Measuring flow: details that matter

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Outline

1. Definition of flow (againÉ).
   Non-flow and flow fluctuations.
   Different methods of measuring flow.

2. Fluctuation in the initial conditions.
   Anisotropic flow as a system respond
   to anisotropic initial conditions.

3. Flow fluctuations in the Gaussian model
   and beyond.

4. Azimuthal correlations and flow (use and
   misuse of flow).

Anisotropic flow. Definitions: vÕs

Anisotropic flow ||\
anisotropies in single particle inclusive distributions (correlations) with respect to the reaction plane

Fourier decomposition of single particle inclusive spectra

$$\frac{d^3N}{dp_t\, dy d\Delta\varphi} = \frac{d^2N}{dp_t\, dy}\frac{1}{2\pi}\left(1 + 2v_1\cos\left(\Delta\varphi\right) + 2v_2\cos\left(2\Delta\varphi\right) + \ldots\right)$$

$\Delta\varphi = \varphi - \Psi_{RP}$

Directed flow

Elliptic flow

Note that the definition of anisotropic flow involves knowledge of the ÒtrueÓ reaction plane.

Term ÒflowÓ Ð not necessarily ÒhydroÓ flow Ð used only to emphasize the collective behavior ←→ multiparticle azimuthal correlation. (event anisotropy?)
ÒCollectiveÓ Ð all particles are correlated to the same reaction plane.

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PRL 103, 042302 (2009) PHYSICAL REVIEW LETTERS week ending 24 JULY 2009

Exposing the Noncollectivity in Elliptic Flow

Jinfeng Liao* and Volker Koch†

Nuclear Science Division, Lawrence Berkeley National Laboratory, MS70R0319, 1 Cyclotron Road, Berkeley, California 94720, USA

(Received 18 March 2009; published 24 July 2009)

We show that backward-forward elliptic anisotropy correlation provides an experimentally accessible observable which distinguishes between collective and noncollective contributions to the observed elliptic anisotropy v_2 in relativistic heavy ion collisions. The measurement of this observable will reveal the momentum scale at which collective expansion seizes and where the elliptic anisotropy is dominated by (semi)hard processes.

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The situation is more complicated for 2-particle spectra. Not discussed here.

Methods Ð just mention a few

No discussion of the detector effects

$$\frac{d^3N}{dp_t\,dy\,d\Delta\varphi} = \frac{d^2N}{dp_t\,dy}\frac{1}{2\pi}\left(1 + 2v_1\cos(\Delta\varphi) + 2v_2\cos(2\Delta\varphi) + \ldots\right)$$

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Directed flow

Elliptic flow

2-particle correlations, $v_n\{2\}$

$$v_n\{2\}^2 = \langle\cos[n(\phi_1 - \phi_2)]\rangle = \langle u_{n,1}u^*_{n,2}\rangle$$

$u_n \equiv e^{in\phi}$ unit flow vector

"Standard" method, $v_n\{EP\}$

$$Q_{n,x} = \sum_i w_i\cos(n\phi_i) = Q_n\cos(n\Psi_n),$$

$$Q_{n,y} = \sum_i w_i\sin(n\phi_i) = Q_n\sin(n\Psi_n),$$

$$v_n^{obs}(p_T, y) = \langle\cos[n(\phi_i - \Psi_n)]\rangle$$

$$\mathcal{R}_n = \langle\cos[n(\Psi_n - \Psi_{RP})]\rangle$$

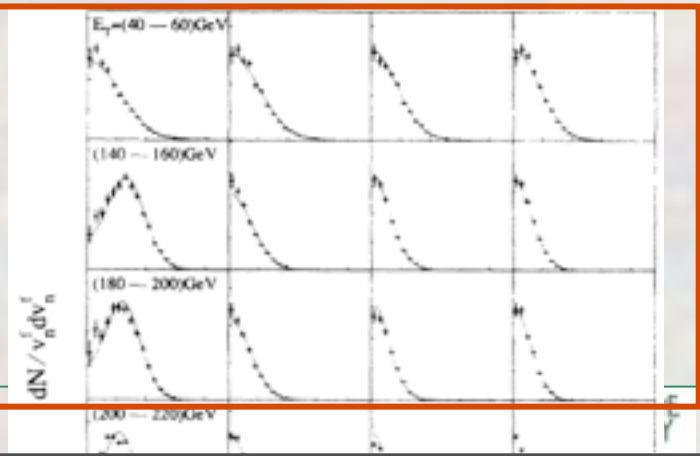
$$\mathcal{R}_{n,sub} = \sqrt{\langle\cos[n(\Psi_n^A - \Psi_n^B)]\rangle}$$

$$v_n = \frac{v_n^{obs}}{\mathcal{R}_n}.$$

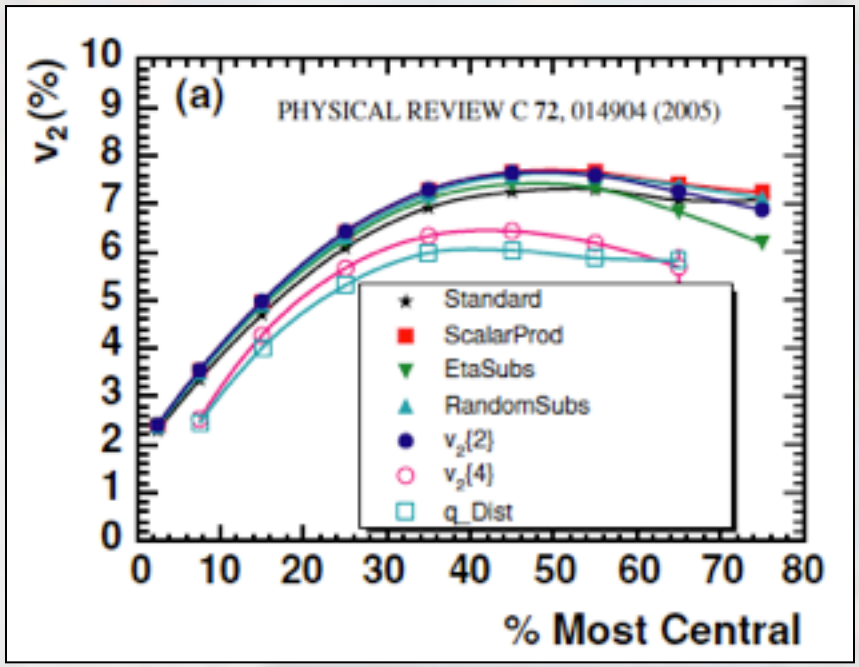
Fitting q-distribution, $v_n\{q\text{-dist}\}$

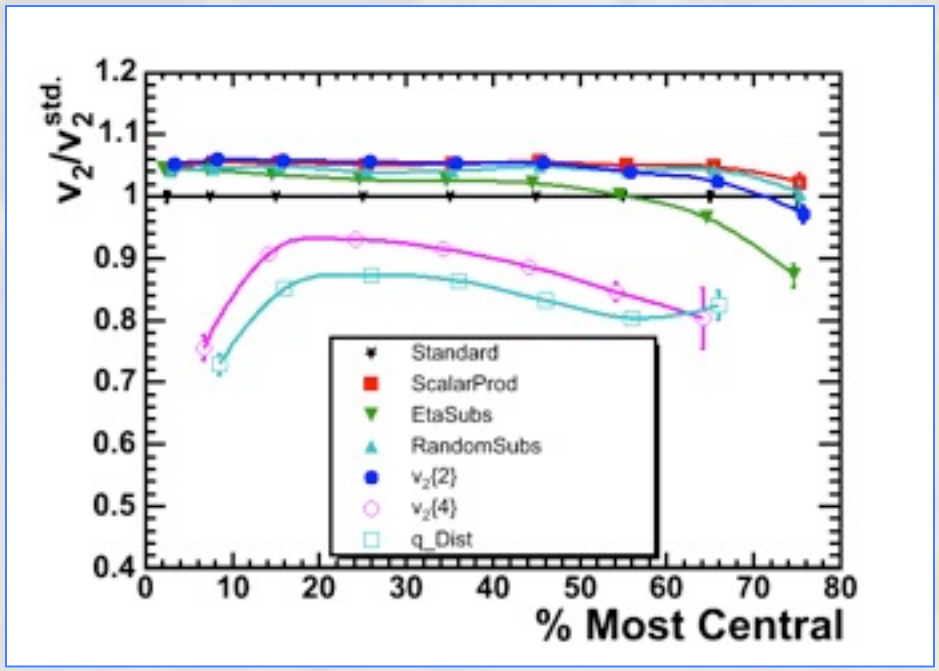
$$\frac{dN}{dq_n} = \frac{q_n}{\sigma_n^2}e^{-\frac{v_n^2 M + q_n^2}{2\sigma_n^2}} I_0\left(\frac{q_n v_n\sqrt{M}}{\sigma_n^2}\right)$$

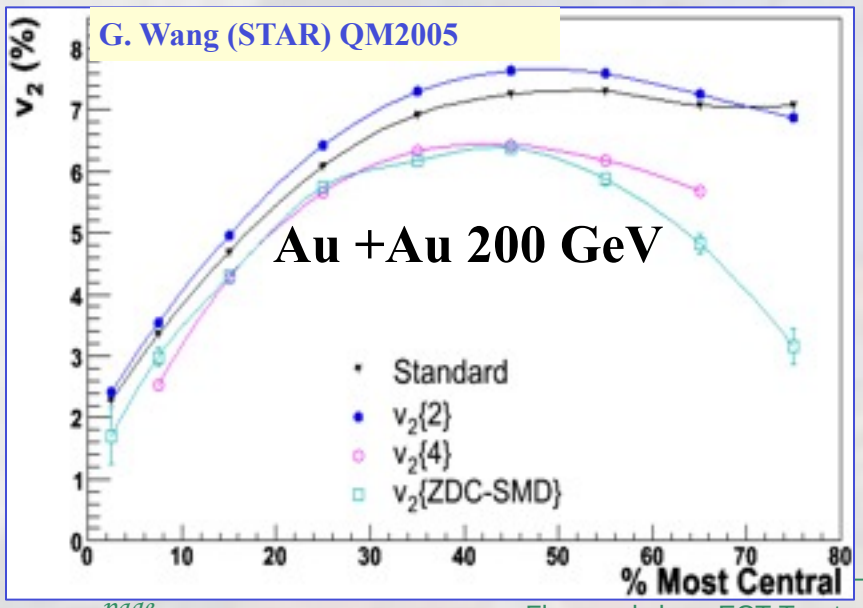
$q_n = Q_n/\sqrt{M}.$



Many methods ➔ many results?







Three bands?
Which one compare to the model?
Note three bands: Òv2[2]Ó, Òv2{EP}Ó, Òv2{4}Ó

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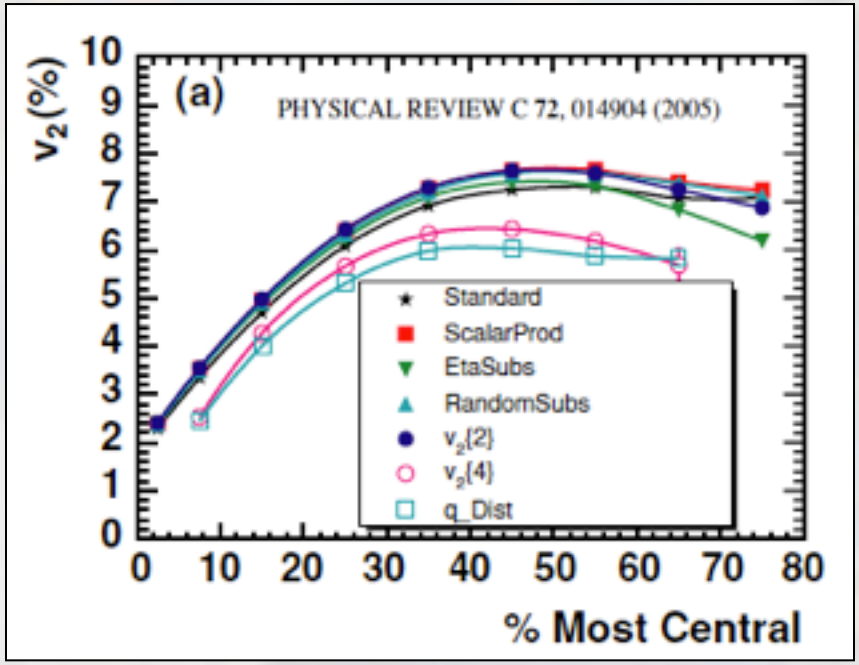

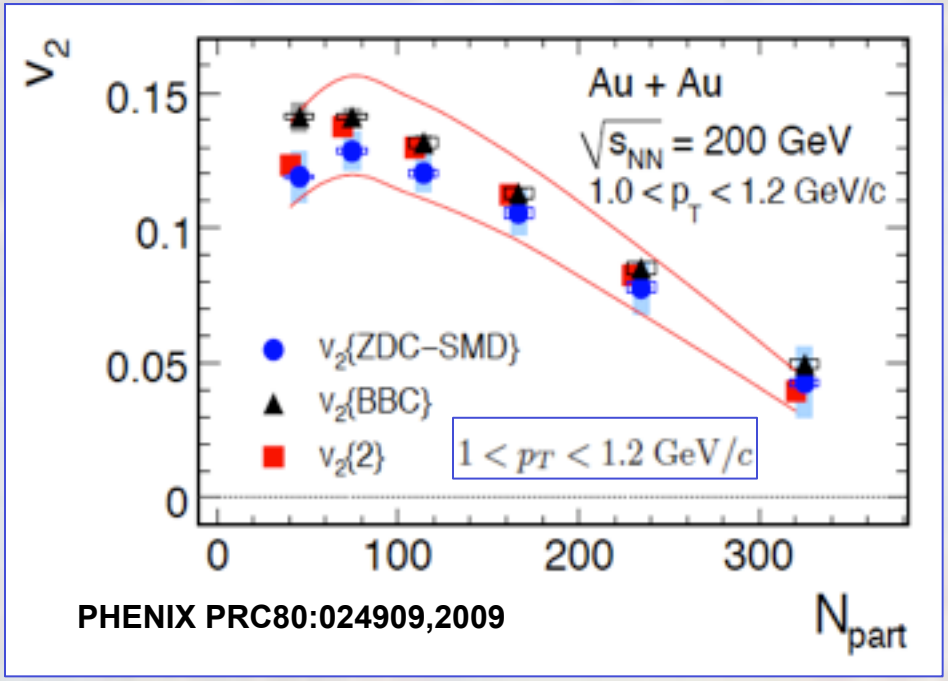

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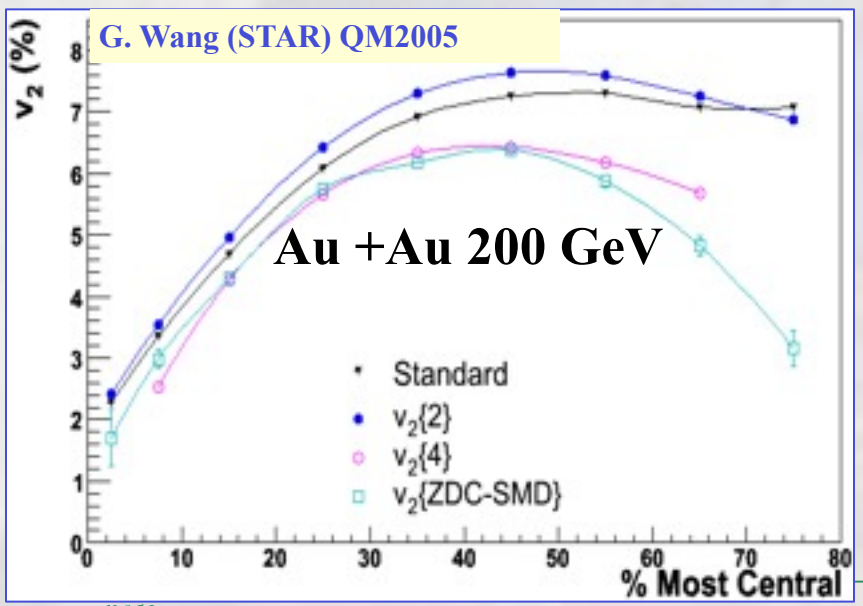

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End of the ideal world: ÒNon-flowÓ, flow fluctuations

$$\left\langle u_{n,a} u_{n,b}^{*} \right\rangle = \left\langle \cos\left[n\left(\phi_a - \phi_b\right)\right]\right\rangle \equiv v_{n,a} v_{n,b} + \delta_n$$

Flow "non-flowÓ

$$\delta_n \ll v_n^2 \quad ?$$

ÒNon-flowÓÐ azimuthal correlations of any other origin except the correlation with respect to the reaction plane.
It combines the possible contributions from resonance decay, inter and intra jet correlations, etc.
If general, two effects do not factorize; then the above equations would serve as a definition of Ònon-flowÓ, with ÒvÓÕs defined as on previous slides.

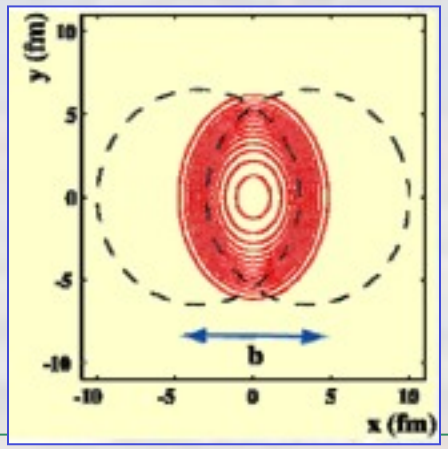
$$\left\langle v_{n,a} v_{n,b} \right\rangle \neq \left\langle v_{n,a} \right\rangle \left\langle v_{n,b} \right\rangle$$

Effect of flow fluctuations

An example: $v_2 \propto \varepsilon \;\rightarrow\; \sigma_v \propto \sigma_\varepsilon$

$$\sigma_\varepsilon^2 = \left\langle \varepsilon^2(b) \right\rangle - \left\langle \varepsilon(b) \right\rangle^2$$

Also, fluctuations in particle density (number of particles, area), etc.

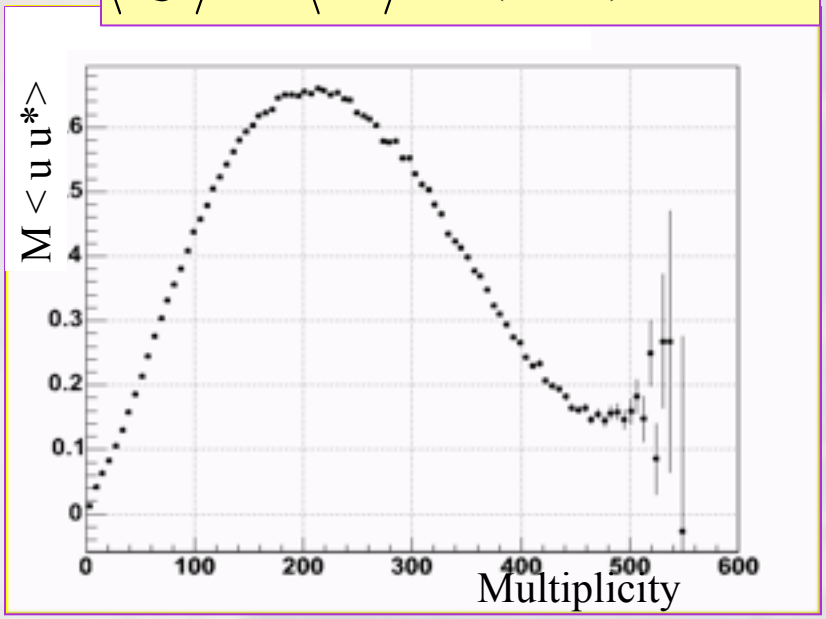


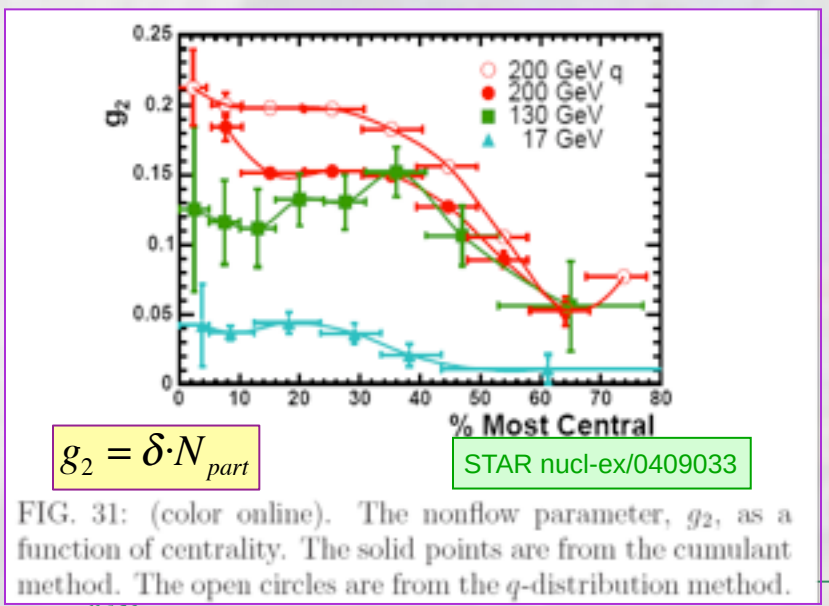
$$v_2 \equiv \left\langle \cos\left(2\left(\varphi_i - \Psi_{RP}\right)\right)\right\rangle$$

$$\varepsilon = \frac{\left\langle y^2 - x^2 \right\rangle}{\left\langle y^2 + x^2 \right\rangle}$$

Non-flow estimates. Centrality dependence.

$$\left\langle uQ^{*}\right\rangle = M\left\langle uu^{*}\right\rangle = M(v^{2}+\delta) = Mv^{2}+\tilde{\delta}$$





$g_2 = \delta \cdot N_{part}$

STAR nucl-ex/0409033

FIG. 31: (color online). The nonflow parameter, g_2, as a function of centrality. The solid points are from the cumulant method. The open circles are from the q-distribution method.

