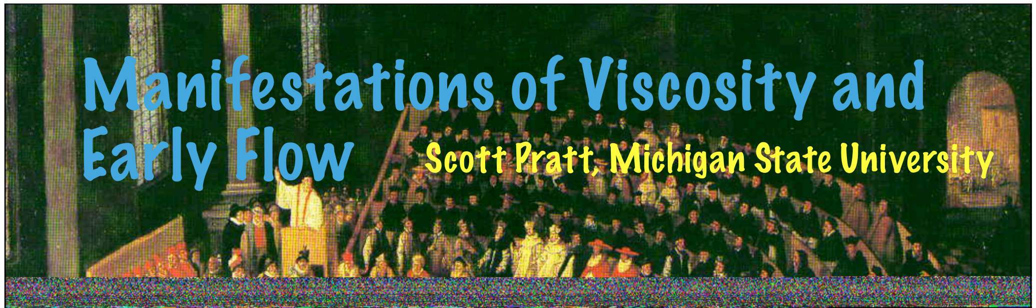


Manifestations of Viscosity and Early Flow

Scott Pratt, Michigan State University

I. Dynamics and Flow

II. Pre-Equilibrium Flow

III. Experimental Observables

IV. Extracting Viscosity from Experiment

Stress-Energy Tensor

$$\Delta T_{ij} = -\eta\left[\partial_i v_j + \partial_j v_i - (2/3)\delta_{ij}\nabla\cdot v\right] - \zeta\nabla\cdot v$$

SHEAR **BULK**

$$\Delta T_{xx} = -\eta\left[(4/3)\partial_x v_x - (2/3)\partial_y v_y - (2/3)\partial_z v_z\right] - \zeta\left[\partial_x v_x + \partial_y v_y + \partial_z v_z\right]$$

reduces pressure along direction of strongest velocity gradient

reduces pressure isotropically

$$\Delta T_{ij} = -\eta\left[\partial_i v_j + \partial_j v_i - (2/3)\delta_{ij}\nabla\cdot v\right]$$

SHEAR

$$\Delta T_{xx} = -\eta\left[(4/3)\partial_x v_x - (2/3)\partial_y v_y - (2/3)\partial_z v_z\right]$$

I. For early times, $\partial_z v_z >> \partial_x v_x \,\&\, \partial_y v_y$:
T_{zz} is reduced, T_{xx} and T_{yy} increase
-> Less cooling, more transverse flow

II. For non-zero b, $\partial_x v_x > \partial_y v_y$:
$T_{xx} > T_{yy}$, elliptic flow reduced

BULK

$$\Delta T_{ij} = -\zeta \nabla \cdot v$$

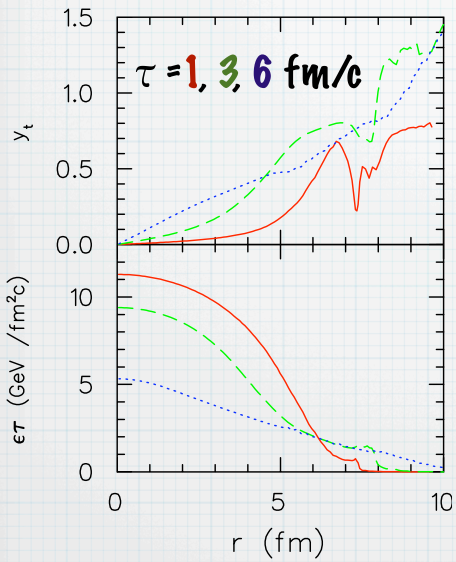
$$\Delta T_{xx} = -\zeta \left[\partial_x v_x + \partial_y v_y + \partial_z v_z \right]$$

I. ς largest near T_c,

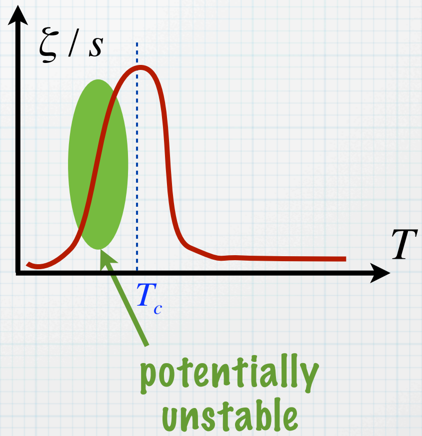
$\nabla \cdot v$ large for early times

II. Can lead to instabilities near T_c

Instabilities from Bulk Viscosity



$$c_{\text{eff}}^2 = \frac{dT_{ii}}{d\epsilon} = c_s^2 - \frac{d\zeta}{d\epsilon}\nabla\cdot v$$



