Relativistic Shock Waves in Viscous Gluon Matter

Ioannis Bouras

with Andrej El, Oliver Fochler, Etele Molnar, Harri Niemi, Zhe Xu, Carsten Greiner and Dirk H. Rischke

I. Bouras et al. PRL 103:032301,2009
I. Bouras, H. Niemi, E. Molnar et al. in preparation

Institut für Theoretische Physik in Frankfurt a. M.

Supported by
Helmholtz Graduate School
HGS-HIRe for FAIR

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

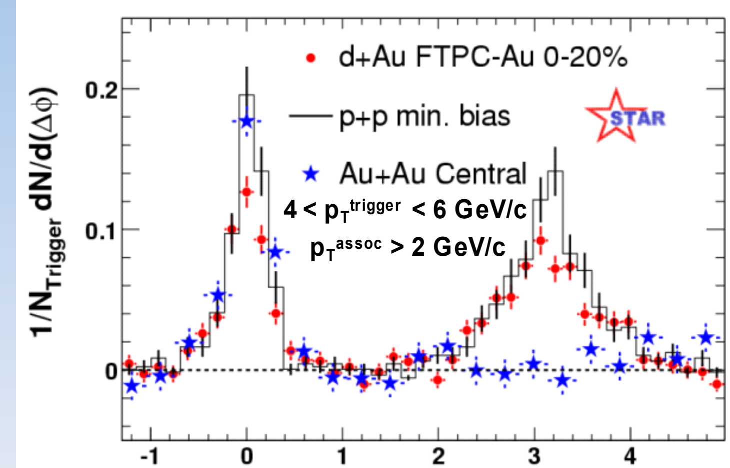
Flow and dissipation in ultrarelativistic Heavy Ion Collisions

ECT/HICforFAIR/CATHIE/Nikhef workshop at ECT* Trento,*

September 14 - September 18, 2009

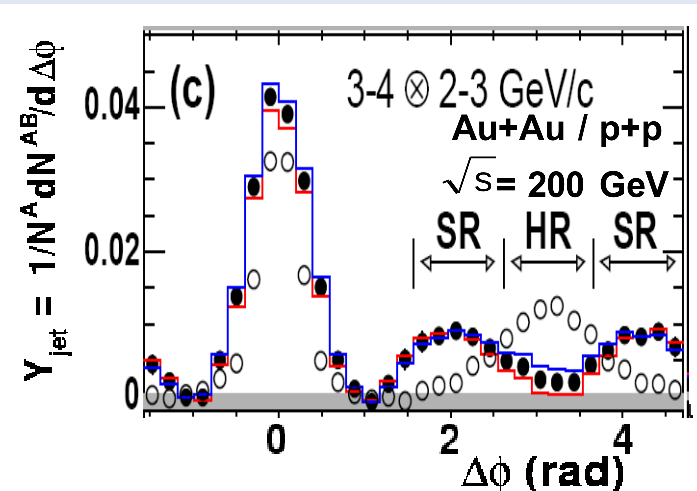
Motivation

- *RHIC data indicates jet-suppression in heavy-ion collisions (signal for QGP)*
- *One possible consequence is the formation of Mach cones*

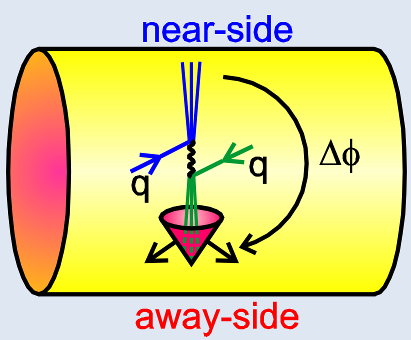


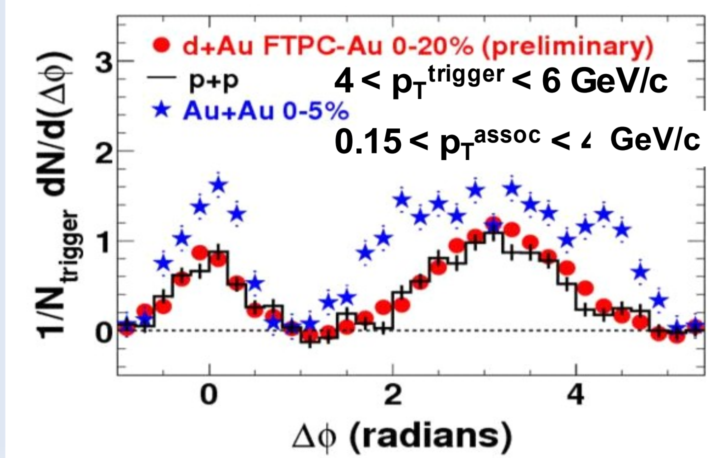
STAR PRL **91** (2003) 072304

*H. Stöcker, Nucl. Phys. A **750**, 121 (2005)*
*J. Ruppert and B. Müller, Phys. Lett. B **618,** 123 (2005)*
*J. Casalderrey-Solana, E.V. Shuryak and D. Teaney,J. Phys. Conf. Ser. **27**, 22 (2005)*
*V. Koch, A. Majumder and X.N. Wang, Phys. Rev. Lett. **96**, 172302 (2006)*
*B. Betz, PRC **79**:034902, 2009*



PHENIX arXiv:0705.3238 [nucl-ex]

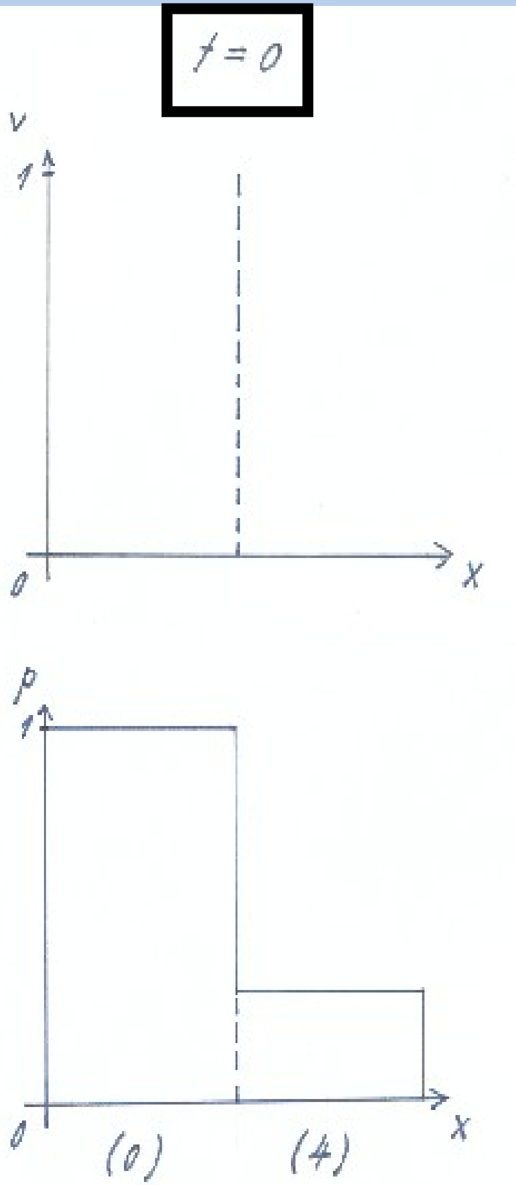


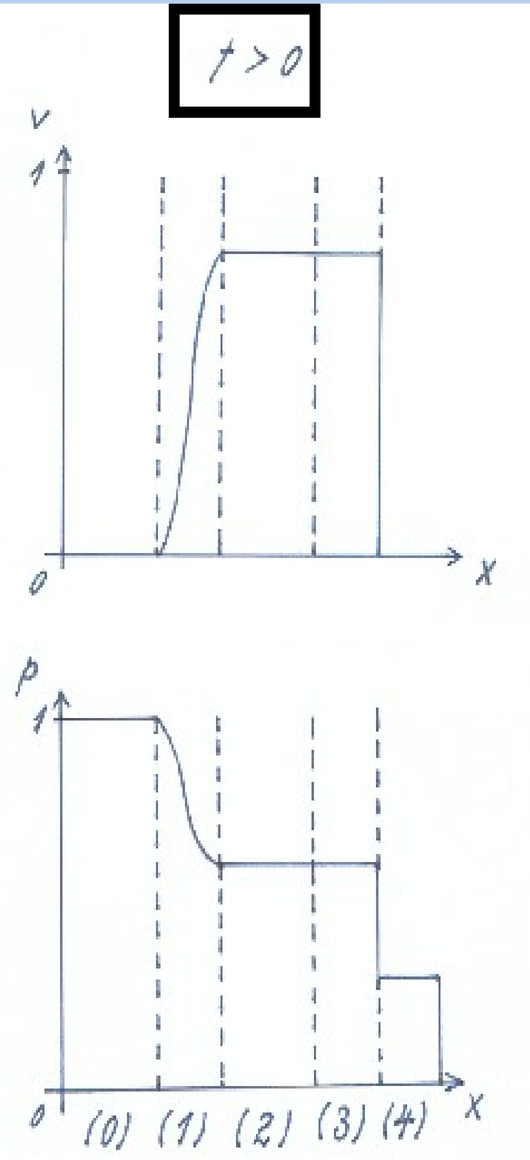


STAR, Nucl. Phys. **A774**, 129 (2006)

The Relativistic Riemann problem

Initial conditions





What happens if you remove the membran?

A shock wave travels to the right with a speed <u>higher</u> than the speed of sound and a rarefaction wave travels to the left with the speed of sound

The Parton Cascade BAMPS

- *Transport algorithm solving the Boltzmann equation using Monte Carlo techniques*

$$p^{\mu}\partial_{\mu}f(x,p)=C(x,p)$$

Boltzmann
Approach for
Multi-
Parton
Scatterings

- *Stochastic interpretation of collision rates*

Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

$$P_{22}=v_{rel}\frac{\sigma_{22}}{N_{test}}\frac{\Delta t}{\Delta^{3}x}$$

See talk by Andrej El and Zhe Xu

- *Boltzmann gas, isotropic cross sections (elastic processes only)*

The Parton Cascade BAMPS

Implementing a constant η/s*, we locally get the cross section* σ_{22}*:*

$$\eta = \frac{4}{15}\frac{\epsilon}{R^{tr}}$$

Transport collision rate R^{tr}

For isotropic elastic collisions:

$$R_{22}^{tr} = n\frac{2}{3}\sigma_{22}$$

$$\epsilon = 3nT$$

$$s = 4n - n\ln(\lambda_{fug})$$

$$\lambda_{fug} = \frac{n}{n_{eq}} \qquad n_{eq} = \frac{g}{\pi^2}T^3$$

g = 16 for gluons

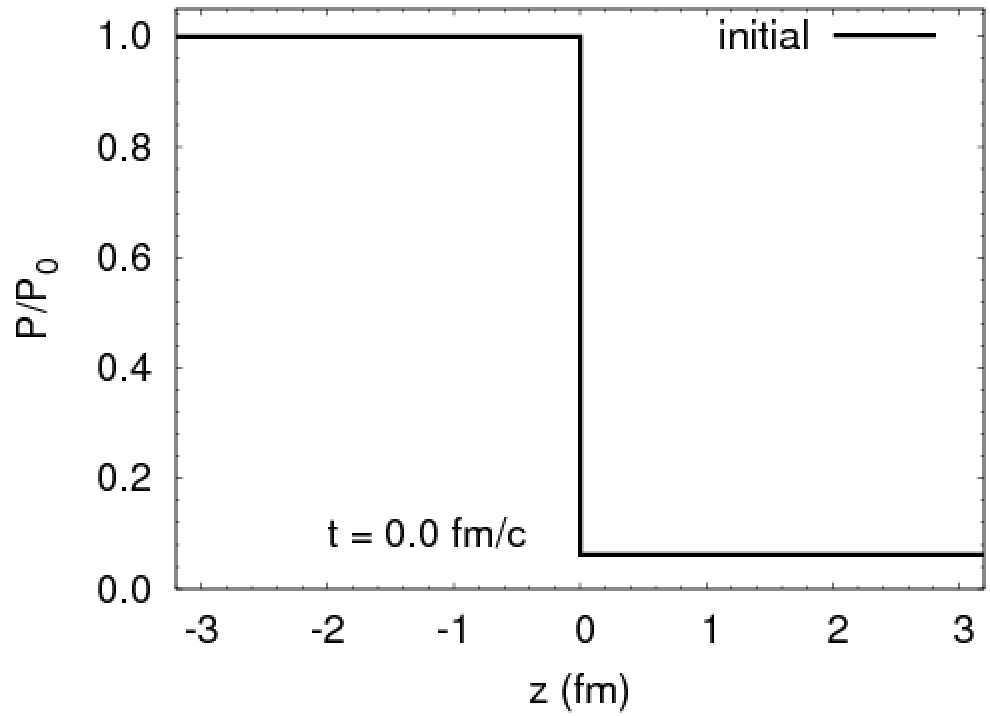
Zhe Xu and Carsten Greiner
Phys. Rev. Lett. 100, 172301 (2008)

