The CGC approach to hydro initial conditions

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Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

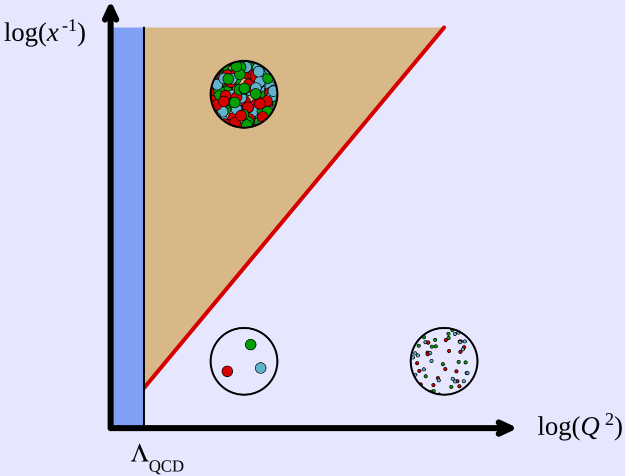
Recombination cross-section :

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Saturation domain



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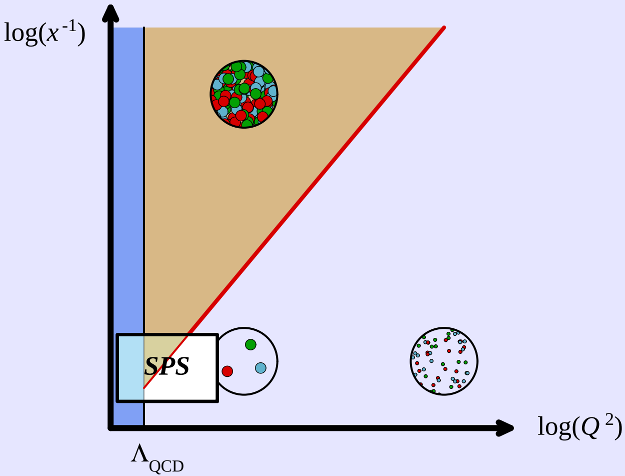

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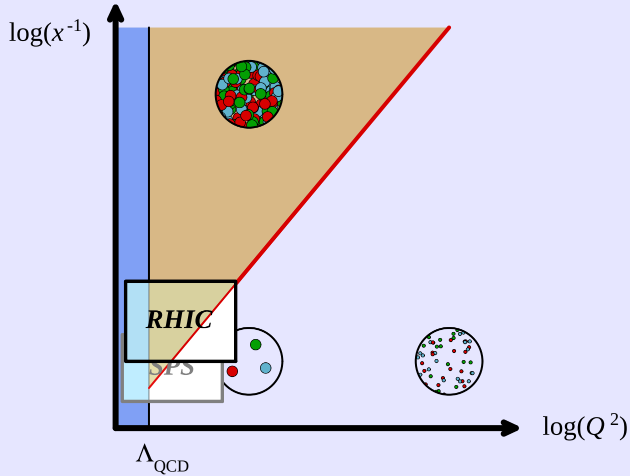

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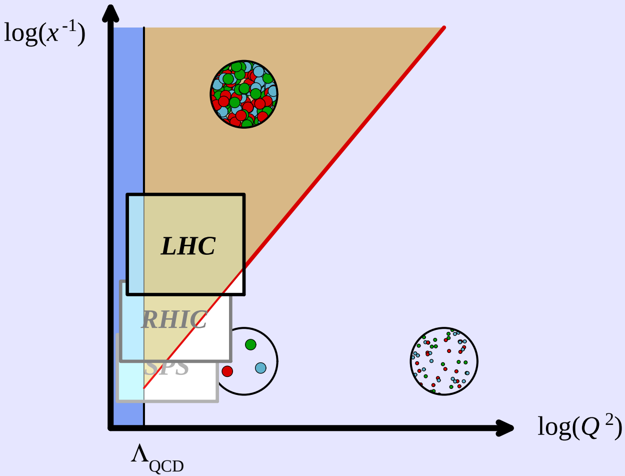

