

# Hybrid Model with the QCD Critical Point

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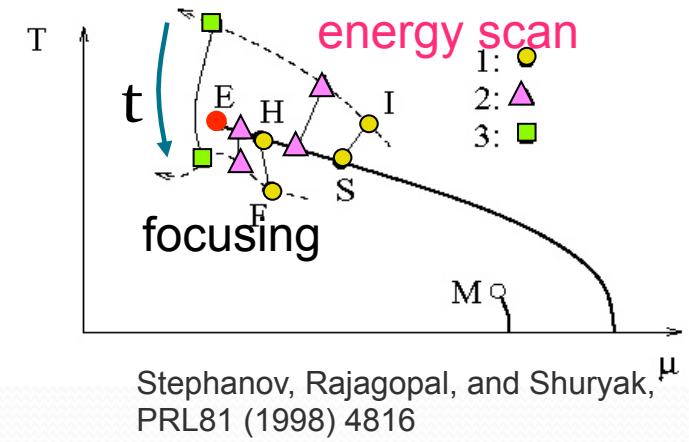
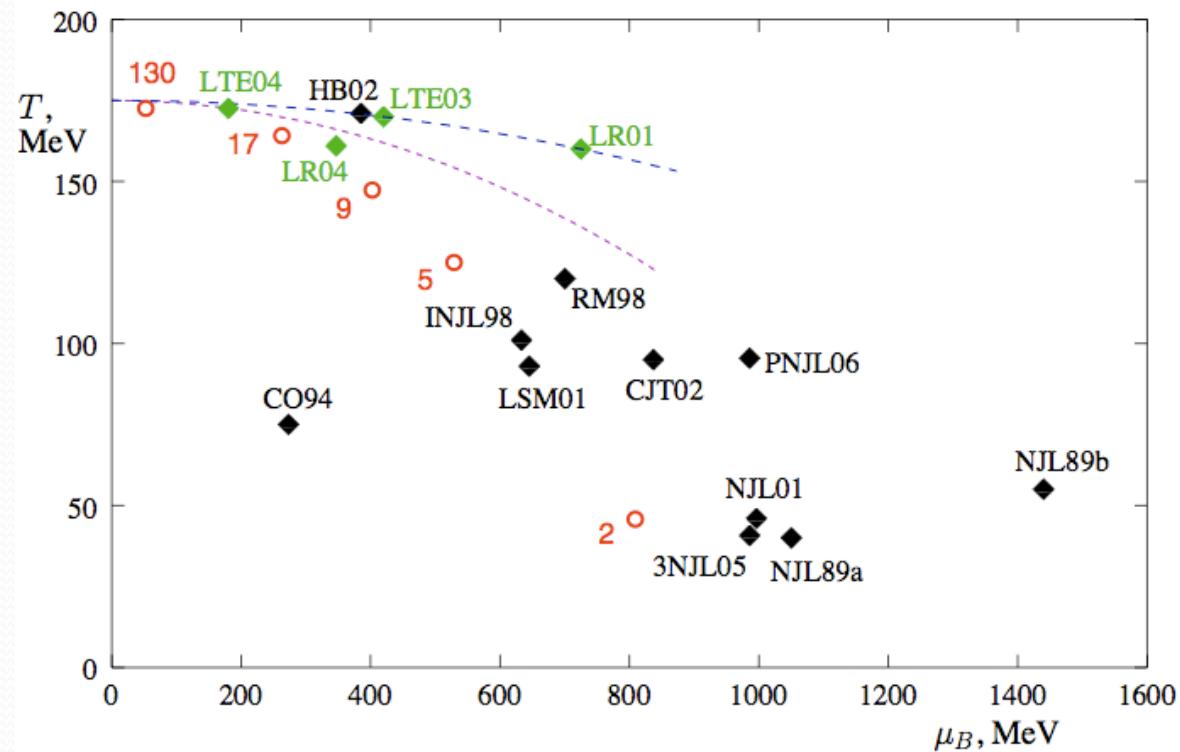
In collaboration with Asakawa, Bass, and Mueller

September 17, 2009@ECT\*

# Where is the QCD Critical Point?

- Lattice QCD, Effective theories....

Stephanov,hep-lat/0701002



Stephanov, Rajagopal, and Shuryak,  
PRL81 (1998) 4816

- QCP search  
in heavy ion collisions
  - Energy scan
  - Experiments and phenomenology

# Towards Quantitative Analyses

- Realistic dynamical model
  - 3D Hydro + UrQMD (hadron base event generator)
- Equation of state with QCD critical point
- Physical observables

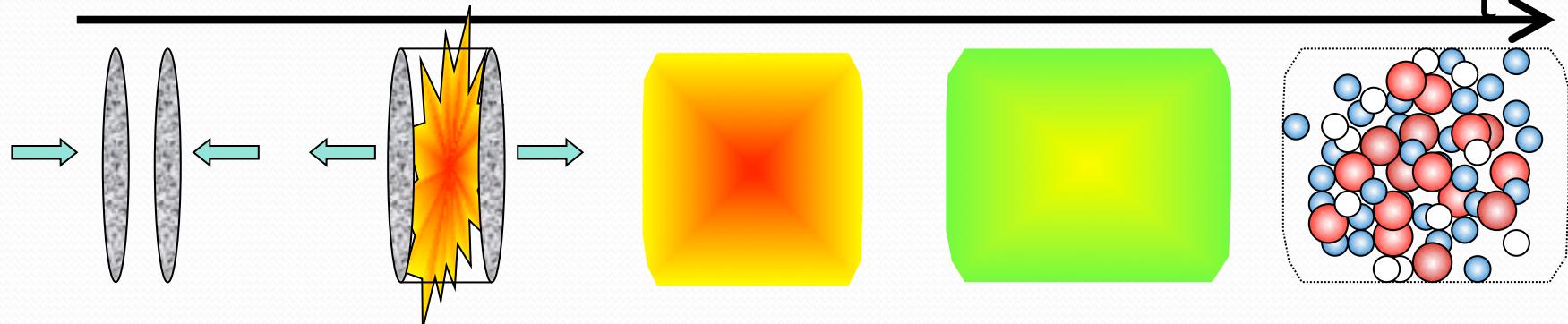
Signals of QCP should survive after freezeout process.

- Fluctuations
- Hadron ratios

# 3D Hydro+UrQMD Model

Nonaka and Bass PRC75:014902(2007)

- Schematic sketch



- 3D Hydro + UrQMD

## Full 3-d Hydrodynamics

EoS :1st order phase transition  
QGP + excluded volume model

## Hadronization

Cooper-Frye  
formula  
Monte Carlo

## UrQMD

final state  
interactions

$T_C$

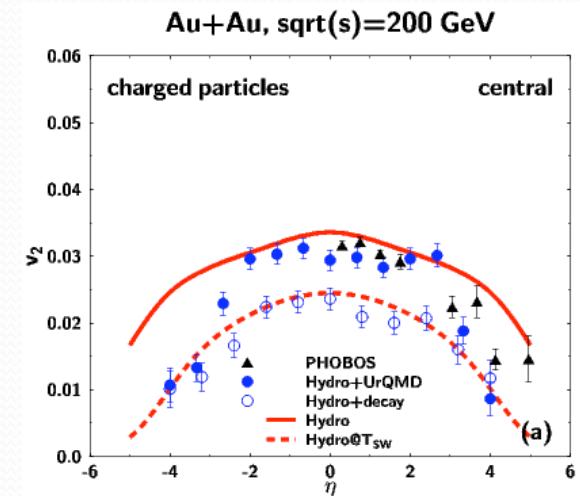
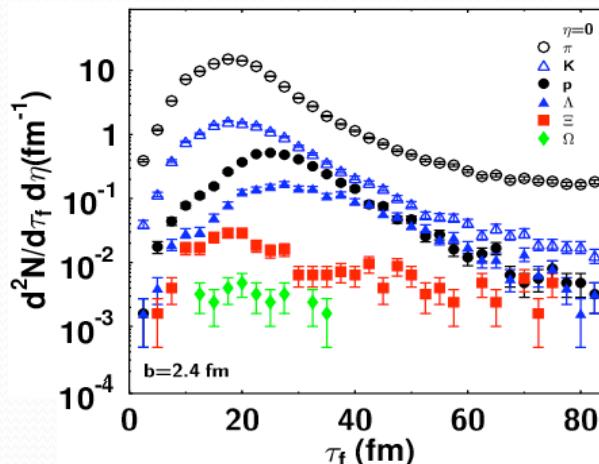
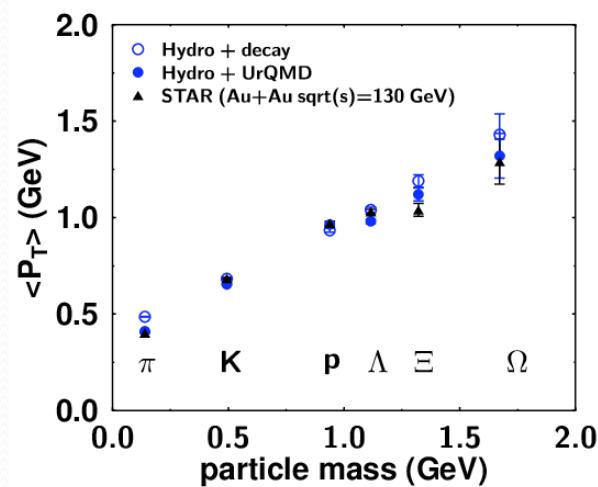
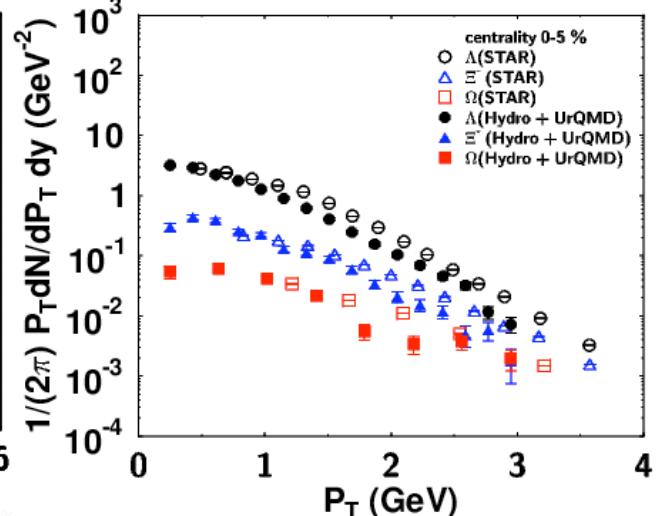
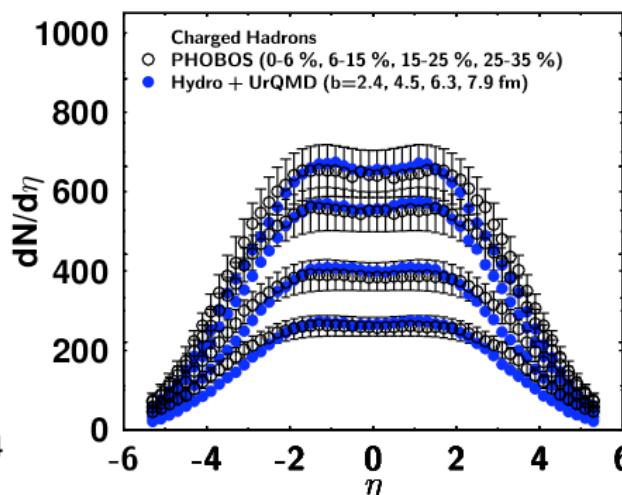
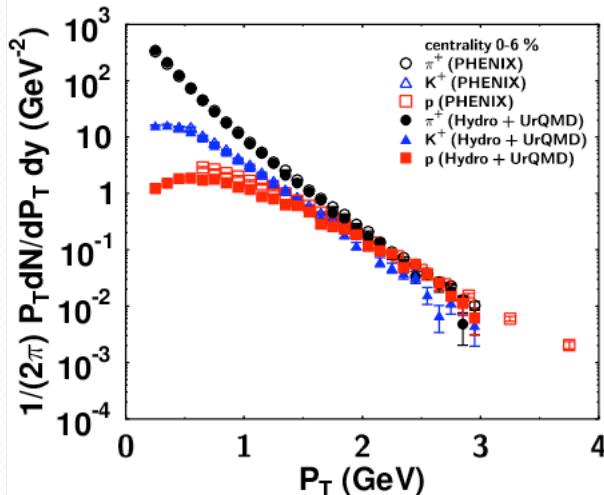
$T_{SW}$