Two Sources of Anisotropy of T_{ij}

1. Shear Viscosity
2. Initial conditions

MODEL	T_{zz}	$T_{\perp}$	$dE/d\eta$	Tr $T_{\alpha\beta}$
Fat Flux Tube	$-\varepsilon$	ε	$\propto\tau$	0
Skinny Flux Tube	0	$\varepsilon/2$	constant	0
Ideal Hydro	$\varepsilon/3$	$\varepsilon/3$	$\propto\tau^{-1/3}$	0
Free-Streaming	0	$\varepsilon/2$	constant	0
CGC (Krasnitz,Nara,Venugopolan)	$-\varepsilon<T_{zz}<0$	$\varepsilon/2<T_{xx}<\varepsilon$	$\approx$constant	0
???????	0	0		

Universal Flow for Early Times

J.Vredevoogd and S.P., PRC 2009

Ignoring Longitudinal Flow

$$T_{xx} = \kappa T_{00}$$

$$\partial_t T_{0x} = -\partial_x T_{xx}$$

$$\text{Flow} \quad \boxed{\frac{T_{0x}}{T_{00}}} = -\kappa \frac{\partial_x T_{00}}{T_{00}} t$$

Bigger κ -> Bigger Flow

Universal Flow for Early Times

J.Vredevoogd and S.P., PRC 2009

Including Longitudinal Flow

$$T_{00} \sim t^{-2+2\kappa}$$

$$T_{xx} = \kappa T_{00}$$

$$\partial_t T_{0x} = -\partial_x T_{xx}$$

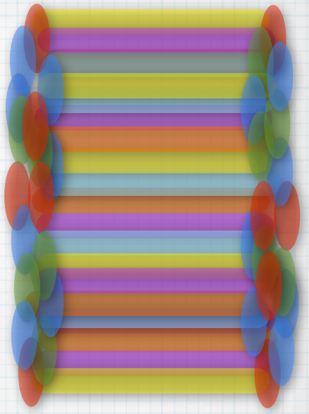
$$\partial_t T_{00} = (-2+2\kappa)\frac{T_{00}}{t}$$

$$\frac{T_{0x}}{T_{00}} = -\frac{1}{2}\frac{\partial_x T_{00}}{T_{00}}t$$

Initial flow independent of κ

(but not velocities, u_x)

Test "Universality with 3 Models"