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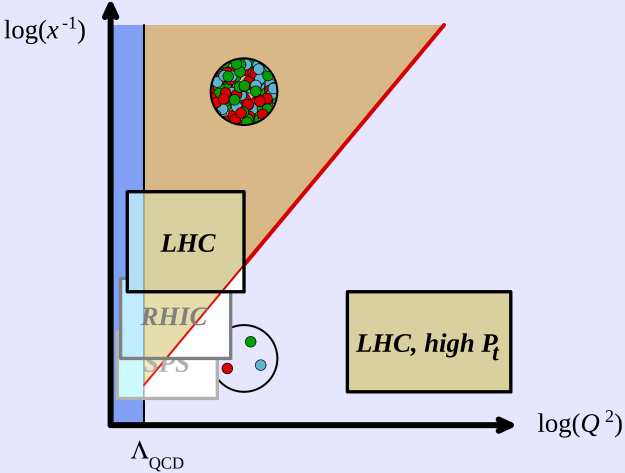
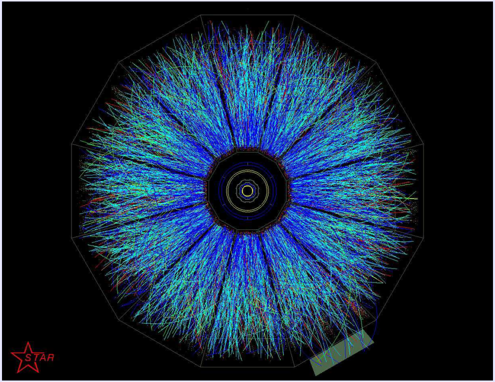


Heavy Ion Collisions

Nucleus-Nucleus collision at RHIC



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- Q_s^2 might be as large as 10 GeV^2 at the LHC ($\sqrt{s} = 5.5$ TeV)

  ▷ saturation expected to play an important role

Effective degrees of freedom

McLerran, Venugopalan (1994)

- The fast partons (large $x > x_0$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu+}\rho(x^-,\vec{\boldsymbol{x}}_\perp) \qquad (x^- \equiv (t-z)/\sqrt{2})$$

Note: $\rho(x^-,\vec{\boldsymbol{x}}_\perp) \propto \delta(x^-)$

- Slow partons (small $x < x_0$) are not static over the time-scales of the collision process
  ▷ must be treated as the usual gauge fields
  ▷ coupled to the current J^μ by a term : $J^\mu A_\mu$

- The color sources ρ are random, with a distribution $W_Y[\rho]$ ($Y \equiv \ln(1/x_0)$) is the rapidity separating “slow” and “fast”)

Color Glass Condensate

Evolution equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H}\; W_Y$$

$$\mathcal{H} = \frac{1}{2}\int_{\vec{x}_\perp,\vec{y}_\perp} \frac{\delta}{\delta\widetilde{\mathcal{A}}^+(\epsilon,\vec{y}_\perp)}\eta(\vec{x}_\perp,\vec{y}_\perp)\frac{\delta}{\delta\widetilde{\mathcal{A}}^+(\epsilon,\vec{x}_\perp)}$$

where $-\partial_\perp^2\,\widetilde{\mathcal{A}}^+(\epsilon,\vec{x}_\perp) = \rho(\epsilon,\vec{x}_\perp)$

- $\eta(\vec{x}_\perp,\vec{y}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

CGC and Nucleus-Nucleus collisions

- For nucleus-nucleus collisions, there are two strong sources that contribute to the color current :

$$J^\mu \equiv \delta^{\mu +}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu -}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$

Average over the sources ρ_1, ρ_2 :

$$\big\langle \mathcal{O} \big\rangle_Y = \int \big[D\rho_1\big]\,\big[D\rho_2\big]\; W_{Y_{\rm beam}-Y}\,\big[\rho_1\big]\; W_{Y+Y_{\rm beam}}\,\big[\rho_2\big]\;\mathcal{O}[\rho_1,\rho_2]$$

1. Gluon saturation
   - Saturation, rescatterings
   - Heavy Ion Collisions
   - Color Glass Condensate

2. **Event averaged initial condition**
   - Reminder on hydrodynamics
   - CGC prediction for $\langle T^{\mu\nu}\rangle$

3. Initial correlations and fuctuations
   - Why does it matter?
   - Sources of correlations
   - Correlations in η and $\vec{\boldsymbol{x}}_\mathrm{t}$

4. Glasma instabilities
   - Unstable fluctuations
   - Anomalous transport
   - Resummation

5. Open issues
   - State of the art
   - Limitations to keep in mind
   - Instabilities, thermalization
   - Initial time (in)dependence