$t \text{ fm}/c$

$T_C$ :critical temperature >  $T_{SW}$ : Hydro → UrQMD

# Highlights of 3D Hydro+UrQMD

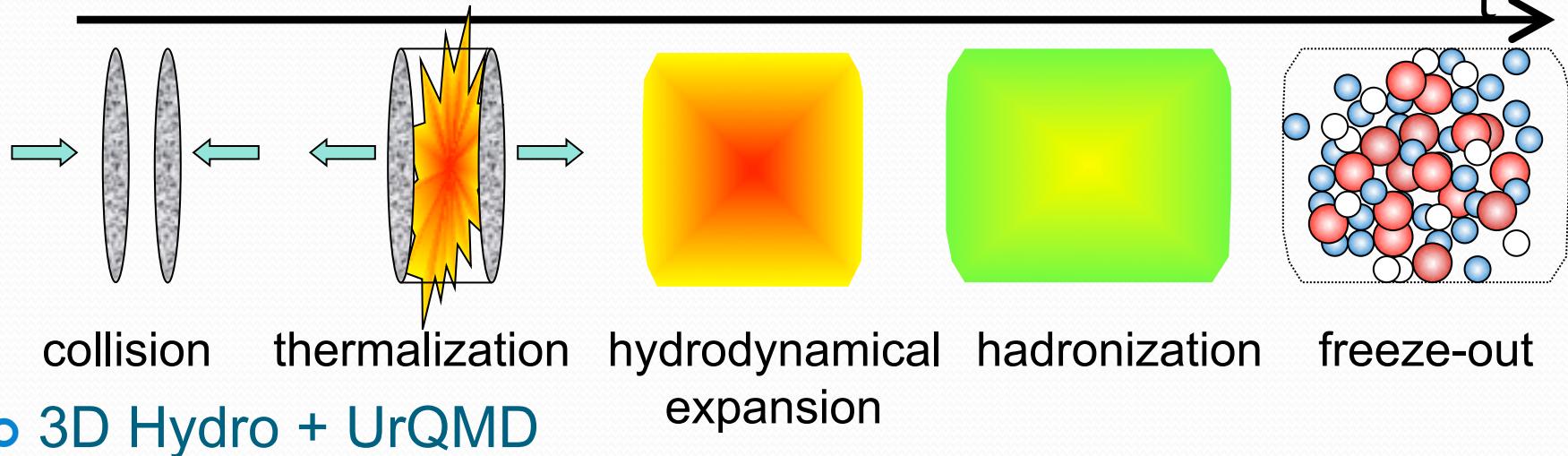
Nonaka and Bass PRC75:014902(2007)



# 3D Hydro+UrQMD Model

C.N. and Bass PRC75:014902(2007)

- Schematic sketch



- 3D Hydro + UrQMD

## Full 3-d Hydrodynamics

EoS with QCD critical point

## Hadronization

Cooper-Frye formula  
Monte Carlo

## UrQMD

final state interactions

Asakawa, Bass, Mueller, C.N. PRL101:122302(2008),  
C.N., Asakawa, Phys.Rev.C71:044904(2005)

$T_C$

$T_{SW}$

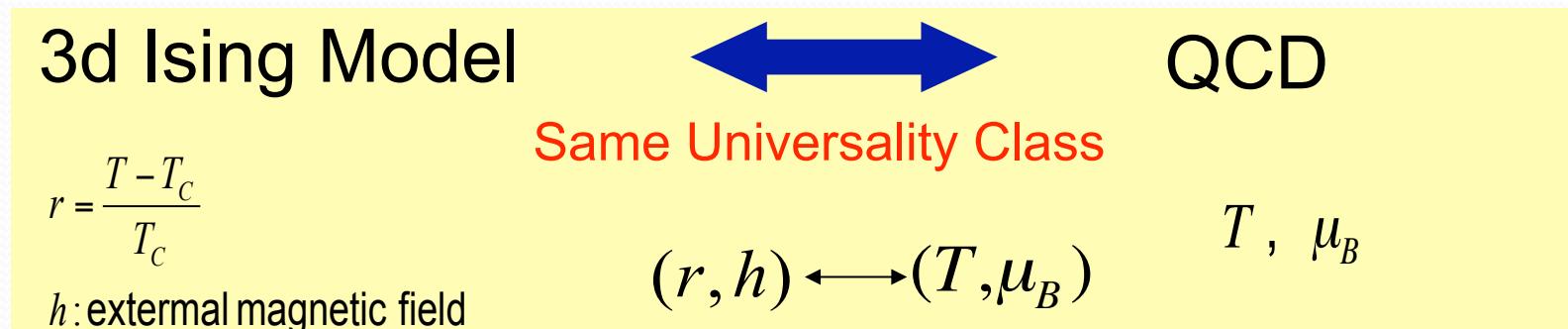
$t \text{ fm/c}$

$T_C$ :critical temperature >  $T_{SW}$ : Hydro → UrQMD

# EOS with QCD Critical Point

C.N. and Asakawa, PRC71,044904(2005)

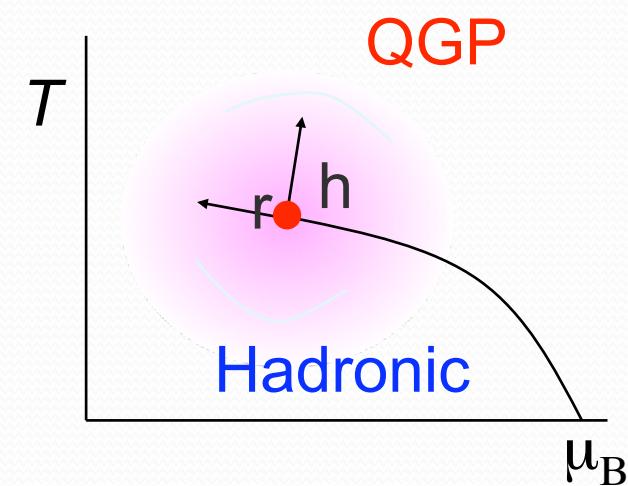
- Singular part near QCD critical point + Non-singular part
  - Singular part



- Mapping:  $(r, h)$  3-d Ising Model



$(T, \mu_B)$  QCD



# EOS of 3-d Ising Model

- Parametric Representation of EOS

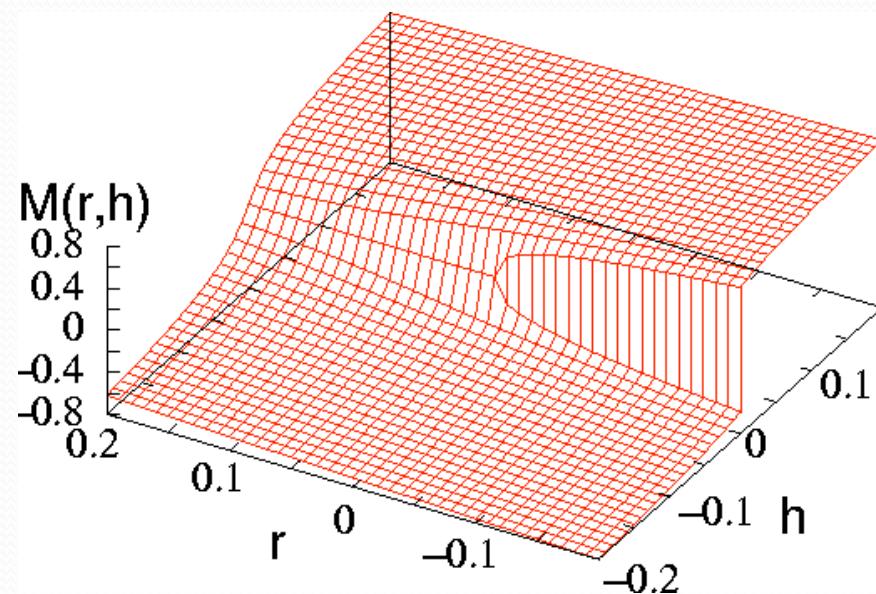
$$\begin{cases} M = M_0 R^\beta \theta \\ h = h_0 R^\beta \tilde{h}(\theta) = h R_0^{\beta\delta} (\theta - 0.76201\theta^3 + 0.00804\theta^5) \\ r = R(1 - \theta^2) \quad (R \geq 0, -1.154 \leq \theta \leq 1.154) \end{cases}$$

$$r = \frac{T - T_c}{T_c}$$

$h$  : external magnetic field

$$\beta = 0.326$$

$$\delta = 4.8$$

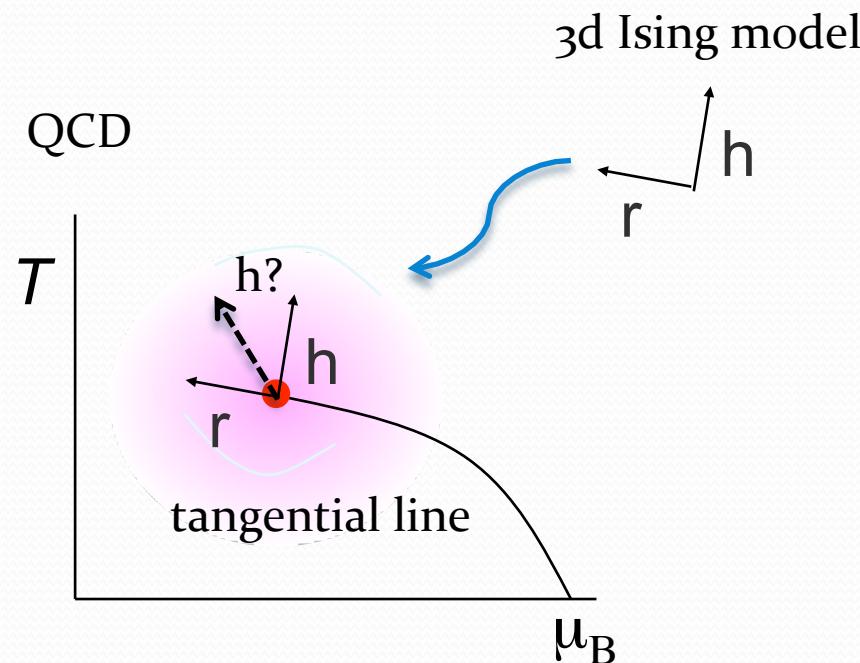


Guida and Zinn-Justin NPB486(97)626

# Mapping : 3D Ising Model → QCD

## No Universality

- $h$  axis ?