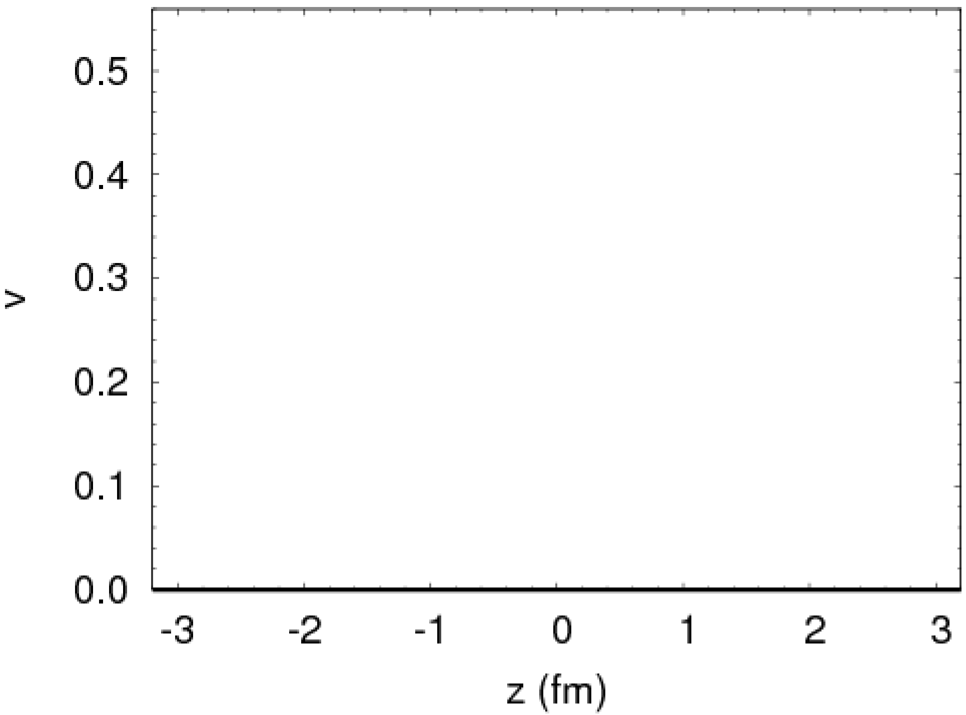
$$\sigma_{22} = \frac{6}{5}\frac{T}{s}\left(\frac{\eta}{s}\right)^{-1}$$

Numerical Results

Boltzmann solution of the relativistic Riemann problem

$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 0$ fm/c

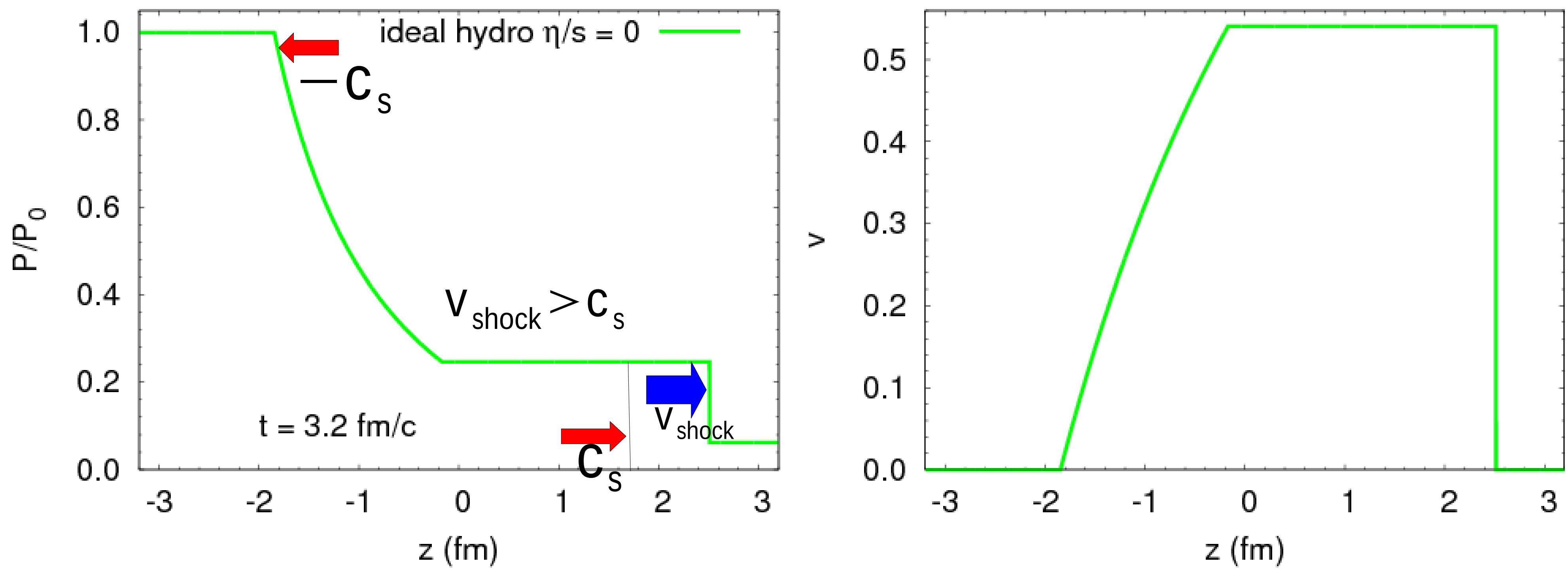




Numerical Results

Boltzmann solution of the relativistic Riemann problem

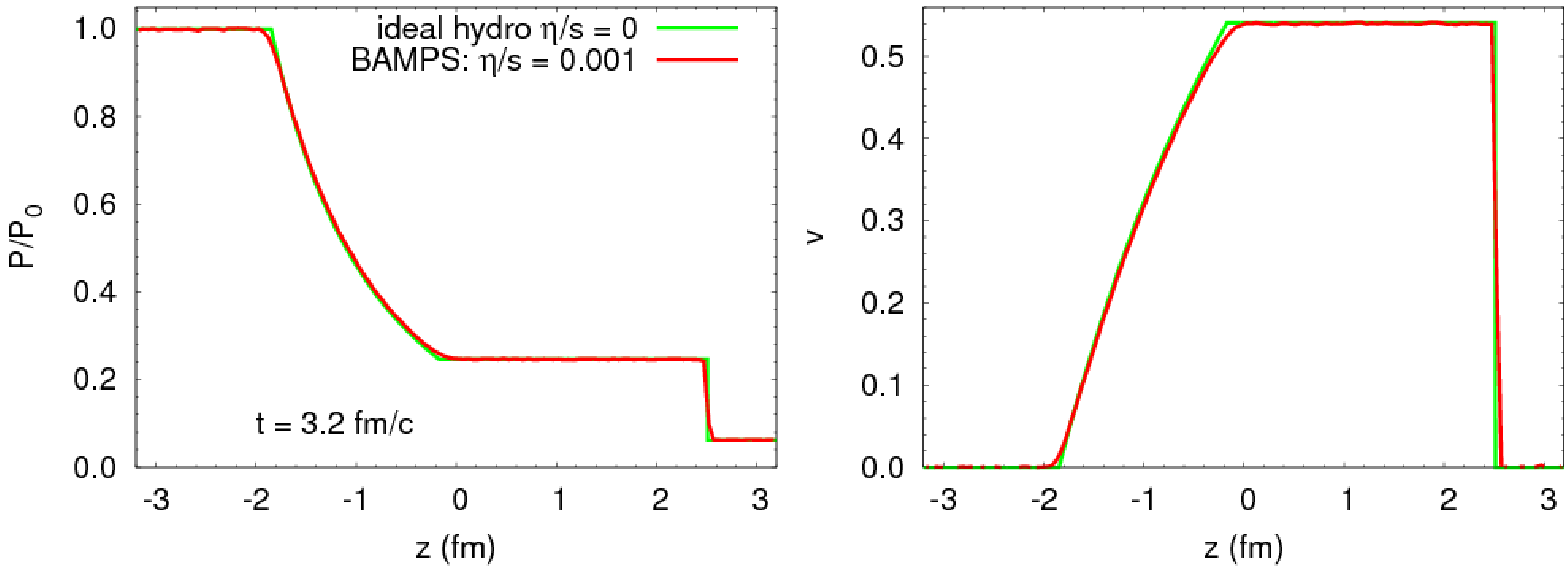
$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c



Numerical Results

Boltzmann solution of the relativistic Riemann problem

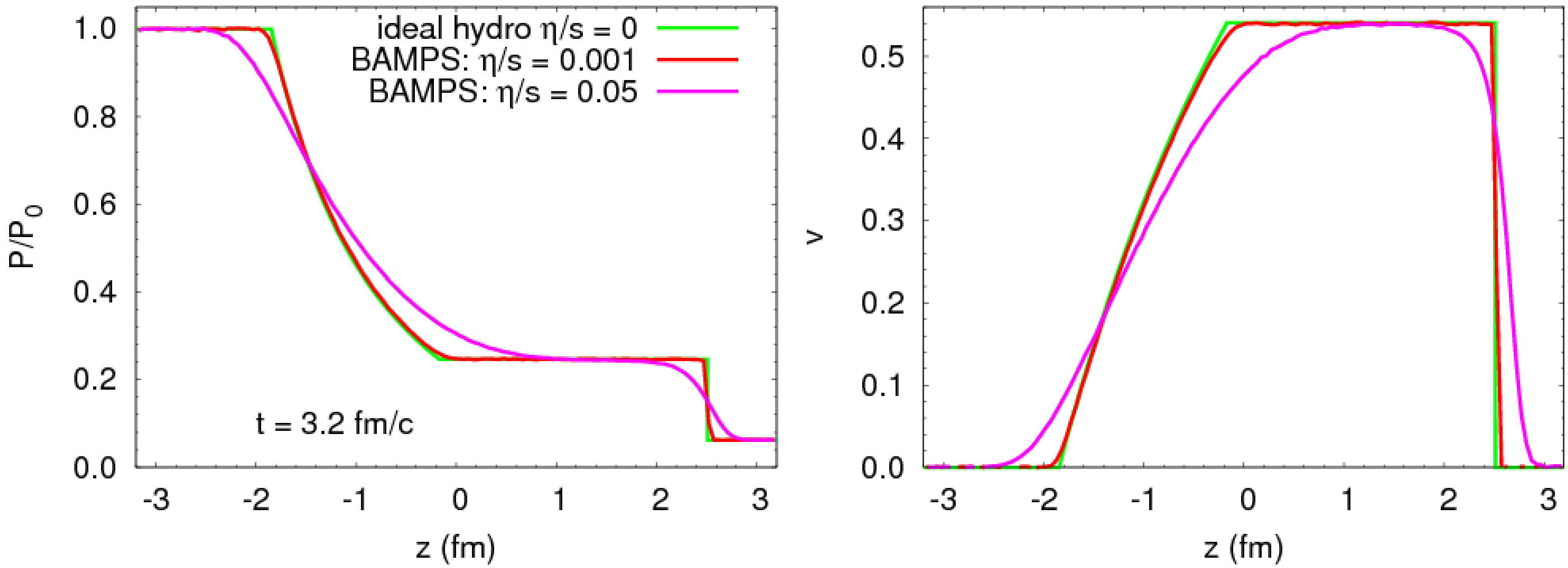
$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c