For a superposition of independent NN collisions non-flow contribution in such a plot is a constant

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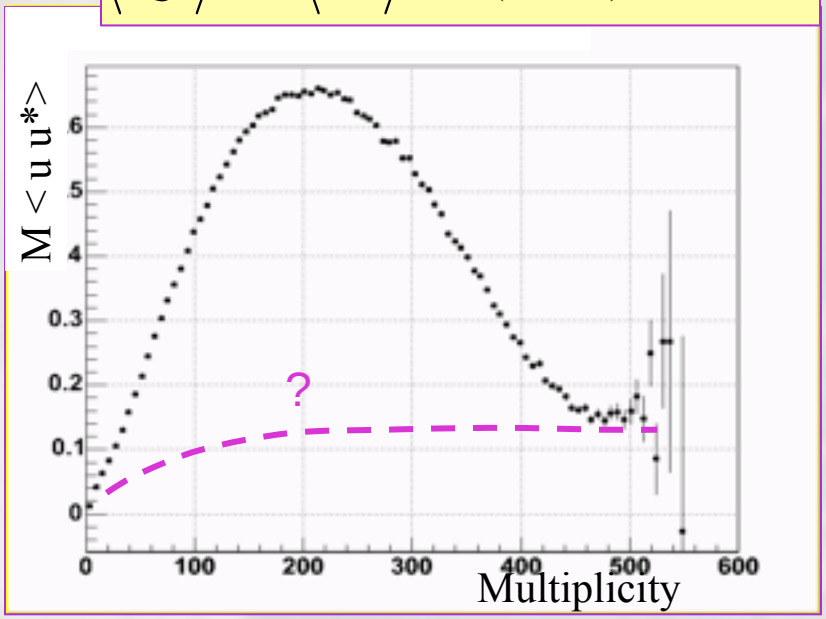

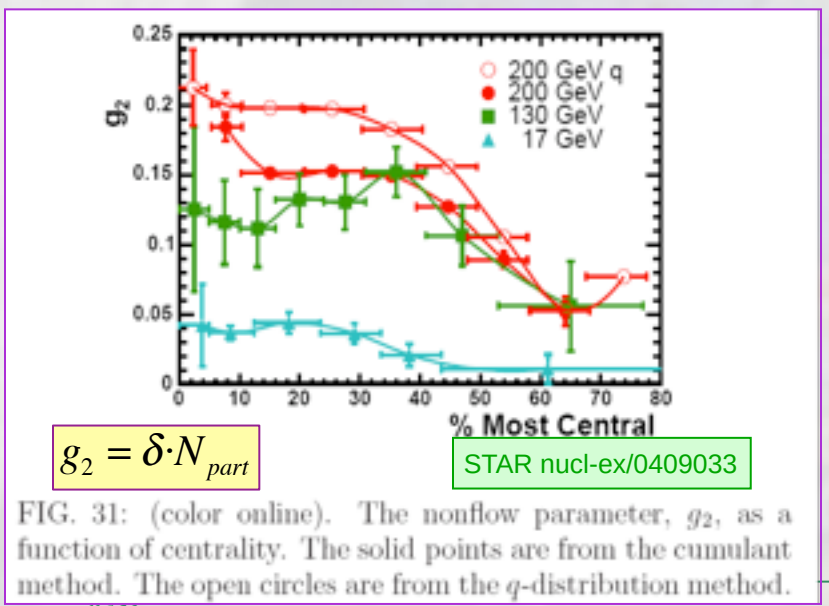

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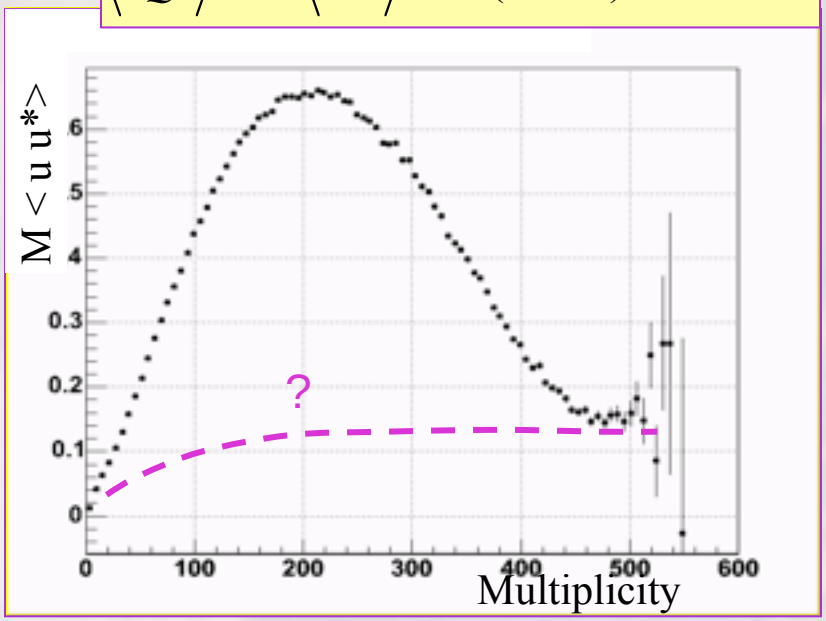

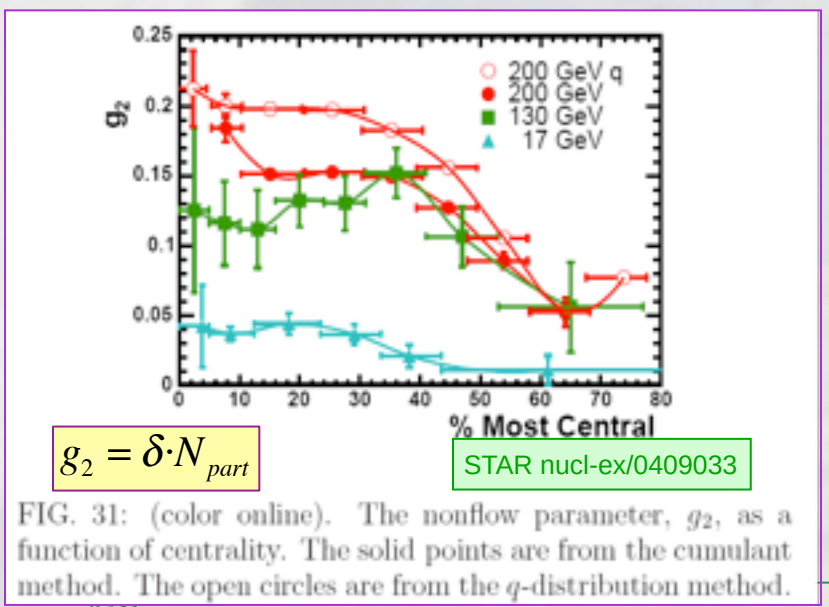

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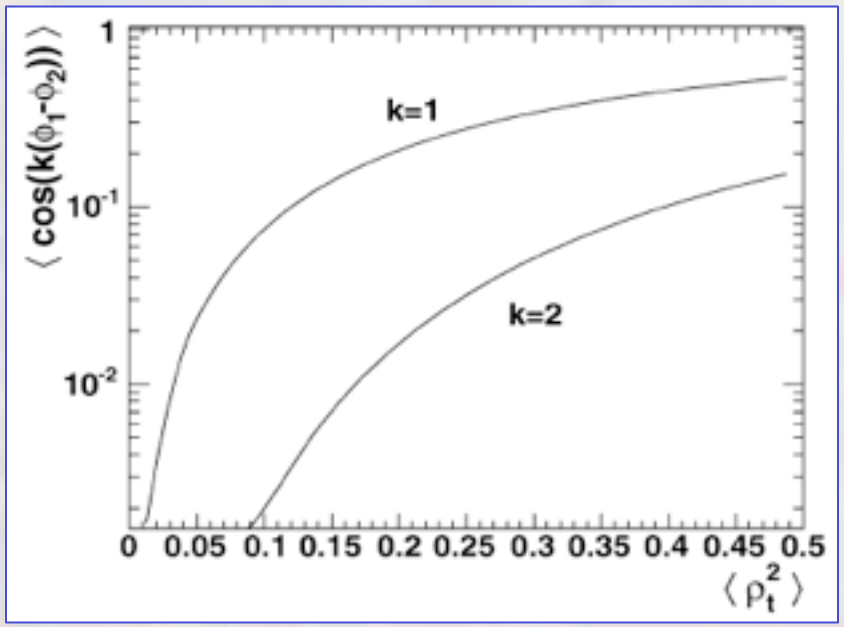
Correlations induced by transverse radial expansion

S.V. nucl-th/0312065

n=1, T=110 MeV



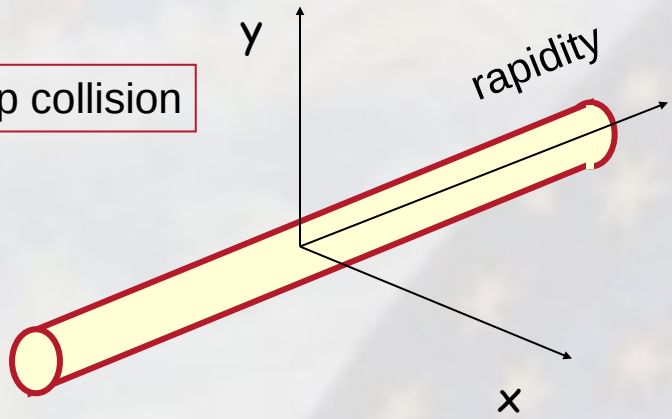
! - the large values of transverse flow, <!^{2}> > 0.25, would contradict Ònon-flowÓ estimates in elliptic flow measurements

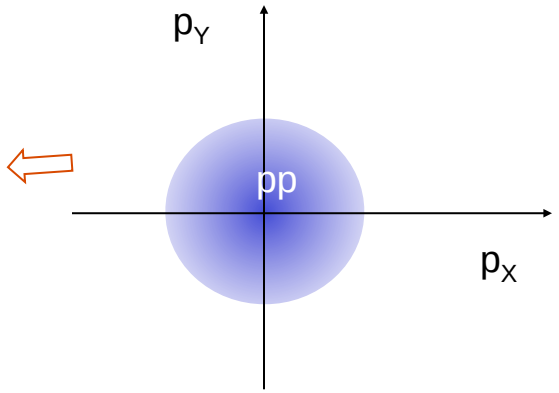
Radial expansion ➔ 2-part azimuthal correlations

[arXiv:nucl-th/0312065]

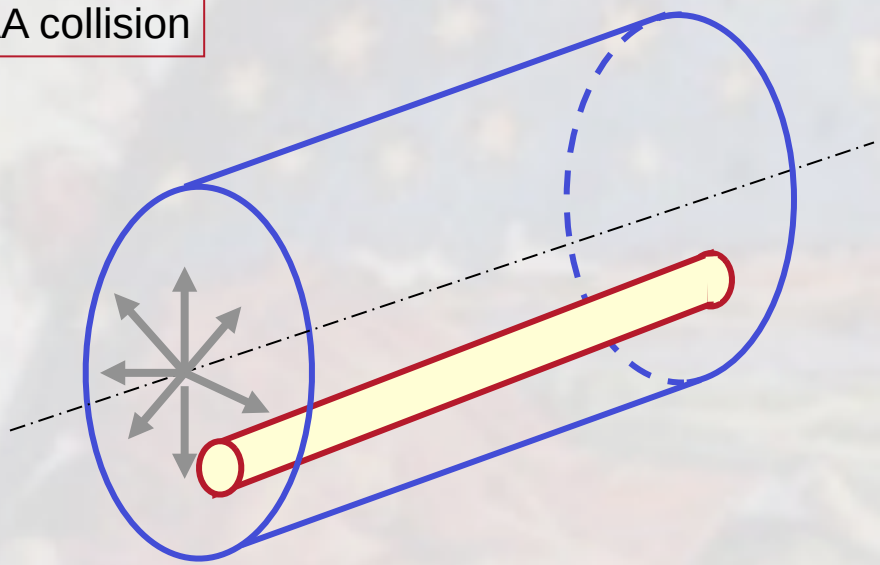
All particles produced in the same NN-collision (qq-string) experience the transverse radial ÒpushÓ that is
(a) in the same direction (leads to correlations in phi)
(b) the same in magnitude (➔ correlations in p_t)

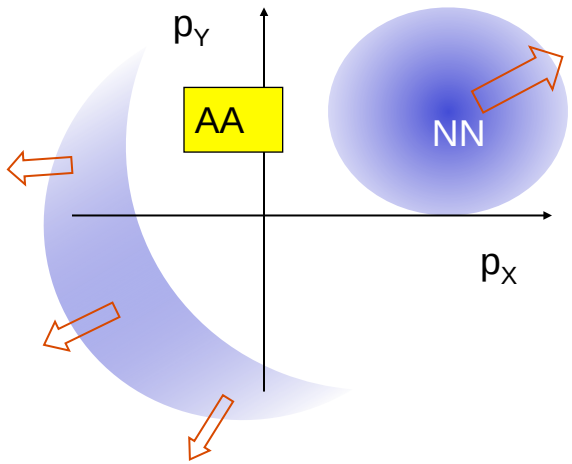
pp collision





AA collision

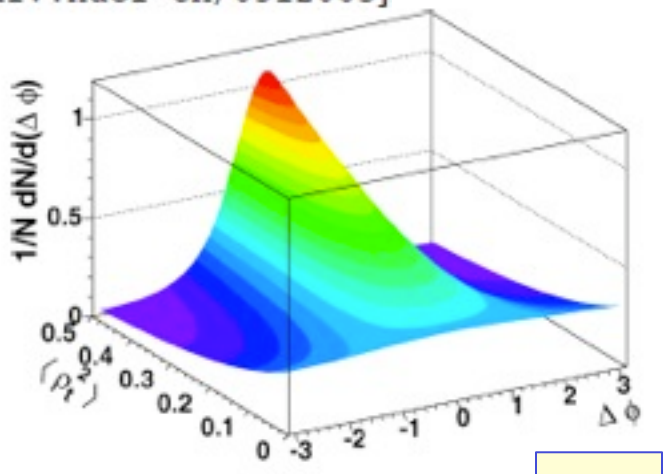




Radial expansion ➔ 2-part azimuthal correlations

S.A. Voloshin / Physics Letters B 632 (2006) 490–494 493

[arXiv:nucl-th/0312065]



$m = m_\pi$

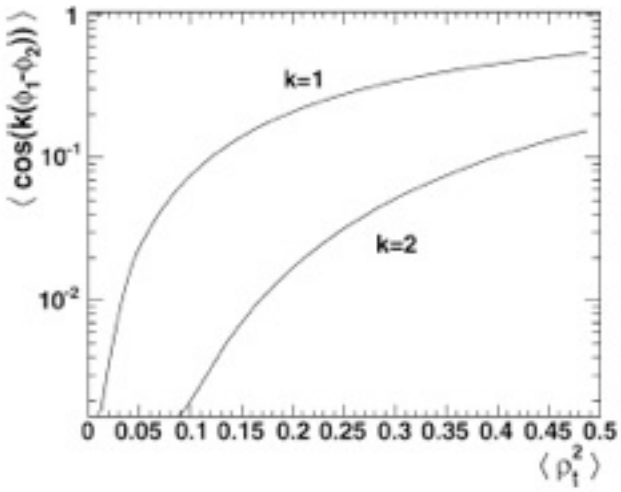
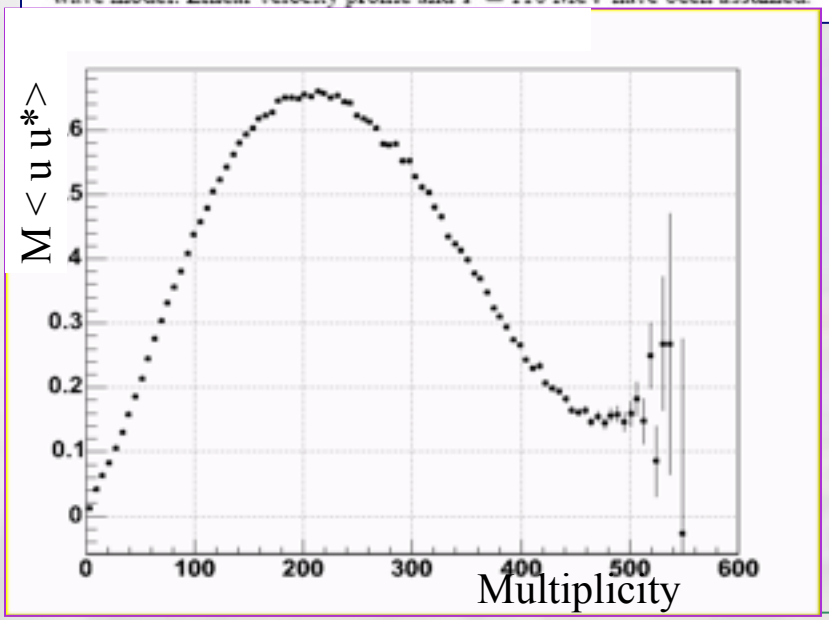


Fig. 3. (Color online.) Two pion $\Delta\phi$ distribution as function of $\langle\rho_t^2\rangle$ in the blast wave model. Linear velocity profile and $T = 110$ MeV have been assumed.

Fig. 4. The average values of $\cos(\Delta\phi)$ and $\cos(2\Delta\phi)$ for the distribution shown in Fig. 3.



Figures are shown for particles from the same NN collision. Dilution factor to be applied!

!!! - the large values of transverse flow, $v_2^2 > 0.25$, would contradict Ònon-flowÓ estimates in elliptic flow measurements

If the momentum conservation effect is approximated by the first harmonic, the amplitude can be estimated from the momentum of the tag particle + ÒassociatesÓ from the same NN collision

Non-flow: pp vs. AA

Consider correlations of particles from some ÒbinÓ with all particles from a ÒpoolÓ

$$Q = \sum_{i \in "pool"} u_i; \quad u_i = e^{i 2\varphi_i}$$

$$\langle u_b Q^* \rangle = (v_b v_p + \delta_{bp}^{AA}) M^{AA}$$

$$\delta_{bp}^{AA} \approx \frac{\delta_{bp}^{pp}}{N_{coll}} \approx \frac{\delta_{bp}^{pp} M^{pp}}{M^{AA}}$$

$$\langle u_b Q^* \rangle^{AA} \approx v_b v_p M^{AA} + \langle u_b Q^* \rangle^{pp}$$

Non-flow looks exactly the same in pp and AA ➔ Results - directly ÒcorrectibleÓ.

Notations:

v_b – Flow in a particular p_t/eta ÒbinÓ

v_p – Average flow in the pt/eta region used to define RP (or ÒpoolÓ of particles)

δ_{bp}^{pp} – Azimuthal correlations in *pp* ($\langle u_a u_b^* \rangle; \quad u \equiv e^{i2\phi}$)

δ_{pr}^{AA} – Non-flow part in 2-part azimuthal correlation in AA

N_{coll} – Number of Òindependent NN collisionsÓ, a la $N_{part}/2$.

Check if non-flow estimates/measurements reported or *Au+Au* are consistent with measurements in *pp*. (Expect the difference of the order of factor of <~2. Extra particles in jets ➔ non-flow contribution increases B-to-B jet suppression Ð non-flow goes down)

Use *pp* data to estimate non-flow effects in *Au+Au* when other methods do not work (high p_t, K and Lambda flow, etc.)

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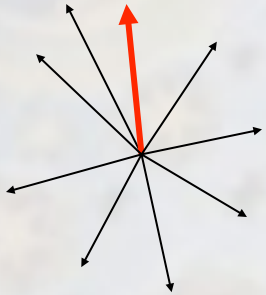

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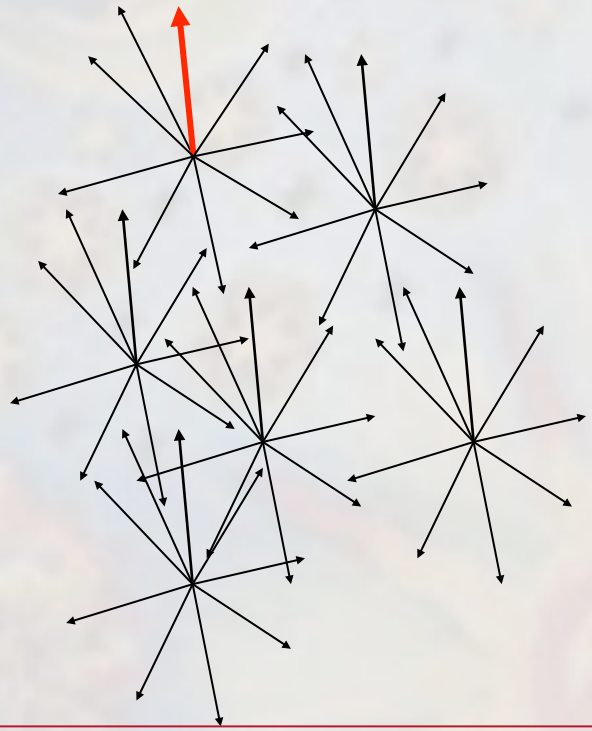

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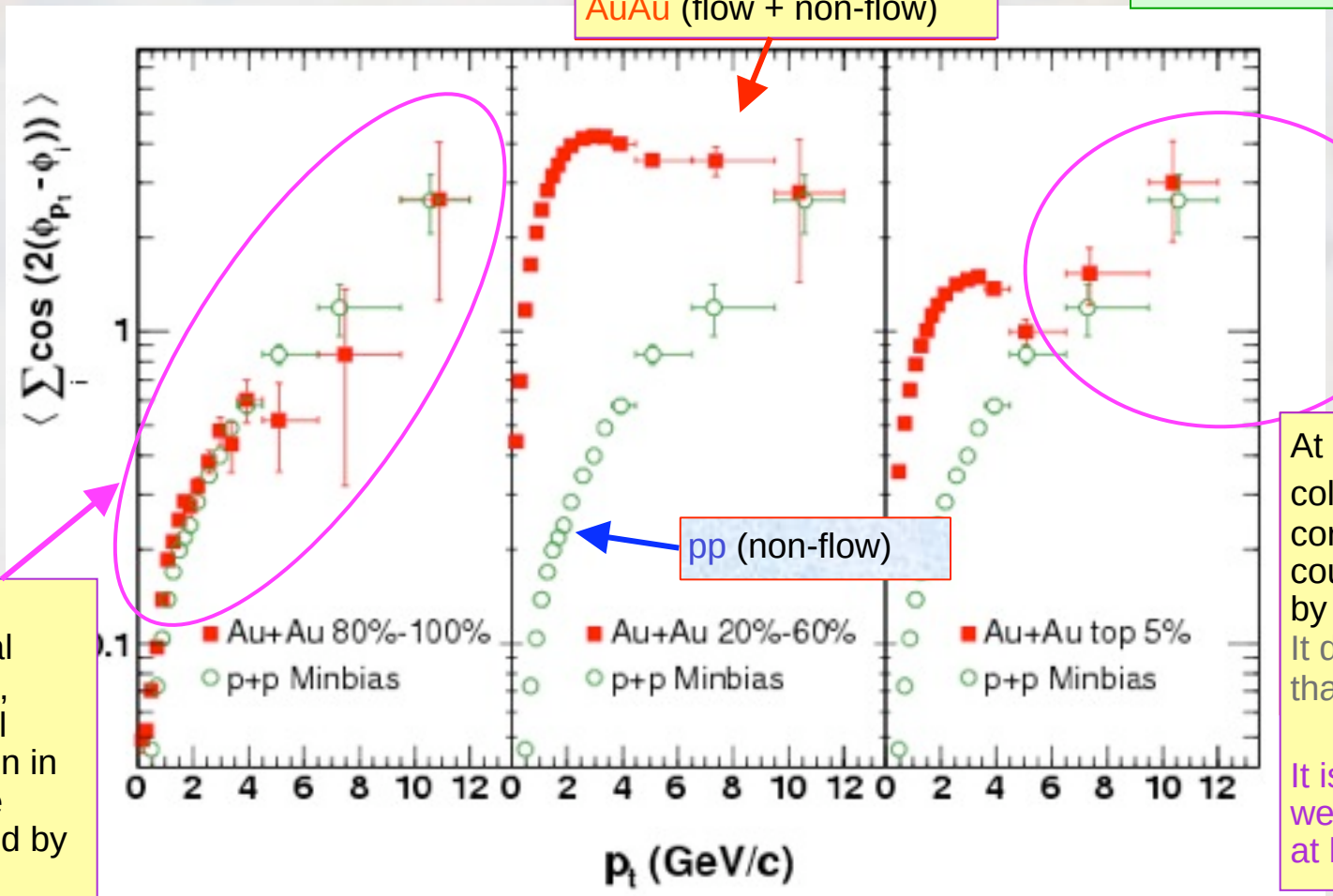
Comparison: pp & AuAu

STAR: PRL 93(2004)252301

AuAu (flow + non-flow)



pp (non-flow)

In VERY peripheral collisions, azimuthal correlation in AuAu are dominated by non-flow.

