Fluid Dynamics with a Critical Point

Marlene Nahrgang

Institut für Theoretische Physik,
Goethe-Universität Frankfurt am Main

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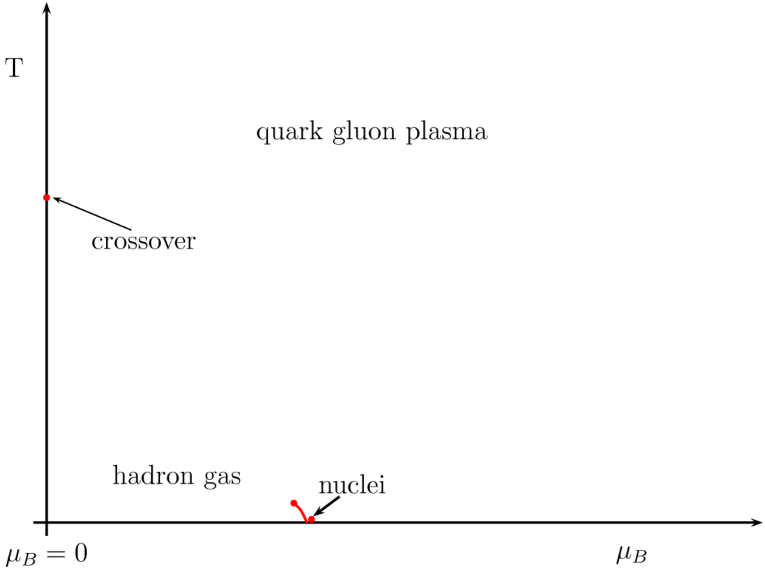
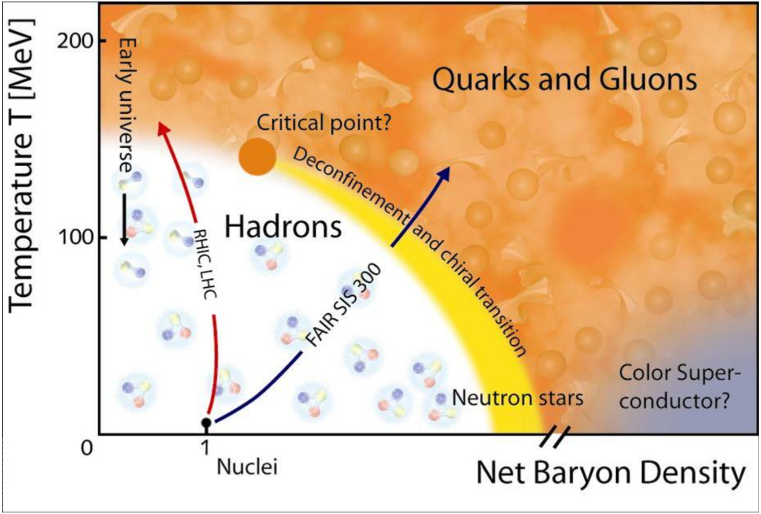
The QCD Phase Diagram



The QCD Phase Diagram
-suggested-



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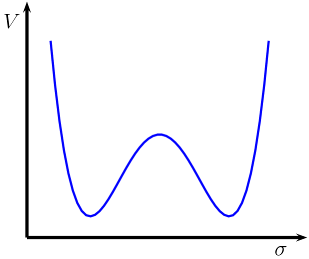
Phase Transitions
thermodynamically

first order phase transition

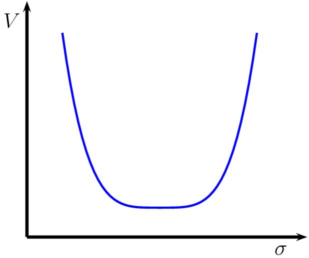
- two degenerate minima separated by a barrier
- nucleation
- spinodal decomposition

(I.N.Mishustin, Phys. Rev. Lett. 82 (4779) 1999; Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports 389 (2004) 263)