Numerical Results

Boltzmann solution of the relativistic Riemann problem

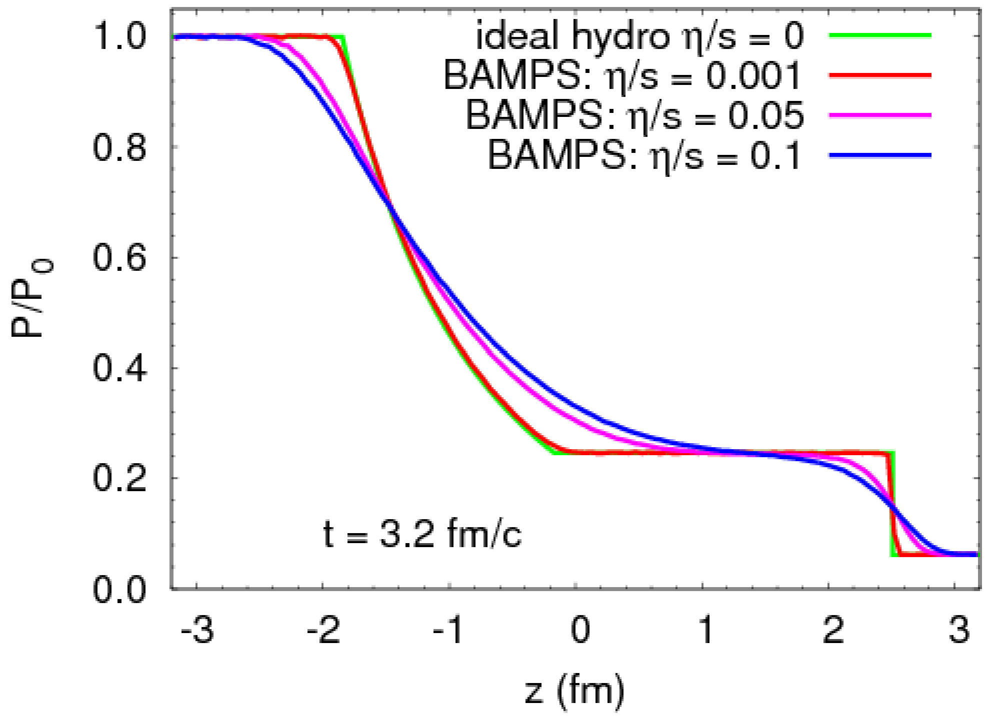
$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c

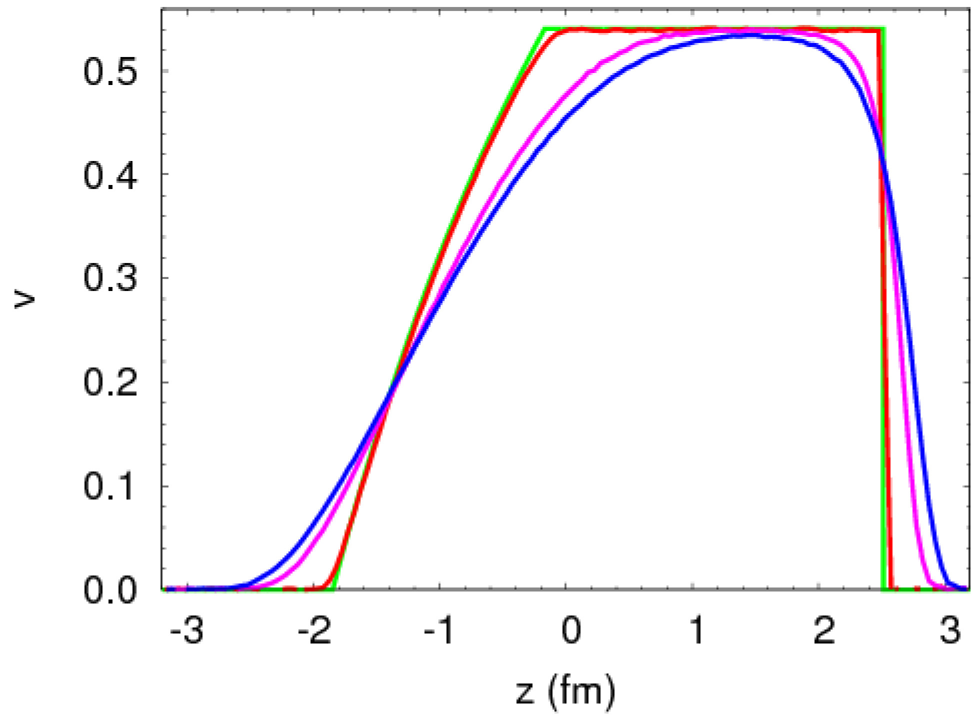


Numerical Results

Boltzmann solution of the relativistic Riemann problem

$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c

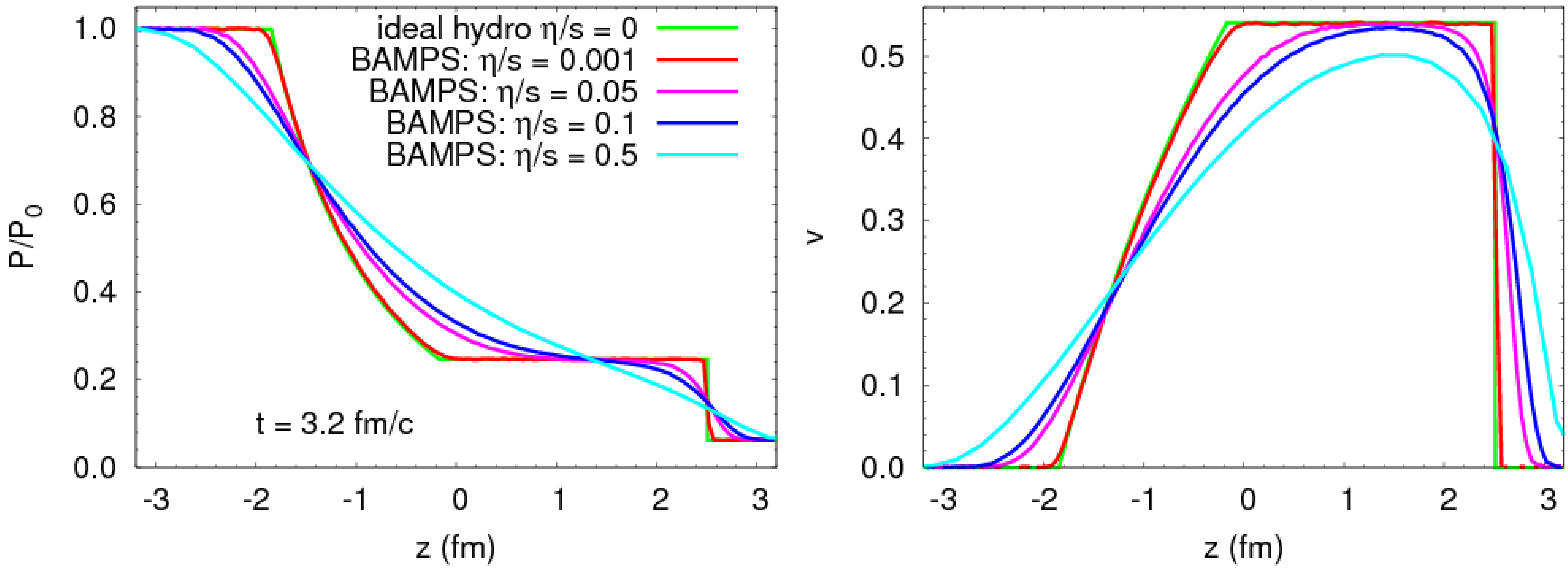




Numerical Results

Boltzmann solution of the relativistic Riemann problem

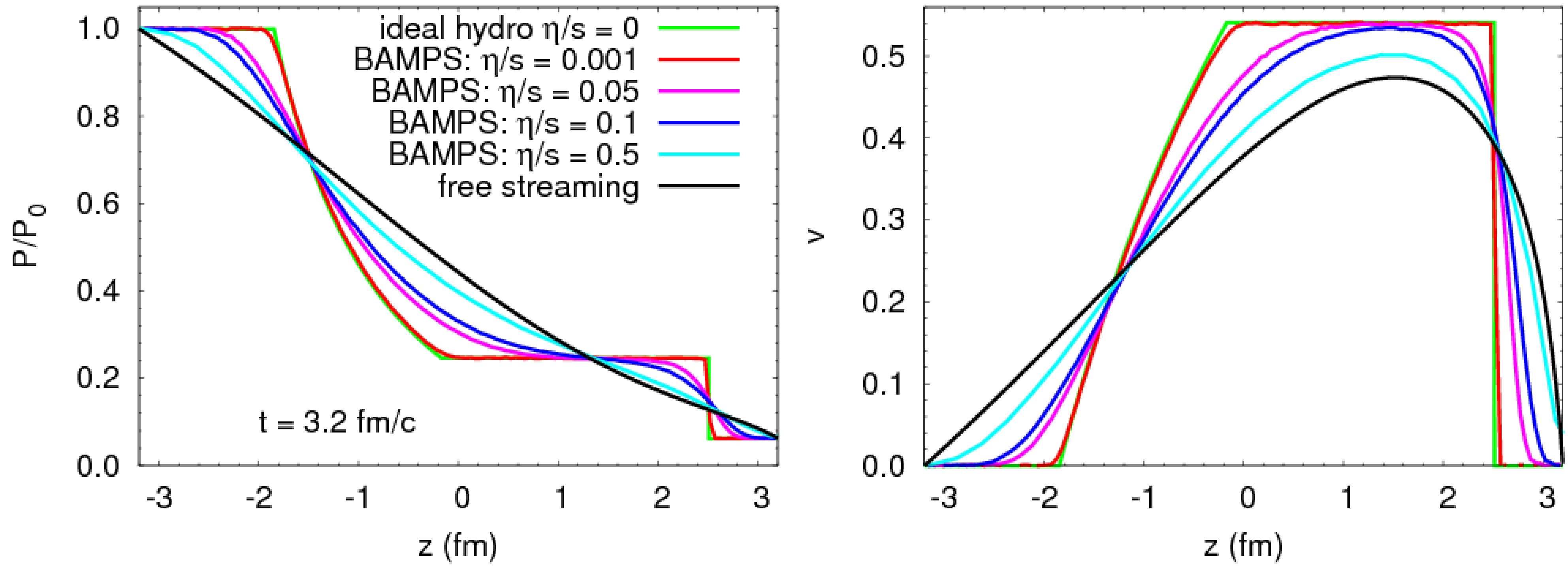
$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c



Numerical Results

Boltzmann solution of the relativistic Riemann problem

$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c



With higher viscous effects:

- *The shock front has a finite (increasing) width*
- *The shock plateau shrinks*
- *The rarefaction wave gets wider and exceeds the speed of sound*