Reminder on hydro

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu J^\mu_{\rm B} = 0$$

Additional inputs :

$$p = f(\epsilon) \quad , \quad \eta, \zeta, \cdots$$

- Required initial conditions :

$$T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp), J^\mu_{\rm B}(\tau = \tau_0, \eta, \vec{x}_\perp)$$

CGC initial conditions (Leading Order)

- In the CGC framework, $J^\mu_{\rm B} = 0$
- In the saturation regime, $\rho_{1,2} \sim g^{-1}$, and we have the following expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\,\Big[c_0 + c_1\,g^2 + c_2\,g^4 + \cdots\Big]$$

- The Leading Order contribution is given by classical fields :

$$T^{\mu\nu}_{\rm LO} \equiv c_0\frac{Q_s^4}{g^2} = \frac{1}{4}g^{\mu\nu}\,\mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\text{with}\quad \underbrace{\big[\mathcal{D}_\mu,\mathcal{F}^{\mu\nu}\big] = J^\nu}_{\rm Yang-Mills\ equation}\quad,\quad \lim_{t\to-\infty}\mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

CGC initial conditions (Leading Log resummation)

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, they contain (possibly large) logarithms of $1/x_{1,2}$:

$$c_1 = d_{10} + d_{11}\ln\left(\frac{1}{x_{1,2}}\right)$$

$$c_2 = d_{20} + d_{21}\ln\left(\frac{1}{x_{1,2}}\right) + \underbrace{d_{22}\ln^2\left(\frac{1}{x_{1,2}}\right)}$$

Leading Log terms

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\, W_{Y_1}\left[\rho_1\right]\, W_{Y_2}\left[\rho_2\right]\, \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed }\rho_{1,2}}$$

with $\partial_Y W = \mathcal{H}W$, $Y_1 = Y_{\rm beam} - \eta$, $Y_2 = Y_{\rm beam} + \eta$

(FG, Lappi, Venugopalan (2008))

Importance of factorization

- A factorization formula divides an observable into a perturbatively calculable part and a non-perturbative part describing the partonic content of nuclei :

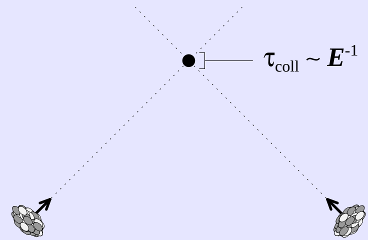
$$\langle \mathcal{O} \rangle = \int W_1 \otimes W_2 \otimes \mathcal{O}_{\mathrm{partonic}}$$

- QCD has no predictive power, unless :

- W does not depend on the observable
- W of one projectile does not depend on the second projectile

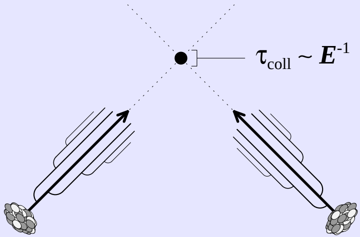
- Factorization is related to log resummations :
  - Loop corrections generate corrections $\sim [g^2 \log(x^{-1})]^n$
  - Universality requires that these logs are resummed by the same evolution equation (JIMWLK in the saturation regime)
  - The summation of these logs drives the x dependence of W

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

Why factorization works: causality



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  ▷ it must happen (long) before the collision

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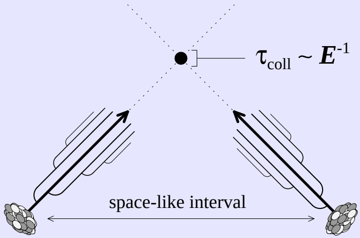


- The duration of the collision is very short: $\tau_{coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Initial correlations matter for hydro

- The equations of hydrodynamics are non-linear. Therefore, running hydro for event averaged initial conditions is not the same as running hydro event-by-event, and averaging observables at the end :

$$\mathrm{HYDRO}\Big[\Big\langle T^{\mu\nu}_{\mathrm{init}}\Big\rangle\Big] \neq \Big\langle \mathrm{HYDRO}\Big[T^{\mu\nu}_{\mathrm{init}}\Big]\Big\rangle$$

- To do hydro event by event, one needs an event generator for $T^{\mu\nu}(\tau_0,\eta,\vec{\boldsymbol{x}}_\perp)$