- Critical Region?

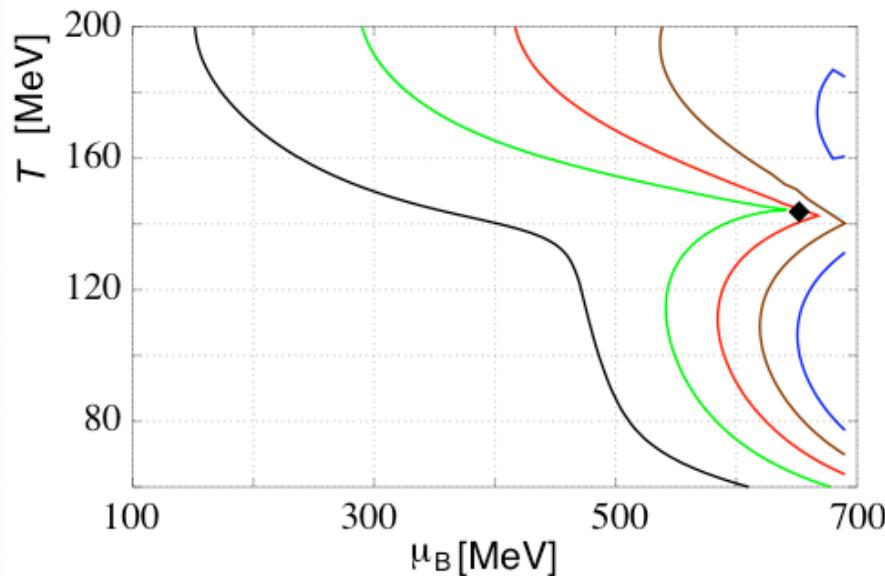
- Lattice QCD
- Effective theory
- Experiments

inputs in our model

# Focusing Effect

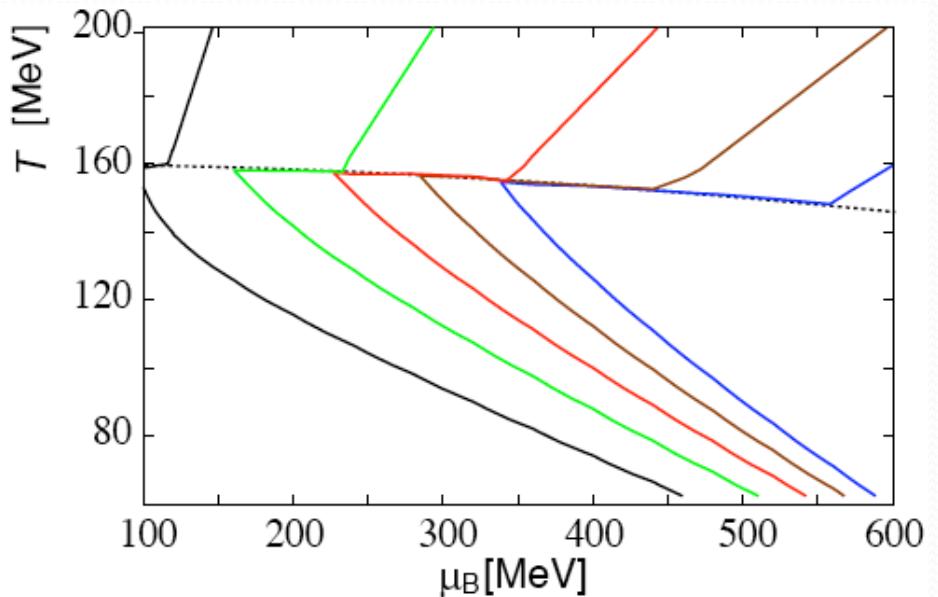
- Isentropic Trajectories on QCD phase diagram

With QCD critical point



*Focused*

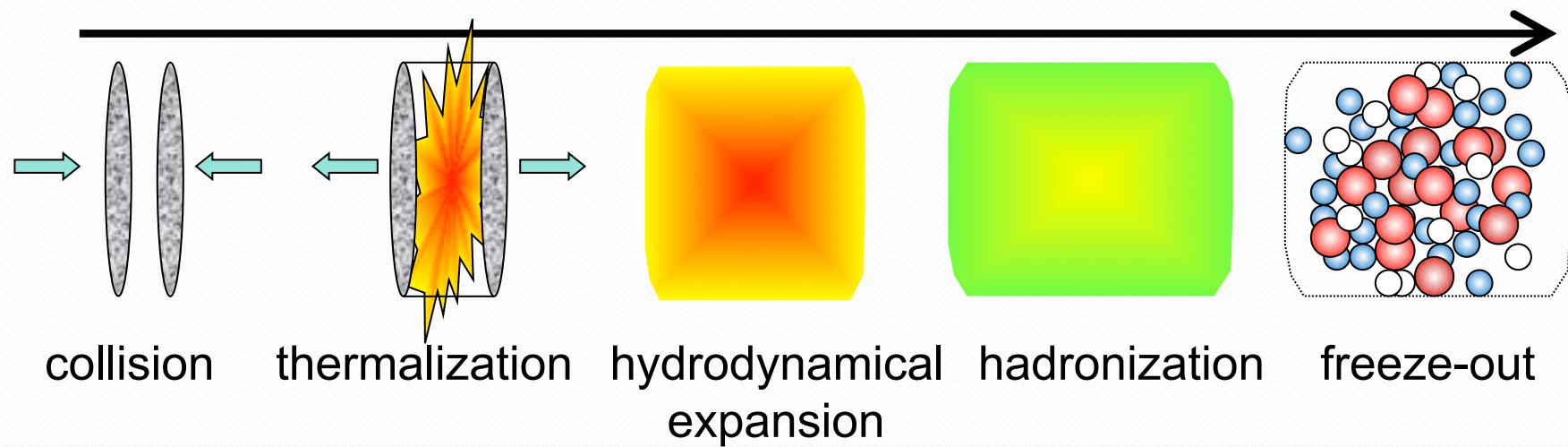
Bag Model +  
Excluded Volume Approximation  
(No Critical Point)



*Not Focused*

# Signal of QCP

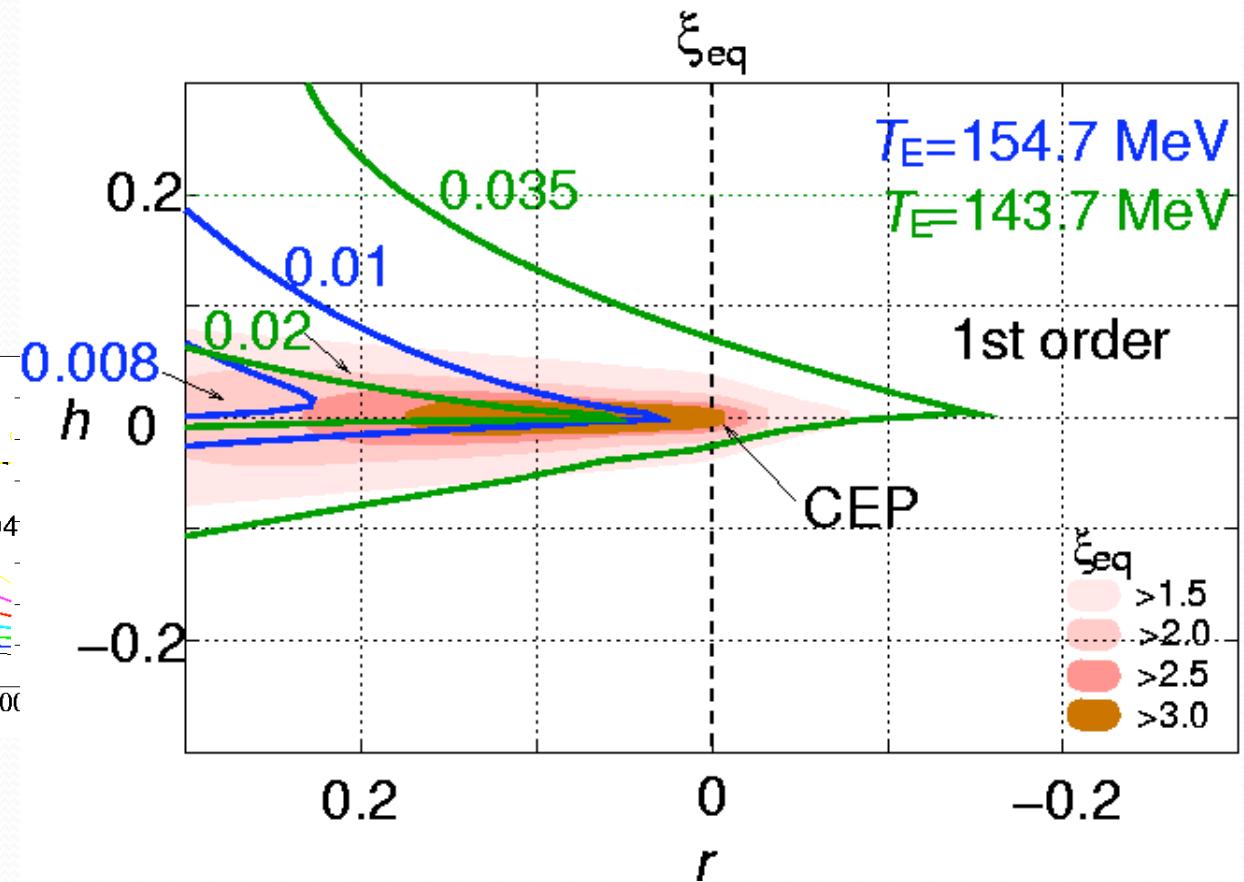
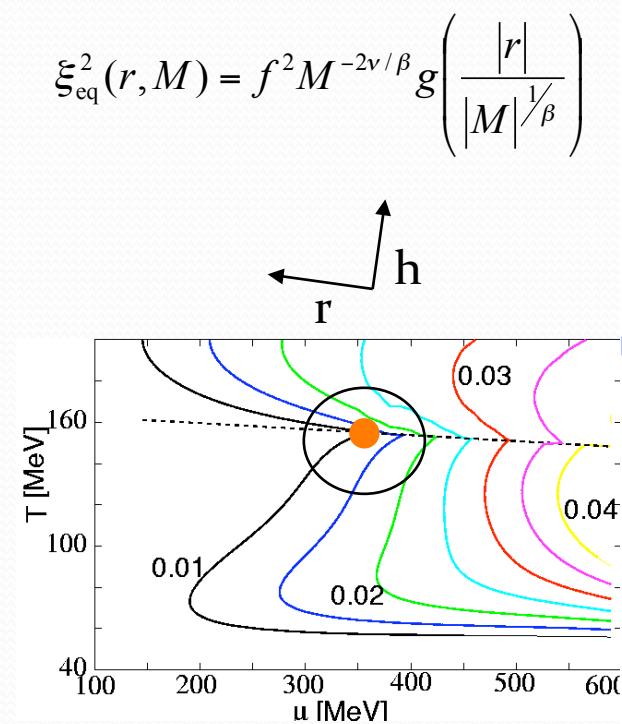
- Signal of QCD critical point should survive even after freezeout process.