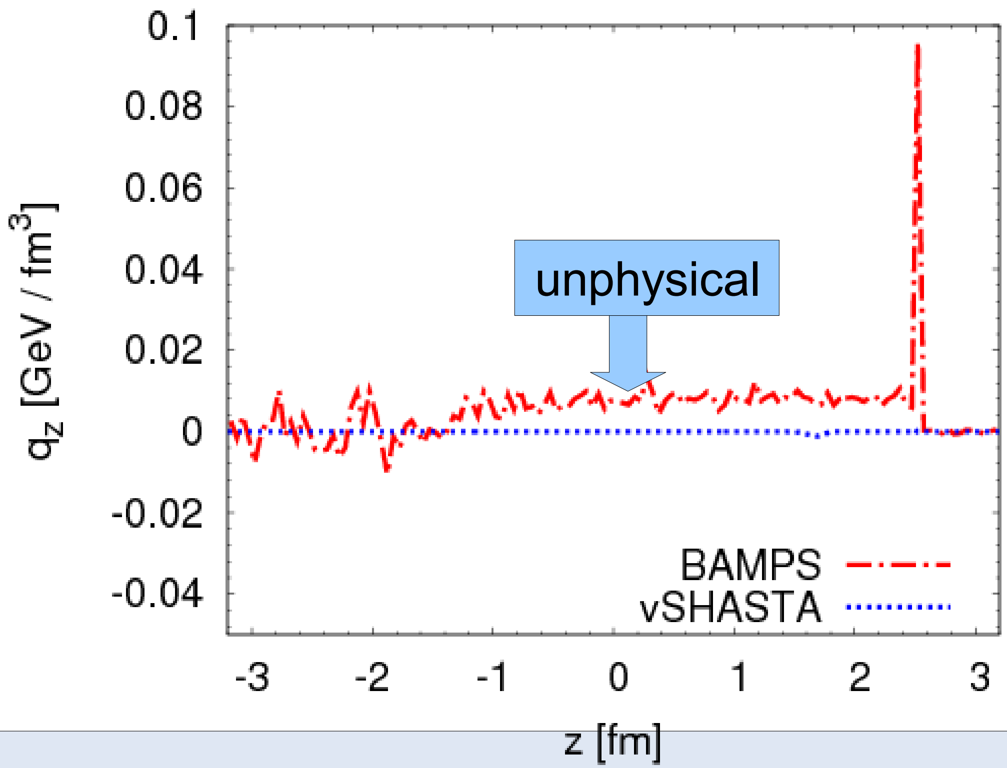
See also Harri Niemi´s talk tomorrow

Ideal Limit

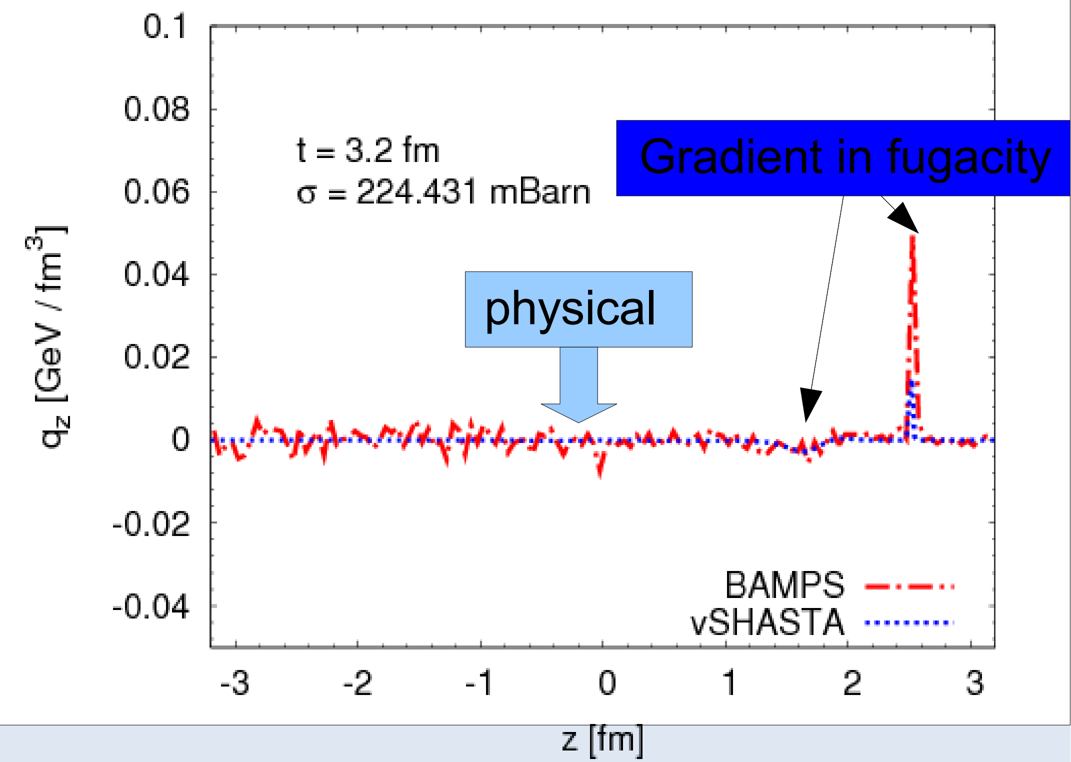
$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c

Heat flow

$\eta / s = 0.001$



$\sigma = 224$ mBarn



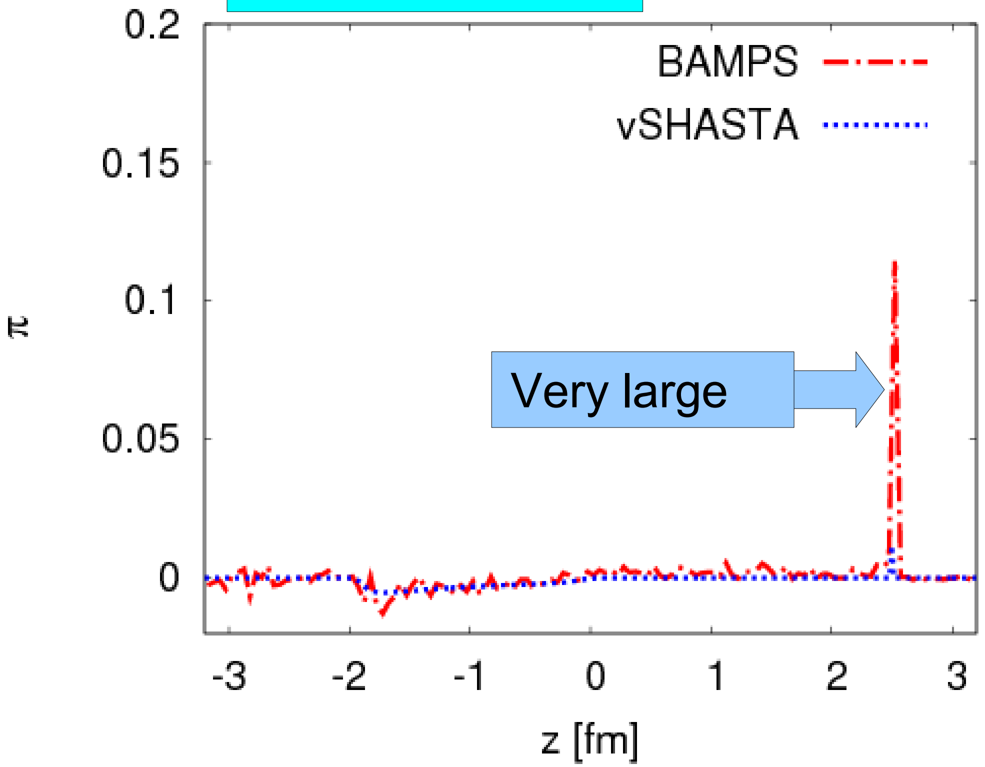
- Numerical statistical fluctuations cause deviations from ideal limit in sensitive physical observables
- Constant cross section eliminates the fluctuations – we reach the **IDEAL LIMIT**

Ideal Limit

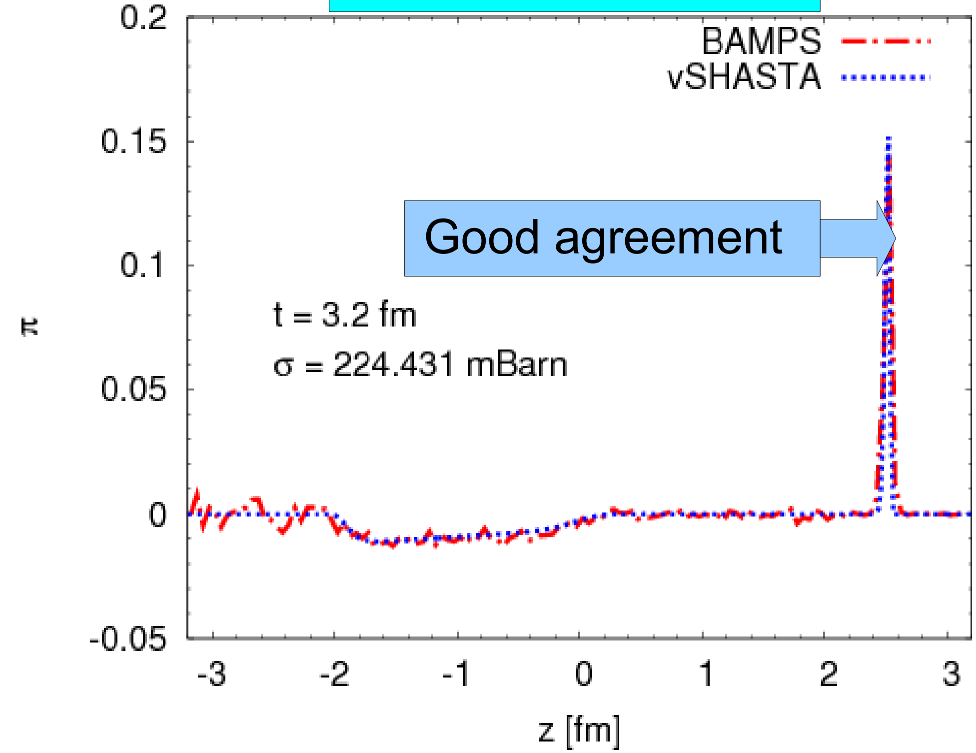
$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c

Shear pressure

$\eta / s = 0.001$



$\sigma = 224$ mBarn

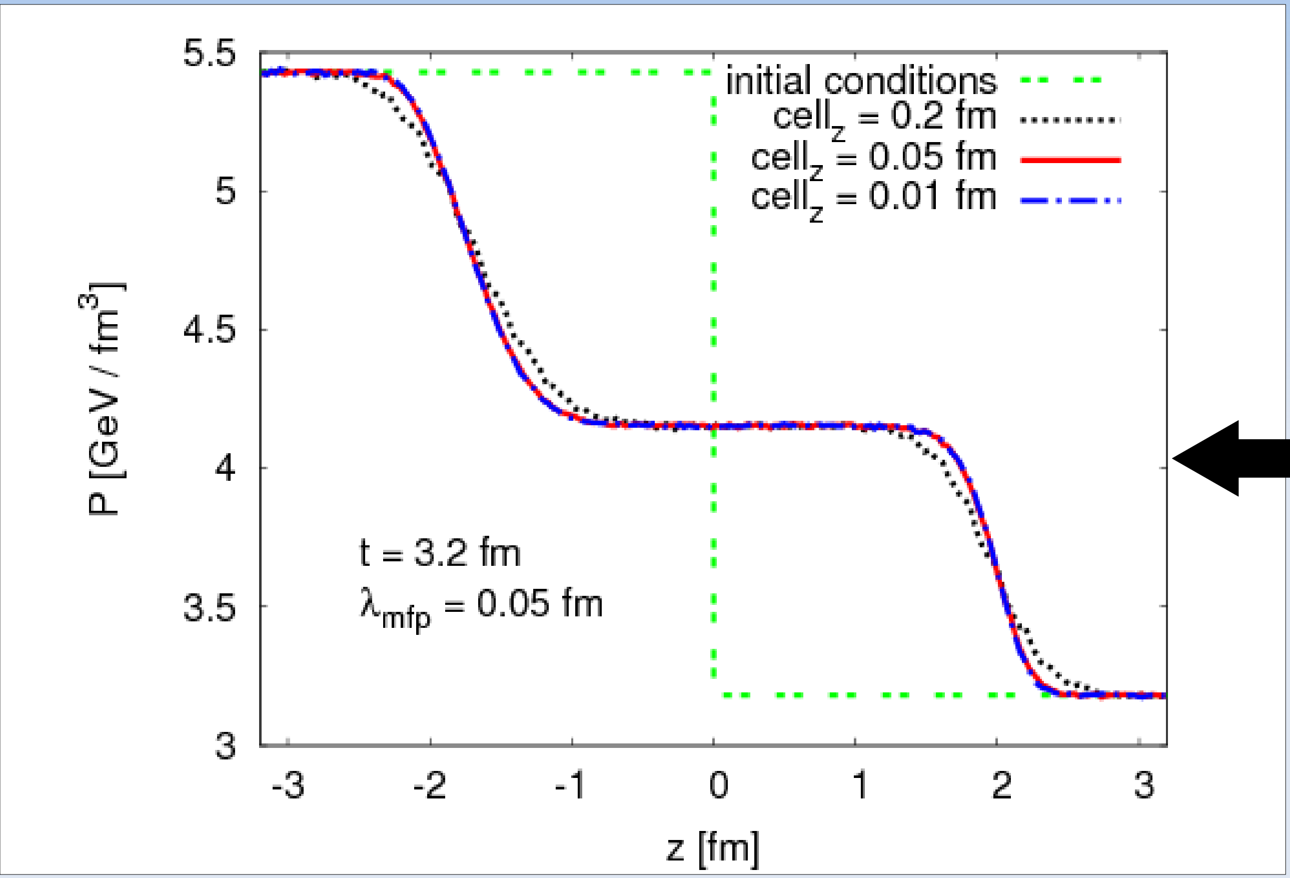


- Constant cross section eliminates the fluctuations – we reach the **IDEAL LIMIT**

For further viscous solutions for the physical observables see the talk of Harri Niemi tomorrow

Cell Dependence

$T_L = 400$ MeV
$T_R = 350$ MeV
$t = 3.2$ fm/c



Convergence!