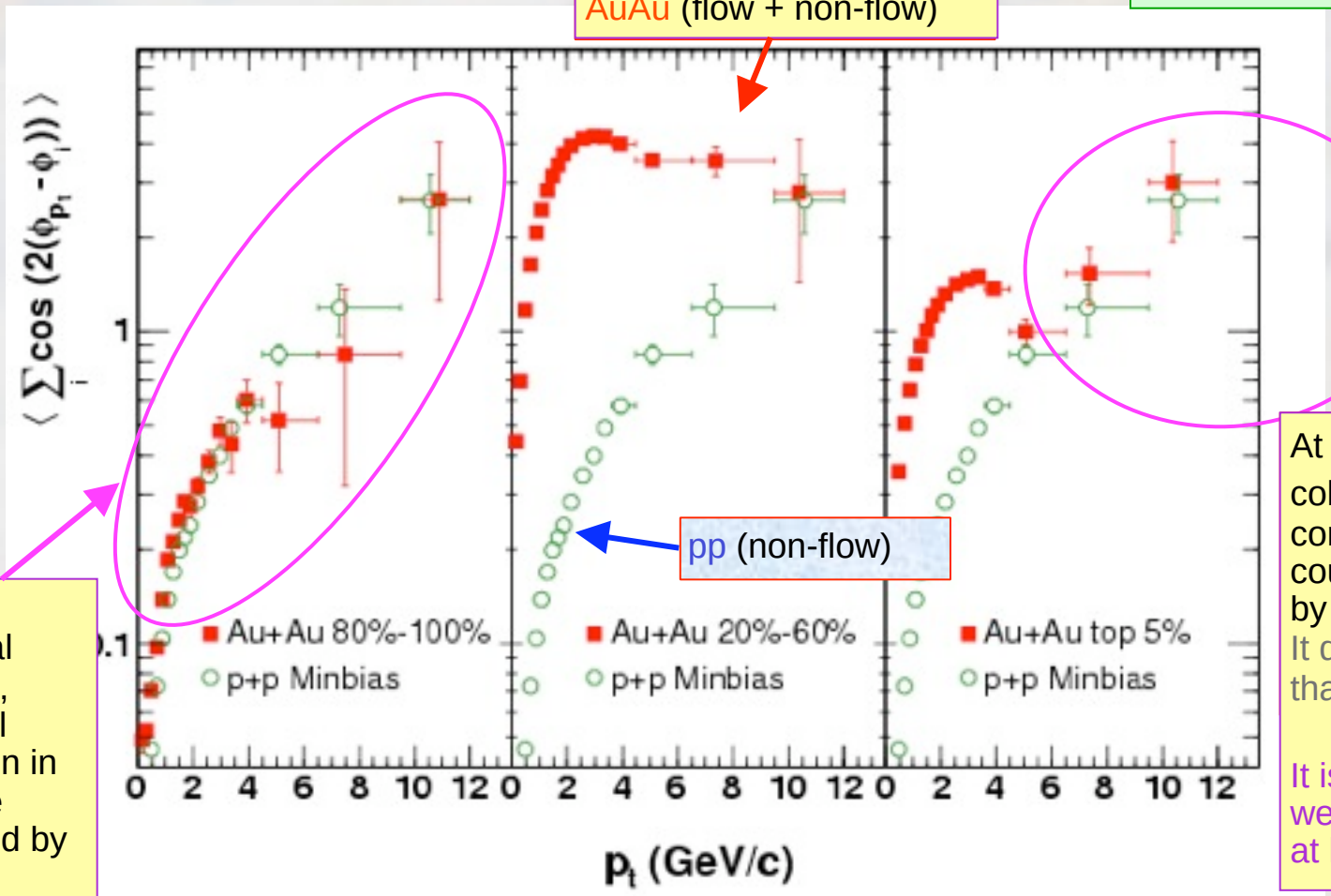
At high p_t in central collisions, azimuthal correlation in AuAu could be dominated by nonflow.
It does not mean that v_2 is zero!

It is remarkable how well they agree at high p_t !

Comparison: pp & AuAu

STAR: PRL 93(2004)252301

AuAu (flow + non-flow)



pp (non-flow)

In VERY peripheral collisions, azimuthal correlation in AuAu are dominated by non-flow.

At high p_t in central collisions, azimuthal correlation in AuAu could be dominated by nonflow.
It does not mean that v_2 is zero!

It is remarkable how well they agree at high p_t !

Analysis has to be continued: charge combination dependence, identified particlesÉ

Suppressing non-flow with multiparticle correlations

$$\langle u_1 u_2^* \rangle = v_2^2 + \delta; \quad u \equiv e^{i2\varphi}$$

$$\langle u_1 u_2 u_3^* u_4^* \rangle = v_2^4 + 2 \cdot 2\delta v_2^2 + 2\delta^2$$

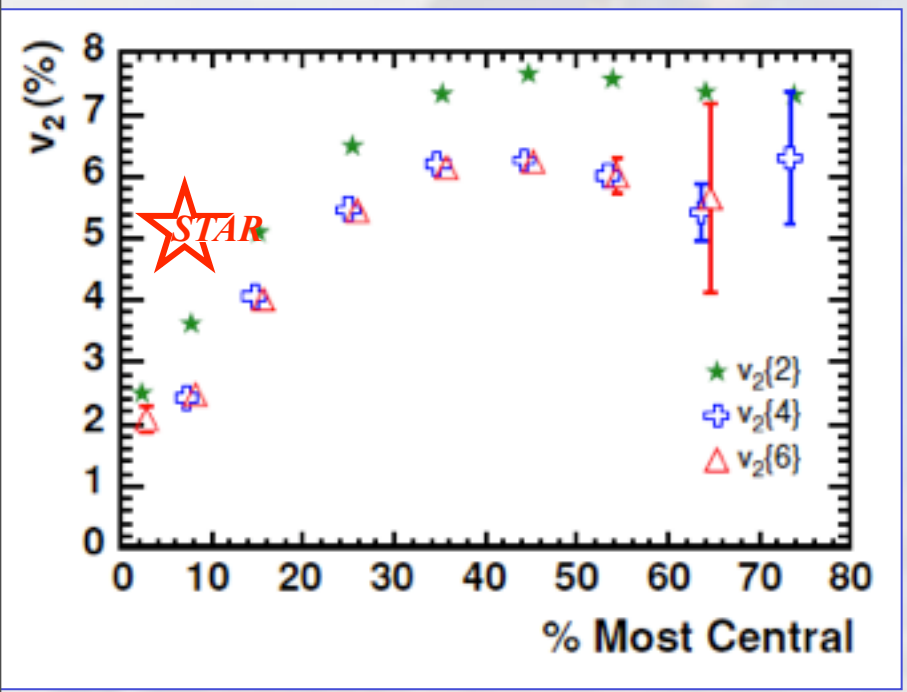
flow*nflow + nflow*flow

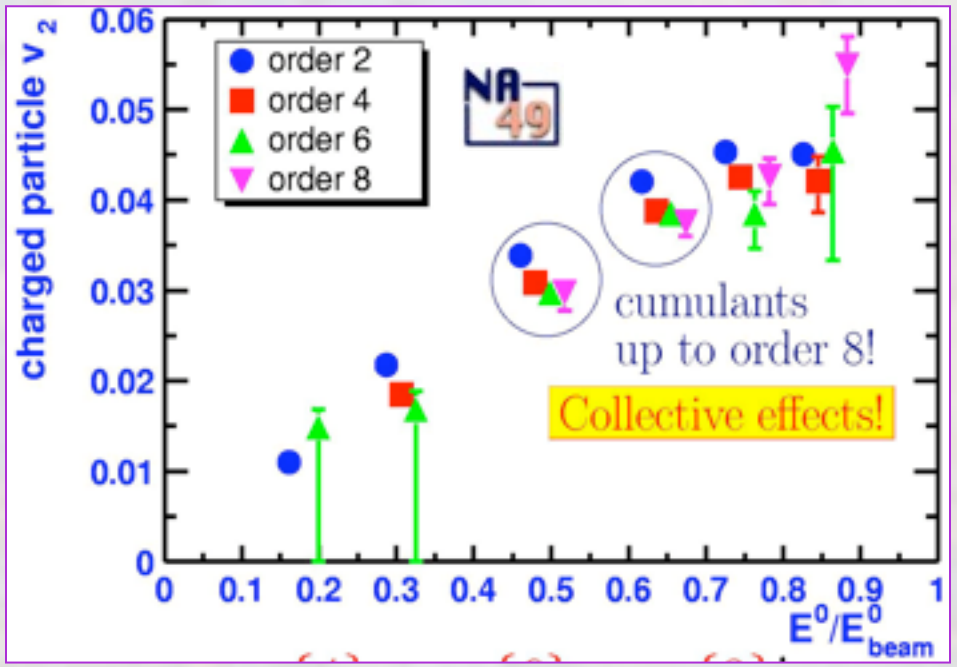
(1,3)(2,4)+(1,4)(2,3)

$$\langle u_1 u_2 u_3^* u_4^* \rangle - 2\langle u_1 u_2^* \rangle = -v_2^4$$

$$\langle u_i u_j u_k^* u_l^* \rangle = \left\langle \frac{1}{N(N-1)(N-2)(N-3)} \left(Q^2 Q^{*2} - N \right) - 2N(N-1) - 4(N-2)(QQ^* - N) - 2(Q^2 Q_2^* - N) + (Q_2 Q_2^* - N) \right\rangle$$

Application: Generating functions (Borghini, Ding, Ollitrault)
+ **many other ways**





Multiparticle correlations are good in removing non-flow, but É flow fluctuationsÉ

Non-flow or Fluctuations?

$$\langle u_1 u_2^* \rangle = v_2^2 + \delta; \quad u \equiv e^{i 2\phi}$$
$$\langle u_1 u_2 u_3^* u_4^* \rangle = v_2^4 + 2 \times 2\delta v_2^2 + 2\delta^2$$

Correct if v is constant in the event sample

Non-flow

$$\langle u_1 u_2^* \rangle = \langle v^2 \rangle = \langle v \rangle^2 + \sigma_v^2;$$
$$\langle u_1 u_2 u_3^* u_4^* \rangle = \langle v_2^4 \rangle$$

Should be used even in a case of "=0

Flow fluctuations

$$v_2\{2\} \equiv \sqrt{\langle uu^* \rangle} = \sqrt{\langle v_2^2 \rangle + \delta} \approx \langle v \rangle \left(1 + \frac{\delta + \sigma_v^2}{2\langle v \rangle^2} \right)$$

$$v_2\{4\} \equiv \sqrt[4]{2\langle uu^* \rangle^2 - \langle uuu^*u^* \rangle} = \sqrt[4]{\langle v_2^2 \rangle^2 - \sigma_{v_2^2}^2} \approx \langle v \rangle \left(1 - \frac{\sigma_v^2}{2\langle v \rangle^2} \right)$$

In this approximation, v is between $v\{4\}$ and $(v\{2\} + v\{4\})/2$.

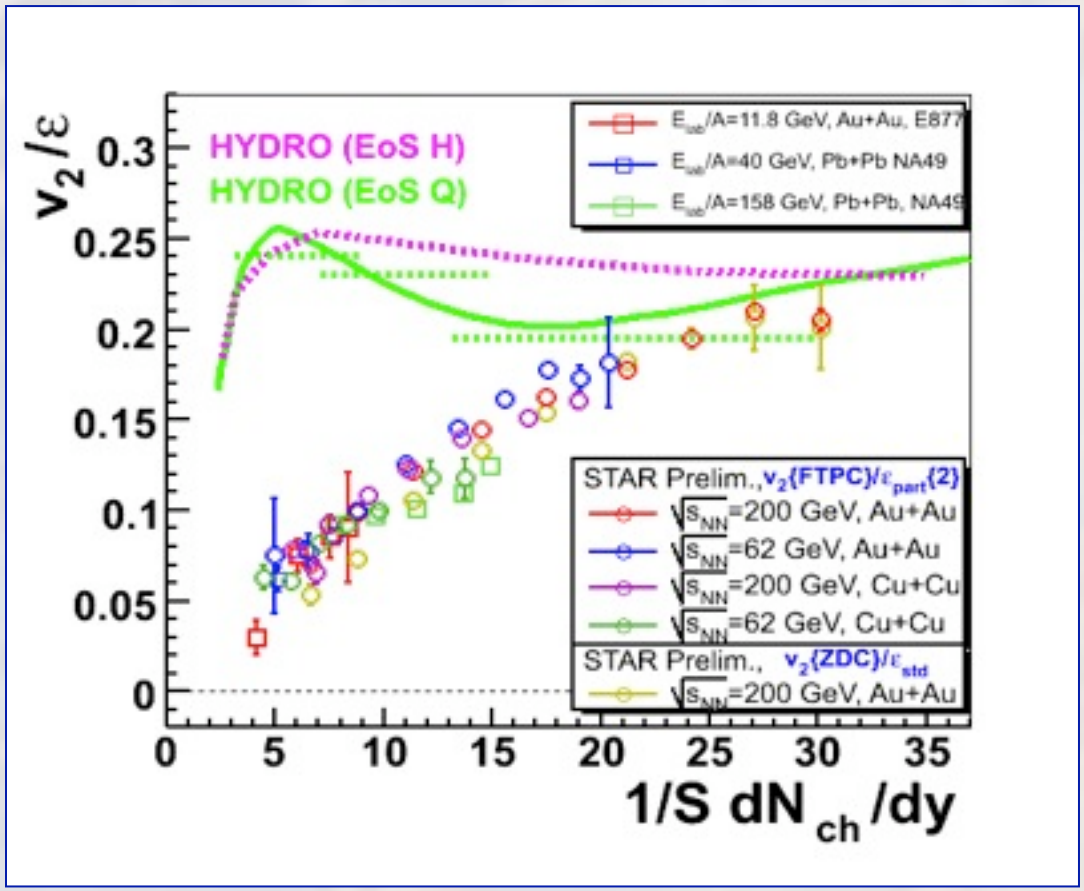
Several reasons for v to fluctuate in a centrality bin:
1) Variation in impact parameter (taken out in STAR 130 GeV PRC flow paper)
2) Real flow fluctuations (due to fluctuations in the initial conditions or in the system evolution)

For each method itÕs own epsilon

If $v_2 \propto \varepsilon$, then $v_2\{2\} \propto \varepsilon\{2\} \equiv \sqrt{\langle \varepsilon^2 \rangle}$

$v_2\{4\} \propto \varepsilon\{4\}$, etc.

$$\varepsilon^2\{2\} \equiv \langle \varepsilon^2 \rangle = \langle \varepsilon \rangle^2 + \sigma_\varepsilon^2$$
$$\varepsilon^4\{4\} \equiv 2\langle \varepsilon^2 \rangle^2 - \langle \varepsilon^4 \rangle$$

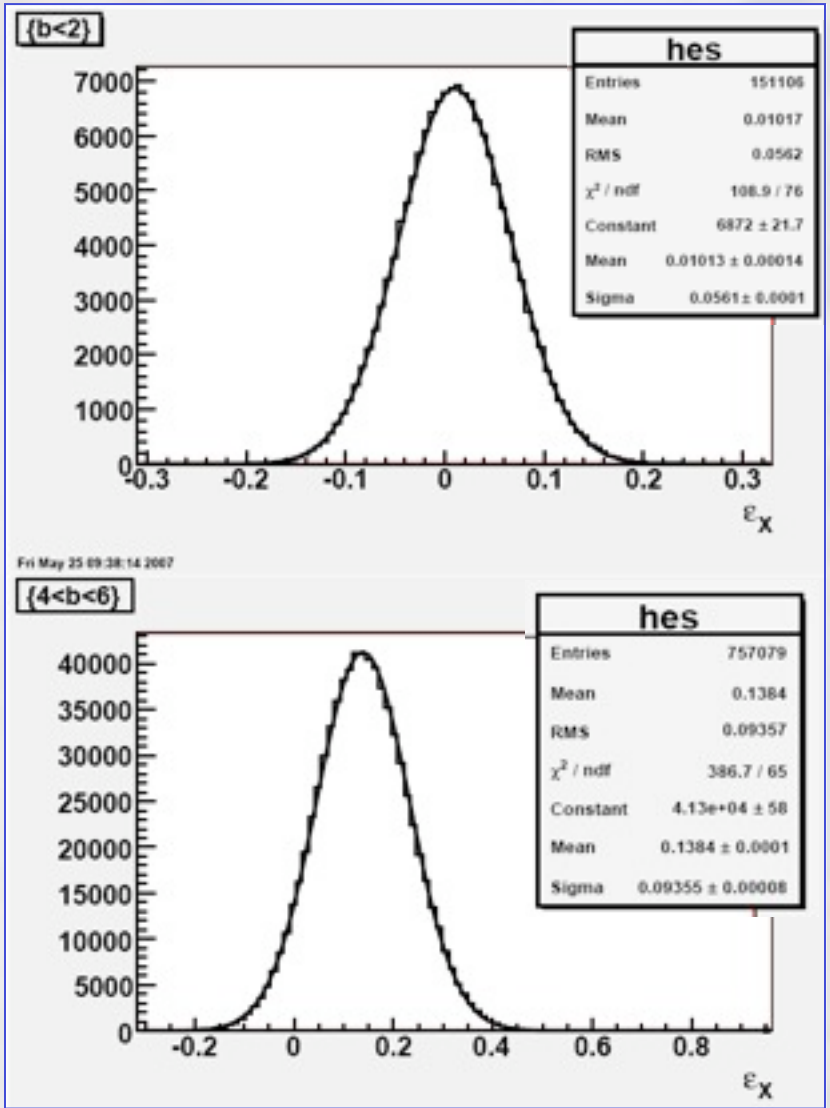


Main idea: use proper $\varepsilon\{n\}$ to rescale corresponding $v_2\{n\}$:

$$v\{n\} = \langle v \rangle \, \varepsilon\{n\} / \langle \varepsilon \rangle$$

Higher order terms in# ?

Distributions in ε_x , ε_y



$$\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part} \right\}$$

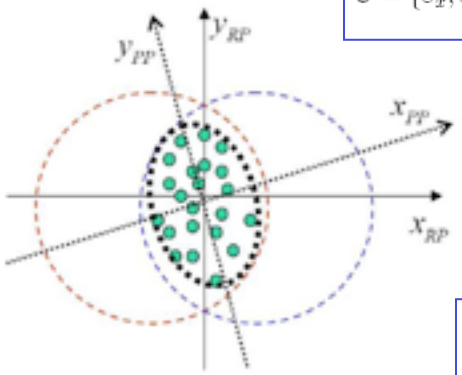
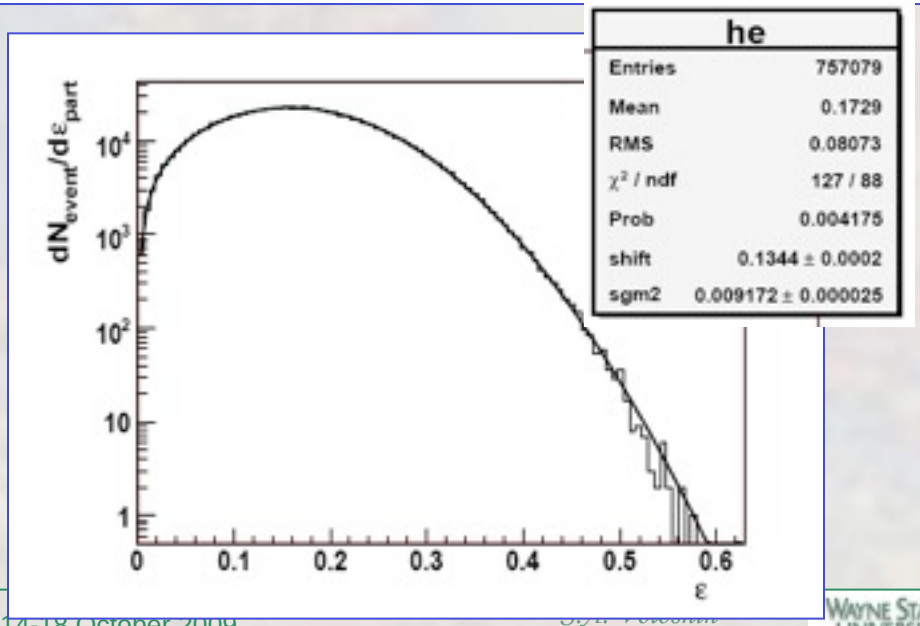


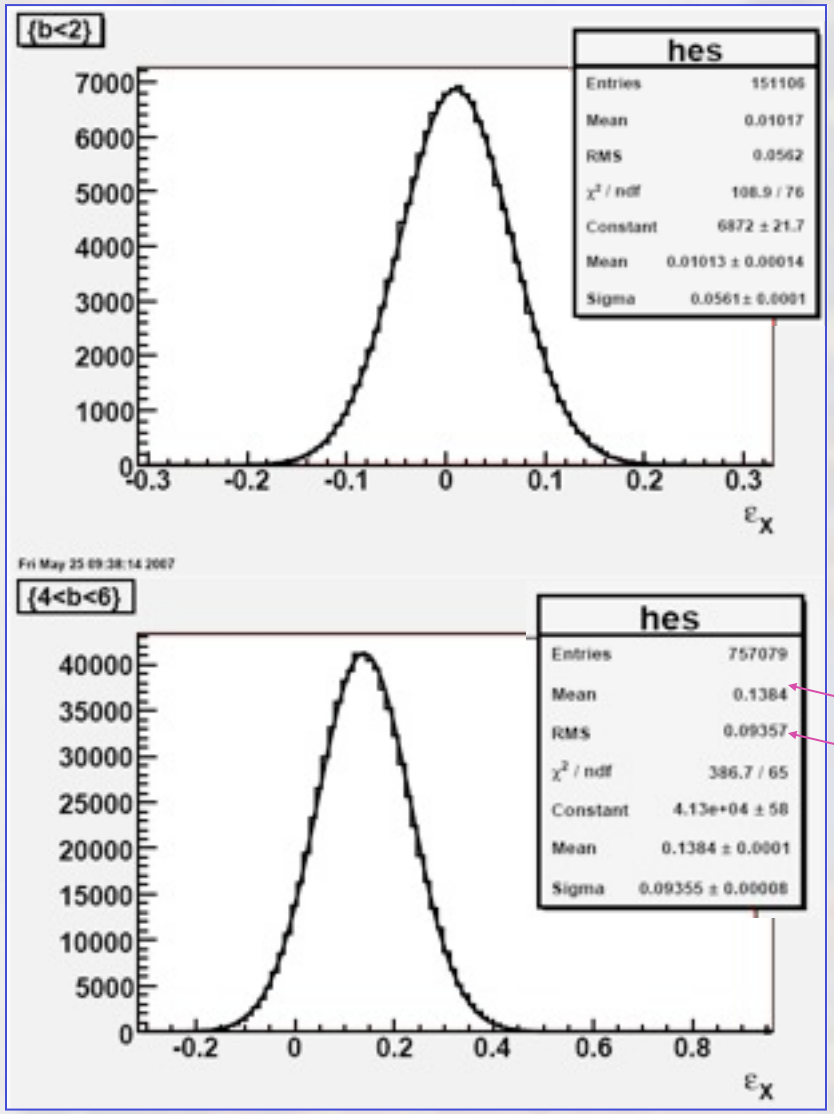
FIG. 1: The definition of the RP and PP coordinate systems.

$$\langle \varepsilon_x \rangle \equiv \bar{\varepsilon} \approx \varepsilon_{opt}.$$

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle\varepsilon_{RP}\rangle}{\sigma_\varepsilon^2}\right) \exp\left(-\frac{\varepsilon_{part}^2 + \langle\varepsilon_{RP}\rangle^2}{2\sigma_\varepsilon^2}\right) \equiv \mathrm{BG}(\varepsilon_{part}; \bar{\varepsilon}, \sigma_\varepsilon),$$



Distributions in ε_x , ε_y



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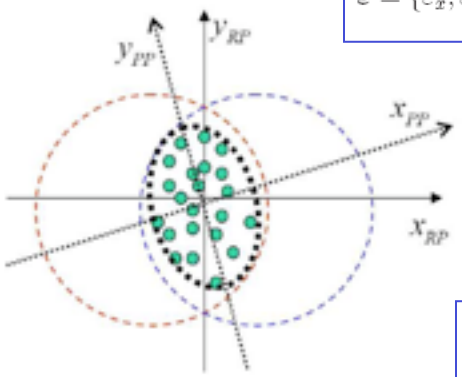
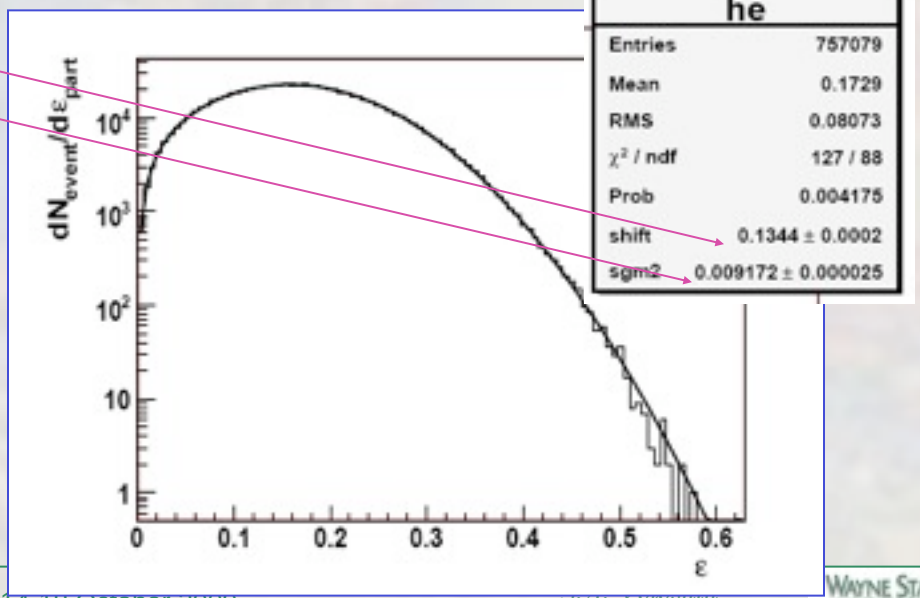


FIG. 1: The definition of the *RP* and *PP* coordinate systems.

$$\langle \varepsilon_x \rangle \equiv \bar{\varepsilon} \approx \varepsilon_{opt}.$$

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0 \left(\frac{\varepsilon_{part} \langle \varepsilon_{RP} \rangle}{\sigma_\varepsilon^2} \right) \exp\left(-\frac{\varepsilon_{part}^2 + \langle \varepsilon_{RP} \rangle^2}{2\sigma_\varepsilon^2} \right) \equiv \mathrm{BG}(\varepsilon_{part}; \bar{\varepsilon}, \sigma_\varepsilon),$$



Gaussian model of eccentricity fluctuations

Sergei A. Voloshin [a,*], Arthur M. Poskanzer [b], Aihong Tang [c], Gang Wang [d] Physics Letters B 659 (2008) 537–541

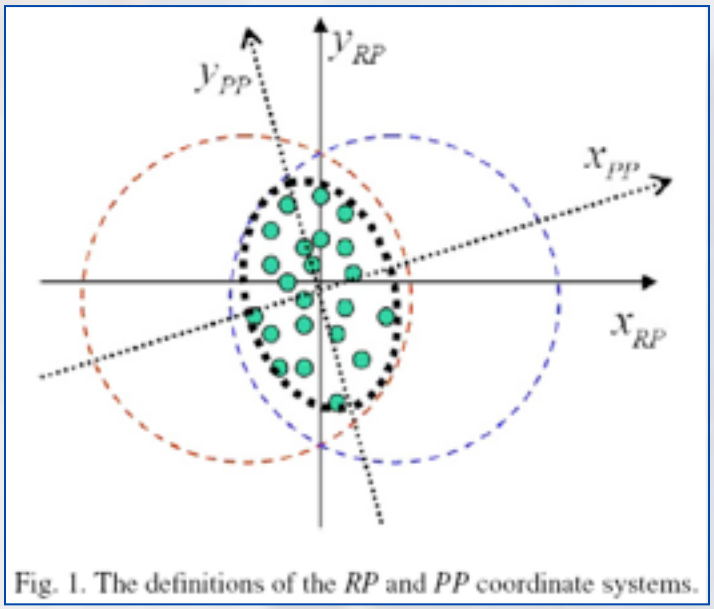


Fig. 1. The definitions of the *RP* and *PP* coordinate systems.

$$v_2 \propto \varepsilon\,; \quad v_2\{2\} = \langle v_2 \rangle \sqrt{\langle \varepsilon^2 \rangle} / \langle \varepsilon \rangle = \langle v_2 \rangle \varepsilon_2\{2\} / \langle \varepsilon \rangle$$

$$\boldsymbol{\varepsilon} = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{\text{part}}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{\text{part}} \right\}$$

Evolution of a concept of anisotropic flow: system response to azimuthally asymmetric initial conditions (one does not need a ÒtrueÓ reaction plane).