- Fluctuations: conserved values ex. charge, baryon number
- Hadron ratios: fixed at chemical freezeout temperature  $T_{\text{ch}}$

# Correlation Length

Widom's scaling law

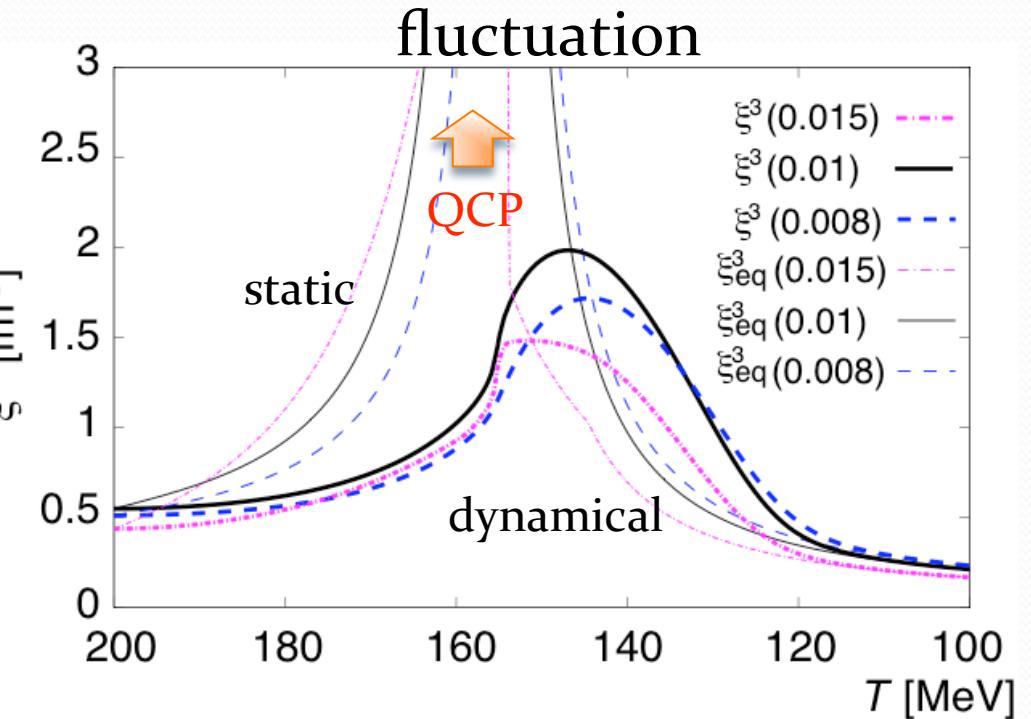
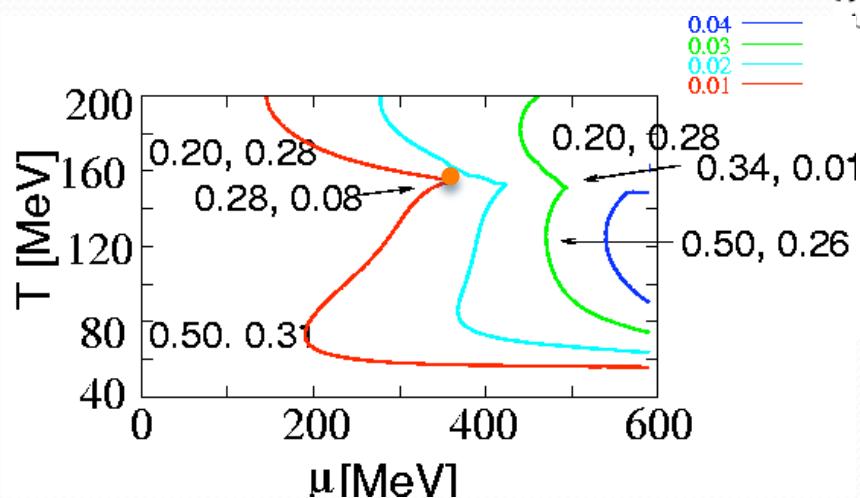


# Fluctuation in 1d Hydro.

- $$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)] \left( m_\sigma(\tau) - \frac{1}{\xi_{\text{eq}}(\tau)} \right)$$

$$\Gamma(m_\sigma) = \frac{a}{\xi_0} (m_\sigma \xi_0)^z, \quad m_\sigma(\tau) = \frac{1}{\xi(\tau)}$$

$z = 3.0$  Model H  
(Halperin RMP49(77)435)

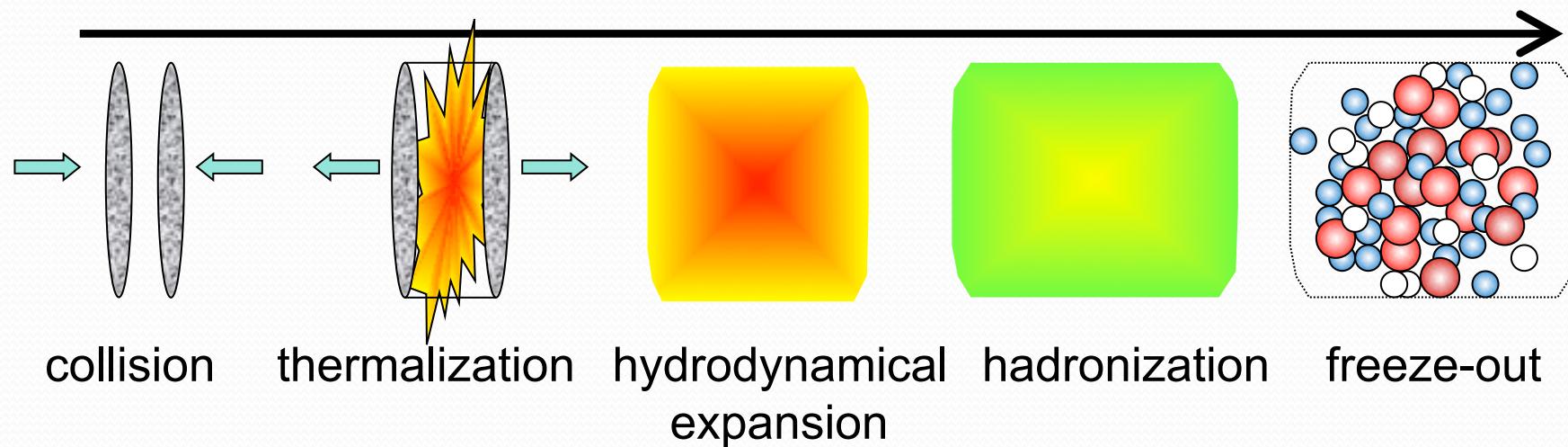


- critical slowing down
- evolution rate

fluctuation : clear signal?

# Signal of QCP

- Signal of QCD critical point should survive even after freezeout process.

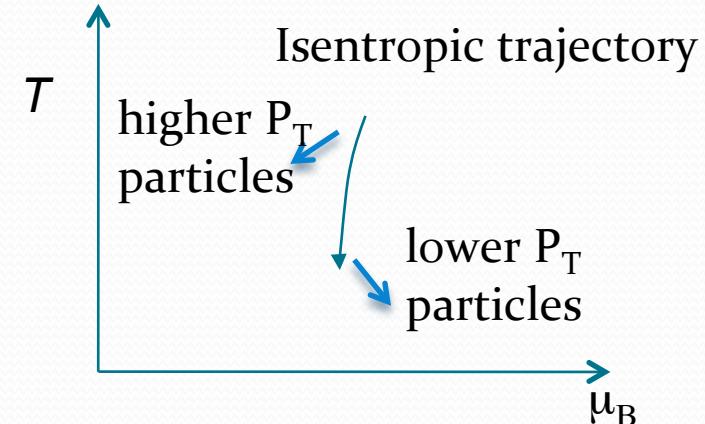
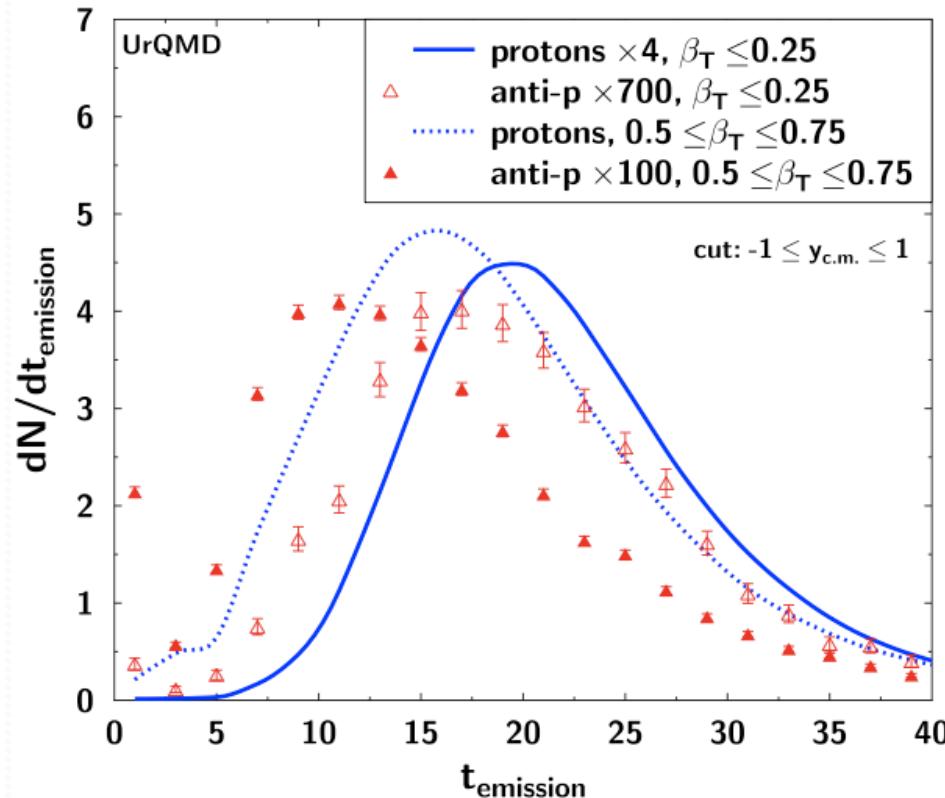


- Fluctuations: conserved values ex. charge, baryon number
  - Hadron ratios: fixed at chemical freezeout temperature  $T_{\text{ch}}$
- Key:**  $T_{\text{ch}}$  depends on transverse momentum

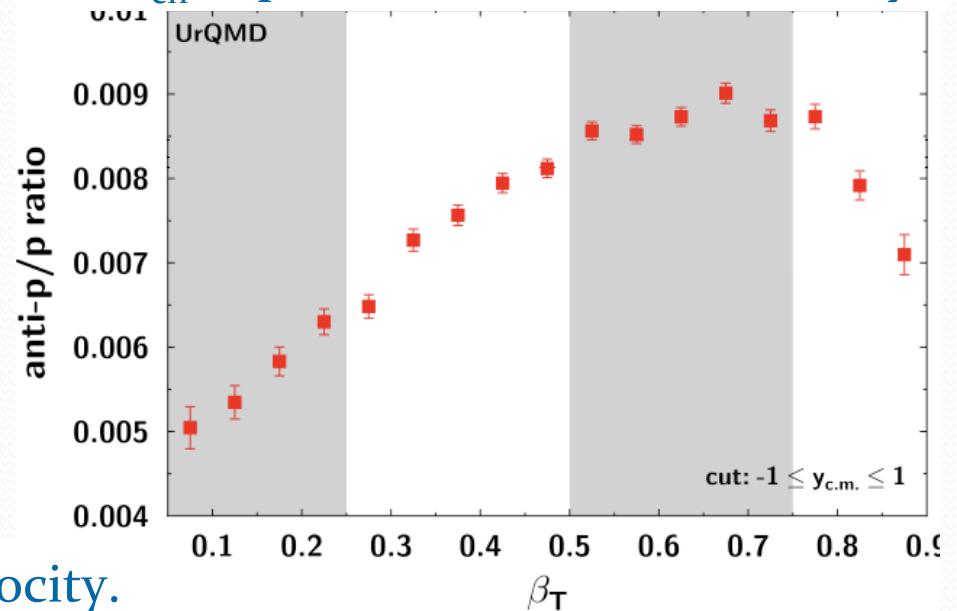
# Hadron Production on $n_B/s$ const. line