If the mean free path λ_{mfp} is equal or smaller than the cell size:

⇨ **Covariant transport model**

Global Knudsen Number

Knudsen number defined as:

$$K = \frac{\lambda_{mfp}}{L}$$

We define the characteristic length L

$$L = t \cdot (v_{shock} + c_s)$$

and use from kinetic theory

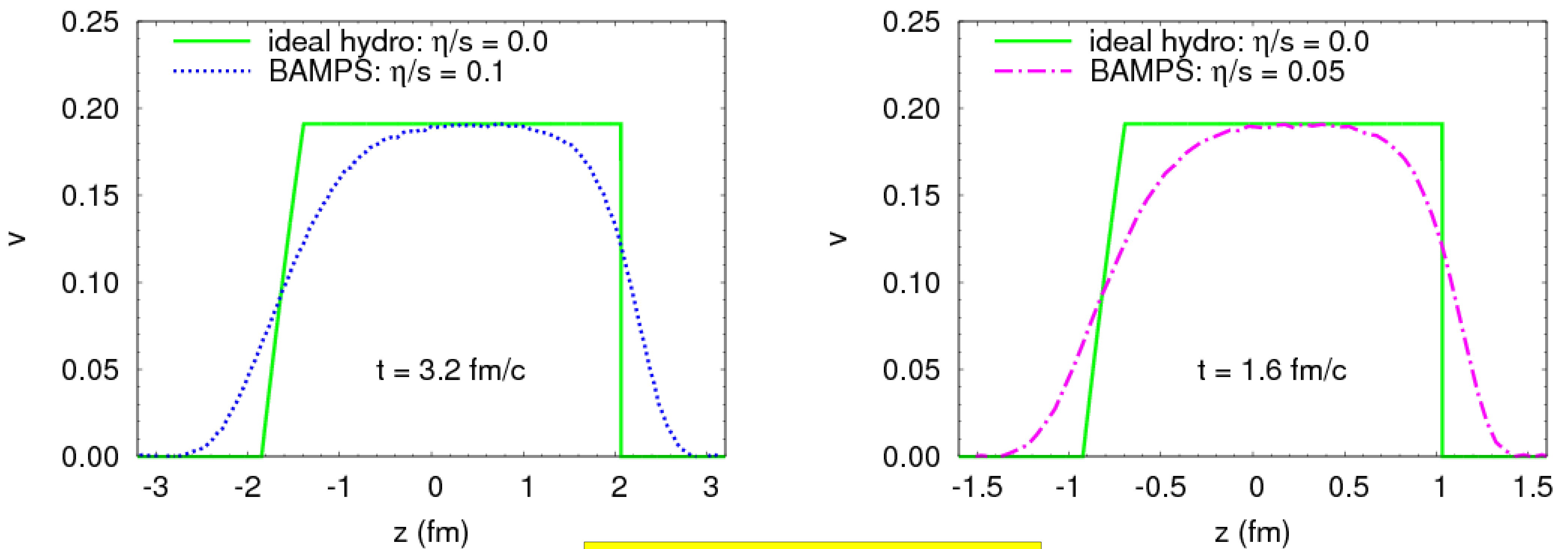
$$\lambda_{mfp} = \frac{10}{3T} \cdot \left(\frac{\eta}{s}\right)$$

$$K = \frac{10}{3} \frac{1}{t \cdot (v_{shock} + c_s) \cdot T} \cdot \left(\frac{\eta}{s}\right)$$

T is the lower temperature of the medium

2 systems behave the same, if they have the same Knudsen number

Scaling Behaviour



$T = 320$ MeV

$\eta/s = 0.1$

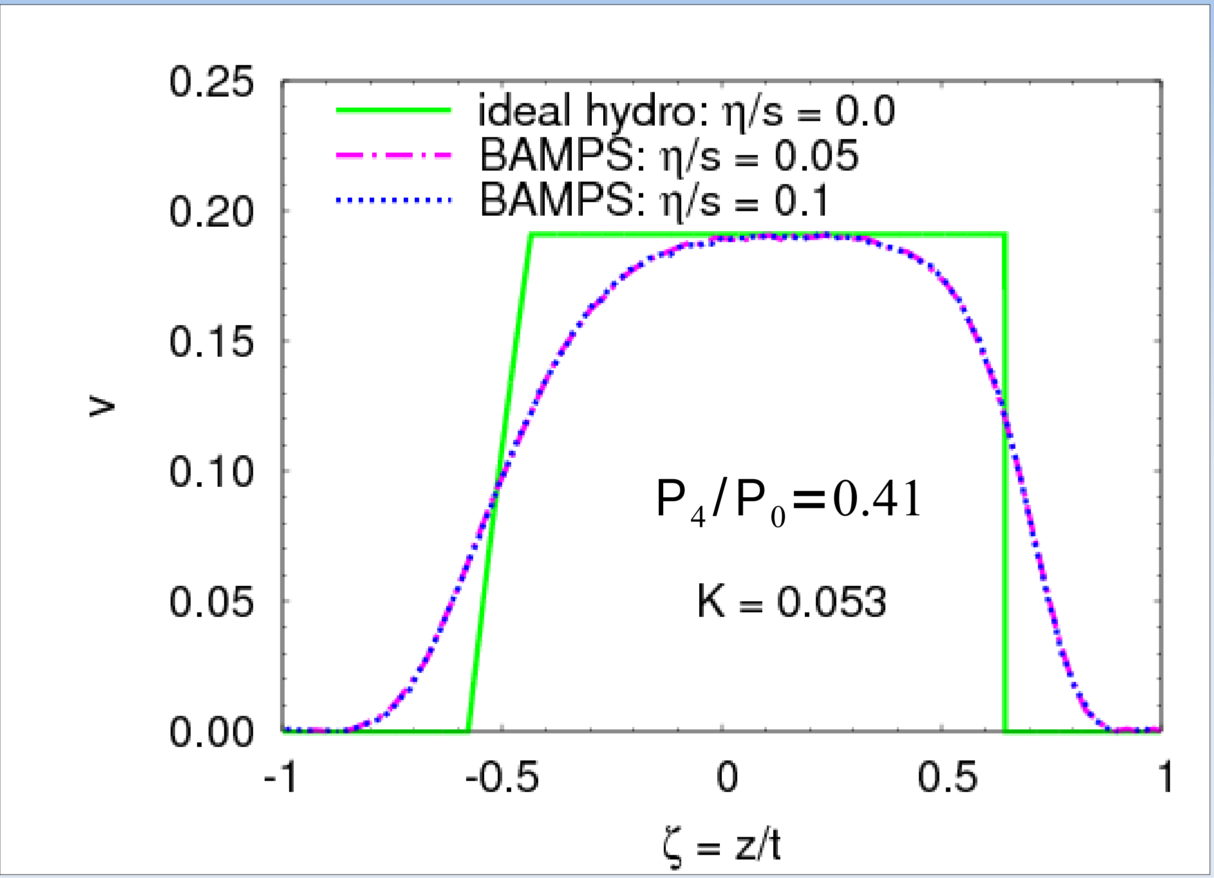
$t = 3.2$ fm/c

$K = 0.053$

$\eta/s = 0.05$

$t = 1.6$ fm/c

Scaling Behaviour



The velocity profile is only a function of

$\zeta = z/t$ *and* K,

$$v(z, t, \eta/s) = F(\zeta, K)$$

and universal for a given ratio P_4/P_0.

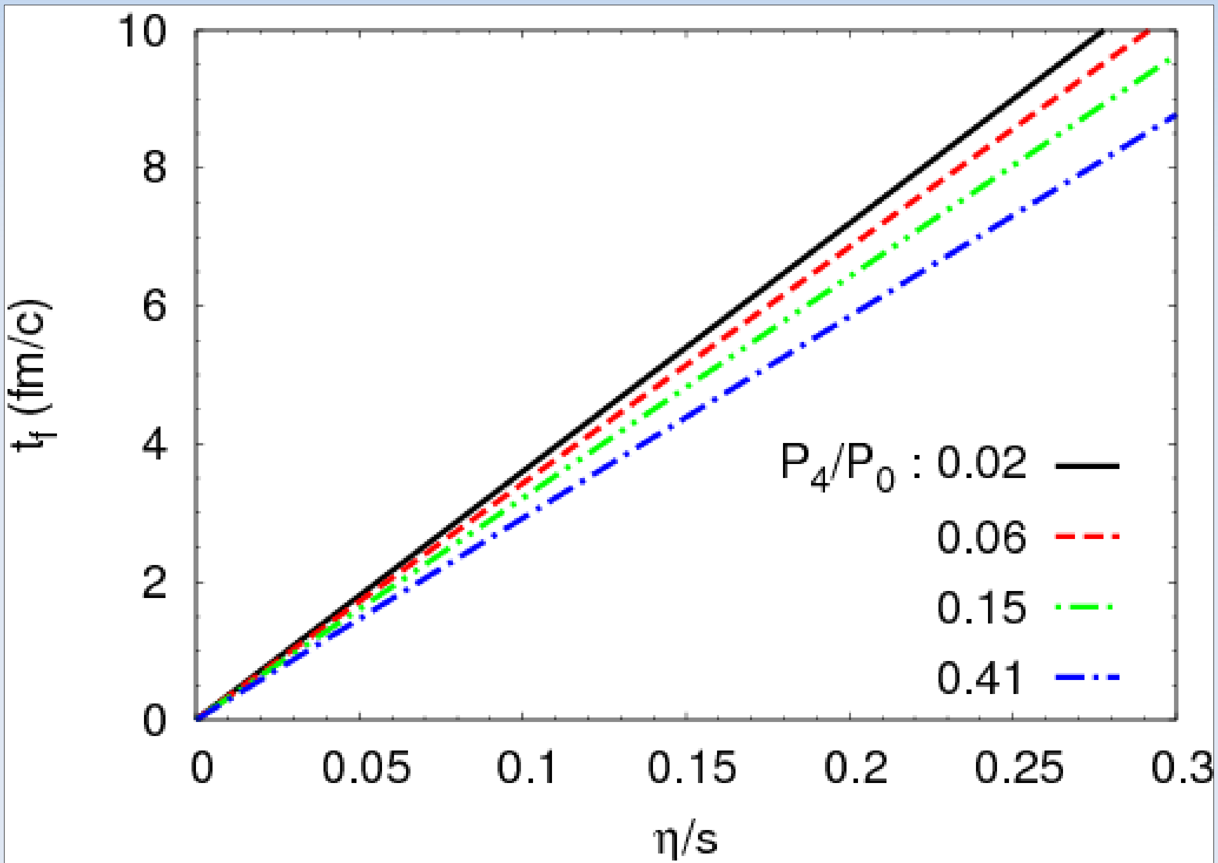
$$K_f = \frac{10}{3} \frac{1}{t_f \cdot (v_{shock} + c_s) \cdot T} \cdot \left(\frac{\eta}{s}\right) = 0.053$$

$$P_4/P_0 = 0.41$$

We call it a shock when a plateau exists !!!

Global Knudsen Number

Is the formation of shocks (Mach cones) possible in gluonic matter?



$$t_f = \frac{10}{3} \frac{1}{K_f \cdot (v_{shock} + c_s) \cdot T} \cdot \left(\frac{\eta}{s}\right)$$

$$T = 350\ \text{MeV}$$

Lifetime QGP ~ 6 fm/c →

The formation of Mach cones is in principle possible if /s < 0.2

Mach Cones

Preliminary Results

In collaboration with F. Lauciello

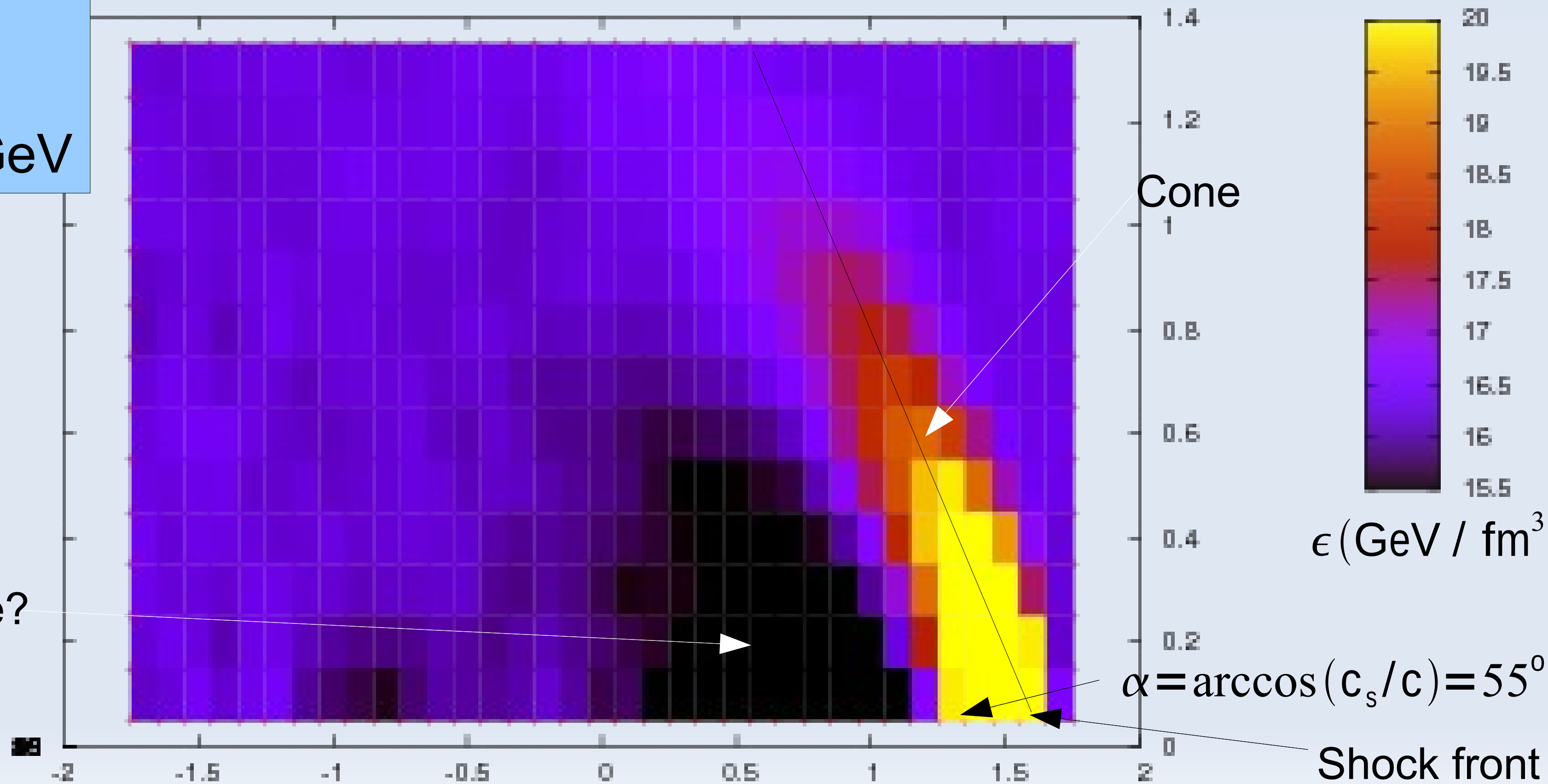
- High-energetic jet moving in z-direction inducing a Mach cone
- The Mach angle agrees well with the expected angle in the ideal fluid case