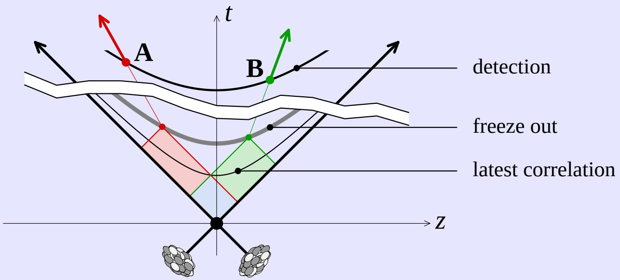
- To achieve this, it is not sufficient to know the average $\Big\langle T^{\mu\nu}(\tau_0,\eta,\vec{\boldsymbol{x}}_\perp)\Big\rangle$. We also need :

$$\Big\langle T^{\mu_1\nu_1}(\tau_0,\eta_1,\vec{\boldsymbol{x}}_{1\perp})T^{\mu_2\nu_2}(\tau_0,\eta_2,\vec{\boldsymbol{x}}_{2\perp})\Big\rangle$$

$$\Big\langle T^{\mu_1\nu_1}(\tau_0,\eta_1,\vec{\boldsymbol{x}}_{1\perp})T^{\mu_2\nu_2}(\tau_0,\eta_2,\vec{\boldsymbol{x}}_{2\perp})T^{\mu_3\nu_3}(\tau_0,\eta_3,\vec{\boldsymbol{x}}_{3\perp})\Big\rangle$$

$$\cdots$$

Initial rapidity correlations matter by themselves



Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\rm correlation} \le t_{\rm freeze\ out}\ \ e^{-\frac{1}{2}|y_A - y_B|}$$

With $t_{\rm freeze\ out} = 10$ fm/c, $|y_A - y_B| = 6$: $t_{\rm correlation} \le 0.5$ fm/c

Sources of correlations

- Nucleons inside a nucleus are correlated by the binding nuclear forces

  ▷ Monte-Carlo generator for the positions of the nucleons inside the nuclei (this information enters in the initial condition for the JIMWLK equation)

- The color sources inside the nucleons acquire correlations when they evolve to smaller values of x

  ▷ two color sources that resulted from the splitting of a common ancestor are correlated. These correlations are included in the solution $W_Y[\rho]$ of the JIMWLK equation

Correlations in η and $\vec{x}_\perp$

- The factorization valid for $\left<T^{\mu\nu}\right>$ can be extended to multi-point correlations :

$$\left<T^{\mu_1\nu_1}(\tau,\eta_1,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta_n,\vec{x}_{n\perp})\right>_{\rm LLog} =$$
$$= \int \big[D\rho_1\, D\rho_2\big]\; W[\rho_1]\; W[\rho_2]$$
$$\times\; T^{\mu_1\nu_1}_{\rm LO}(\tau,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\vec{x}_{n\perp})$$

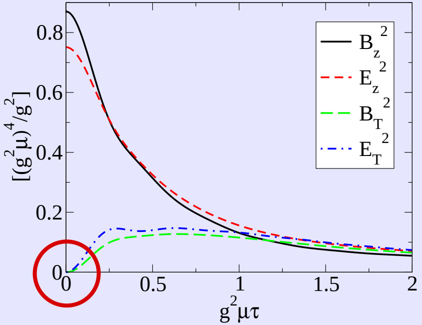
- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$
  ▷ they are a property of the pre-collision initial state

- The most difficult part here is to solve the JIMWLK equation to obtain the evolved W's

Initial classical fields, Glasma

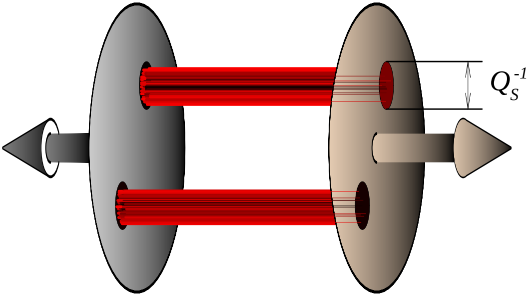
Lappi, McLerran (2006)

- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are purely longitudinal and boost invariant :



Glasma flux tubes

- The initial chromo-$\vec{E}$ and $\vec{B}$ fields form longitudinal "flux tubes" extending between the projectiles:



- The color correlation length in the transverse plane is Q_s^{-1}
  ▷ flux tubes of diameter Q_s^{-1}, filling up the transverse area
- The correlation length in the η direction is $\Delta\eta \sim \alpha_s^{-1}$