UrQMD : no QCD critical point

Au+Au,  $E_{\text{lab}}=40 \text{ GeV/A}$



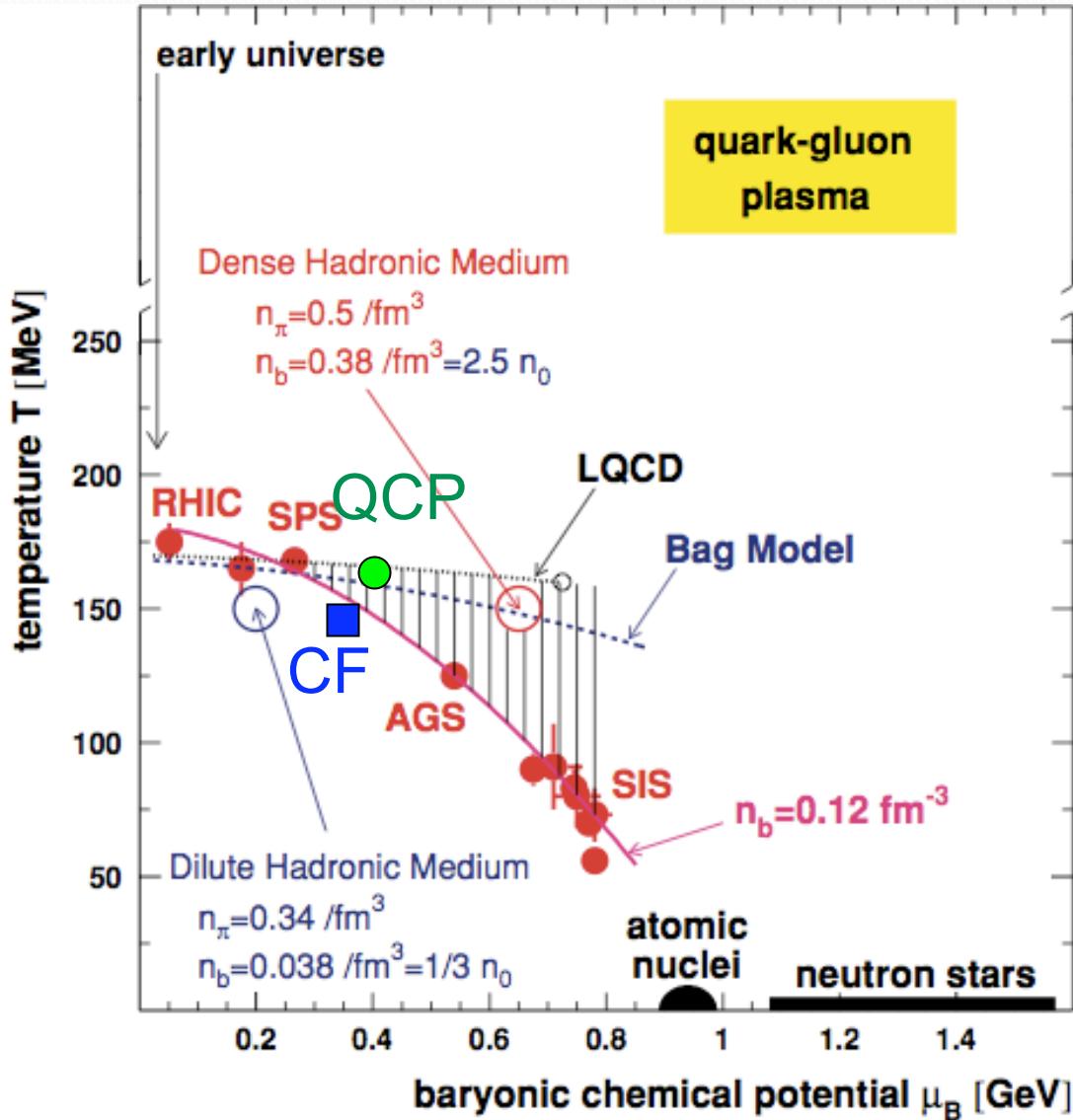
- $T_{\text{ch}}$  depends on transverse velocity.



- Hadron ratios depend on transverse velocity.

Chiho Nonaka

# Demonstration

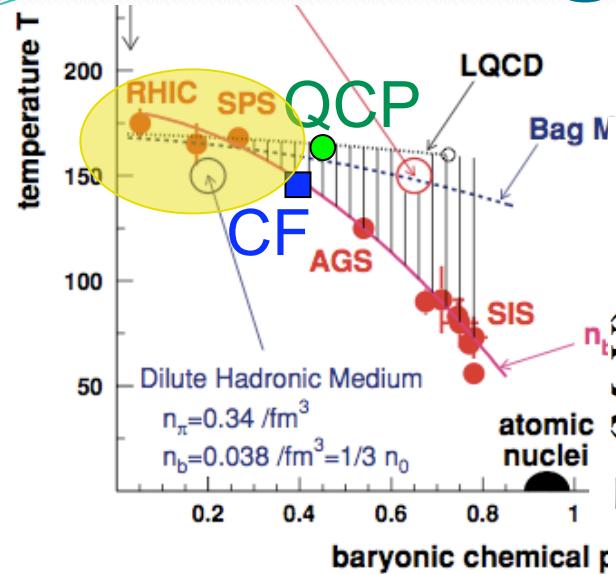


Search of  
the QCD critical point  
from experiments

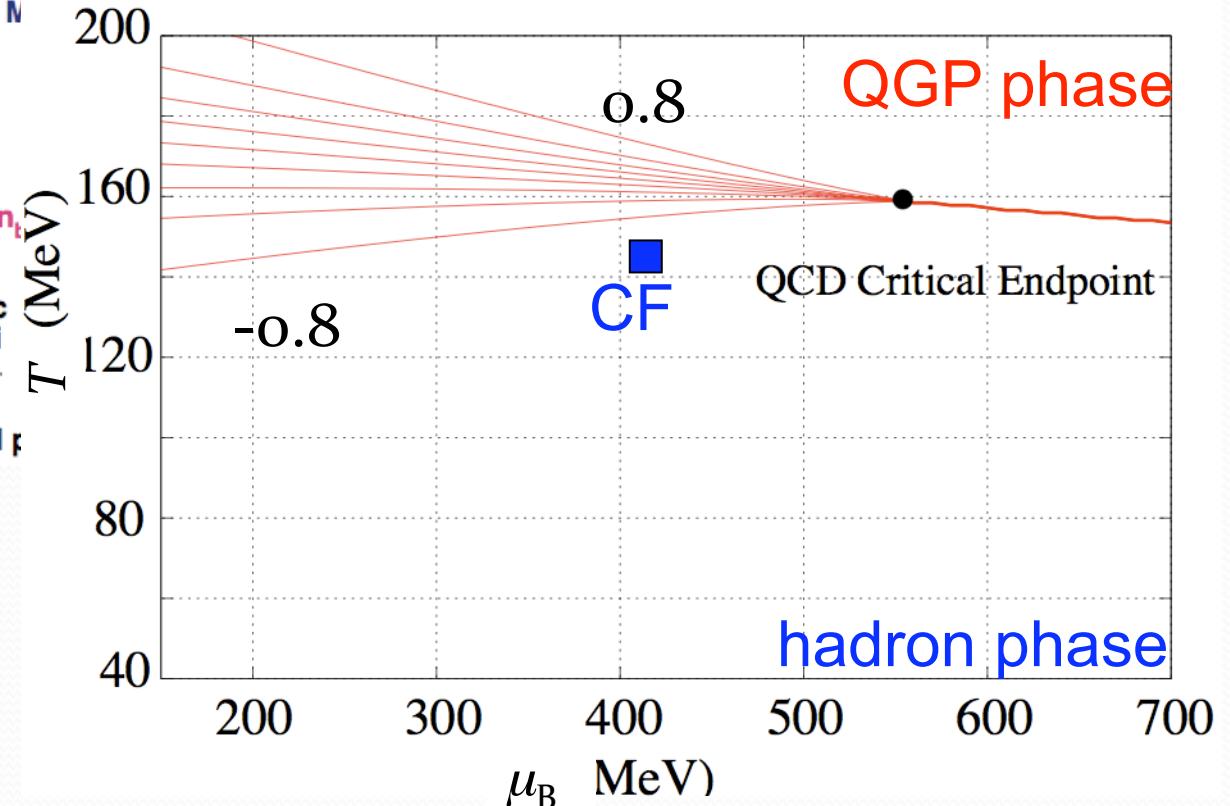
SPS

- Location of QCP  
 $(\mu_B, T) = (550, 159)$
- Critical Region parameter
- Chemical freezeout point  
 $(\mu_B, T) = (406, 145)$   
from statistical model

# Critical Region



QCD critical point  
 $(\mu_B, T) = (550, 159)$



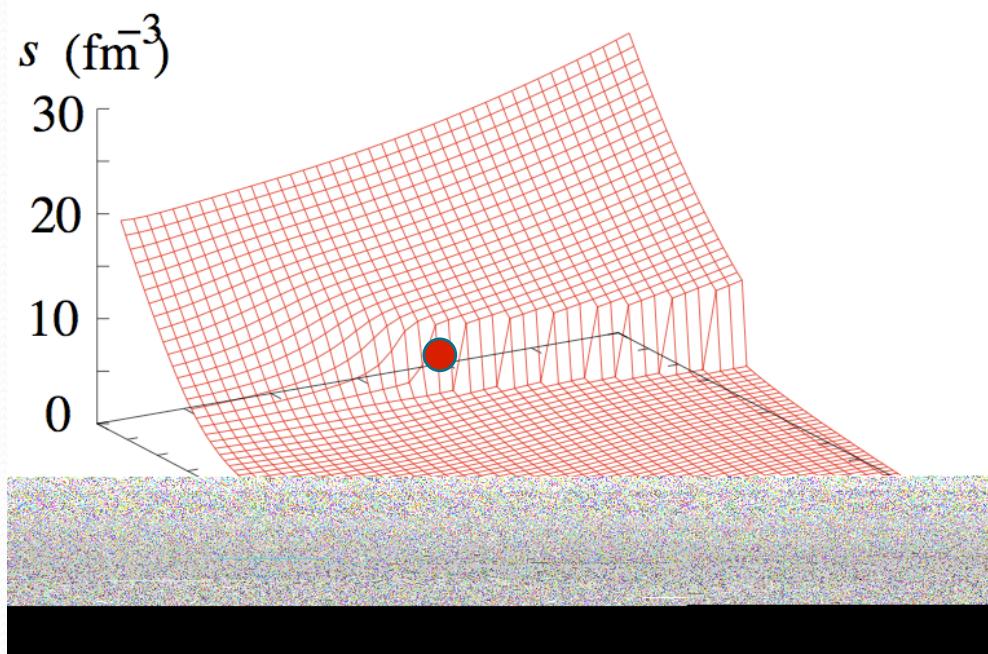
- Entropy density

$$S_{\text{real}}(T, \mu_B) = \frac{1}{2} \left\{ 1 - \tanh[S_c(T, \mu_B)] \right\} S_H(T, \mu_B) + \frac{1}{2} \left\{ 1 + \tanh[S_c(T, \mu_B)] \right\} S_Q(T, \mu_B)$$