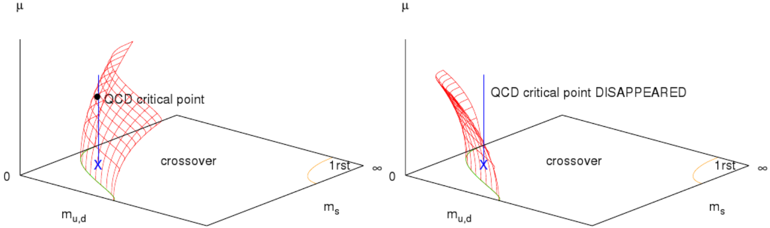
critical point

- $m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \longrightarrow 0$
- correlation length diverges $\xi = \frac{1}{m_\sigma} \longrightarrow \infty$
- universality classes (for QCD: 3d Ising model) $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- critical opalescence



strictly valid only for $\mu_B = 0$
there are methods to explore the $T - \mu_B$–plane

- reweighting (Fodor, Katz): $\mu_B^c = 360 \pm 40$ MeV
- imaginary μ_B (de Forcrand, Philipsen): $\mu_B^c > 500$ MeV



(de Forcrand, Philipsen, hep-lat/0607017)

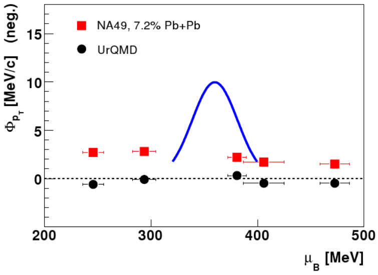
- radius of convergence of the Taylor expansion of the pressure (Gavai, Gupta, RBC-Bielefeld): 250 MeV $< \mu_B^c <$ 400 MeV

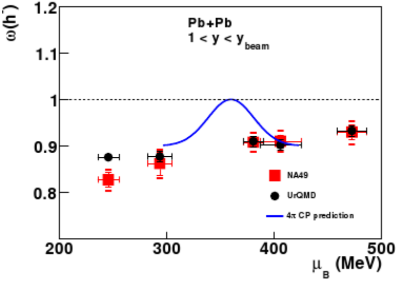
coupling to the order parameter of chiral symmetry (σ-field)
$\Rightarrow$ non-monotonic fluctuations in pion and proton multiplicities

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

(M.Stephanov, K.Rajagopal, E.Shuryak, Phys. Rev D60,114028,1999),

experimental signatures:





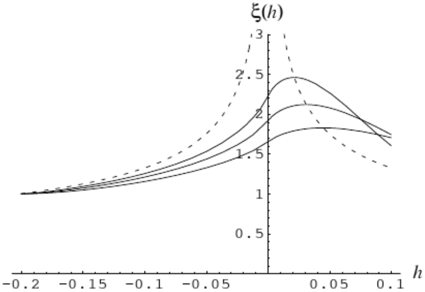
(NA49 collaboration J.Phys.G35:104091,2008)

near a critical point: long relaxation times
$\Rightarrow$ the system is driven out of equilibrium

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)})$$

with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
(dynamic) critical exponent

$\Rightarrow \xi \sim 1.5 - 2$ fm



(B.Berdnikov, K.Rajagopal, Phys. Rev. D61,105017,2000; D.T.Son, M.Stephanov, Phys. Rev. D70:056001,2004; M.Asakawa, C.Nonaka, Nucl. Phys. A774,753-756,2006)

Motivation

experimental situation:

- finite system
- finite life time
- non-equilibrium effects
- dynamics
- observables in a finite phase space

$\Rightarrow$ couple a hydrodynamic quark fluid to field equations of the order parameter
$\Rightarrow$ study dynamic fluctuations through the phase transition