$\eta/s = 1/4\pi$

$T_{medium} = 0.4\,GeV$

t = 2.5 fm\c

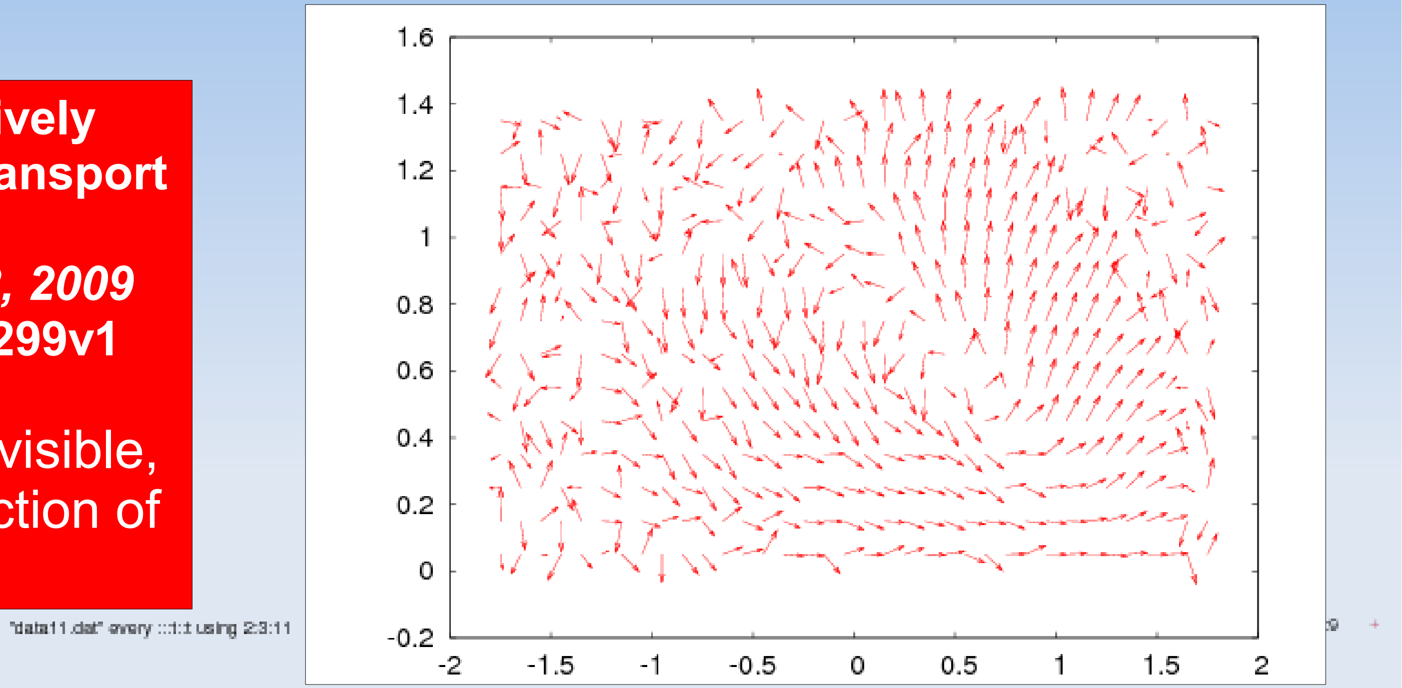
E-Jet = 200 GeV



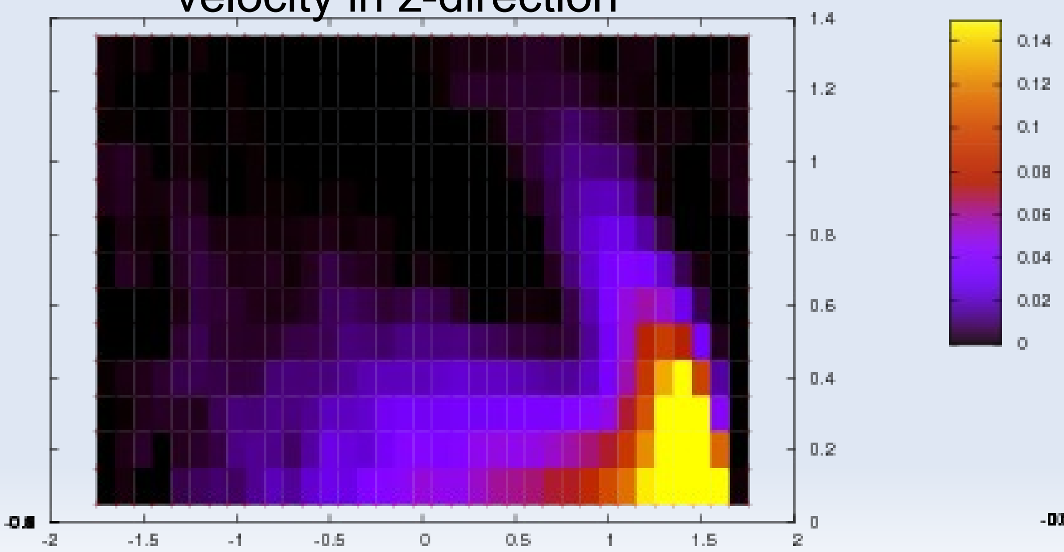
Preliminary Data from 3-D Simulations of Mach Cones

- **The results agree qualitatively with hydrodynamic and transport calculations**
  --- *B. Betz, PRC 79:034902, 2009*
  --- D. Molnar, arXiv:0908.0299v1

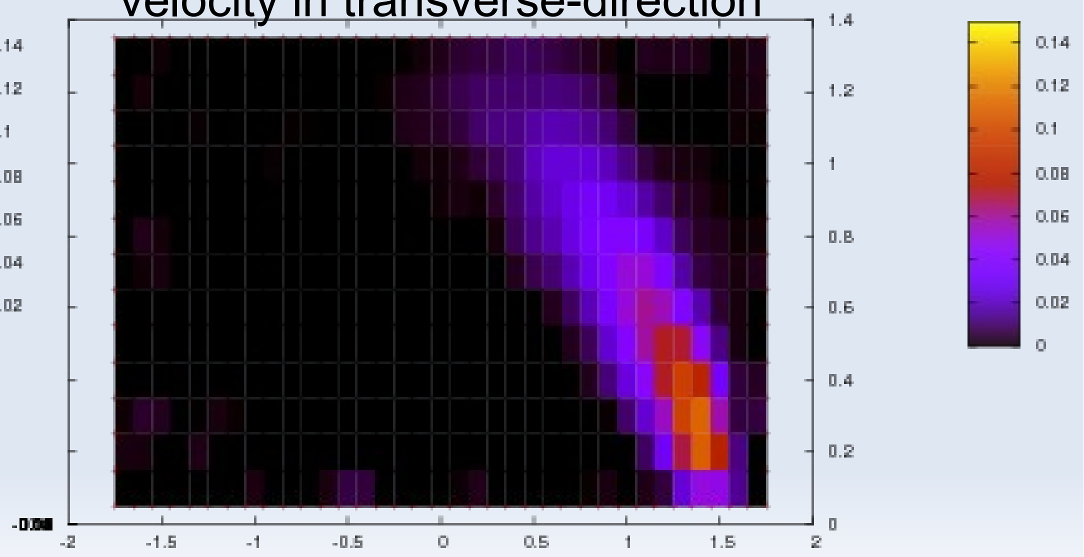
- A diffusion wake is also visible, momentum flows in direction of the Jet



velocity in z-direction



velocity in transverse-direction



Conclusion and Outlook

- *We solve the relativistic Riemann problem using BAMPS from ideal hydro to free streaming*
- *We see a good agreement between BAMPS and vSHASTA (Harri Niemis talk)*
- *The formation of Mach cones in gluonic matter at RHIC is in principle possible for* $\eta/s < 0.2$
- *Mach Cone formation is observed in BAMPS*

Outlook:

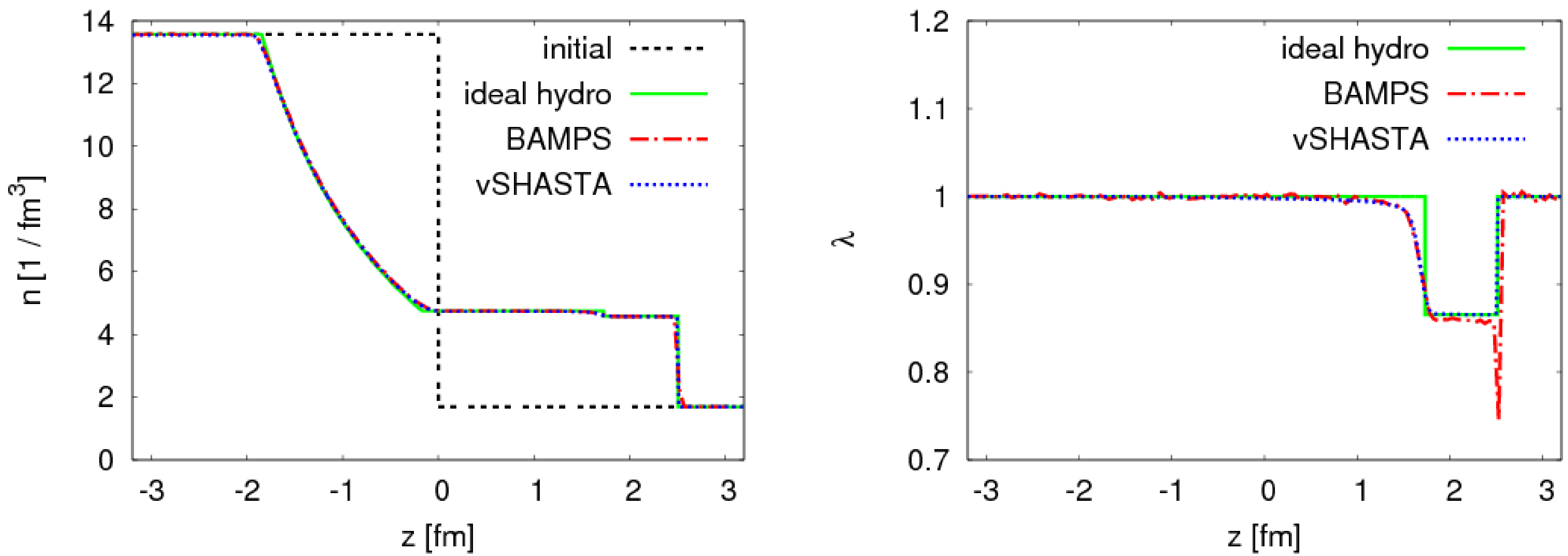
- *Investigate the viscous effects on the formation and evolution of Mach cones as well as the 2/3 particle correlations within BAMPS*