# Equation of State

- QCD critical point  $(\mu_B, T) = (550, 159)$

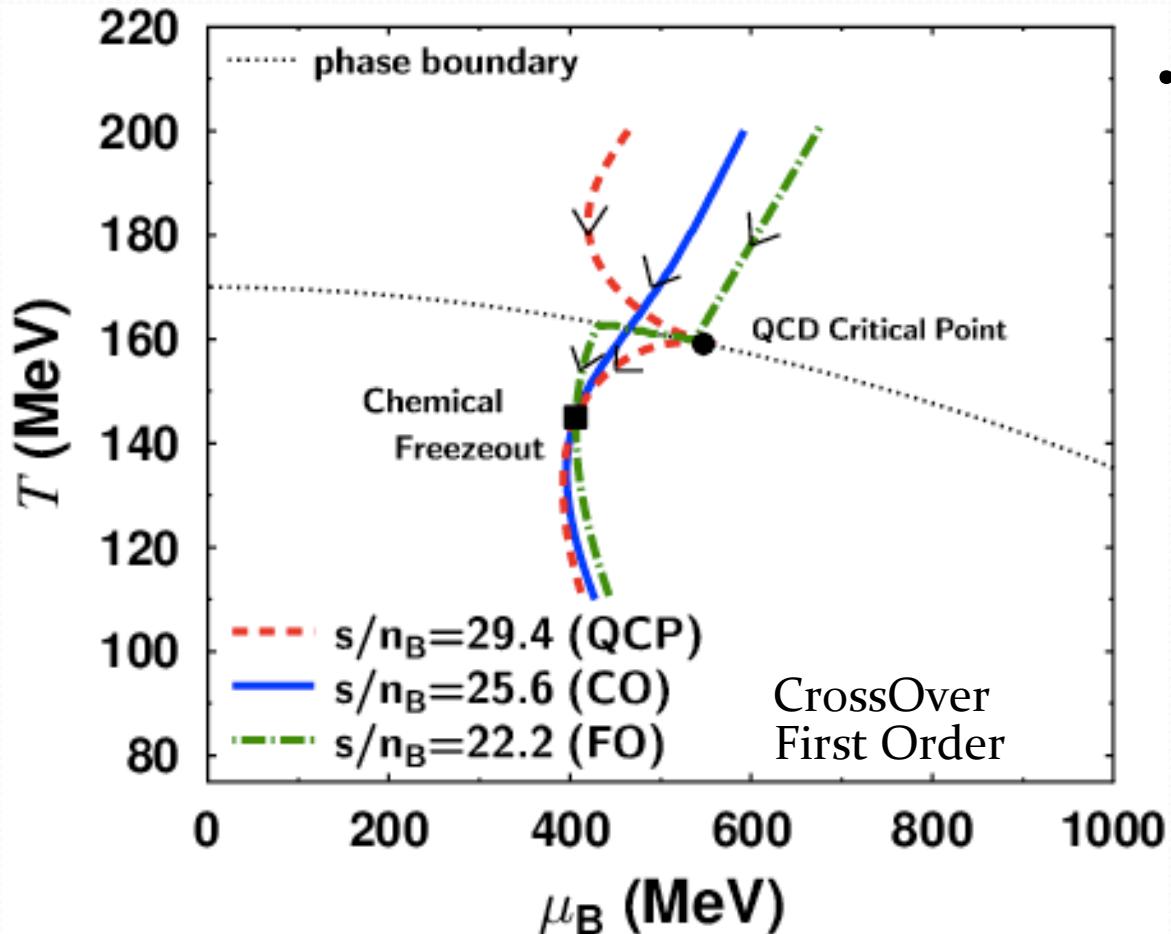
entropy density



trajectories



# ISENTROPIC TRAJECTORIES

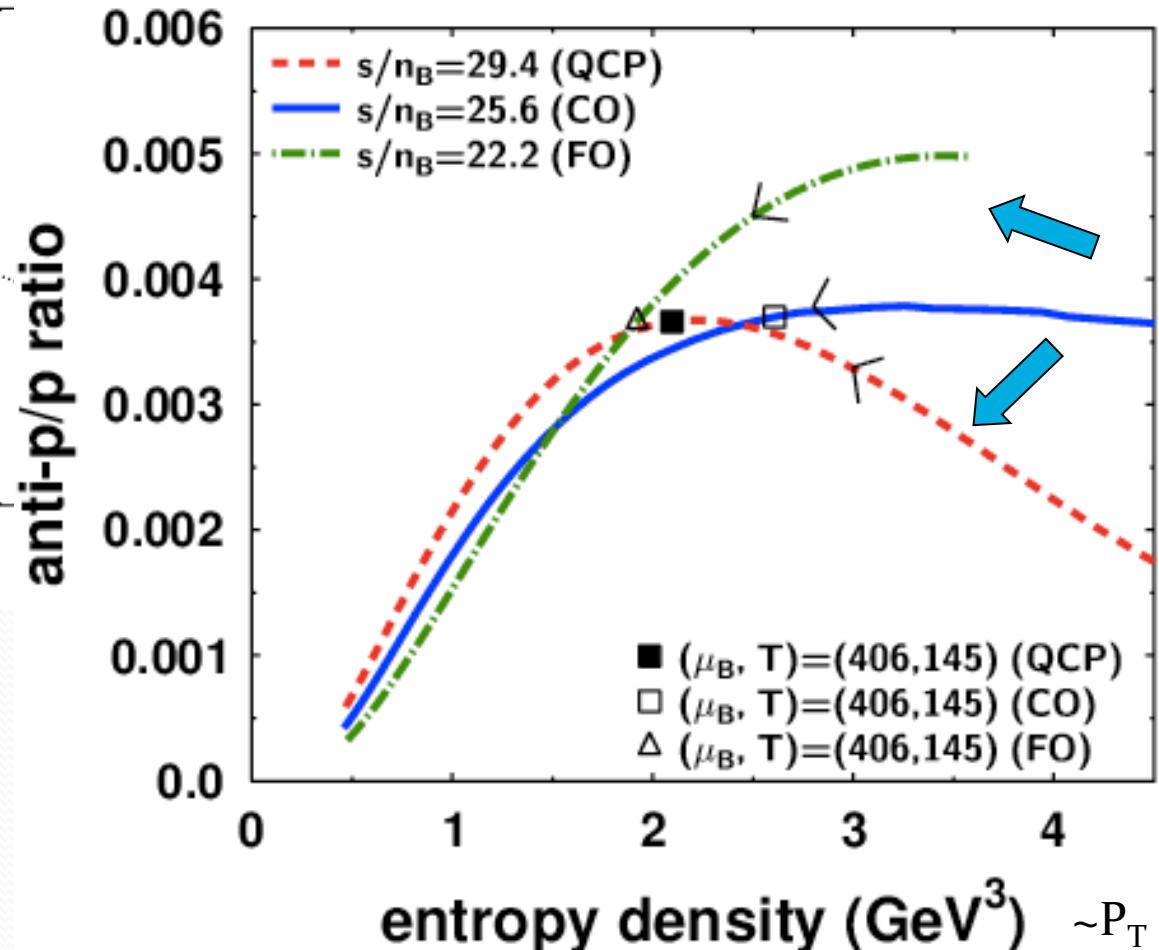
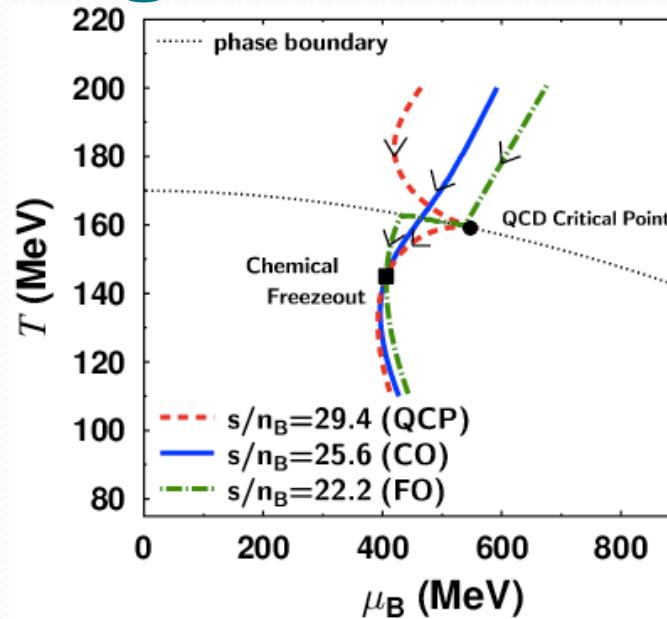


- Hadronization occurs between the phase boundary and chemical freezeout point

$$\left\{ \begin{array}{l} \bullet \text{FO, CO} \quad \nearrow \frac{\mu_B}{T} \\ \bullet \text{QCP} \quad \longrightarrow \end{array} \right. \quad \rightarrow \bar{p}/p \text{ ratio}$$