Model assumes Gaussian form for the distributions in $\#_x$ and $\#_y$, (which is a very good approximation of MC Glauber calculations).

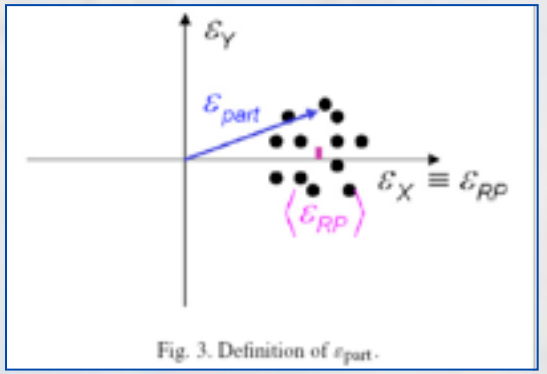


Fig. 3. Definition of ε_{part}.

$$v_2\{2\}^2 \equiv \left\langle \cos\left(2(\varphi_1 - \varphi_2)\right) \right\rangle = \left\langle v_2^2 \right\rangle + \delta = \left\langle v_2 \right\rangle^2 + \sigma_v^2 + \delta$$

$$v_2\{4\}^4 \equiv 2\left\langle \cos\left(2(\varphi_1 - \varphi_2)\right) \right\rangle^2 - \left\langle \cos\left(2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\right) \right\rangle \approx 2\left\langle v_2^2 \right\rangle^2 - \left\langle v_2^4 \right\rangle$$

$$v_2\{2\}^2 = \kappa^2\left(\langle \varepsilon_{RP} \rangle^2 + 2\sigma_\varepsilon^2\right) + \delta = \langle v_{RP} \rangle^2 + 2\sigma_{vx}^2 + \delta$$

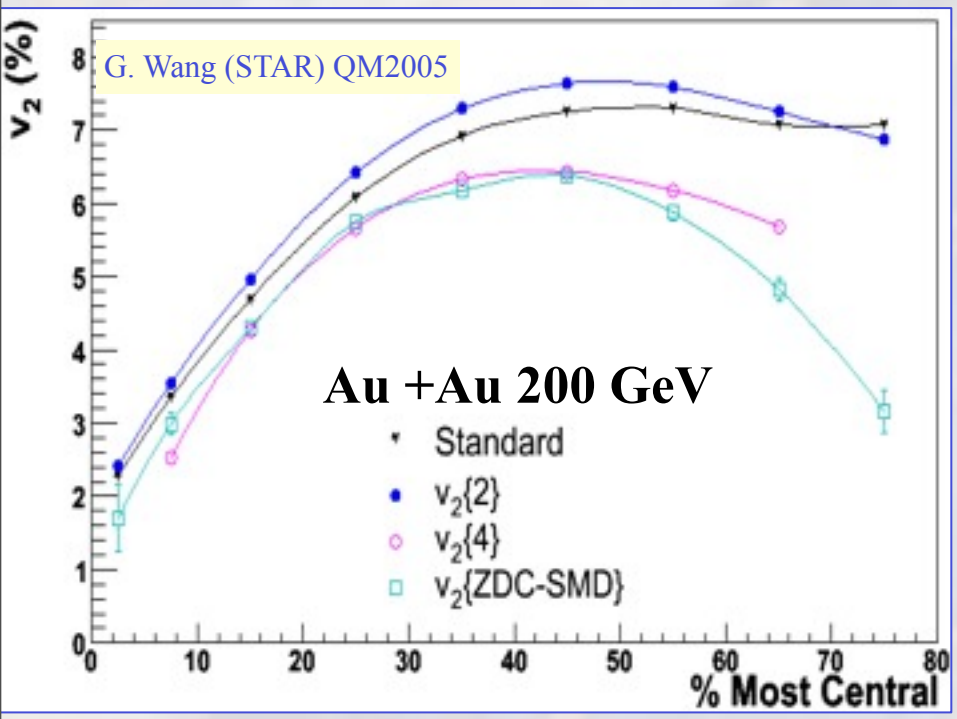
$$v_2\{4\}^4 = 2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle = \bar{v}_2^4 = \langle v_{RP} \rangle^4$$

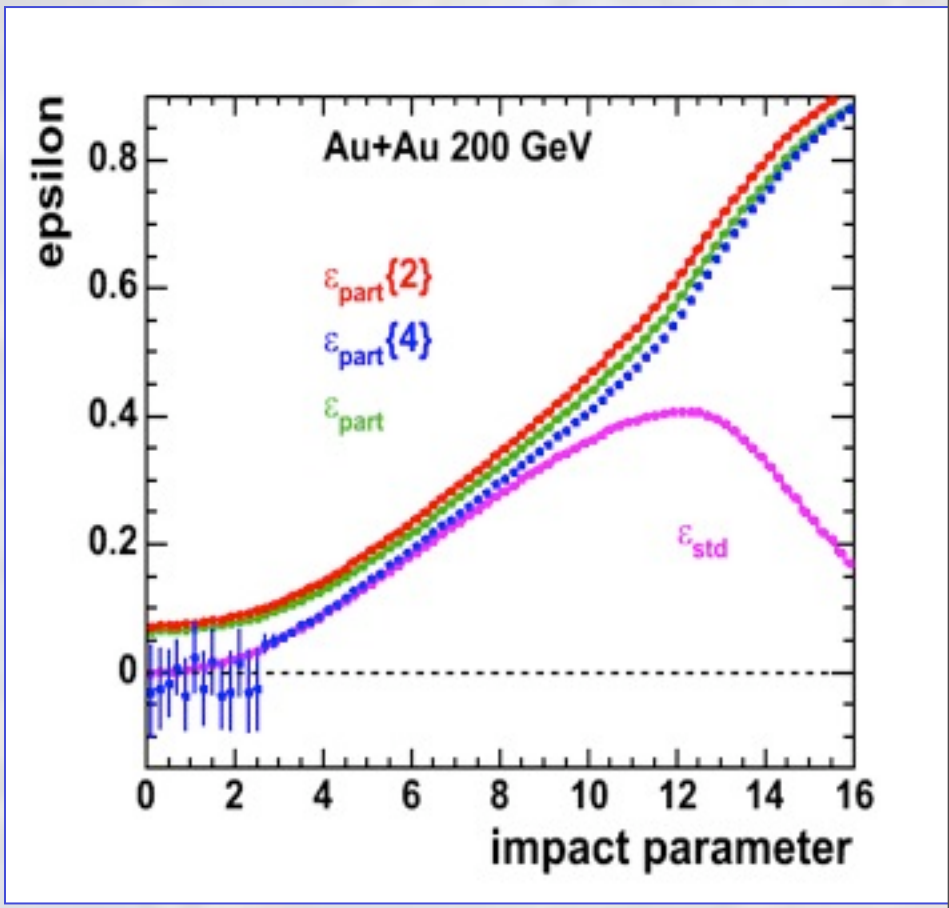
$$v_2\{6\}^6 = \left(\langle v_2^6 \rangle - 9\langle v_2^4 \rangle \langle v_2^2 \rangle + 12\langle v_2^2 \rangle^3\right)/4 = \langle v_{RP} \rangle^6$$

In this model it is not possible to separate flow fluctuations and non-flow effects (this can be traced to the fact that the Gaussian distribution has all cumulants higher than rank 2 equal to zero)

➔ $v_2\{4\}$ (and higher cumulants, v2{LYZ}, v2{q-dist}) measures ÒtrueÓ elliptic flow (wrt reaction plane) Ð exactly what is needed for comparison with theory!

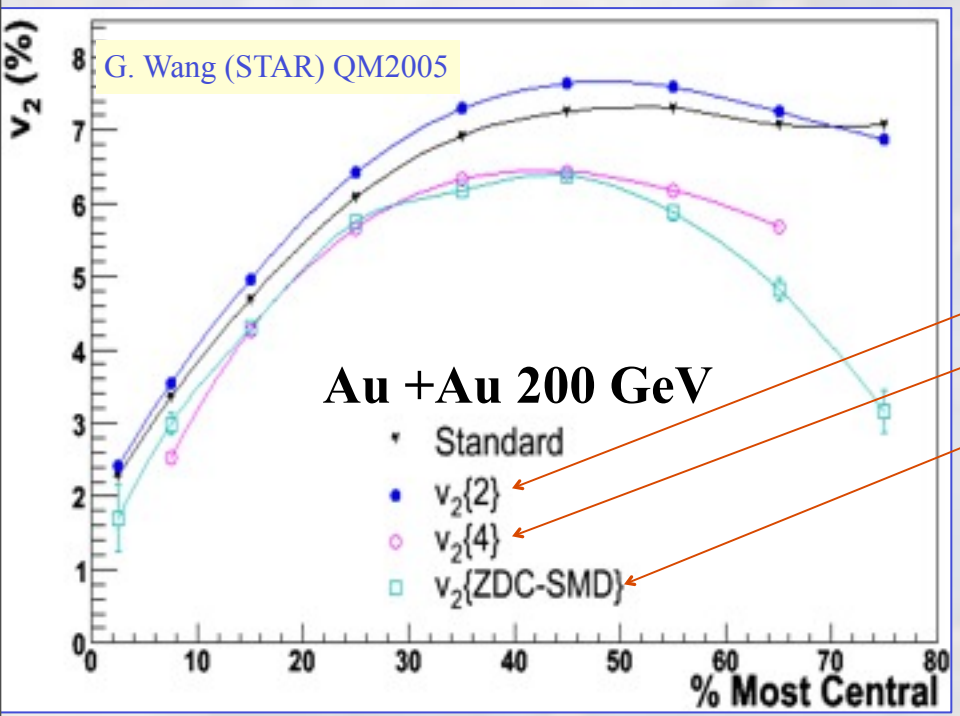
v_2\{ZDC-SMD\} and eccentricity

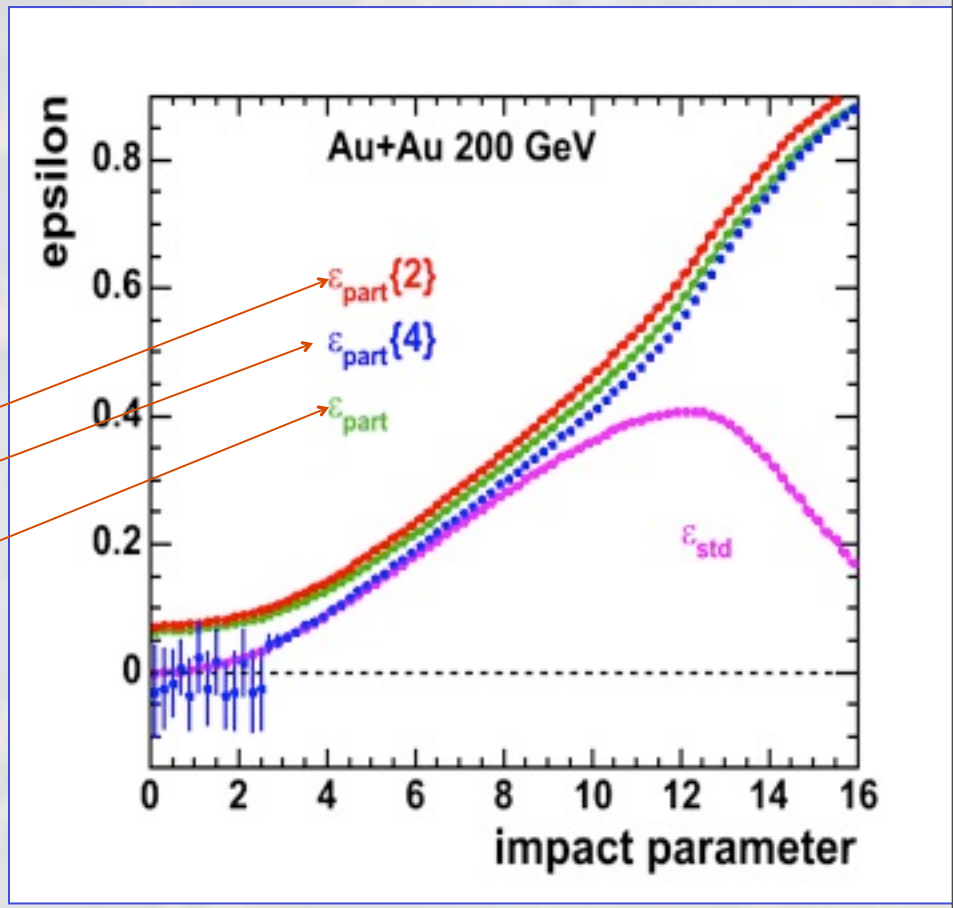




Note that $\#_{part}\{4\}$ approaches $\#_{std}$.

v_2\{ZDC-SMD\} and eccentricity





Note that $\#_{part}\{4\}$ approaches $\#_{std}$.

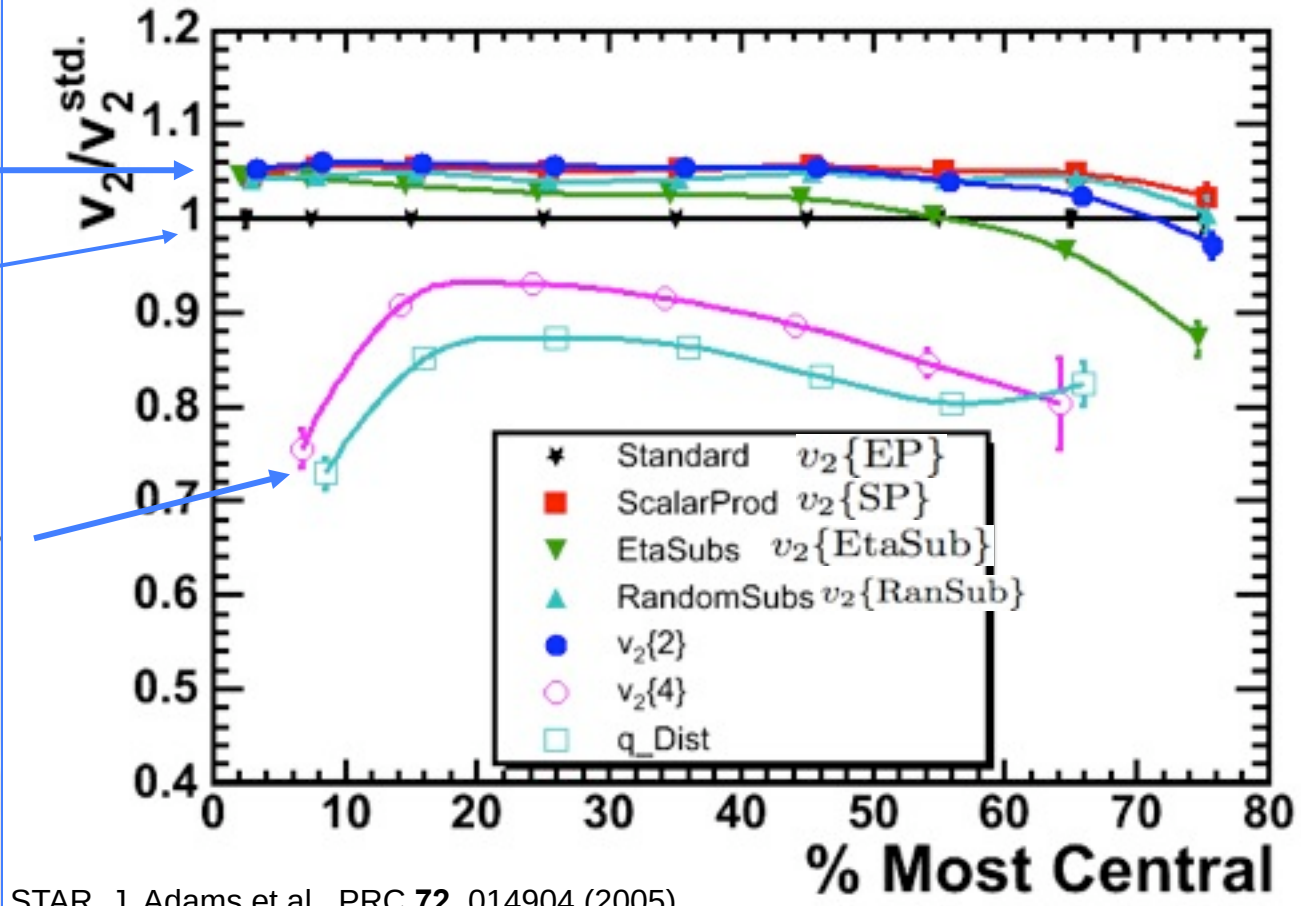
What is so special about $v_2\{EP\}$?

2-part. methods

std. method

5% low!

multi-part. methods



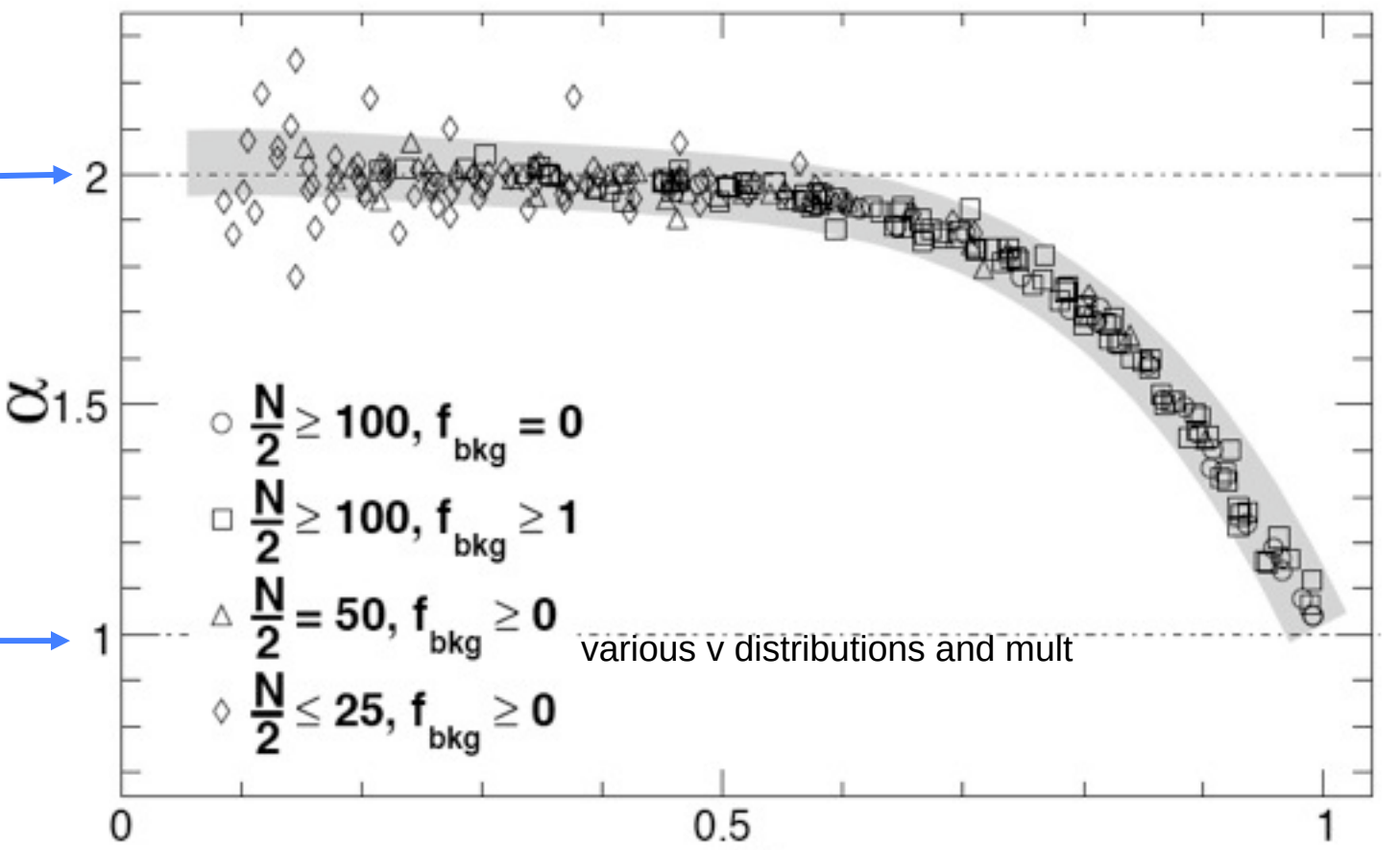
STAR, J. Adams et al., PRC **72**, 014904 (2005)