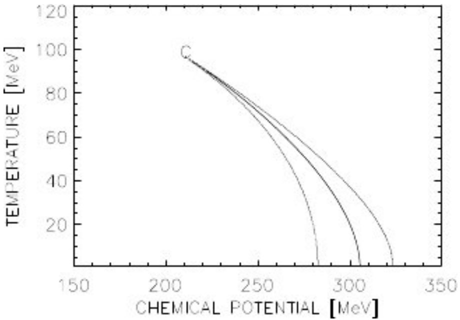
The Linear Sigma Model

$$\mathcal{L} = \overline{q}\left[i\gamma^\mu\partial_\mu - g\left(\sigma + i\gamma_5\tau\vec{\pi}\right)\right]q + 1/2\left(\partial_\mu\sigma\right)^2 + 1/2\left(\partial_\mu\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

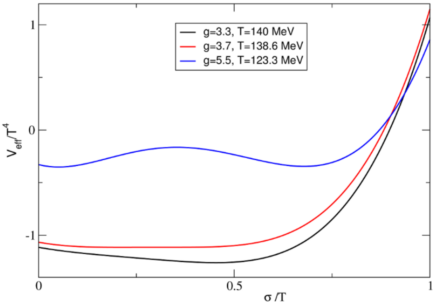
(M.Gell-Mann, M.Levy, Nuovo Cim. 16, 705,1960)

chiral symmetry spontaneously broken in the vacuum, where $\langle\sigma\rangle = f_\pi = 93\text{MeV}$ and $\langle\vec{\pi}\rangle = 0$ and explicitly broken by $h_q = f_\pi m_\pi^2$



(O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, Phys. Rev. C64, 045202,2001) grand-canonical partition function

$$\mathcal{Z} = \int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\vec{\pi} \exp\left[\int_0^{1/T} \mathrm{d}(it)\right] \int_V \mathrm{d}^3 x \mathcal{L}$$



grand-canonical potential at $\mu_B = 0$

$$V_{\text{eff}} = \Omega = -{}^T/_V \log \mathcal{Z}$$

$$= -d_q T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \log(1 + e^{-E/T})$$

$$+ U(\sigma, \vec{\pi})$$

with $E = \sqrt{p^2 + g^2\phi^2}$

classical equation of motion for the fields: $\phi = (\sigma, \vec{\pi})$

$$\partial_\mu \partial^\mu \phi + \frac{\delta U}{\delta \phi} = -g\rho_\phi$$

with the (pseudo-)scalar density

$$\rho_\phi = g\phi d_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{E} f_{FD}(p)$$

solved by a staggered leap-frog algorithm equations of relativistic fluid dynamics:

$$\partial_\mu(T^{\mu\nu}_{\text{fluid}} + T^{\mu\nu}_{\text{field}}) = 0 \;\Rightarrow\; \partial_\mu T^{\mu\nu}_{\text{fluid}} = g\rho_\phi\partial^\nu\phi$$

with the stress-energy tensor for an ideal fluid

$$T^{\mu\nu}_{\text{fluid}} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$$

equation of state from self-consistency conditions

$$\begin{aligned} e(\phi, T) &= T\frac{\partial p(\phi, T)}{\partial T} - p(\phi, T)\,, \\ p(\phi, T) &= -V_{\text{eff}}(\phi, T) + U(\phi) \end{aligned}$$

(K.Paech, A.Dumitru,H.Stöcker, Phys.Rev.C68:044907,2003)

Initial Conditions

energy density:

$$e(\vec{r}, t=0) = \begin{cases} e_{\mathrm{eq}} & \text{for } b^2x^2 + a^2y^2 < (ab)^2 \text{ and } |z| < l_z \\ 0 & \text{for } b^2x^2 + a^2y^2 > (ab)^2 \text{ or } |z| > l_z \end{cases}$$

Here $a = 3.5$ fm, $b \approx 5.8$ fm and $l_z = 6$ fm.
velocity profile:

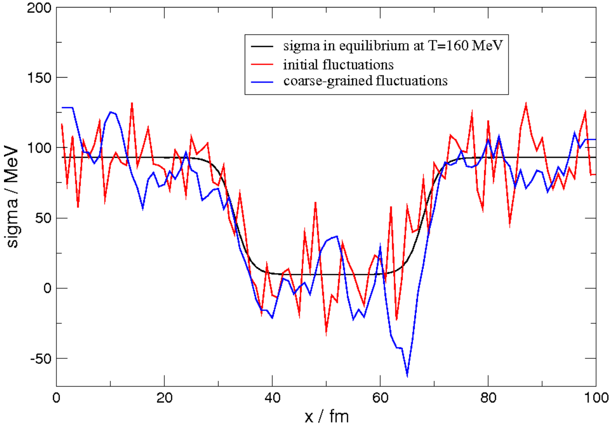
$$v_z(\vec{r}, t=0) = |z|/l_z \cdot v_{\mathrm{max}}$$

where $v_{\mathrm{max}} = 0.2$.
sigma field:

$$\sigma(\vec{r}, t=0) = f_\pi + \frac{\sigma_{\mathrm{eq}} - f_\pi}{(1+\exp((\tilde{r}-\tilde{R})/\tilde{a}))(1+\exp\left(((|z|-l_z)/\tilde{a}\right))}$$

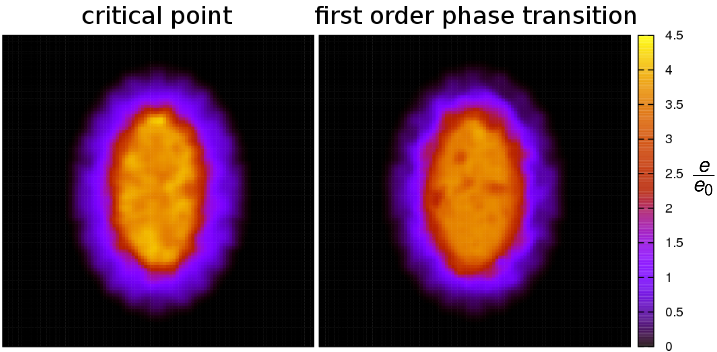
where $\tilde{a} = 0.3$ fm, $\tilde{r} = \sqrt{x^2+y^2}$, $\tilde{R} = ab\tilde{r}/\sqrt{b^2x^2+a^2y^2}$ for $\tilde{r} \neq 0$ and $\tilde{R} = a$ for $\tilde{r} = 0$.