Classical prediction for $T^{\mu\nu}$

- The $T^{\mu\nu}$ predicted from the classical color fields is far from equilibrium:
  - $T^{\mu}{}_{\mu} = 0$ (classical equations are scale invariant)
  - At $\tau = 0^+$, $T^{xx}, T^{yy} > 0$, while $T^{zz} < 0$:

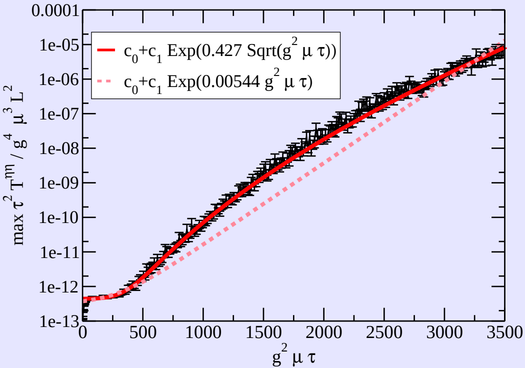
$$\langle T^{\mu\nu}(\tau = 0^+)\rangle = \begin{pmatrix} \epsilon_0 & & & \\ & \epsilon_0 & & \\ & & \epsilon_0 & \\ & & & -\epsilon_0 \end{pmatrix}$$

- Using these initial conditions for hydro requires some "voodoo"... Implicitly, one assumes the existence of a mechanism that brings $T^{\mu\nu}$ closer to the equilibrium form
- Open issue: is there such a mechanism in the CGC framework?

Unstable fluctuations

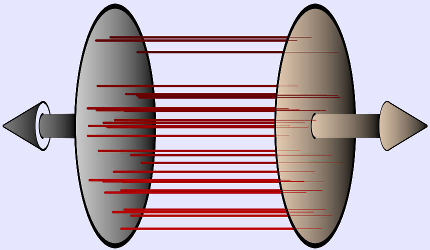
Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\sqrt{\mu\tau})$ $(\mu \sim Q_s)$ until the non-linearities become important :



Unstable fluctuations

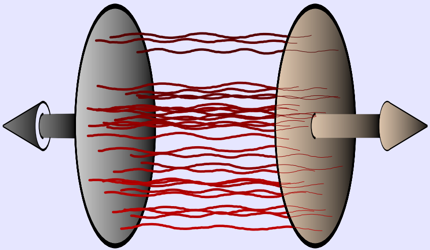
- Leading order magnetic fields at $\tau = 0^+$:



- At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal
- They are also boost invariant in the longitudinal direction (independent of η)

Unstable fluctuations

- Leading order + quantum fluctuations at $\tau = 0^+$:

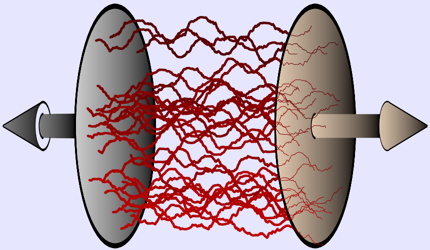


- Loop corrections bring quantum fluctuations in this picture
- In the weak coupling regime, they are small corrections

François Gelis

Unstable fluctuations

- Effect of the instability :



- η-dependent perturbations grow quickly in time $\sim e^{\sqrt{Q_s \tau}}$
- Breakdown of the CGC approach at $\tau_{\rm max} \sim Q_s^{-1} \ln^2(g^{-2})$?
- Outcome : disordered configurations of color fields?

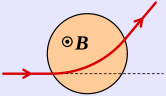
Anomalous transport

Asakawa, Bass, Muller (2006)

- Consider a domain of size Q_s^{-1}, in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain

$$\frac{d\vec{\boldsymbol{p}}}{dt} = g\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

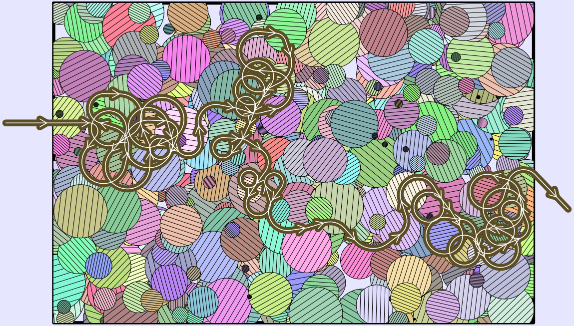
time spent in the domain : $\delta\tau \sim Q_s^{-1}$



▷ The Lorenz force deflects its trajectory by an angle of order unity

Anomalous transport

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1}, i.e. as low as permitted by the uncertainty principle ▷ fast thermalization?