PHOBOS Simulations

$$v_2\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$$

root-mean-square

mean



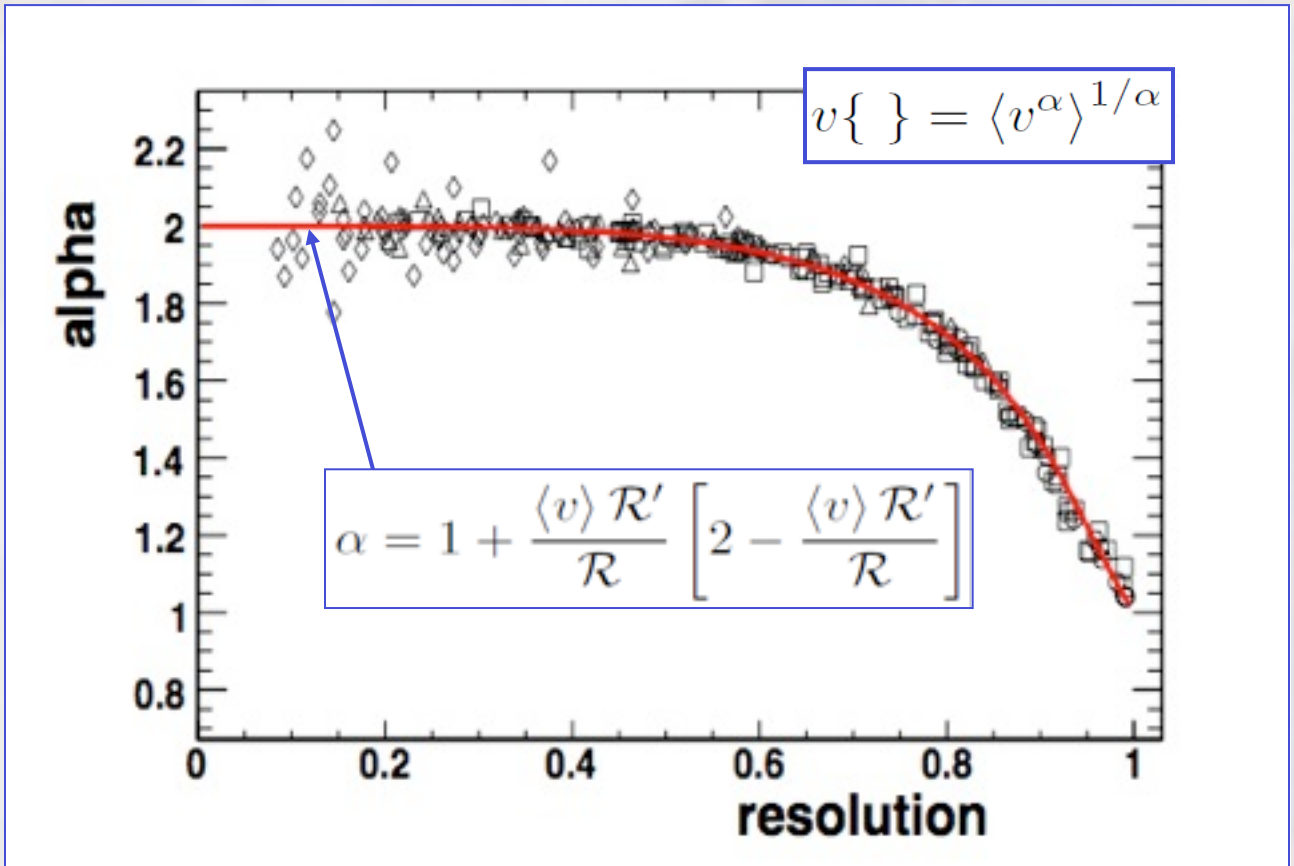
PHOBOS+Heinz, PRC **77**, 014906 (2008)

event plane resolution

v{EP}

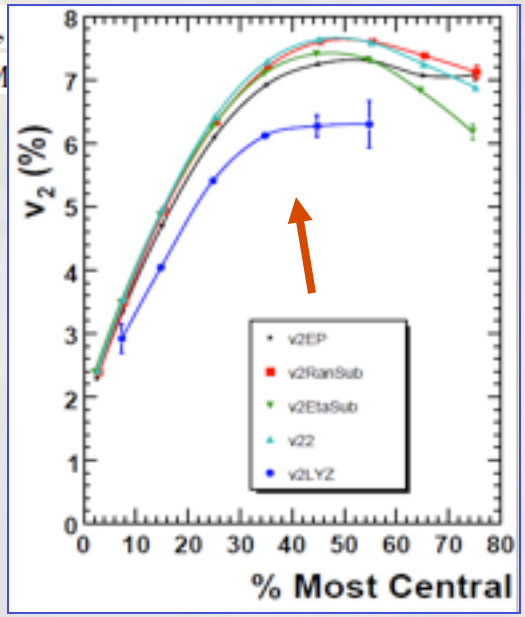
PHYSICAL REVIEW C 80,

Jean-Yves Ollitrault,[1] Arthur M



$$v\{\ \} = \langle v^{\alpha} \rangle^{1/\alpha}$$

$$\alpha = 1 + \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \left[2 - \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \right]$$



$$v\{\mathrm{EP}\} \equiv \frac{\langle \cos(\phi - \Psi_R) \rangle}{R}$$

$$R = \sqrt{\langle \cos(\Psi_A - \Psi_B) \rangle}$$

$$q \cos \Psi_R = \frac{Q}{\sqrt{N}} \cos \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \cos \phi_j$$

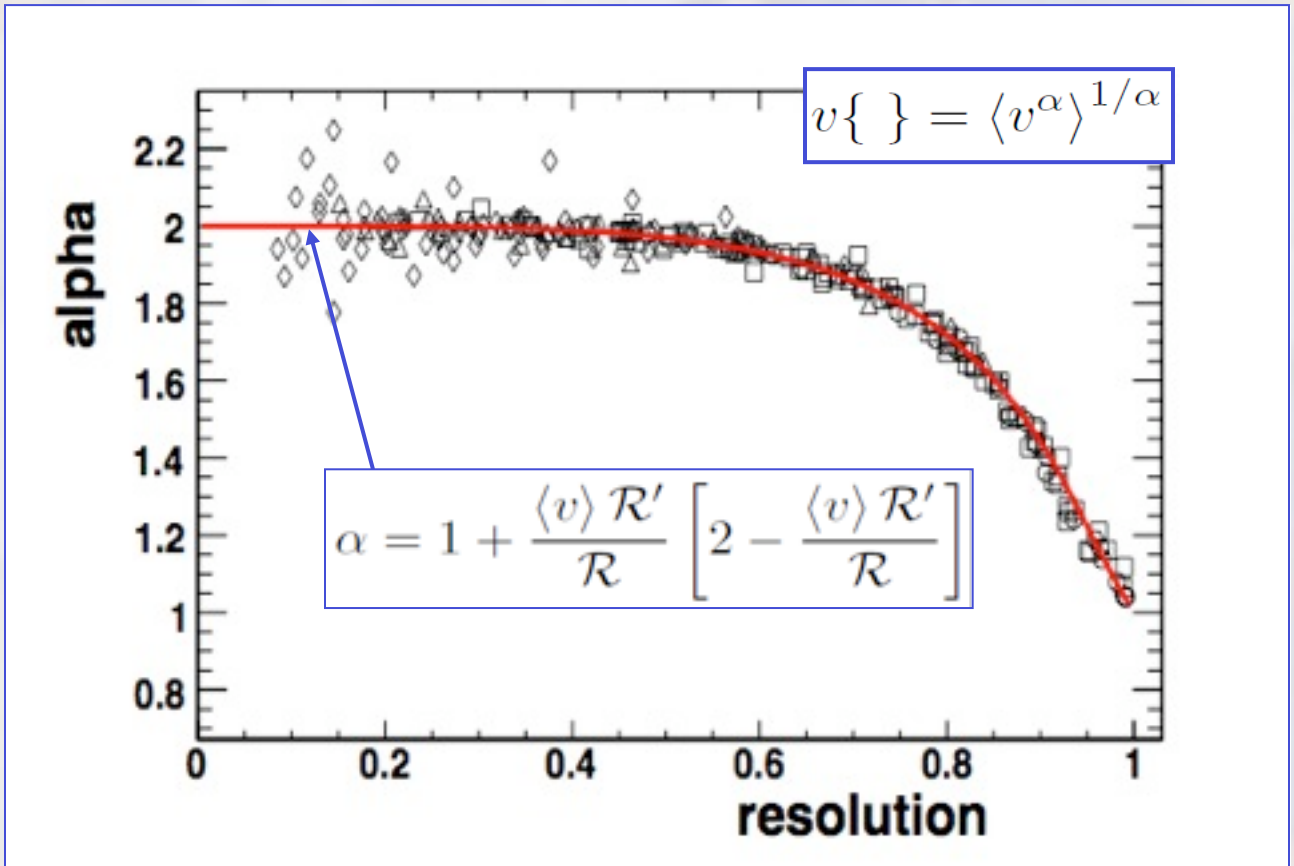
$$q \sin \Psi_R = \frac{Q}{\sqrt{N}} \sin \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sin \phi_j$$

$$\mathcal{R}(\chi) = \frac{\sqrt{\pi}}{2} e^{-\chi^2/2} \chi \left[I_0\left(\frac{\chi^2}{2}\right) + I_1\left(\frac{\chi^2}{2}\right) \right] \qquad \chi_s = v\sqrt{N_s}$$

v{EP}

PHYSICAL REVIEW C 80,

Jean-Yves Ollitrault,[1] Arthur M



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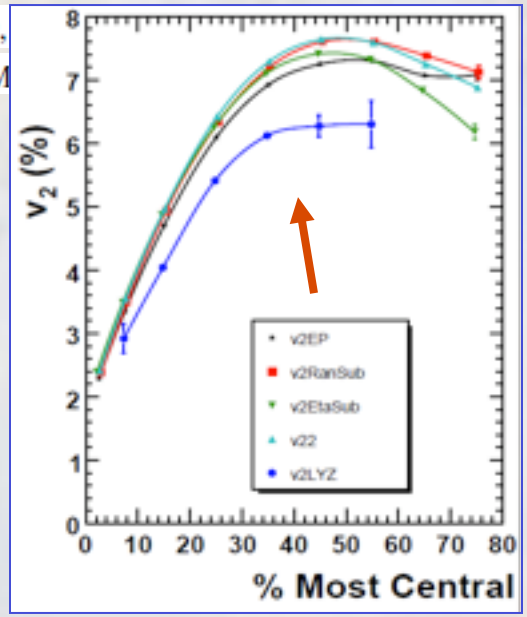
$$\alpha = 1 + \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \left[2 - \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \right]$$



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É but it is still not possible to separate the effect of fluctuations from non-flowÉ

Analytic Correction

to flow fluctuations

$\chi = v_2 \sqrt{M}$ $I_{0,1} = I_{0,1}(\chi^2/2)$ $i_{0,1} = I_{0,1}(\chi_s^2/2)$

$v_2\{2\}^2 = \langle v \rangle^2 + \sigma_v^2$ $v_2\{\mathrm{subEP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{4\, i_1^2}{(i_0 + i_1)^2}\right) \sigma_v^2$

$$v_2\{\mathrm{EP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{2(I_0 - I_1)}{I_0 + I_1}\left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{i_0^2 - i_1^2}\right)\right)\sigma_v^2$$

and non-flow

$v_2\{2\}^2 = \langle v \rangle^2 + \delta$

$v_2\{4\} = \langle v \rangle$

$$v_2\{\mathrm{EP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{(I_0 - I_1)}{(I_0 + I_1)}\left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)}\right)\right)\delta$$

$$v_2\{\mathrm{subEP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{2i_1^2}{(i_0 + i_1)^2}\right)\delta$$

$\langle v \rangle = v$

Differences of Measured v_2 Values

$$
\begin{aligned}
v_2\{2\}^2 - v_2\{4\}^2 &= \delta + 2\sigma_v^2 \\
v_2\{2\}^2 - v_2\{\mathrm{EP}\}^2 &= \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)} \right) (\delta + 2\sigma_v^2) \\
v_2\{2\}^2 - v_2\{\mathrm{subEP}\}^2 &= \frac{2i_1^2}{(i_0 + i_1)^2} (\delta + 2\sigma_v^2) \\
v_2\{\mathrm{subEP}\}^2 - v_2\{\mathrm{EP}\}^2 &= \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)} - \frac{2I_1^2}{(I_0^2 - I_1^2)} \right) (\delta + 2\sigma_v^2)
\end{aligned}
$$

All differences proportional to $\sigma_{\mathrm{tot}}^2 = \delta_2 + 2\sigma_{v2}^2$

Without additional assumptions
can not separate non-flow and fluctuations

How it works under simple assumption

Art Poskanzer

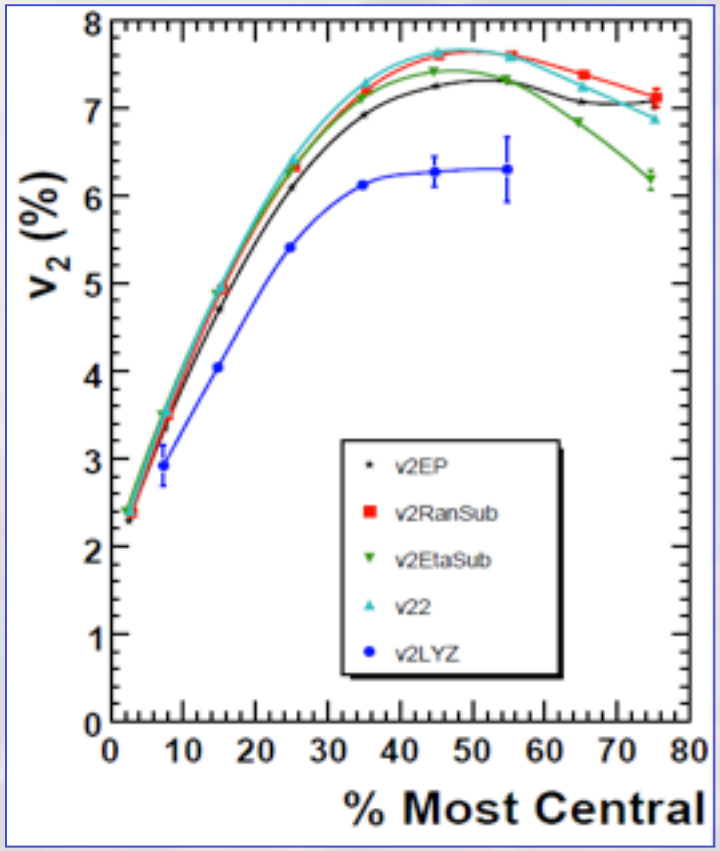
$$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle\varepsilon\rangle}\langle v_2\rangle$$

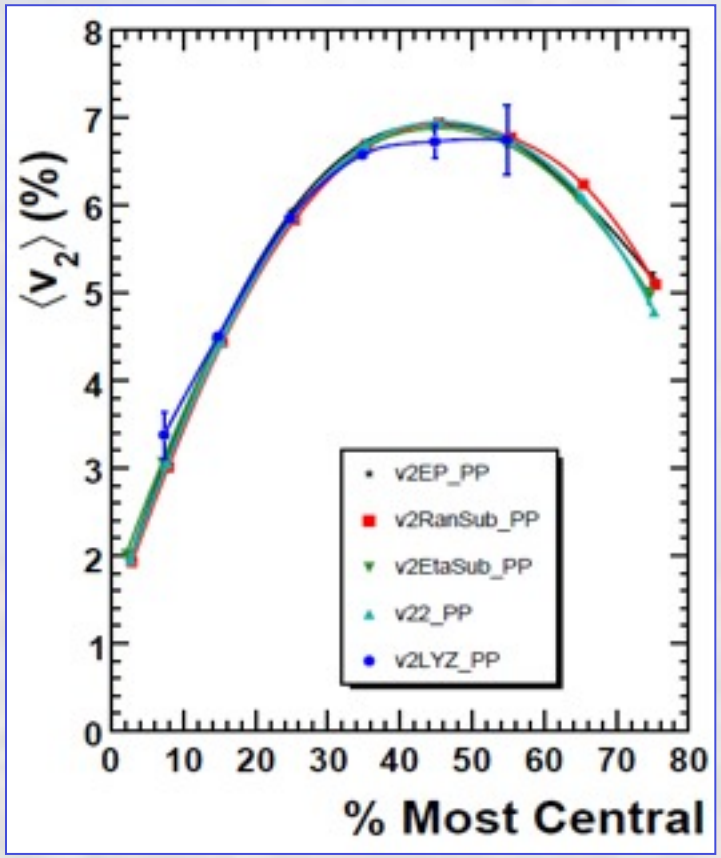
MC Glauber ε participant

$$\delta_2 = 2\ \delta_{pp}/N_{\rm part} \qquad \delta_{pp} = 0.0145$$

$$\delta_{\rm etaSub} = 0.5\ \delta_2$$

less nonflow





Fluctuating initial conditions: further details

J. Takahashi *et al.*,arXiv:0902.4870v1

J.Takahashi et al. arXiv:0902.4870v1

NEXUS + SPHERIO

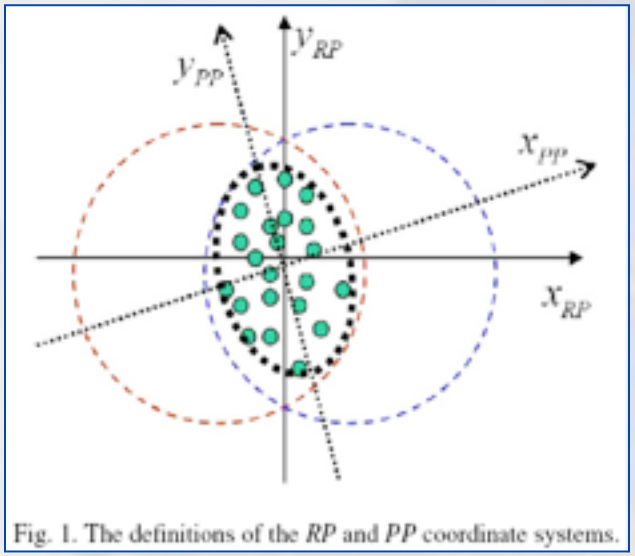
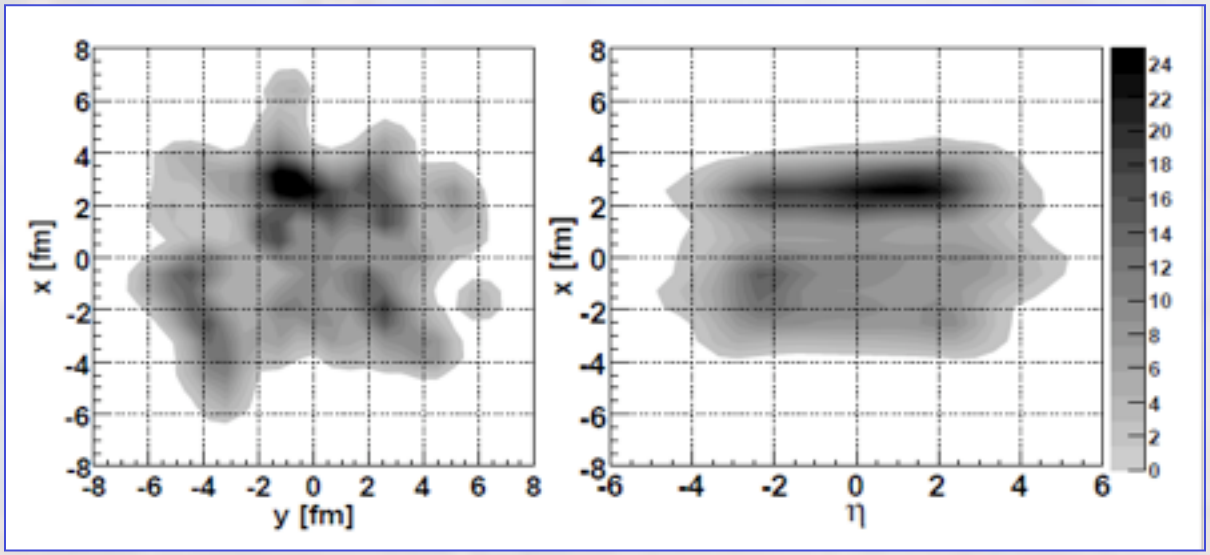
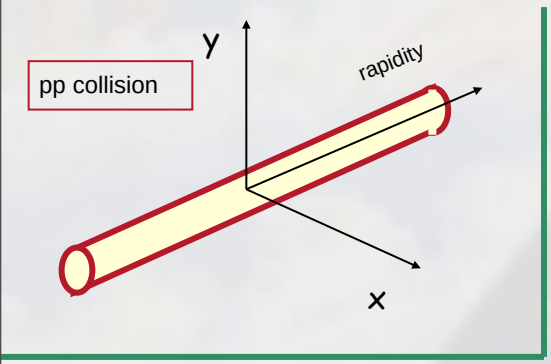


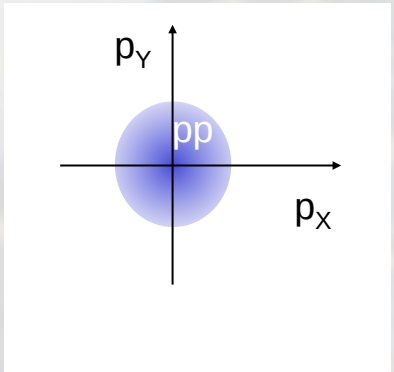
Fig. 1. The definitions of the *RP* and *PP* coordinate systems.



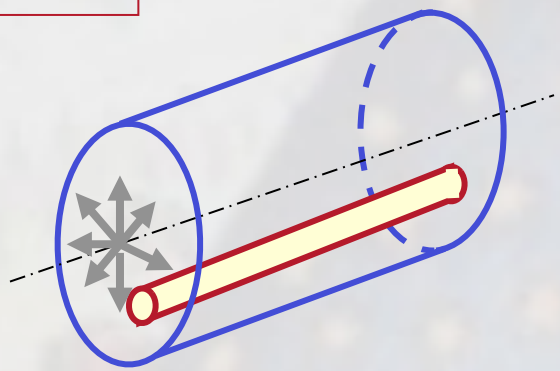
Radial expansion ➔ 2-part azimuthal correlations

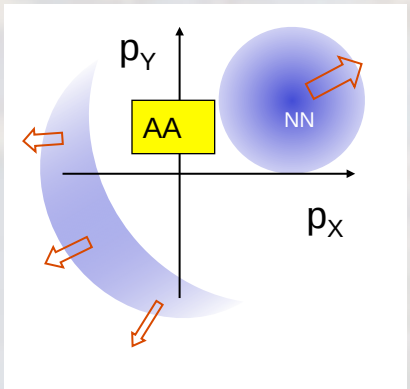
[arXiv:nucl-th/0312065]





AA collision

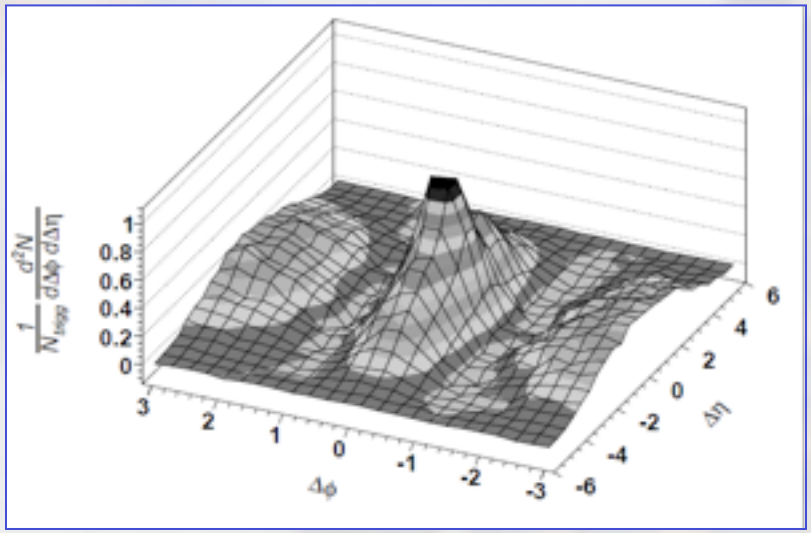




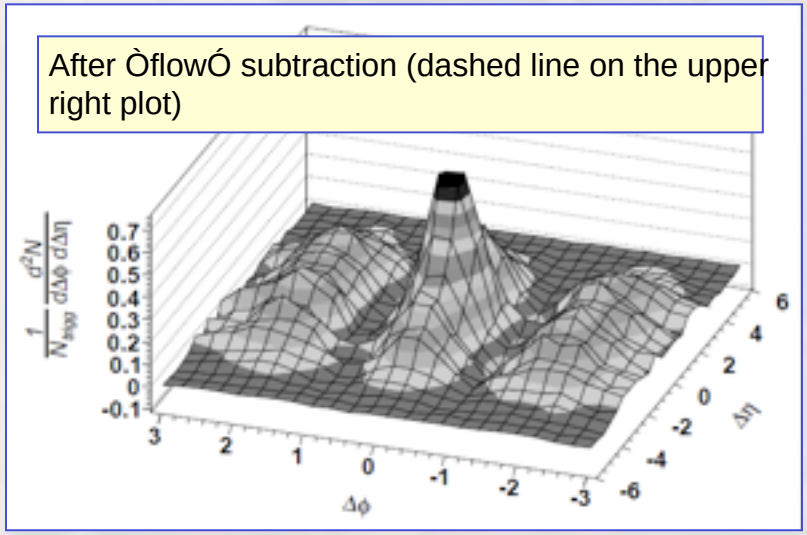
Particle correlations existed in pp Ð become modified.