start with Gaussian initial fluctuations in the σ field



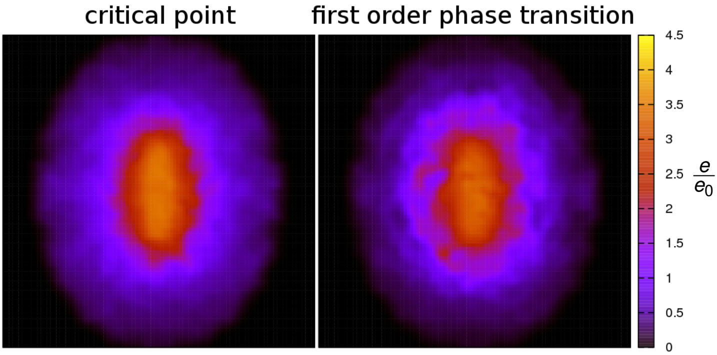
Energy Density

$t = 2$ fm



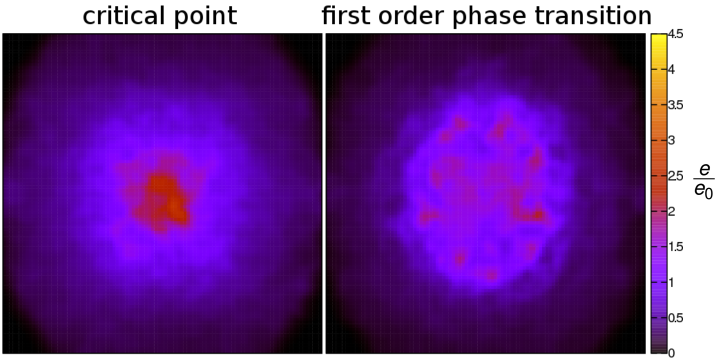
Energy Density

$t = 4.4$ fm



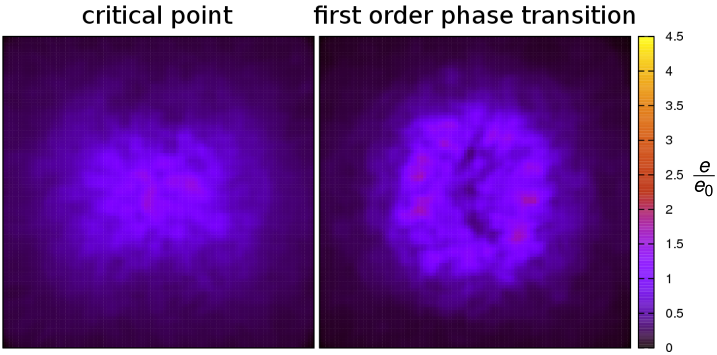
Energy Density

$t = 7.2$ fm

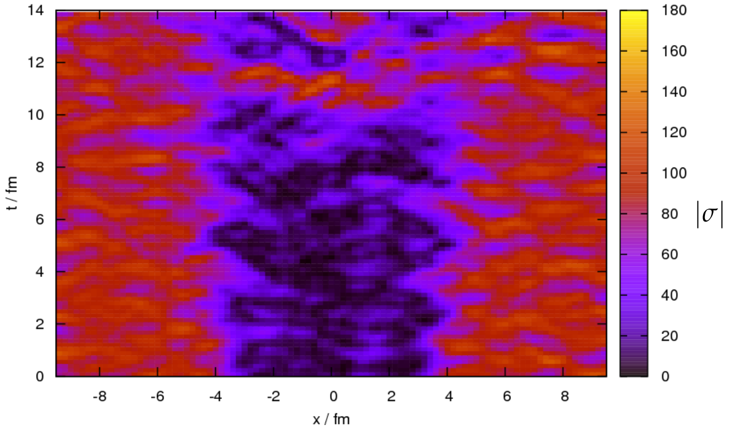


Energy Density

$t = 9.6$ fm

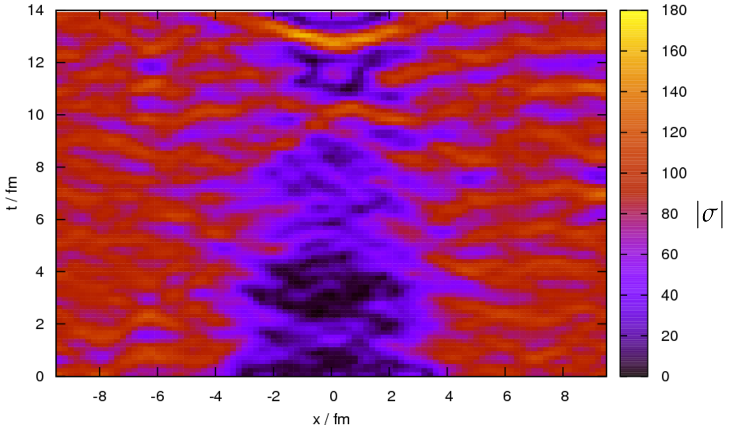


Chiral Field
first order phase transition



for a first order phase transition (g=5.5), ($m_q = g|\sigma|$)

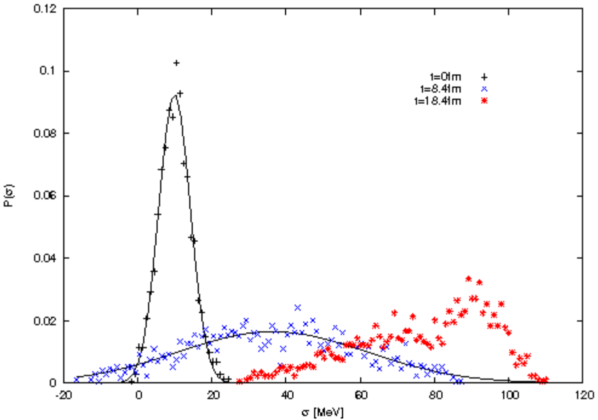
Chiral Field
critical point



near a critical point (g=3.7), ($m_q = g|\sigma|$)