Resummation of the unstable terms

- To go beyond the time $\tau_{\max}$, one must resum all the fastest growing instabilities $\sim [g^2 e^{\sqrt{\mu\tau}}]^n$

- Summing both the large logs of $1/x_{1,2}$ and these unstable terms, we get (FG, Lappi, Venugopalan (2008)) :

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle \underset{\substack{\text{LLog}\\\text{resummed}}}{=} \int \big[D\rho_1\, D\rho_2\big]\; W_{Y_1}\big[\rho_1\big]\, W_{Y_2}\big[\rho_2\big]$$
$$\times \int \big[Da\big]\; F\big[a\big]\;\; T^{\mu\nu}_{\text{LO}}\big[\underbrace{\mathcal{A}+a}_{\text{initial field}}\big]$$

- $F[a]$ is the spectrum of the initial color field fluctuations. It can be calculated analytically (in some approximation) (Fukushima, FG, McLerran (2006))

Numerical state of the art

- Solution of the Yang-Mills equations :
  Krasnitz, Nara, Venugopalan (1999-2003)
  Lappi (2003)
  Romatschke, Venugopalan (2005) (in 3-dimensions)

- Solution of the JIMWLK equation for $W_Y[\rho]$:
  Rummukainen, Weigert (2004) (not reproduced since then, very heavy computationnally)

- Solution of JIMWLK in the mean field approximation :
  ▷ Balitsky-Kovchegov equation
  Many independent codes exist for solving the BK equation

- Average over the spectrum of initial fluctuations : work in progress, complicated by strong ultraviolet divergences

Limitations to keep in mind

- All this is based so far on Leading Log calculations

  ▷ as usual, the K-factor for going from LLog to NLLog could be rather large. Maybe up to $K \sim 2$

  ▷ some efforts are ongoing to extend the JIMWLK evolution equation to NLLog (Kovchegov, Weigert, Balitsky,...), but that's only half of the story

- As with any evolution equation, the initial condition (for $W[\rho]$) is not predicted by pQCD

  ▷ it must be constrained by data. *eA* collisions would be the best. *pA* collisions are the next best thing

Instabilities, fluctuations and thermalization

- Why is it difficult to resum the initial fluctuations ?
  - The spectrum of these fluctuations contains arbitrarily high momentum modes (origin : these fluctuations amount to loop corrections, in which the momentum that circulates has no upper bound)
  - The energy momentum tensor has dimension $(\mathrm{momentum})^4$. It diverges like the power four of the hardest momentum in the fluctuation spectrum

- Strategy 1 : put a cutoff $\Lambda \sim Q_s$ by hand

- Strategy 2 : do all the calculations with a cutoff Λ. Try to perform the cancellation of the UV divergences numerically
  ▷ delicate cancellations between large numbers

- Strategy 3 : work out the structure of the UV divergences analytically, in order to get finite analytical expressions, that are then evaluated numerically

Initial time (in)dependence

- The CGC provides the value of $T^{\mu\nu}$ at some initial time τ_0
- Hydrodynamics is used to describe the system at $\tau \geq \tau_0$
- Some observable $\mathcal{O}$ is measured at a late time τ_f
- The value of $\mathcal{O}(\tau_f)$ should be independent of the time τ_0 at which one switches from the CGC description to the hydro description (at least in a reasonable window)
- Analogy : τ_0 is similar to the factorization scale used in fragmentation functions

CGC	$\longleftrightarrow$	hard process
Hydro	$\longleftrightarrow$	soft physics

$\triangleright$ Varying the factorization scale (i.e. τ_0) and checking the effect of this variation on the final result is a good way of assessing the uncertainty of the whole chain

Summary

Things that work right now: Leading Log framework

- Evolution equations for the distribution of sources (JIMWLK, BK)
- Initial $W[\rho]$ (some constraints from DIS and RHIC)
- $\left\langle T^{\mu\nu}(\tau,\eta_1,\vec{\boldsymbol{x}}_{1\perp})T^{\rho\sigma}(\tau,\eta_2,\vec{\boldsymbol{x}}_{2\perp})\cdots\right\rangle$

Things that need serious work

- Resummation of the unstable modes ($\triangleright$ positive P_L ?)

Things that should be looked at eventually

- Independence w.r.t. the time at which one starts hydro
- Next to Leading Log