→ Long range rapidity correlations become narrow in phi Ð ÒridgeÓ develops
→ Stronger 2-particle pt correlation in narrow phi bins.
→ Narrowing of the charge balance function ($\Delta p_z \approx m_t \sinh(\Delta y)$ -- increase in m_t → decrease in rapidity separation)
→ Charge correlations become narrow in phi.
Azimuthal Balance function
→ stronger in-plane than out-of-plane, etc.

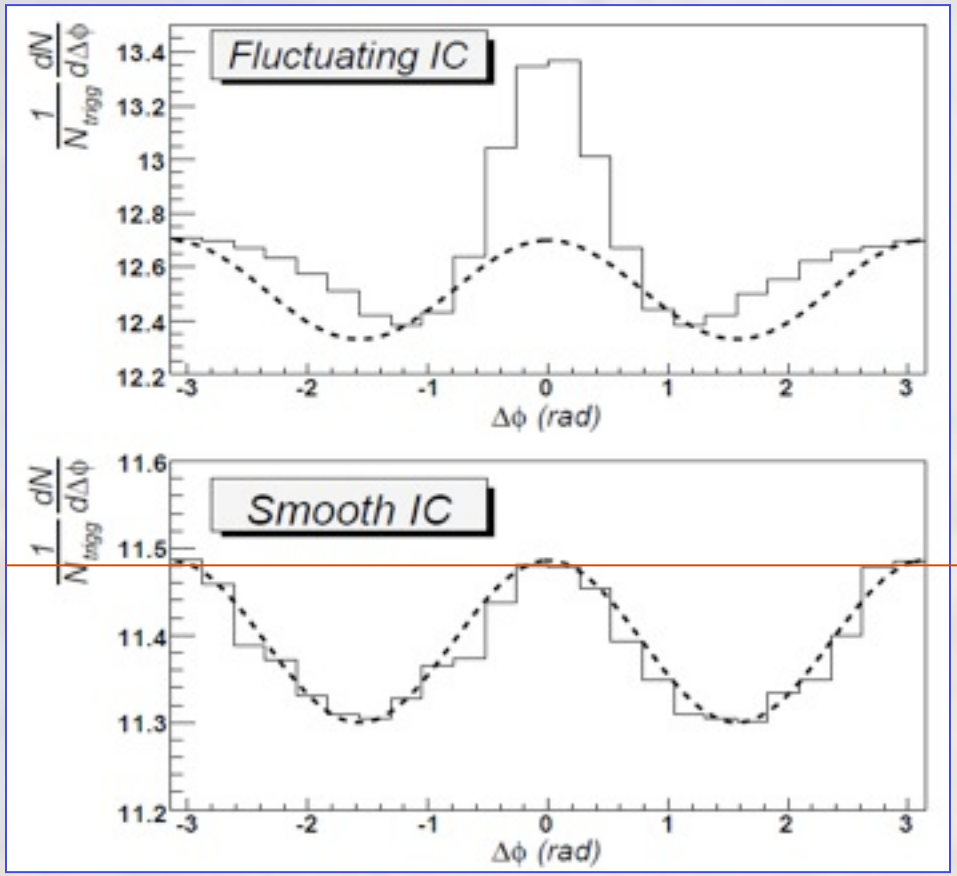
Correlation function



After ÒflowÓ subtraction (dashed line on the upper right plot)

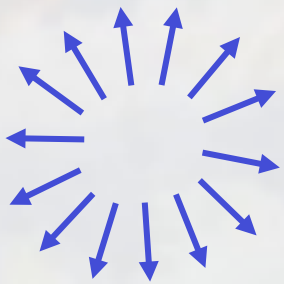


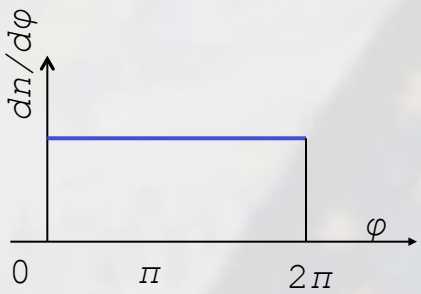
J. Takahashi *et al.*,arXiv:0902.4870v1



Correlations (particle interactions) modify background (ÒflowÓ).
Ignoring such modification produces perfect ÒMachÓ cone

Simple ÒmodelÓ - does not fit to the two component picture





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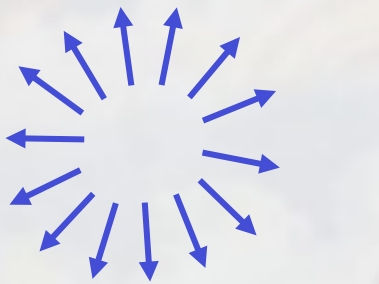
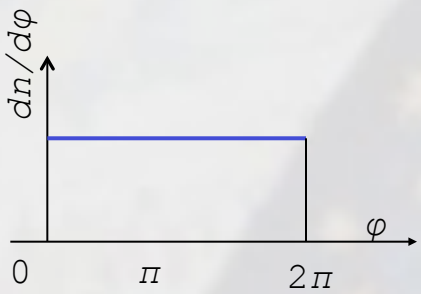
Simple ÒmodelÓ - does not fit to the two component picture





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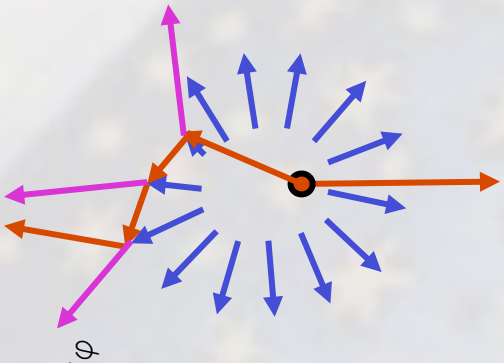

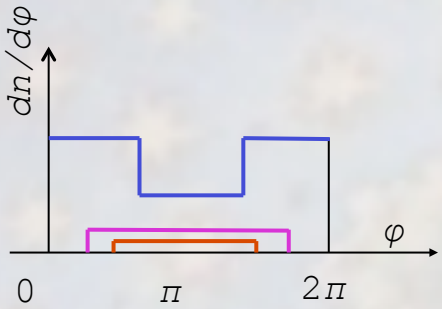

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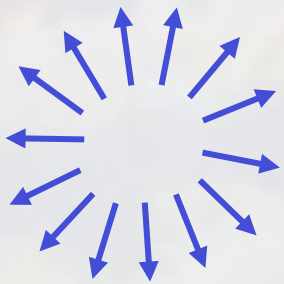

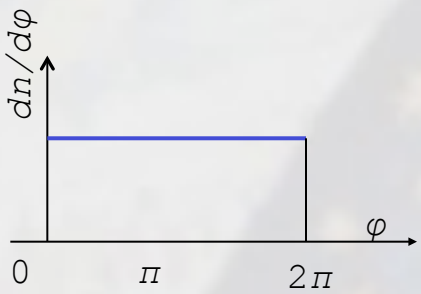

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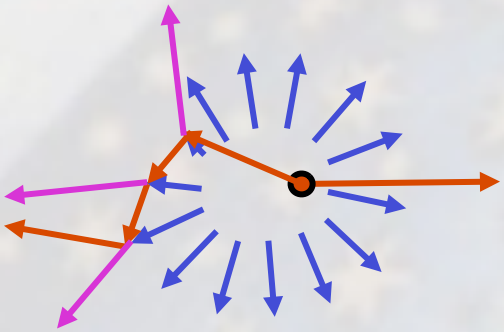

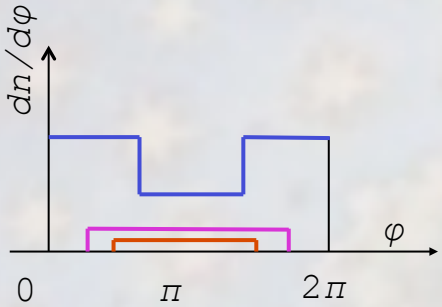

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The resulting distribution. The question is: how one can split all these particles into 2 components?

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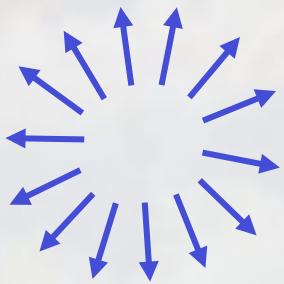

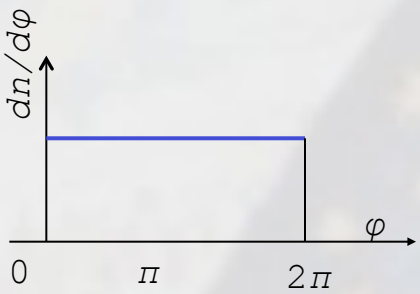

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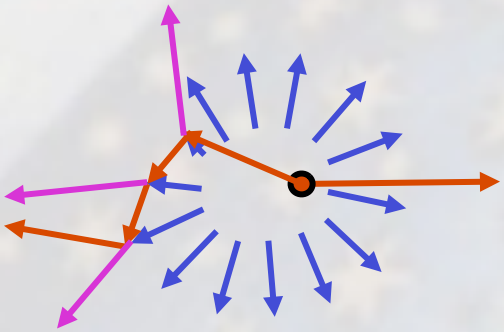

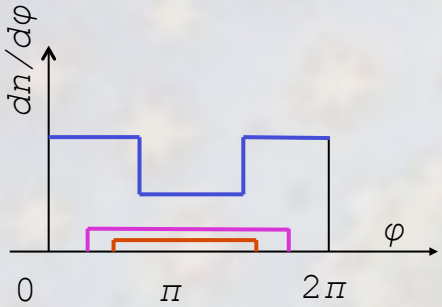

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Conclusion: Modified Òjets Ó ↔ modified flow ÒbackgroundÓ

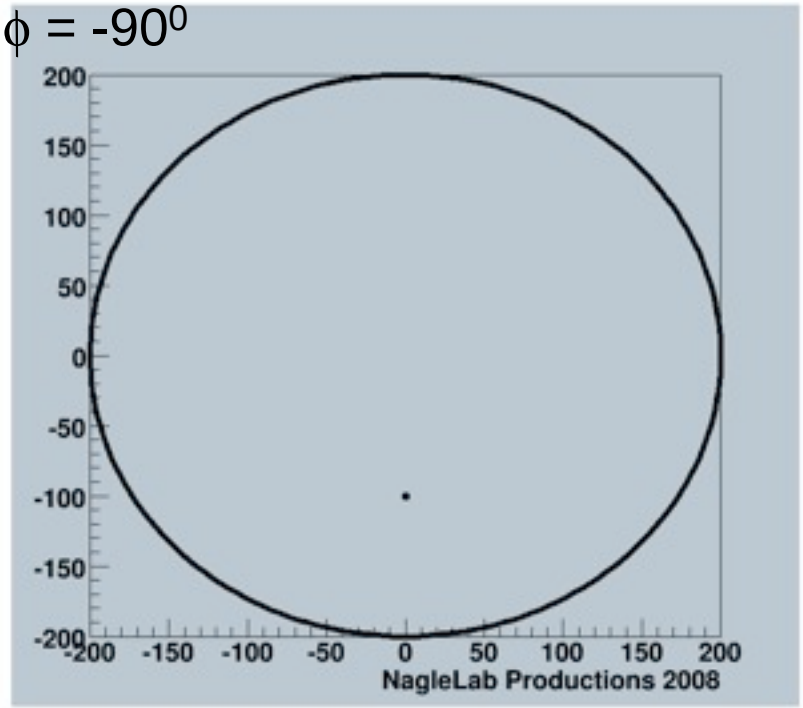
Note that in all simulations ÒsupportingÓ ZYAM the two component model was assumed from the very beginning.

Next 2½ slides are taken from QM09 talk

Mach Cone Dream

Wave Energy at Circle Boundary

$\phi = -90^0$



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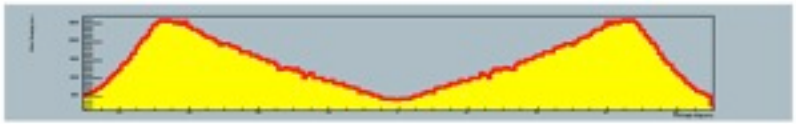

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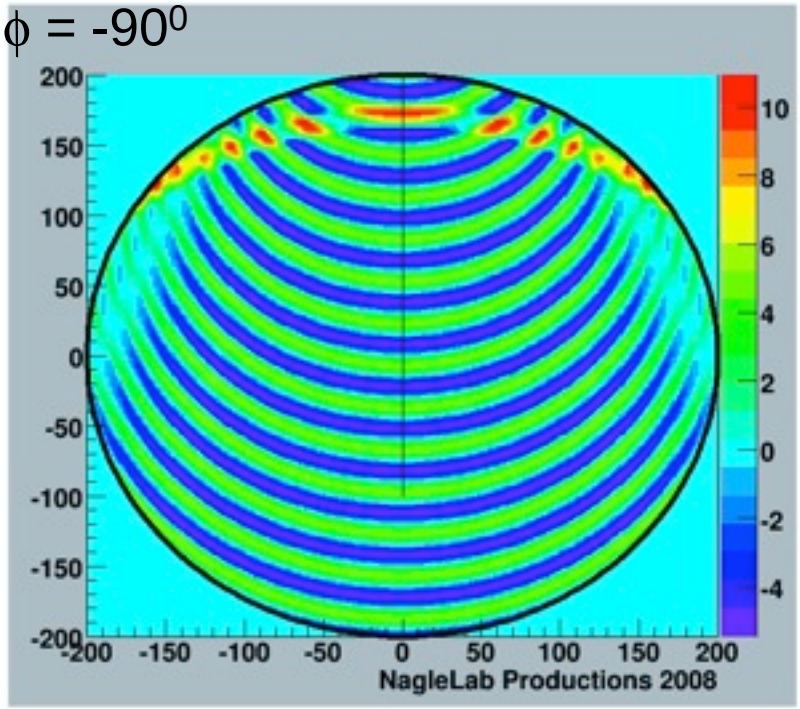
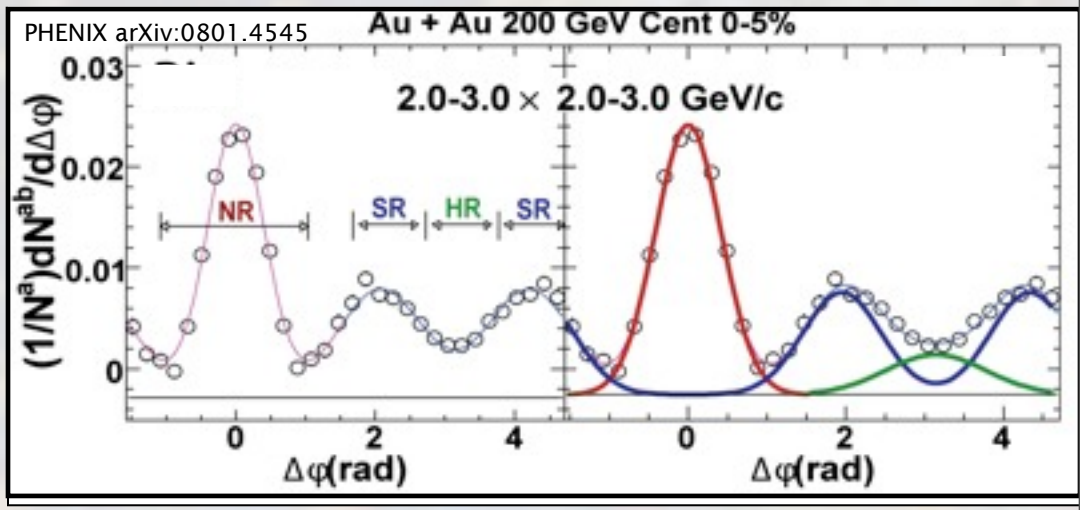
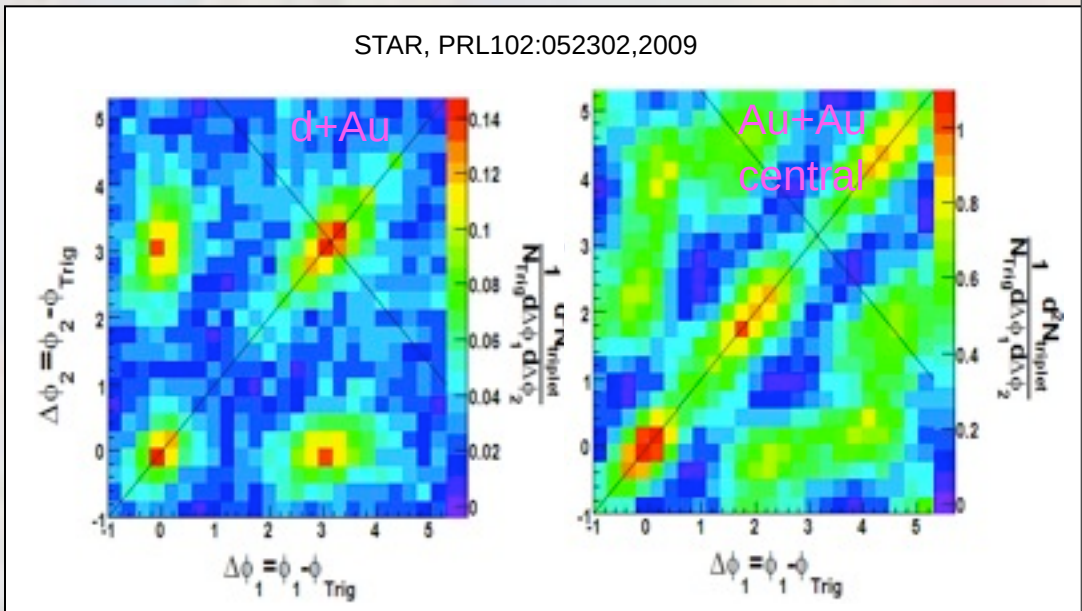


Mach Cone Dream

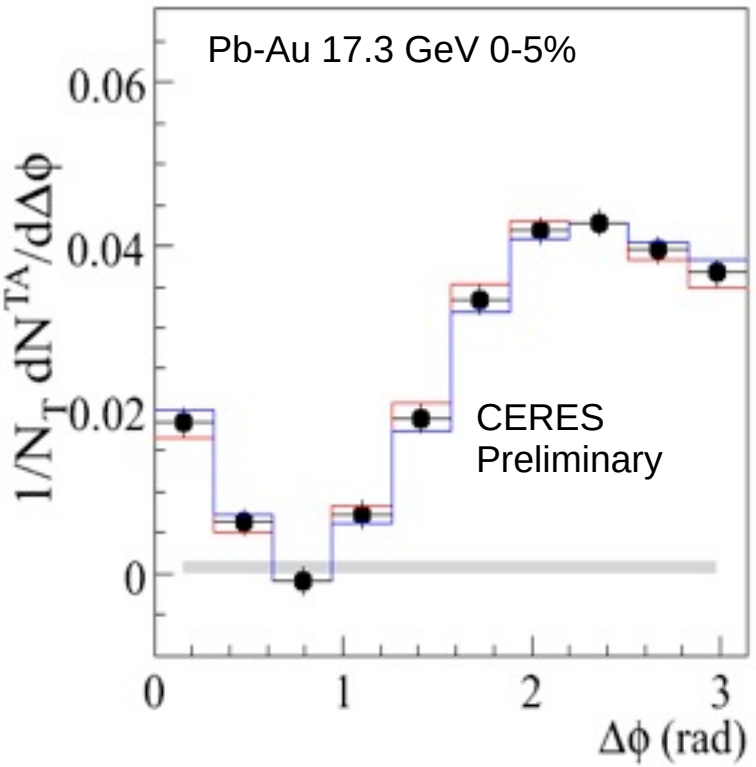


$\theta_M = 1.37 \pm 0.02$ (stat.)$\pm$ 0.06 (syst.)



PERFECT FLUID RESPONSE!

Time to Wake Up

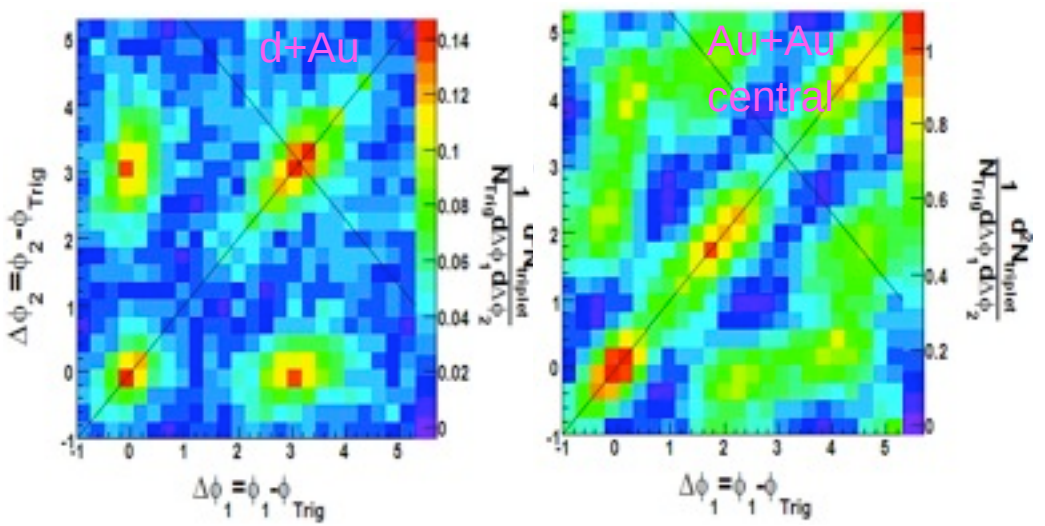


Mach Cone Dream

PHENIX arXiv:0801.4545

$\theta_M = 1.37 \pm 0.02$ (stat.)$\pm$ 0.06 (syst.)

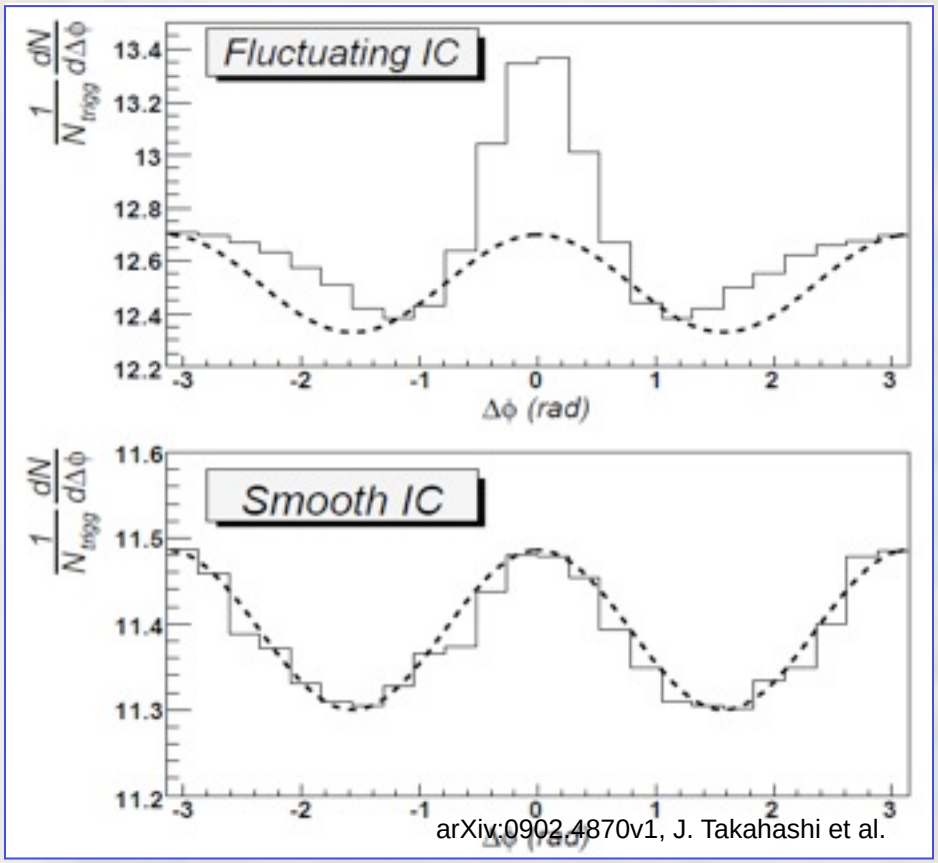
STAR, PRL102:052302,2009



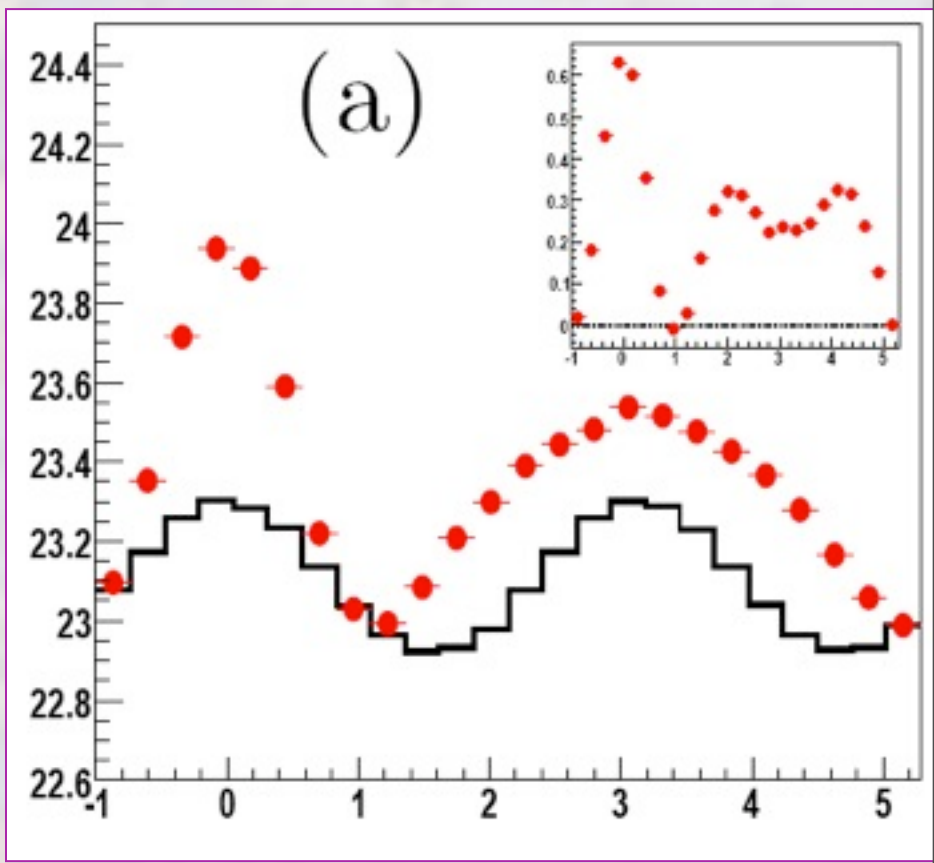
PERFECT FLUID RESPONSE!