$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

# Signature of QCP



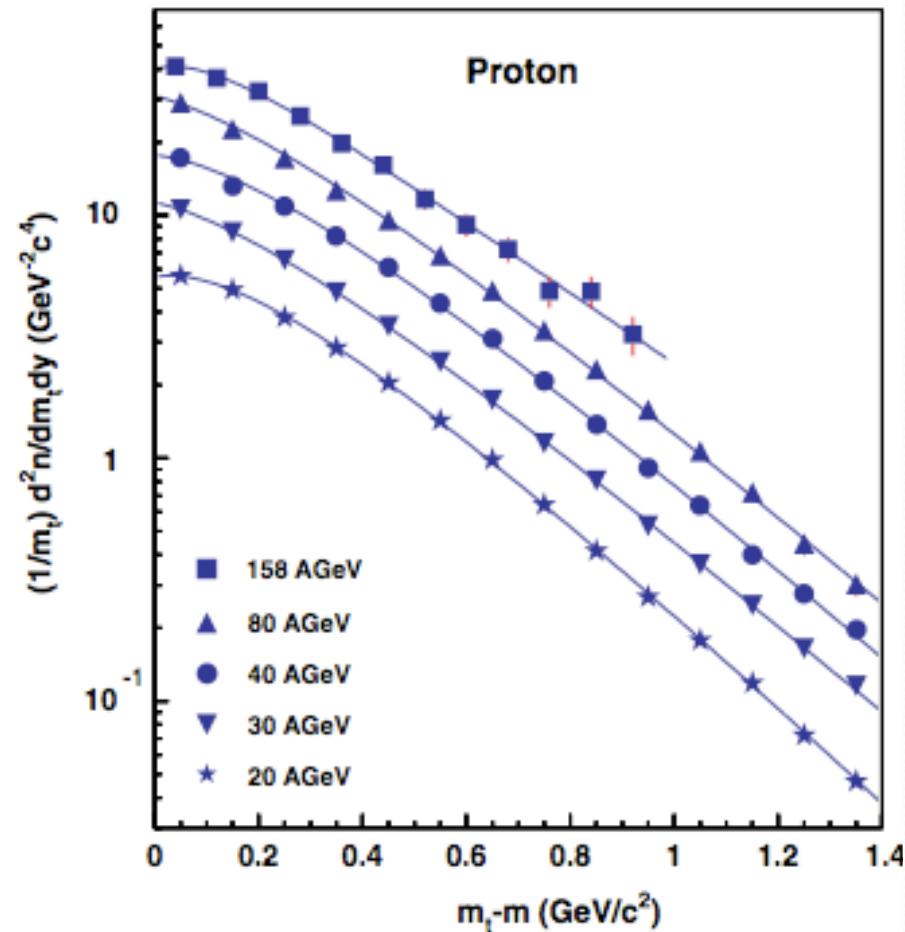
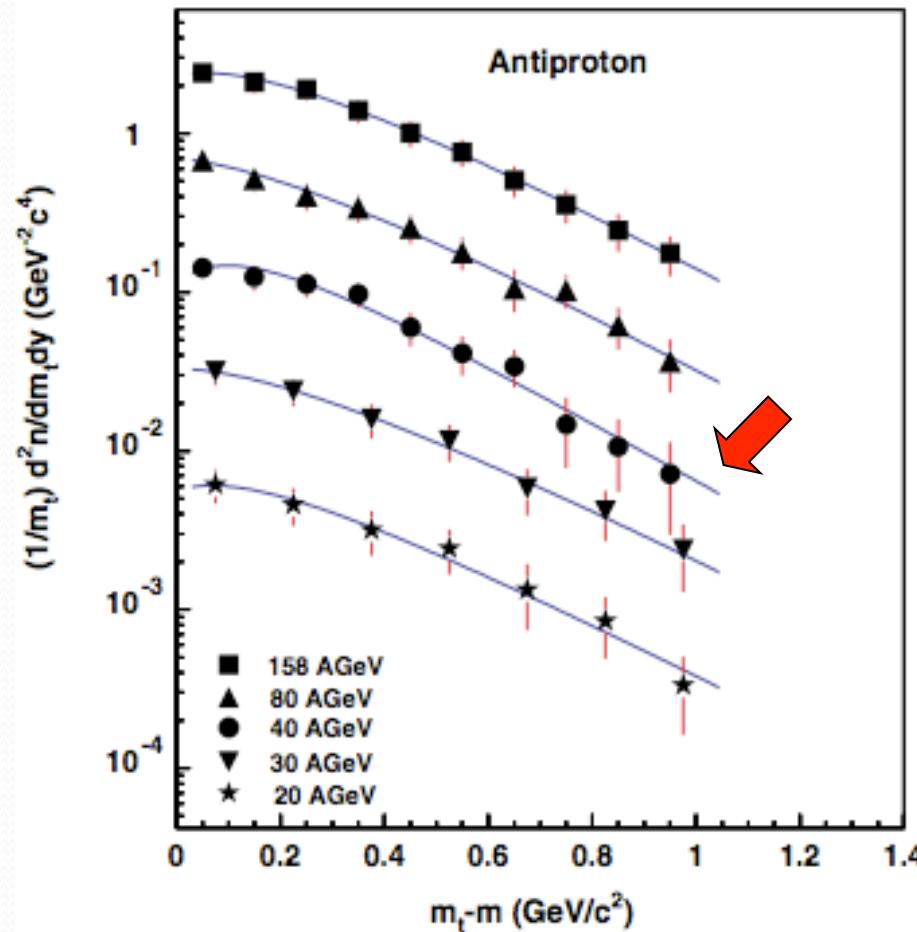
$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

- decreases (FO, CO)
- increases (QCP)

**entropy density ( $\text{GeV}^3$ )  $\sim P_T$**   
**with QCP**  
**steeper  $\bar{p}$  spectra at high  $P_T$**

# QCD Critical Point?

NA49



steeper  $\bar{p}$  spectra at high  $P_T$

NA49, PRC73,044910(2006)

# 3D Hydro+UrQMD with QCP

- Initial Conditions

- Energy density

$$\epsilon(x,y,\eta) = \epsilon_{\max} W(x,y;b) H(\eta)$$

- Baryon number density

$$n_B(x,y,\eta) = n_{B\max} W(x,y;b) H(\eta)$$

- Parameters

$$\begin{cases} \tau_0 = 0.6 \text{ fm/c} \\ \eta_0 = 0.5 \quad \sigma_\eta = 1.5 \end{cases}$$

- Flow

$$v_L = \eta \text{ (Bjorken's solution); } v_T = 0$$

- EOS: QCP, Bag Model

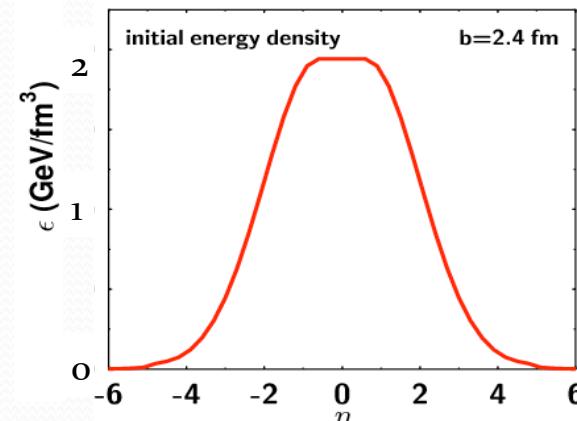
- Switching temperature

$$T_{\text{sw}} = 150 \text{ [MeV]}$$

QCP:  $T_E = 159 \text{ MeV}, \mu_E = 550 \text{ MeV}$

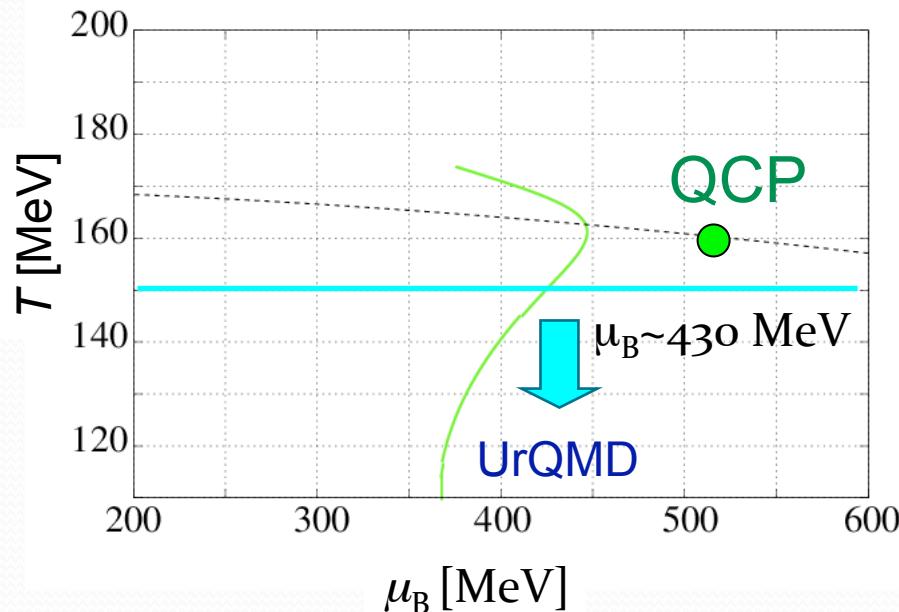
$T_c = 170 \text{ MeV}$  at  $\mu_B = 0 \text{ MeV}$