Thank you for your attention

Ideal Limit

$T_L = 400$ MeV
$T_R = 200$ MeV
$t = 3.2$ fm/c

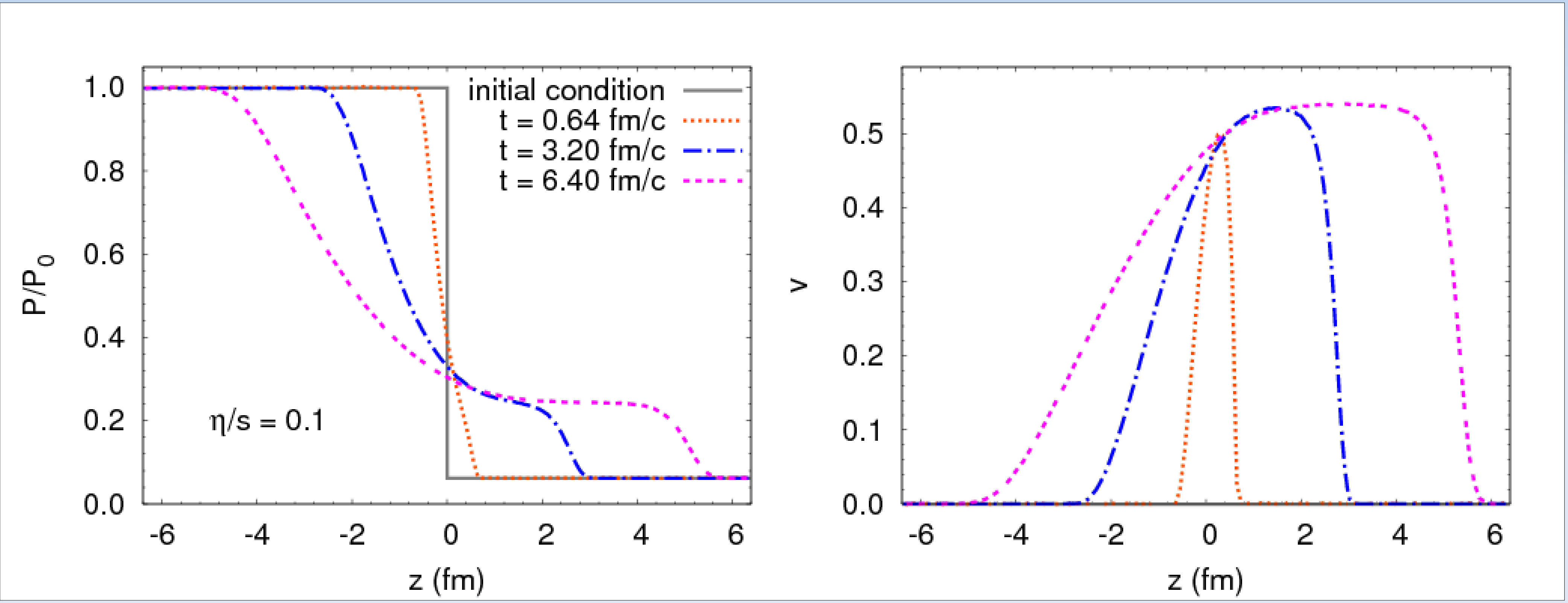
$\eta / s = 0.001$



Numerical Results

$T_L = 400$ MeV
$T_R = 200$ MeV
$\eta / s = 0.1$

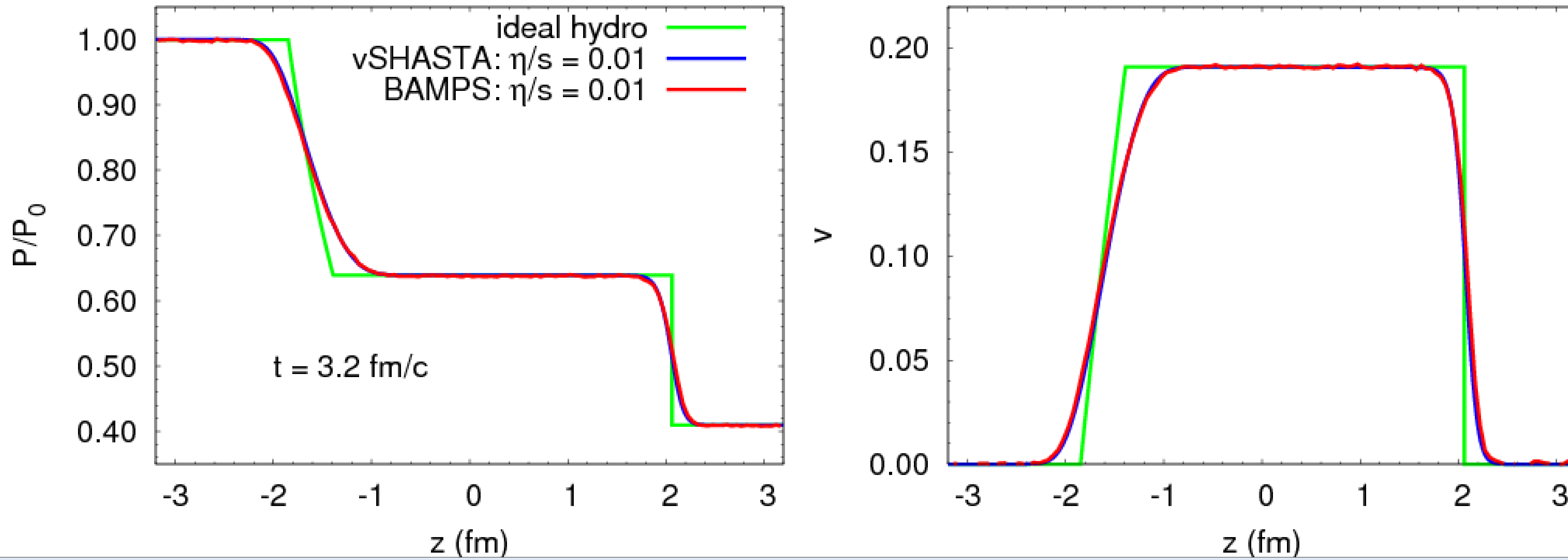
Evolution from free streaming to a shock



Numerical Results

Comparison between BAMPS and vSHASTA

$T_L = 400$ MeV
$T_R = 320$ MeV
$t = 3.2$ fm/c



vSHASTA

1 + 1 dimensional viscous hydro model
using the Israel-Stewart equations

Etele Molnar and Harri Niemi
arxiv:0807.0544
Etele Molnar and Harri Niemi
arxiv:0807.0544

/s = 0.01

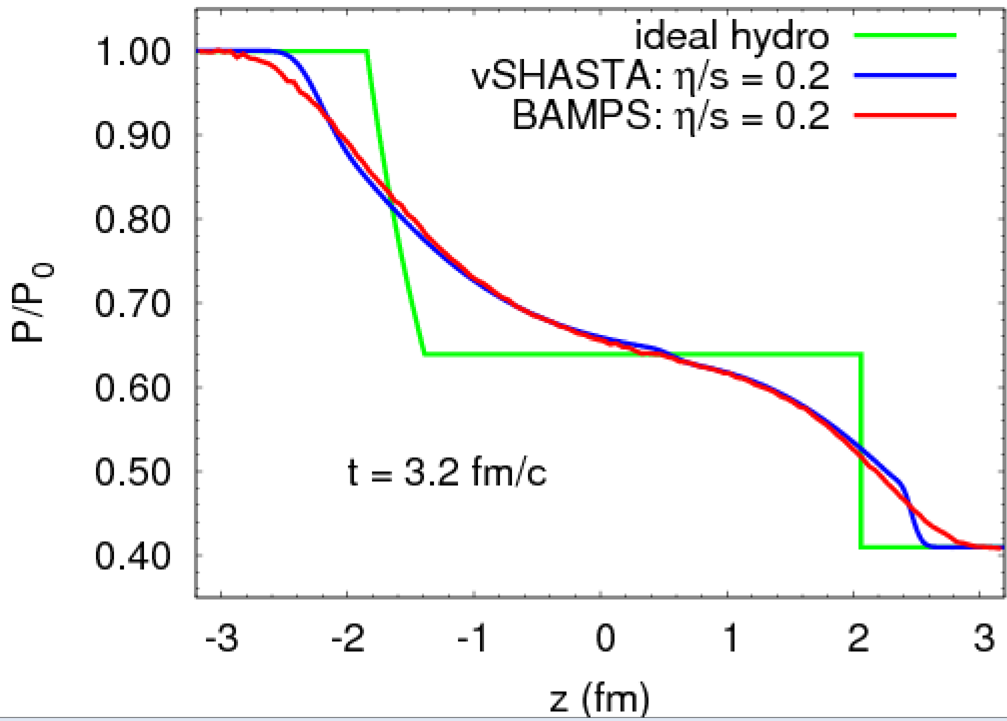
$K_{oca} = \lambda_{mfp} \partial_\mu u^\mu$

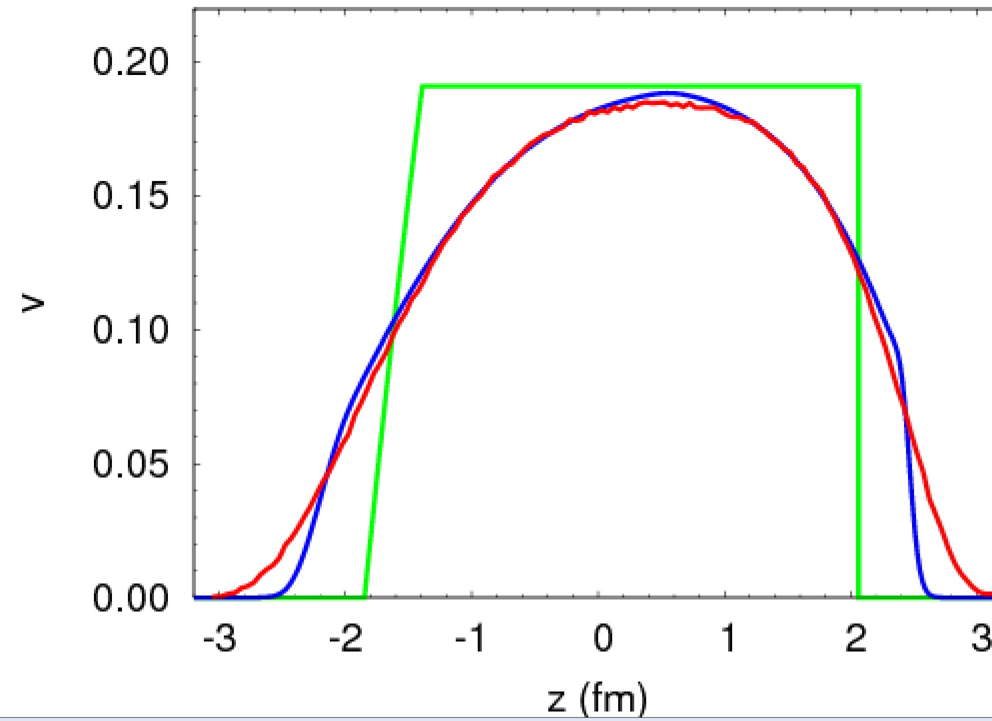
is ***SMALL*** *in the region of the shock front*