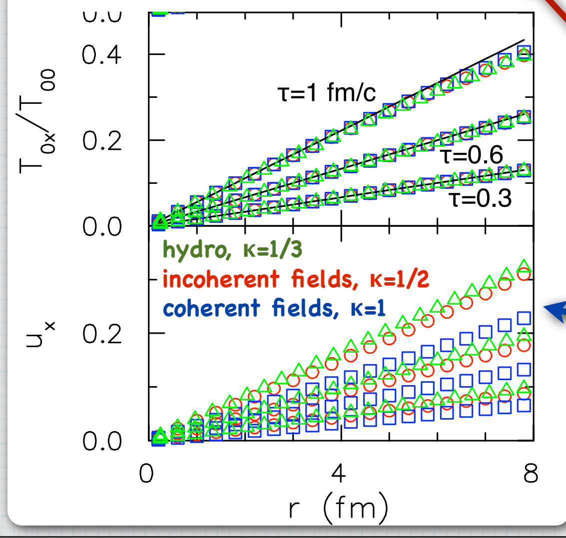


1. Hydrodynamics (κ =1/3)
   Solve relativistic hydro

2. EM Fields from two receding capacitor plates (κ =1)
   (one positive one negative)
   Solve Maxwell's eq.s

3. Same as (2) but with point-like random charges (κ =1/2)
   Similar to free-streaming
   Solve Maxwell eq.s again

Three Models

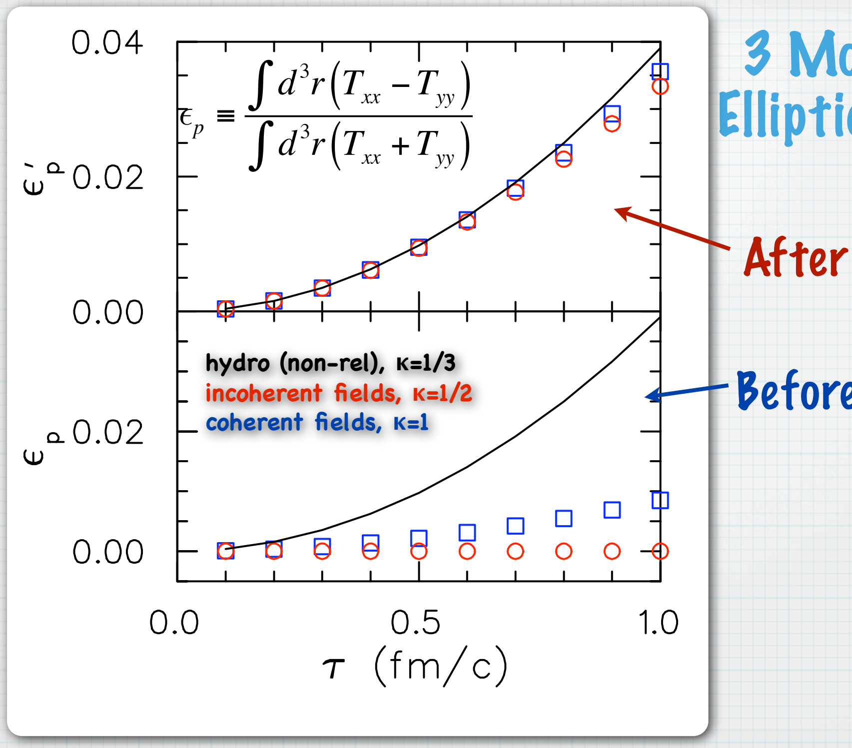


Switch to hydro at τ
Preserve T_{00} & T_{0x}

After thermalization

Before thermalization

3 Models: Elliptic Flow



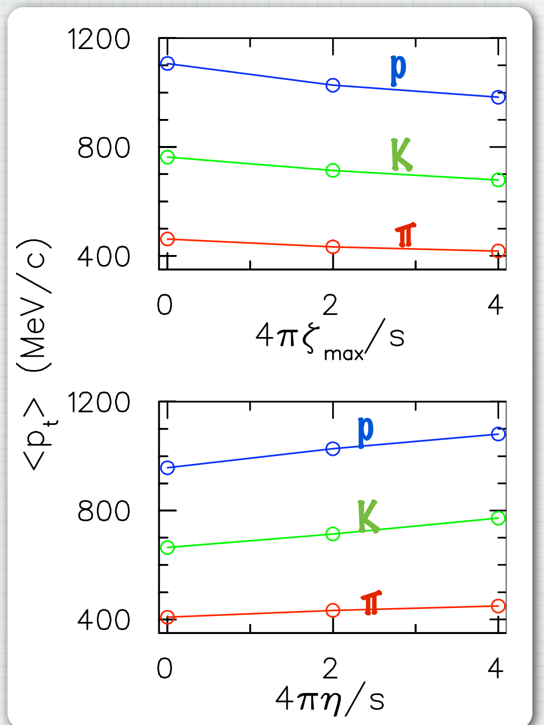
"Universality" Summary

- Only for initial flow, T_{0x}/T_{00}.
- Universality requires:
  Traceless T_{ij}, κ depends only on τ, Boost invariance
- For very different models, first fm/c flows vary at 10-20% level only
- One can confidently initialize viscous hydro if NS conditions are satisfied by 1.0 fm/c
- Different IS, 2nd-order formalisms shouldn't matter