Field Distributions

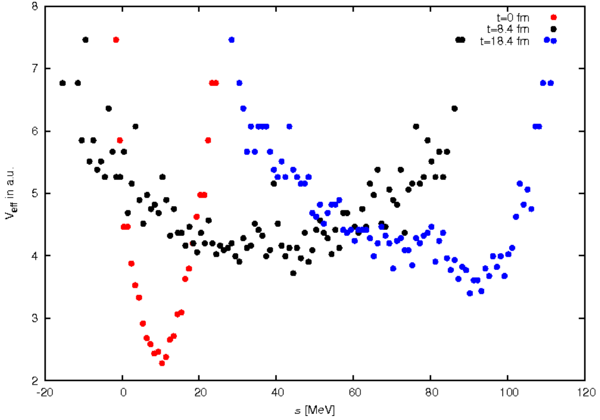
critical point



at $t = 0\mathrm{fm}$: Gaussian distribution with $v = 4.2\mathrm{MeV}$
at $t = 8.4\mathrm{fm}$: Gaussian distribution with $v = 25.5\mathrm{MeV}$
at $t = 18.4\mathrm{fm}$: not Gaussian anymore

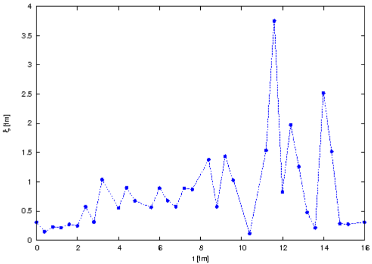
Dynamic Effective Potential
critical point

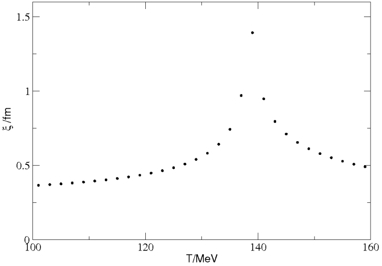
$$V_{eff}^{dyn} = a_0 + a_1\tilde{\sigma} + a_2\tilde{\sigma}^2 + ... + a_n\tilde{\sigma}^n \quad \text{with} \quad \tilde{\sigma} = \sigma - \sigma_{eq}$$



$$\frac{1}{\xi^2} = \frac{\partial^2 V_{eff}}{\partial \tilde{\sigma}^2}\Big|_{\tilde{\sigma}=0}$$

- ξ stays finite
- at $t = 0\mathrm{fm}$:
  $\xi \sim 0.31\mathrm{fm} \Rightarrow$
  $m_\sigma \sim 636.5\mathrm{MeV}$
- at $t = 8.4\mathrm{fm}$:
  $\xi \sim 1.4\mathrm{fm} \Rightarrow$
  $m_\sigma \sim 142.9\mathrm{MeV}$
- at $T = 139$ MeV:
  $\xi \sim 1.4\mathrm{fm}$





The Search for the Critical Point

Conclusions

- formation of high energy density droplets due to a first order phase transition
- fluctuations of the sigma field for trajectories near a critical point
- effects of critical slowing down, $\xi \sim 1.5$ fm
- the dynamical and non-equilibrium effects of a chiral phase transition can be investigated within a hydrodynamic model

Outlook

- pion spectra from the fluid and from the decay of the sigma field
- dissipative and Langevin terms in the dynamics of the sigma field
- confinement-deconfinement phase transition (Polyakov loop)

upcoming experiments:

- energy scan @RHIC (**2010**)
- CBM@FAIR (2015)

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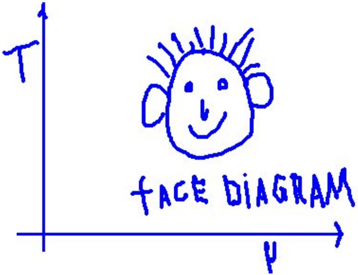

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