Numerical Results

Comparison between BAMPS and vSHASTA

$T_L = 400$ MeV
$T_R = 320$ MeV
$t = 3.2$ fm/c





vSHASTA

1 + 1 dimensional viscous hydro model using the Israel-Stewart equations

E. Molnar, H. Niemi and D. Rischke
Eur.Phys.J.C60:413-429,2009
arXiv:0907.2583

/s = 0.2

$K_{\text{oca}} = \lambda_{mfp} \partial_\mu u^\mu$

*is **LARGE** in the region of the shock front*

Numerical Results

$T_L = 400$ MeV
$t = 1$ fm/c

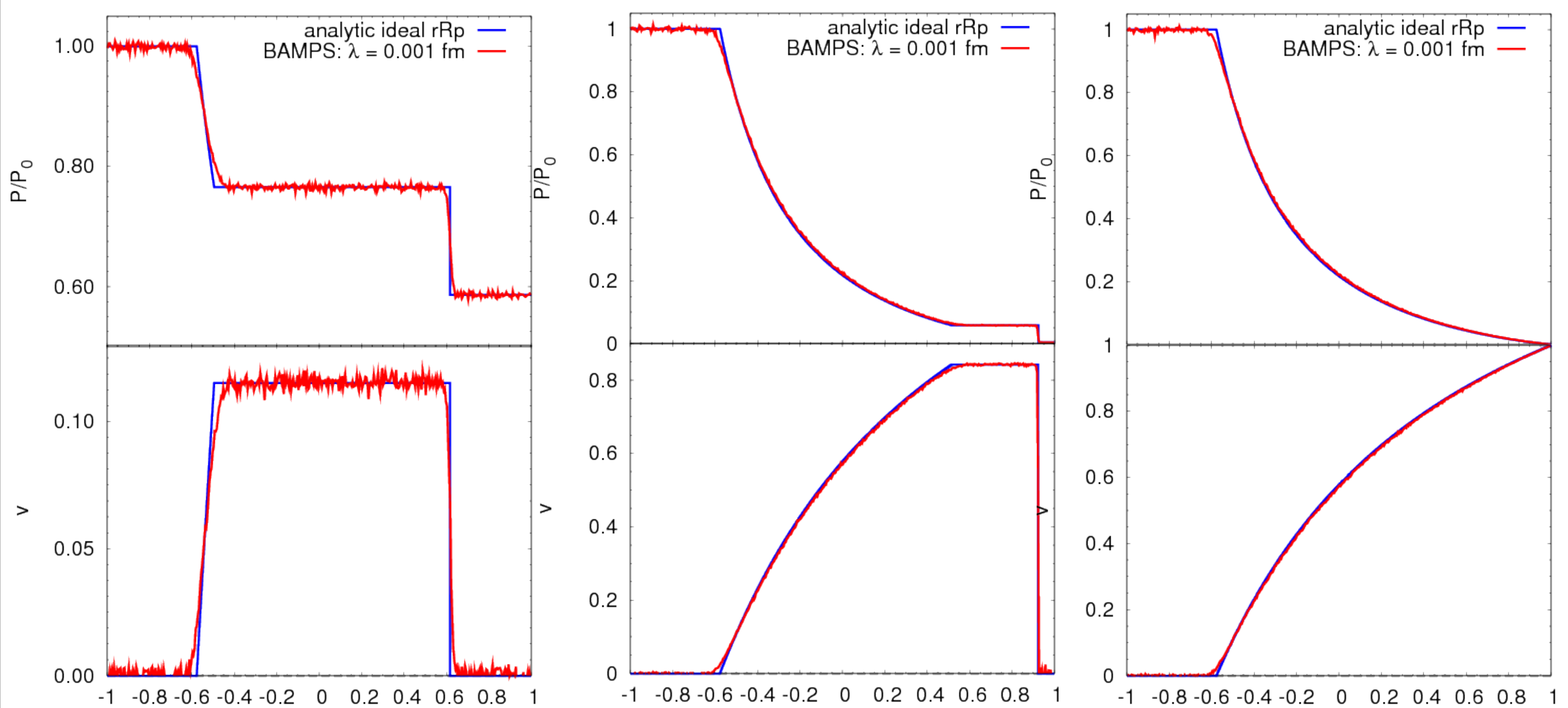
Expansion into the medium

$T_R = 350$ MeV

$T_R = 100$ MeV

Expansion into the vacuum

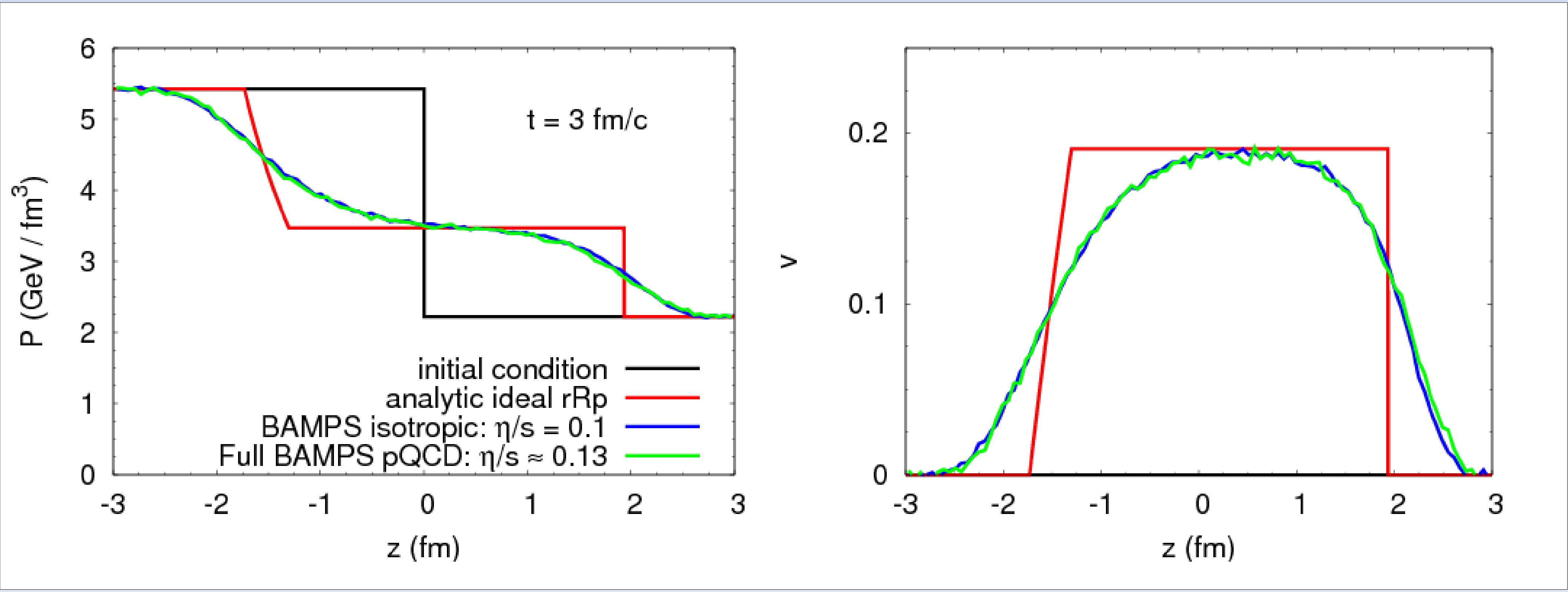
$T_R = 0$ MeV



Numerical Results

BAMPS with pQCD crossections

$T_L = 400$ MeV
$T_R = 320$ MeV
$t = 3$ fm/c



Shocks exist of course within pQCD simulations
Shocks exist of course within pQCD simulations

The relativistic Riemann problem

The relativistic hydrodynamic equations

- The local conservation of charge, energy and momentum

$$\partial_\mu \, !^{\mu} = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

with

$$T^{\mu\nu} = (\epsilon + p) u^{\mu} u^{\nu} - p\, g^{\mu\nu}$$
$$!^{\mu} = n u^{\mu}$$
$$g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$
$$u^{\mu} = (\gamma, \gamma v) \quad \text{″ith} \quad u^{\mu} u_{\mu} = 1$$
$$\gamma = 1/\sqrt{1 - v^2}$$

- The equations of relativistic hydrodynamics of an ideal fluid in one dimension

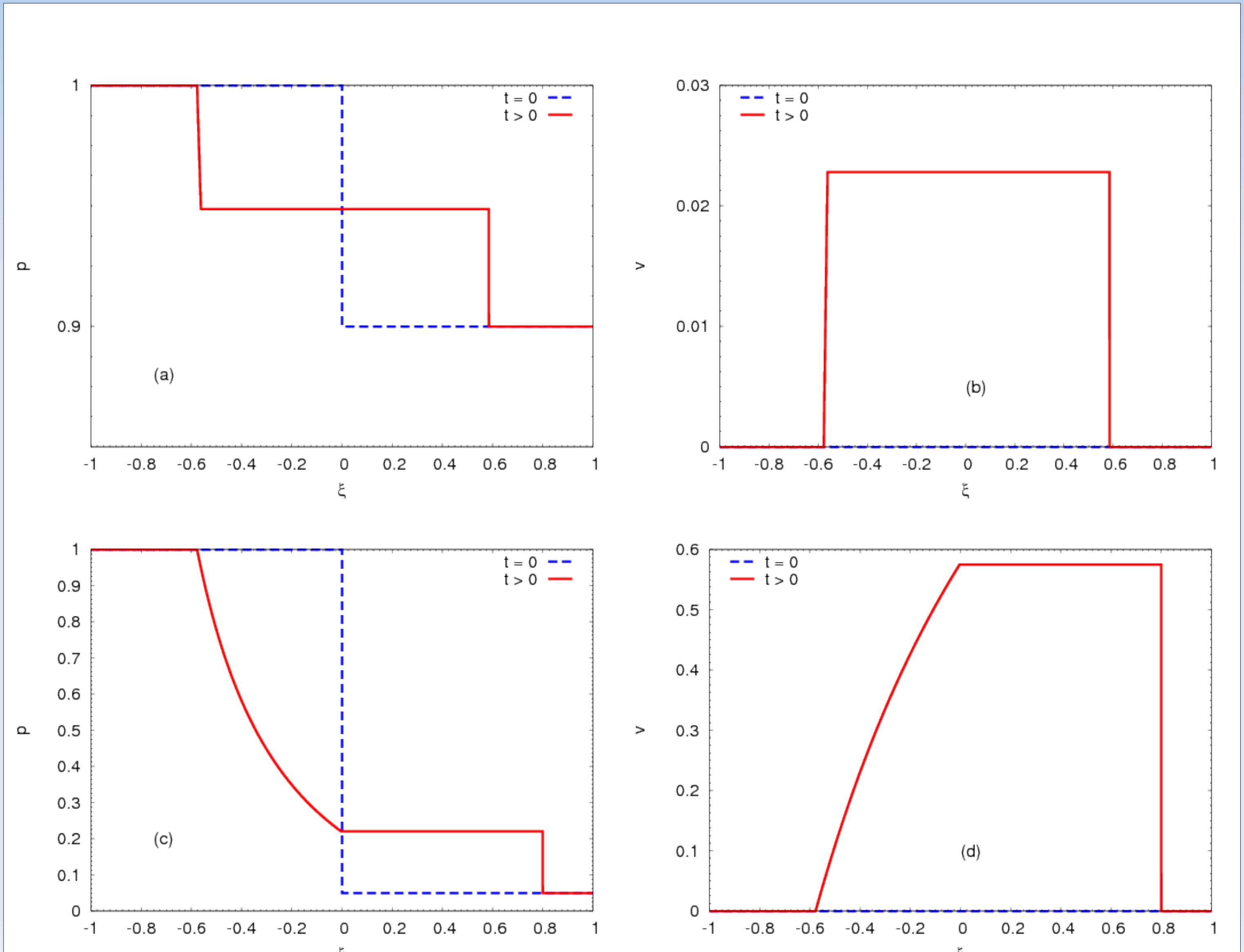
$$\partial_t \, !^{0} + \partial_z (v_z \, !^{0}) = 0$$
$$\partial_t T^{0z} + \partial_z (v_z T^{0z}) = -\partial_z(p)$$
$$\partial_t T^{00} + \partial_z (v_z T^{00}) = -\partial_z(v_z p)$$

$$!^{0} = \gamma n$$
$$T^{00} = \gamma^2 (\epsilon + p) - p$$
$$T^{0z} = \gamma^2 (\epsilon + p) v$$

Equation of state $\quad p = p(\epsilon, n)$

The relativistic Riemann problem

Some examples for the analytical solutions



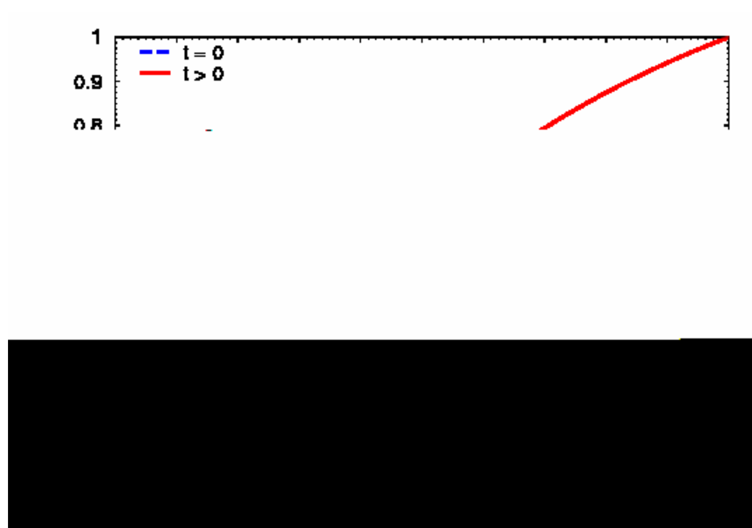
$p_0 \# p_4 = 1 \# 0.9$

$v_{shock} = 0.585$

$p_0 \# p_4 = 1 \# 0.05$

$v_{shock} = 0.7999$

The relativistic Riemann problem



$p_0 \# p_4 = 1 \# 0.002$

$v_{shock} = 0.941$

Expansion into the vacuum

$p_0 \# p_4 = 1 \# 0$

$v_{shock} = 1.0$