• longitudinal direction:  $H(\eta)$

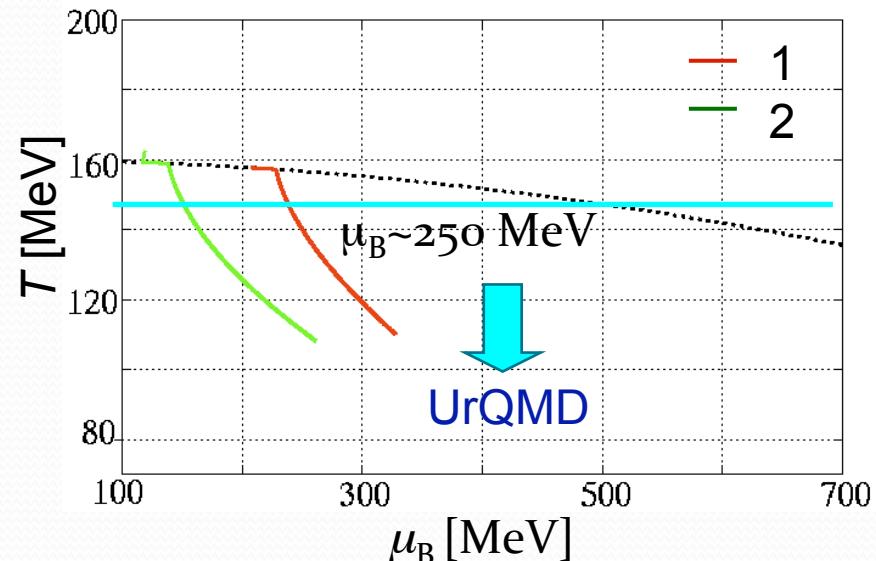


# 3D Hydro + UrQMD

Hydro with QCD critical point



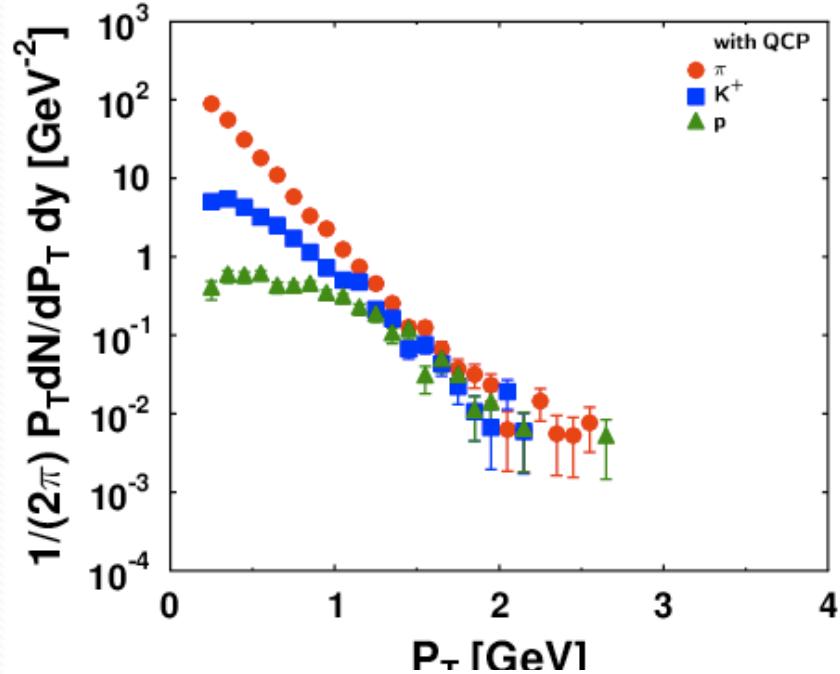
Hydro with Bag Model



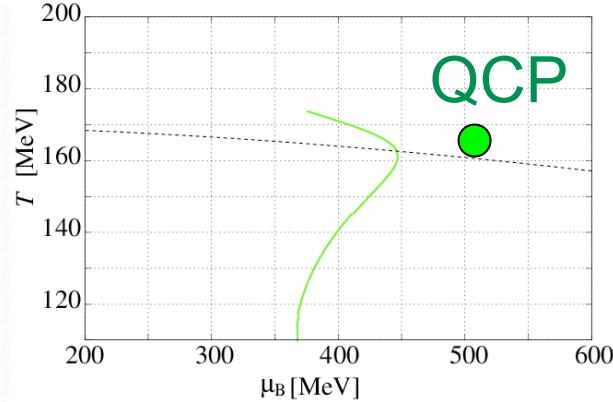
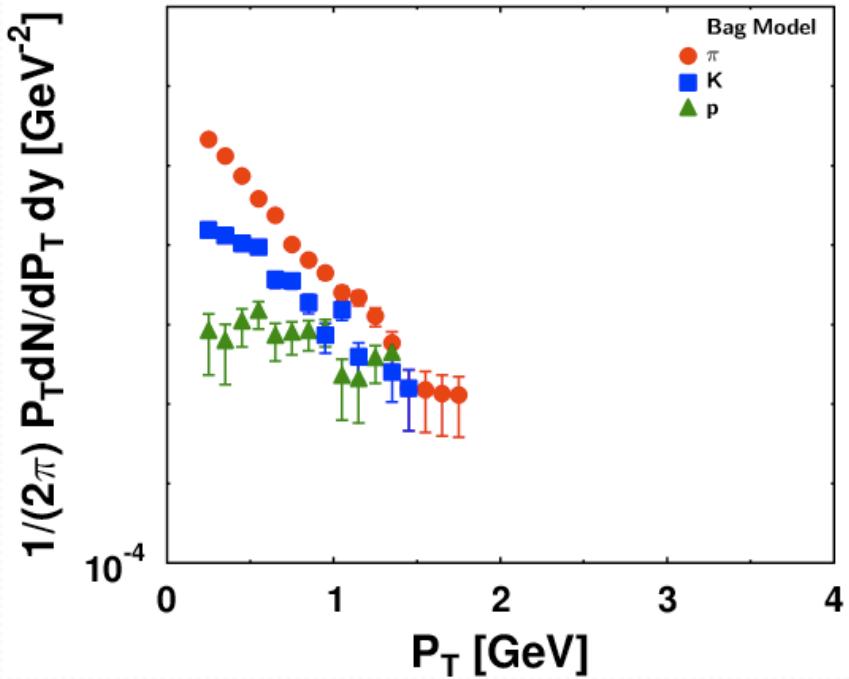
Switching temperature: 150 MeV

# $P_T$ Spectra

QCD critical point



Bag Model

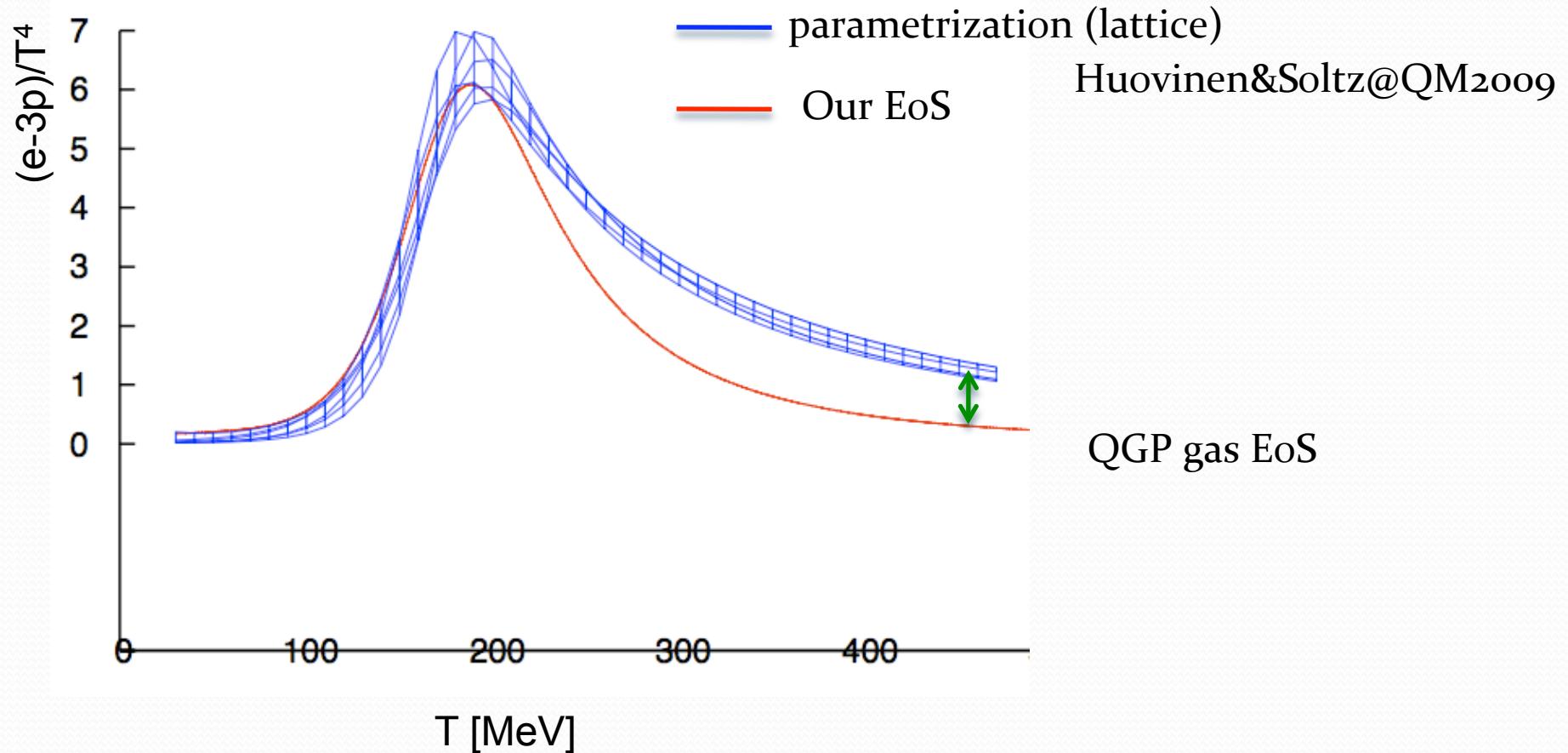


Because of focusing effect

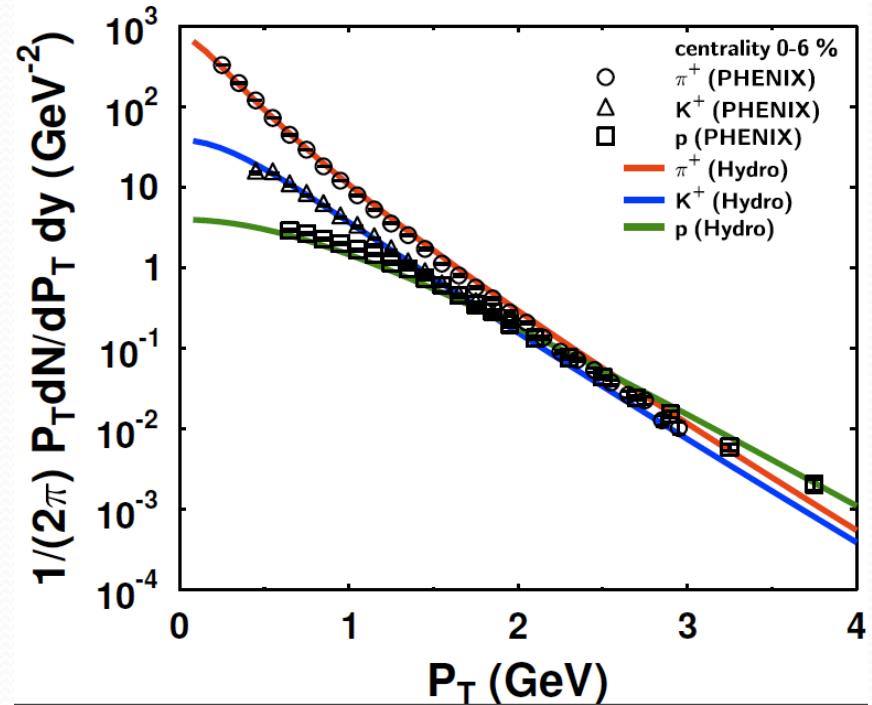
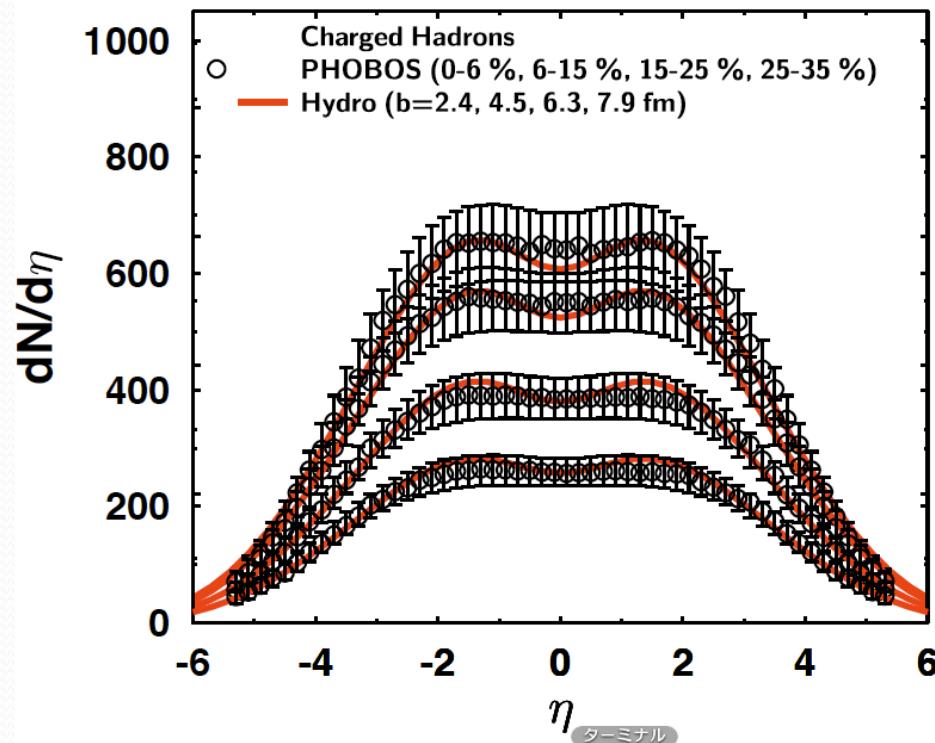
$$\text{At } T_{\text{SW}} \quad \langle \mu_B \rangle_{\text{QCP}} > \langle \mu_B \rangle_{\text{BG}} \iff \frac{p}{\pi_{\text{QCP}}} > \frac{p}{\pi_{\text{BG}}}$$

# At RHIC

- Smooth connection to lattice data at  $\mu=0$



# At RHIC



# Summary

Signal of QCD critical point:  
 $\bar{p}/p$  ratio as a function of transverse momentum

- Towards quantitative analyses
  - EoS with QCP  
Focusing effects in isentropic trajectories
- 3D Hydro + UrQMD model
  - $P_T$  spectra, hadron ratios