Monday, September 14, 2009

ÒMachÓ cone

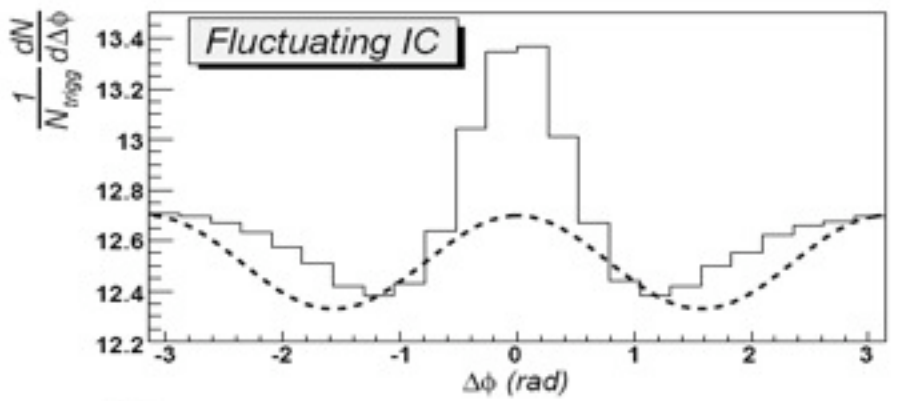


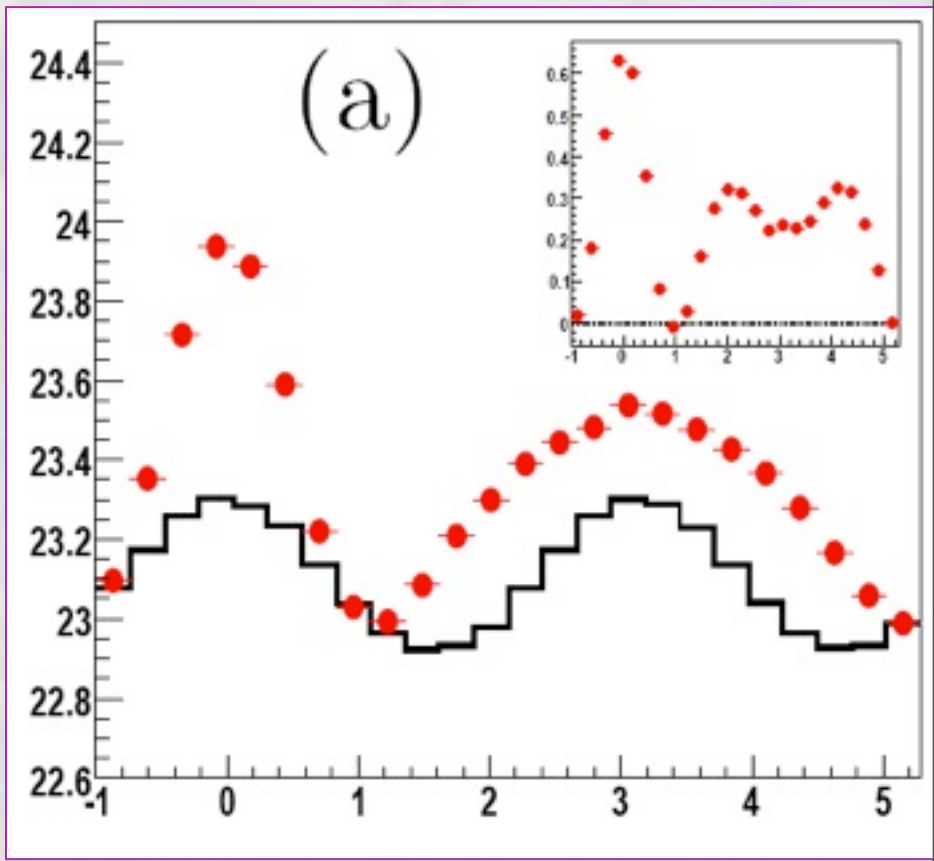
arXiv:0902.4870v1, J. Takahashi et al.



Striking similarity with recent data analysis!

ÒMachÓ cone





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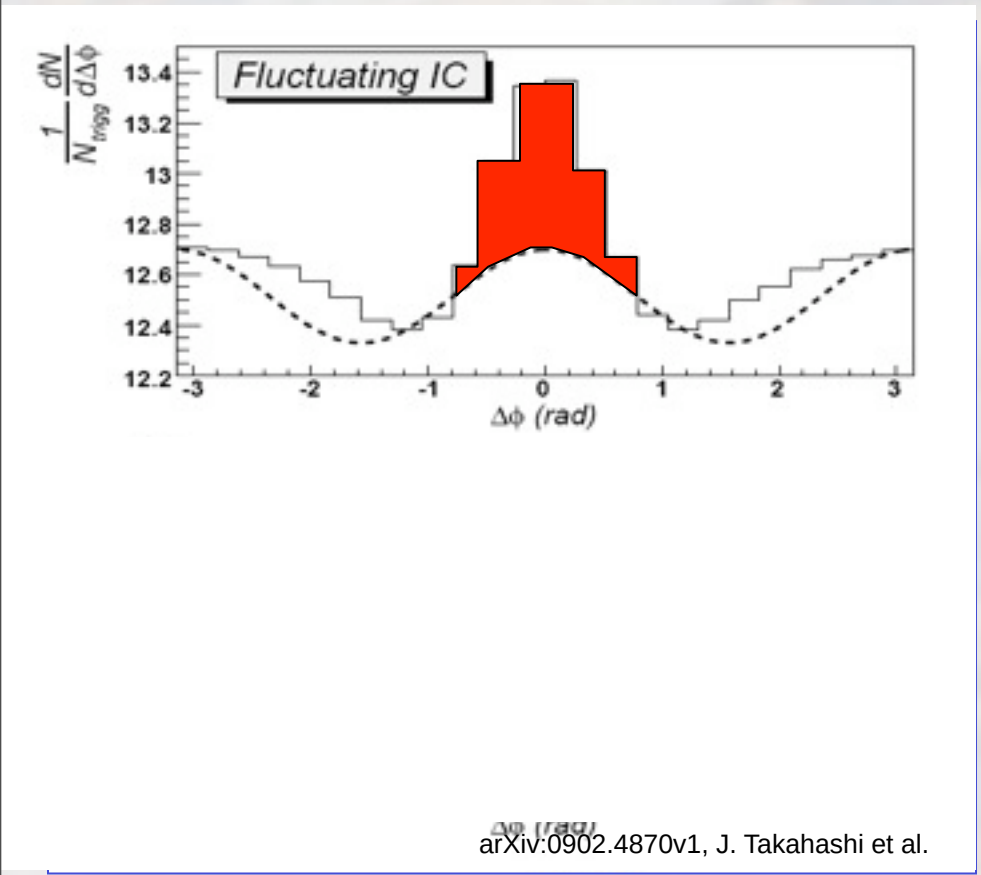

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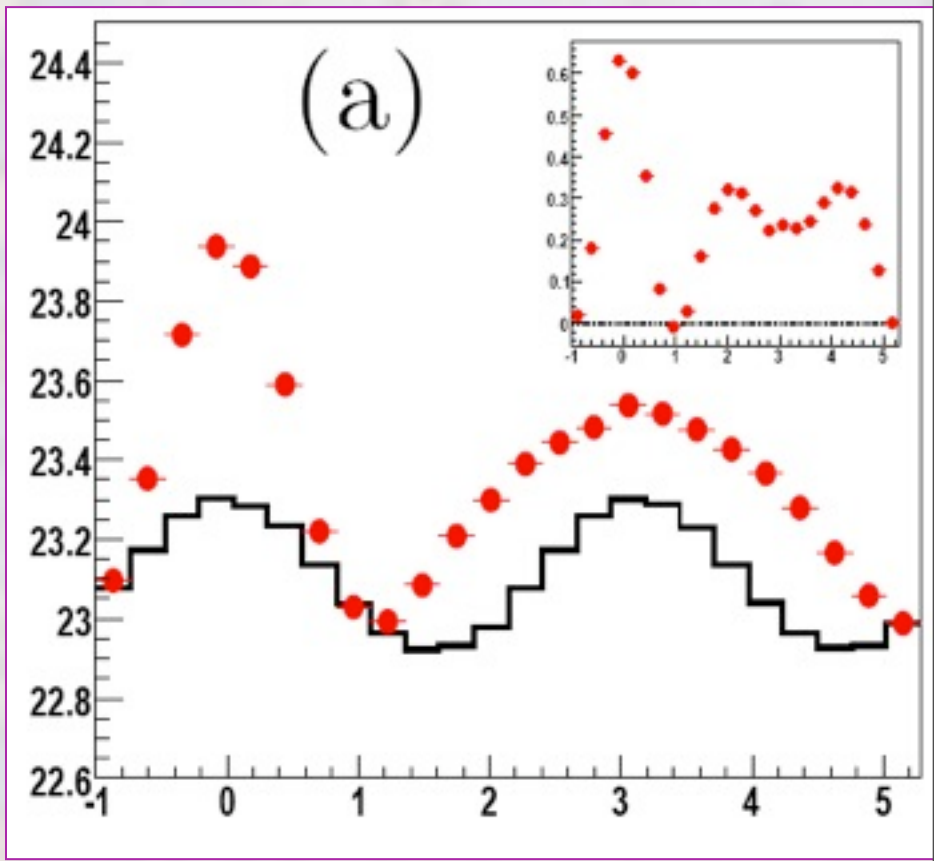

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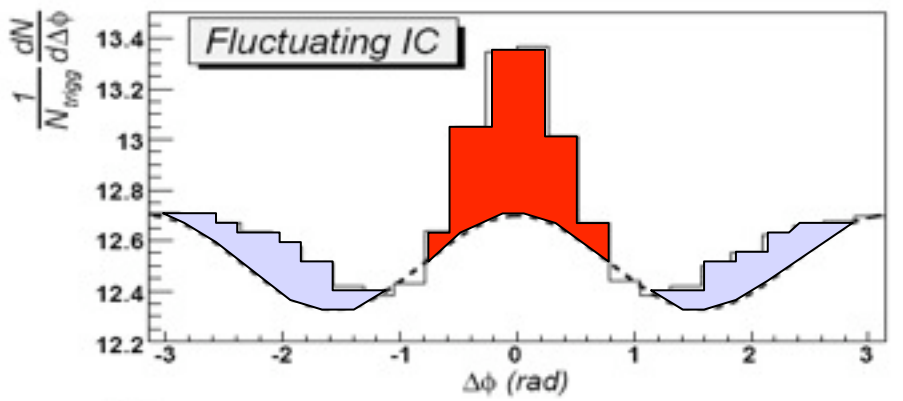

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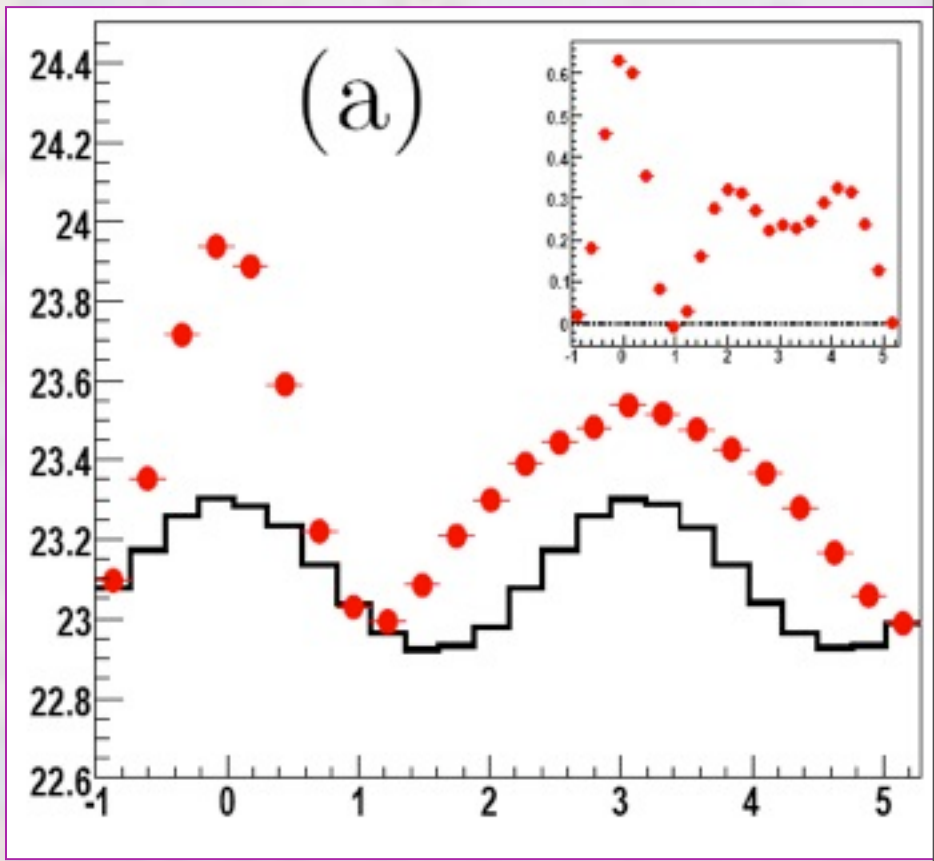

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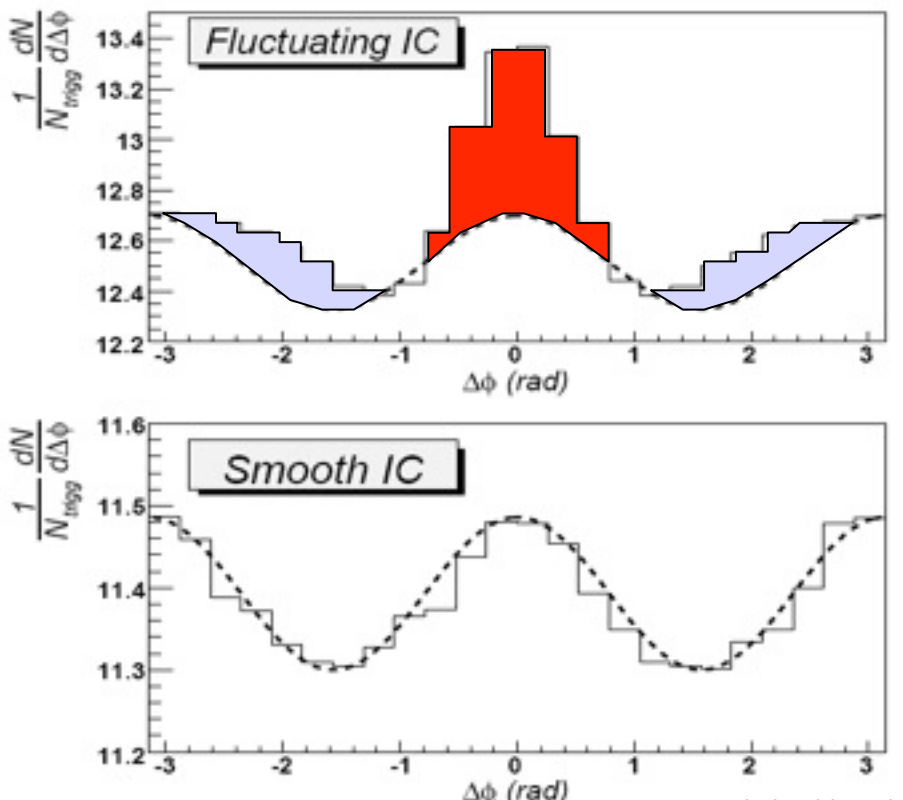

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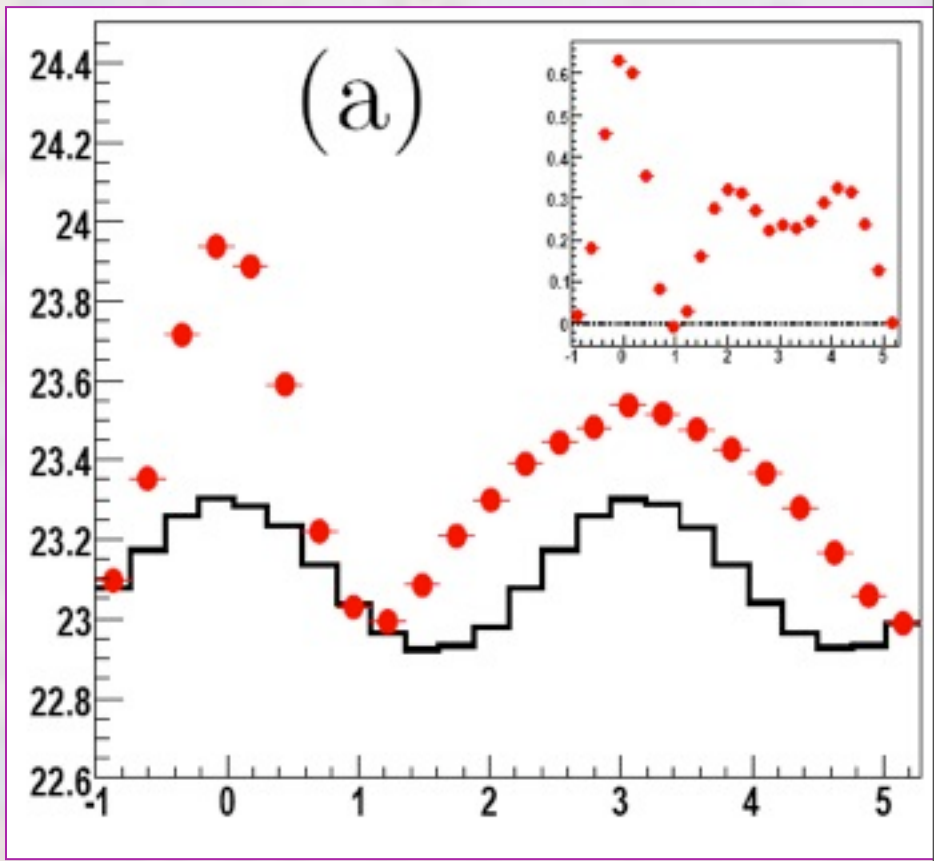


Striking similarity with recent data analysis!

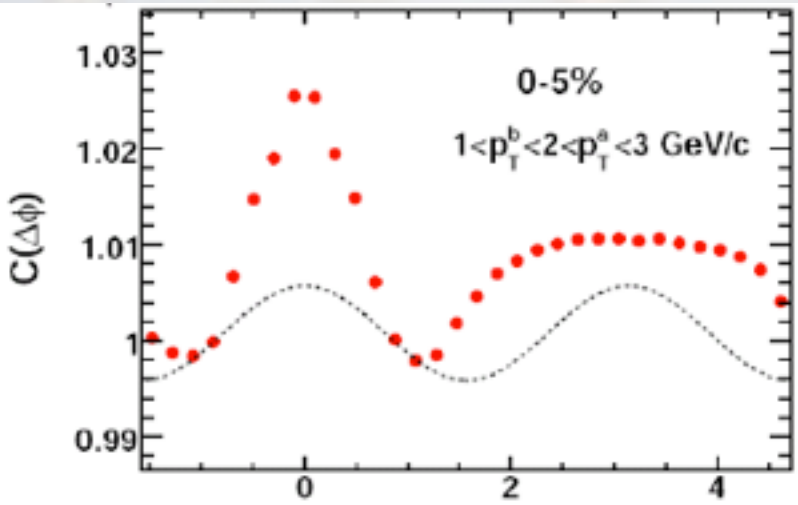
More Differential Data

Data versus reaction plane, 3 particle correlations, 2+1, É

Slide from M. Gyulassy

The "Mach" signal is tiny ~ 1% in correlation func

Systematic errors need much more attention



Recent progress on absolute normalization is an attempt to measure background levels, but even then the Assumption of two independent Jet + QGP sources is dubious.

I believe experimentalists are paying much more attention.
ZYAM is not the issue.
If components do not factorize, then
we should publish the full correlations (every time).

Summary

PAY ATTENTION TO DETAILS

1. Let us talk the same language (and use terms as they have been defined)

2. When comparing to experimental results Ð pay attention how those have been derived. (But do not be scared by different results obtained with different methods Ð all of them has its own value).

3. Clear separation of effects of non-flow and flow fluctuations is still not possible, but the Gaussian model provides good guidelines.

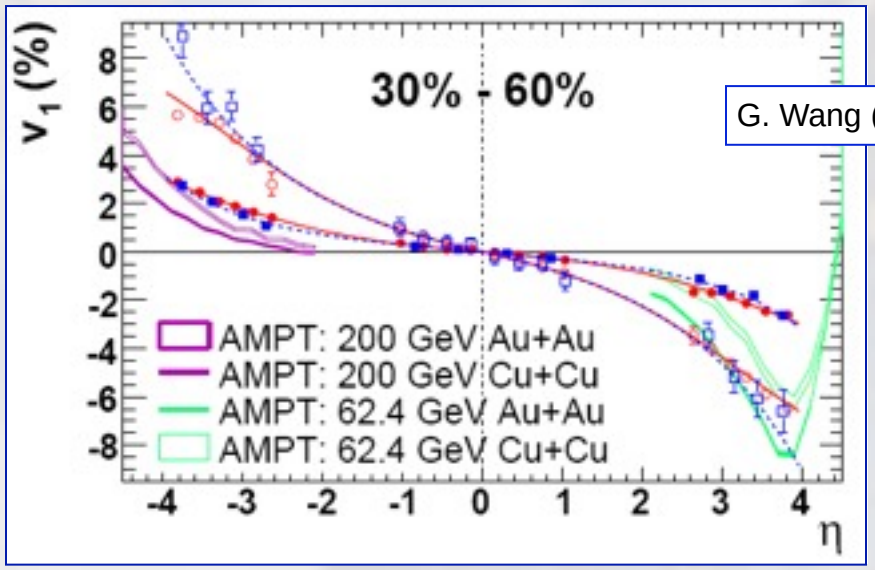
4. Use but do not misuse flow results!

