 0$$

Isotropization
Pressure Density

$$\boxed{\Delta p\,(\tau) = 1 - \frac{p_{\parallel}\,(\tau)}{p_{\perp}\,(\tau)}}$$





$v + w = A)$: $\tanh(z^2) - \tan(z^2)$ $\qquad$ $B)$: $\tanh(z^2 + z^8/6) - \tan(z^2)$

Conclusions and Prospects

Conclusions:

- Gauge-Gravity Correspondence
  A promising way to study Boost-Invariant Dynamics
- Late-time (Hydro)Dynamics
  Scaling, “almost-pefect” fluid, Einstein *vs.* Navier-Stokes
- Early-time Dynamics
  No scaling, singularity at all times, thermalization studies

Prospects

- More Numerical Work
  Classifying the thermalization solutions
- More “Translation” Work
  Relation with initial conditions
- More Theoretical Work
  Going beyond boost-invariance
- From S^4QCD to S^0QCD ?
  Approaching the “Gravity Dual” of QCD

Why Einstein Eqs. may govern the QGP?

EXTRA SLIDES

Boost-Invariant Viscosity and Relaxation time

R.Janik, R.Janik and M.Heller;

- Shear Viscosity equation (first order)

$$\boxed{\dot{\tau}\epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\eta}{\tau^2}}$$

- Asymptotic Expansion of the Black Hole Solution

$$\boxed{a(\tau,z), b(\tau,z), c(\tau,z) \Rightarrow \sum_n \lambda_n^{a,b,c}(v)\ \tau^{-2n/3}}$$

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \Rightarrow \sum_n \mathfrak{R}^2{}_n\ \tau^{-2n/3}$$

- Results

$\boxed{\frac{\eta}{S} = \frac{1}{4\pi}}$ Universal viscosity (needs $n \to 2$)

$\boxed{\tau_{Rel} = (1 - \log 2)/2\pi T}$ Relaxation Time (needs $n \to 3$)

EMERGENCE of the 5d BLACK HOLE

Balasubramanian, de Boer, Minic; Myers; Janik, R.P.

– 4d Perfect Fluid "on the brane"

$$\boxed{\langle T_{\mu\nu}\rangle \propto g^{(4)}_{\mu\nu} = \begin{pmatrix} 3/z_0^4 = \epsilon & 0 & 0 & 0 \\ 0 & 1/z_0^4 = p_1 & 0 & 0 \\ 0 & 0 & 1/z_0^4 = p_2 & 0 \\ 0 & 0 & 0 & 1/z_0^4 = p_3 \end{pmatrix}}$$

– Holographic Renormalisation (Resummed)

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2} dt^2 + (1 + z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

– $\Rightarrow 5d$ Black Brane with horizon at $z_0 \sim T_0^{-3}$

$$\boxed{ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4}\frac{d\tilde{z}^2}{\tilde{z}^2}}$$

$$z \rightarrow \tilde{z} = \; z/\sqrt{1+\frac{z^4}{z_0^4}}$$