Azimuthal correlations analyses of non-central collisions established to be one of the most informative direction in HIC studies (+ Global polarization, local parity violation *) studies, etc.)
but as everything else cam not be treated blindly...

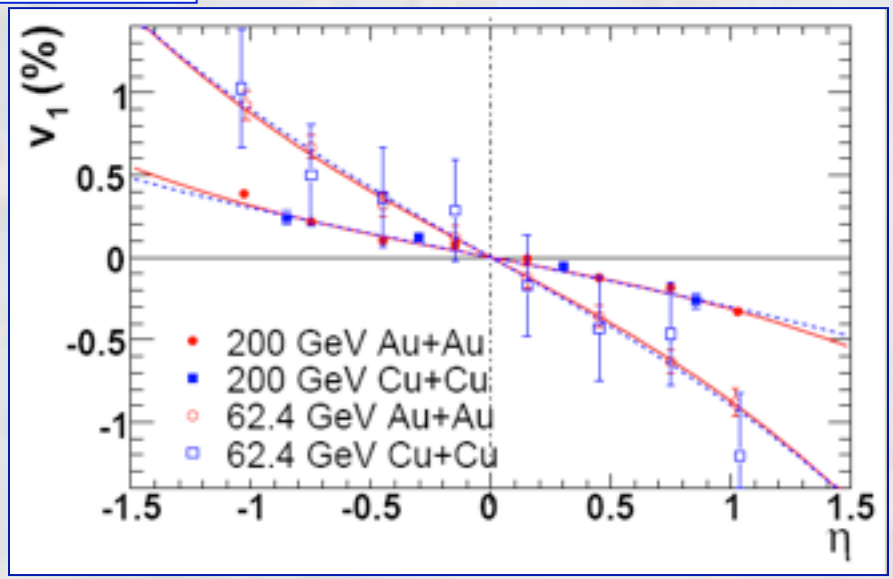
*) For recent STAR results see arXiv: 0909:1717 !!!

Backup slides

Directed flow



G. Wang (STAR), QM2006



Not quantitatively explained by any model (even close)!
Error-bars at the level of $< 10^{-3}$!

AuAu and CuCu are very similar at the same centrality!

(Magenta curves are polynomial fits to guide the eye)

High accuracy of these measurements achieved by
- using STAR ZDC-SMD (Òspectator neutronsÓ)
- 3 Ðparticle correlations (mixed harmonics)

3 particle correlations measure the difference in correlations projected into the reaction plane and out-of-plane directions making use of strong elliptic flow to define the plane..

$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$$
$$= \langle \cos(\phi_\alpha - \Psi_{RP}) \cos(\phi_b - \Psi_{RP}) \rangle - \langle \sin(\phi_\alpha - \Psi_{RP}) \sin(\phi_b - \Psi_{RP}) \rangle$$
$$\approx v_{1,a} v_{1,b}$$

2-particle correlations relative to the Reaction Plane

2-particle correlations wrt RP

$$\frac{d^2N}{dx_1 dx_2}(x_1, x_2; \Psi_{RP})$$

x - azimuthal angle, transverse momentum, rapidity, etc.

Selection of one (or both) of particles in- or out- of the reaction plane "distorts" the RP determination

(J. Slivova, P. Wurm, K. Filimonov S. Esumi, S.V.)

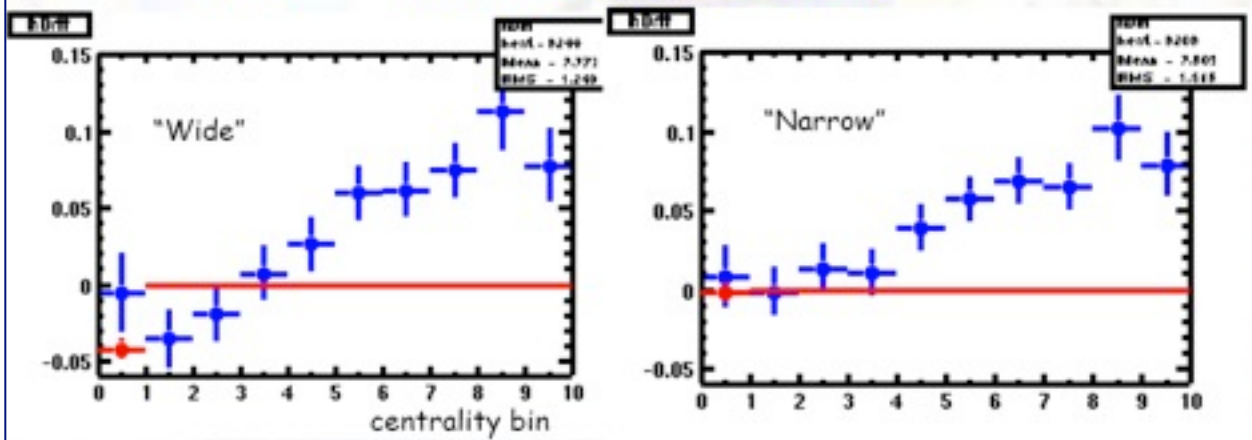
$$\frac{dN^{flow}_{pairs}}{d\Delta\phi_{a,b}} \propto 1 + 2v_{2,b}\, v^{in,out}_{2,a} \cos(2\Delta\phi_{a,b})$$

"a" == "trigger particle"

$$v_2^{in} = \frac{\pi v_2 + 2}{\pi + 4v_2} \qquad v_2^{out} = \frac{\pi v_2 - 2}{\pi - 4v_2}$$

page 31 Columbia University,

"Same" - "Opposite". AuAu and pp



"Trigger" == abs(eta) < 0.5; 3 < pt < 5 GeV
"Associated" == 1.8 < pt < 5 GeV

"Same" == cos(phi1 - phi2) > 0.5
"Opposite" == cos(phi1-phi2) < -0.5

"Wide: == abs(eta1-eta2) < 1.5
"Narrow" == abs(eta1-eta2)<0.5 if eta*eta2> 0 and abs(eta-eta2)>1.0 if eta*eta2<0

page 28 Columbia University, May, 2003 S.A. Voloshin

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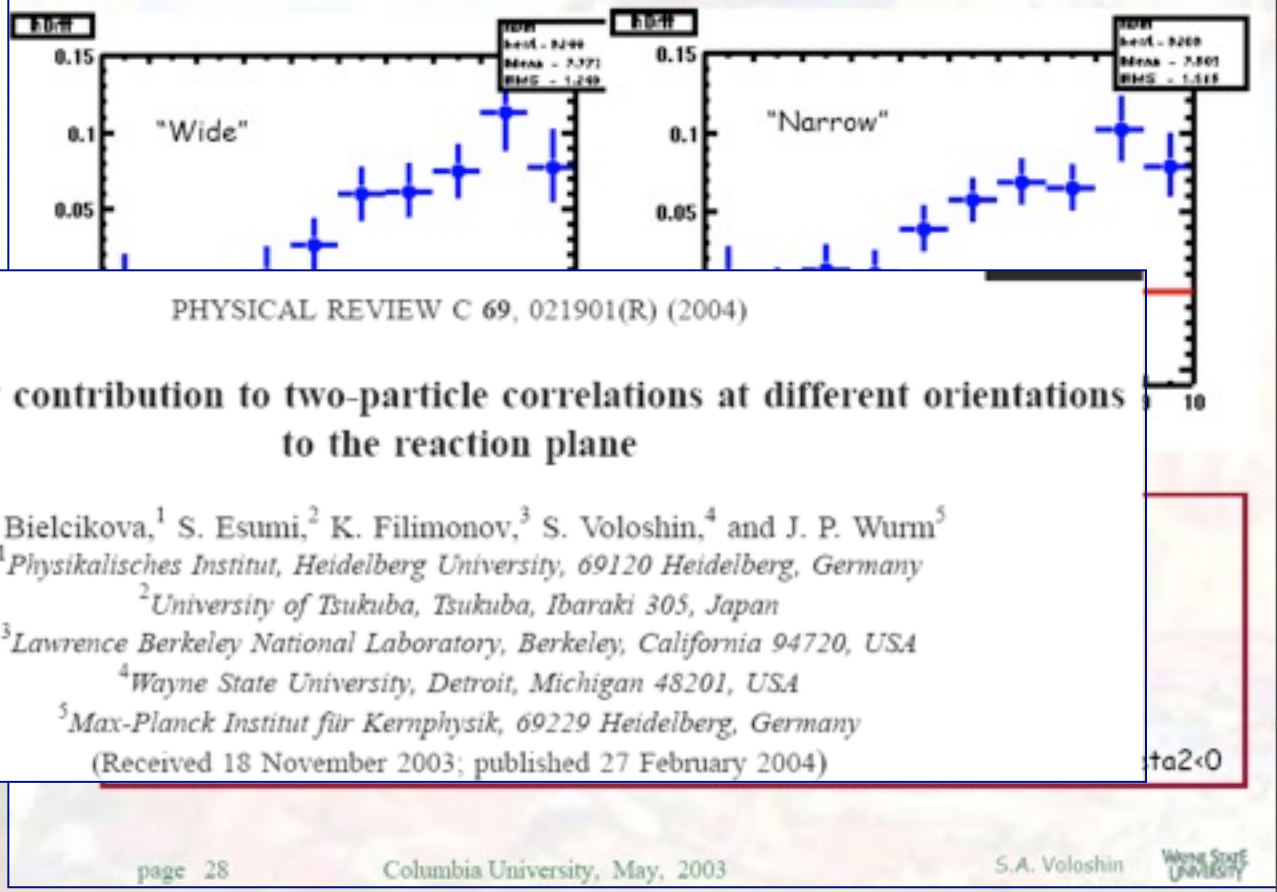
page 31

"Same" - "Opposite". AuAu and pp



page 28 Columbia University, May, 2003 S.A. Voloshin

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Elliptic flow contribution to two-particle correlations at different orientations to the reaction plane

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Correlation functions (cumulants)

$$C_2(x_1,x_2)=\rho_2(x_1,x_2)-\rho_1(x_1)\rho_1(x_2)$$

$$R_2(x_1,x_2)=\frac{C(x_1,x_2)}{\rho_1(x_1)\rho_1(x_2)} \qquad \left[B(y_1,y_2)=\frac{C_2(y_1,y_2)}{\rho_1(y_1)}\right]$$

$$\int dx\rho_1(x)=\langle n\rangle$$

$$\int dx_1\int dx_2\rho_2(x_1,x_2)=\langle n(n-1)\rangle$$

$$C_3=\rho_3(x_1,x_2,x_3)-C_2(x_1,x_2)\rho_1(x_3)-C_2(x_1,x_3)\rho_1(x_2)-C_2(x_2,x_3)\rho_1(x_1)-\rho_1(x_1)\rho_1(x_2)\rho_1(x_3)$$

Note that cumulants are the only "true" indicators of correlations. If cumulant is zero, there is no way to prove that correlations (e.g due to clustering or temperature fluctuations) exists (though they might be present).

Correlation functions (cumulants)

$$C_2(x_1,x_2) = \rho_2(x_1,x_2) - \rho_1(x_1)\rho_1(x_2)$$

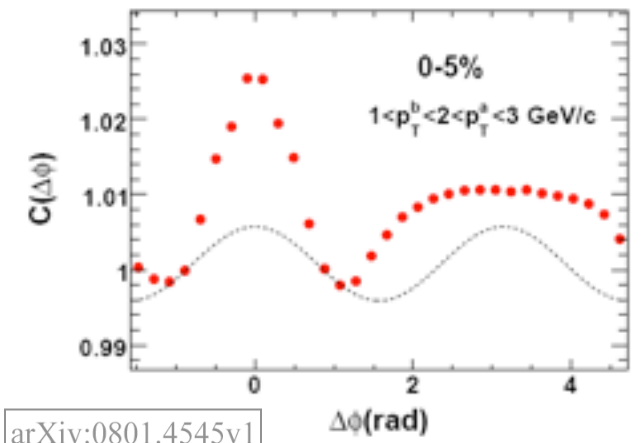
$$R_2(x_1,x_2) = \frac{C(x_1,x_2)}{\rho_1(x_1)\rho_1(x_2)} \qquad \left[B(y_1,y_2) = \frac{C_2(y_1,y_2)}{\rho_1(y_1)} \right]$$

$$\int dx \rho_1(x) = \langle n \rangle$$

$$\int dx_1 \int dx_2 \rho_2(x_1,x_2) = \langle n(n-1) \rangle$$

$$C_3 = \rho_3(x_1,x_2,x_3) - C_2(x_1,x_2)\rho_1(x_3) - C_2(x_1,x_3)\rho_1(x_2) - C_2(x_2,x_3)\rho_1(x_1) - \rho_1(x_1)\rho_1(x_2)\rho_1(x_3)$$

Note that cumulants are the only "true" indicators of correlations. If cumulant is zero, there is no way to prove that correlations (e.g due to clustering or temperature fluctuations) exists (though they might be present).



arXiv:0801.4545v1

FIG. 2: (Color online) The correlation function for $2 < p_T^a < 3$, $1 < p_T^b < 2$ GeV/c in 0-5% Au+Au collisions. The dashed line represents the estimated elliptic flow modulated combinatoric background using zero yield at minimum (ZYAM) method (see Section.III E).

Would the interpretation of the figure on the left change
(as leading to the two bump structure on the away side)?

Not much, in a sense that if one compare the cumulant
(close to shown in red points) to the cumulant expected
from "only elliptic flow" (different from dash curve only
in normalization), the difference will exhibits the same
two bumps. But one will be free from confusing picture

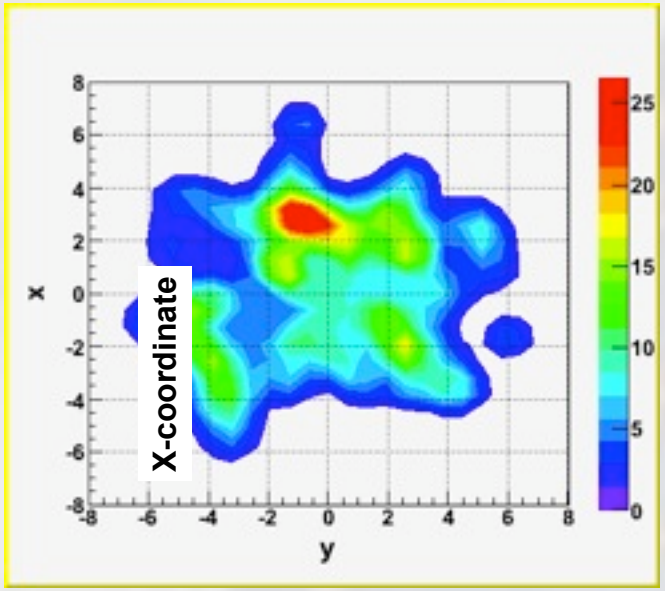
"Consequences" – possible misinterpretations

– If the "background" is not correctly identified (and from the above I would conclude that in many cases
we just do not know how to do it) we can fool ourselves (and others) talking about the pt spectra of associated particles, Mach cones, etc.

For example, having two back-to back jets with, e.q. 10 GeV each, and redistributing this 10 GeV among
100 particles but counting above "background" only 5, we would conclude that we have a component
carrying 2 GeV per particle, which would have nothing to do with reality. Similar can be said about baryon/meson ratios in the "correlated part", etc.

conclusions

1. We should take ZYAM and similar with **much** of caution….
2. We should always include in publications "raw" data, namely single, two, and three particle densities
   (+ cumulants, etc) before we apply any manipulations to them. I put "raw" in quotation marks, as those,
   are to be corrected for efficiencies, etc. Otherwise we can be at risk that in a few years the publications
   can be useless.


X-coordinate
x
y
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2
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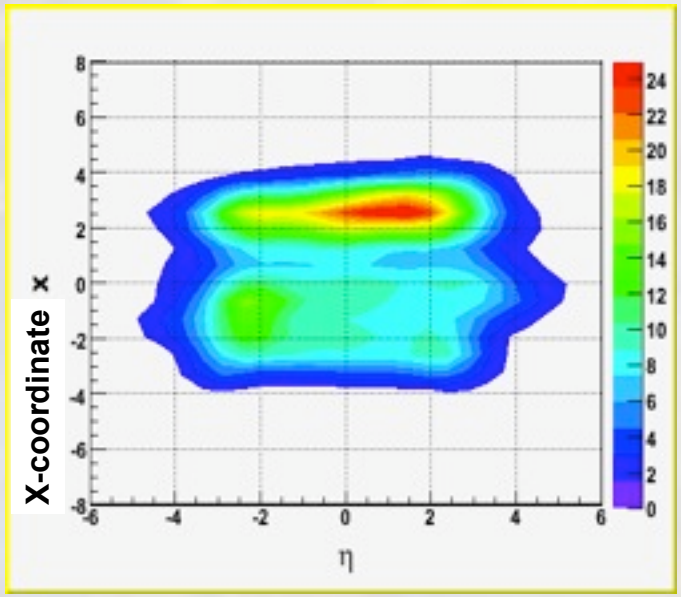
X-coordinate
x
η
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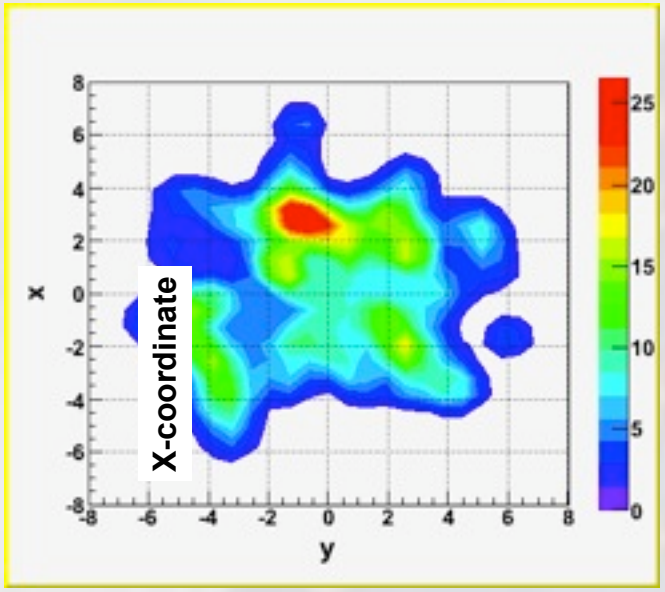
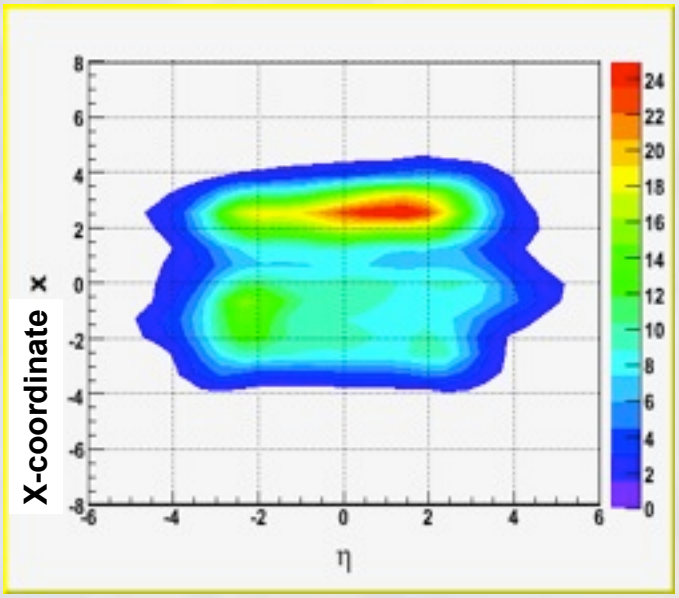
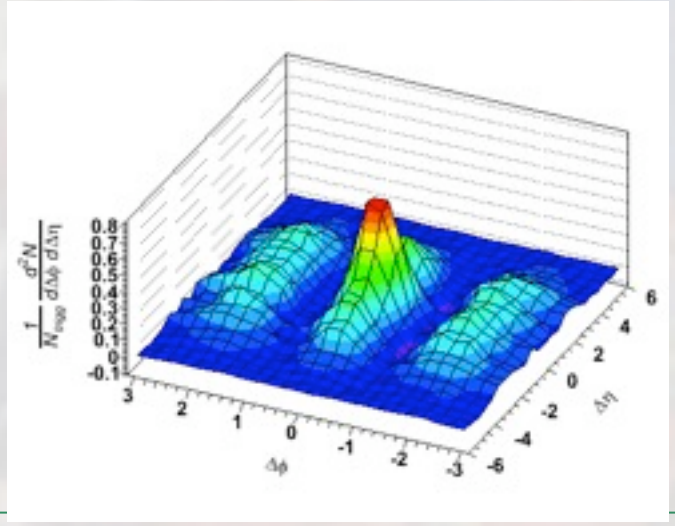

X-coordinate
x
y


X-coordinate
x
η


Δφ
Δη

Reaction, ÒparticipantÓ, and event (flow vector) planes

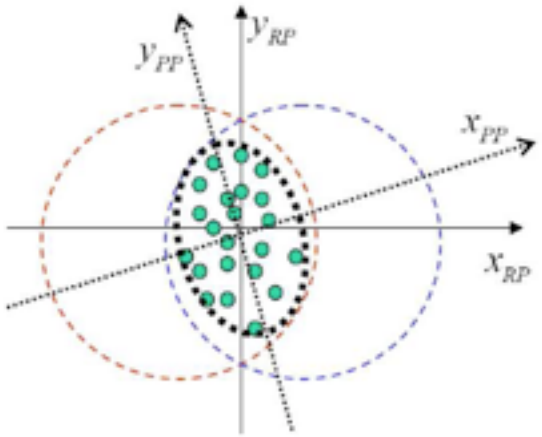


FIG. 1: The definition of the RP and PP coordinate systems.

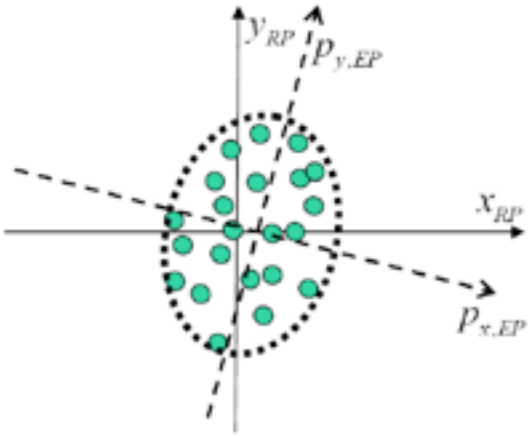


FIG. 2: The definitions of the EP coordinate system

$$v = \kappa\varepsilon$$

$$\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part} \right\}$$

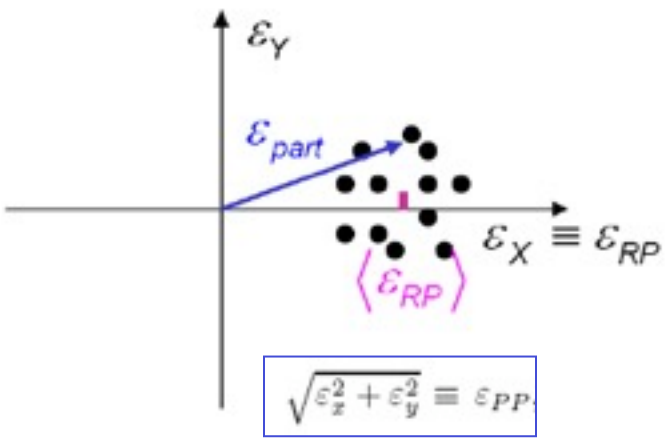


FIG. 3: Definition of ε_{part}.

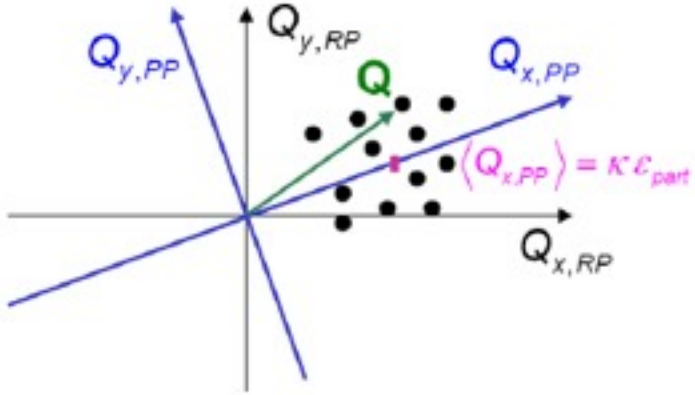


FIG. 4: Flow vector distribution in events with fixed ε