Experimental Manifestations of Viscosity

1. Spectra
2. Elliptic Flow
3. HBT

Viscosity & Spectra

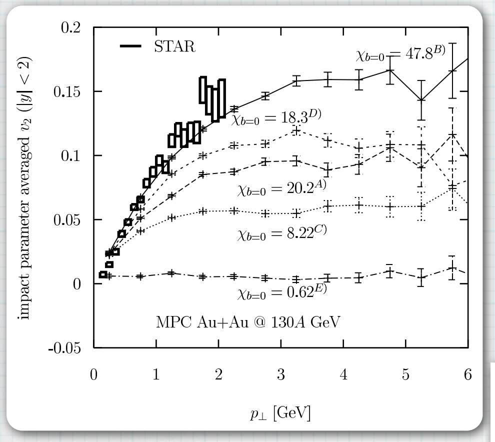


S.P. and J. Vredevoogd, PRC 2009

Larger η -> larger flow

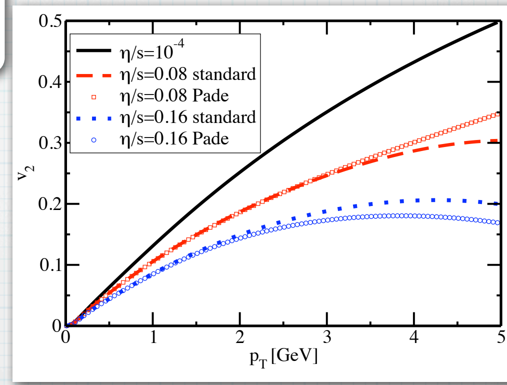
Larger ζ -> smaller flow

Viscosity & Elliptic Flow

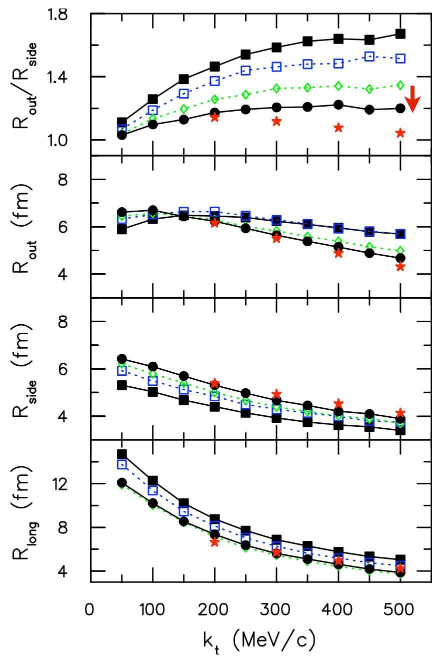


Molnar and Gyulassy, 2002

Luzum and Romatschke, 2009



More η -> less v_2



S.Pratt, PRL 2009

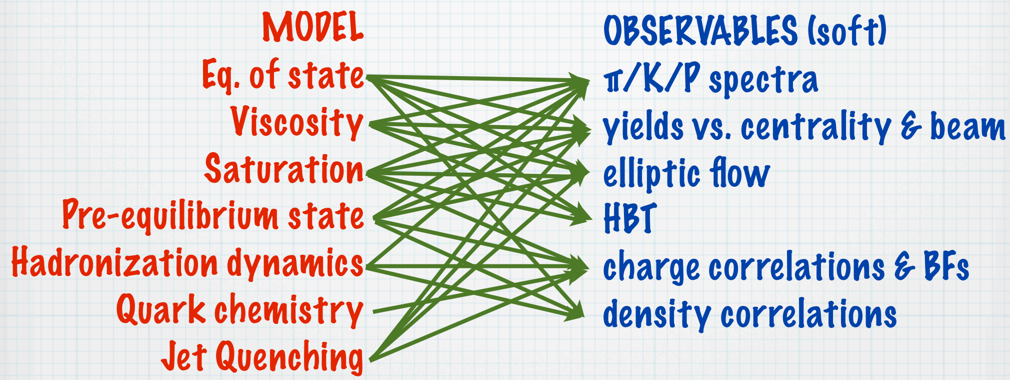
Viscosity & HBT

R_{out}/R_{side} affected at 10% level by:

1. Stiffening EoS
2. Adding initial flow
3. Shear
4. Changing initial profile

Bulk has little effect on HBT but changes spectra

Model/Observable Sensitivities



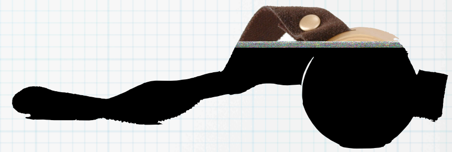
Uncertainties and Parameters

Initial State	6	Energy density, profile shape, rapidity width, pressure, anisotropy of T_{ij}, quark/gluon content
Hadronic Boltzmann	2-4	Mass changes
Eq. of State / Viscosity	3-8	Might be constrained by lattice, hadron gas
Chemical	3-6	Quark density, relaxation rates, hadronic scattering reduction
Jet Quenching	2-4	Dissipation rates
Systematic Experimental	?	Efficiencies, calibrations...

≈ 30 parameters
Some are unimportant
Some combinations are unimportant

Turn ALL the knobs!!!!

MaDAI Collaboration
Models and Data Analysis Initiative



Michigan State University

- RHIC Physics: Scott Pratt
- Supernova: Wolfgang Bauer
- Astrophysics: Brian O'Shea and Mark Voit
- Atmospheric Modeling: Sharon Zhong
- Statistics: Dan Dougherty

Duke University

- RHIC Physics: Steffen Bass and Berndt Müller
- Statistics: Robert Wolpert

University of North Carolina & Renaissance Computing Institute

- Visualization: Xunlei Wu

Will be funded by NSF CDI program (Cyber-Enabled Discovery Initiative)

Bayesian Analysis

WIKIPEDIA: Bayesian inference is statistical inference in which evidence or observations are used to update or to newly infer the probability that a hypothesis may be true. The name "Bayesian" comes from the frequent use of Bayes' theorem in the inference process.

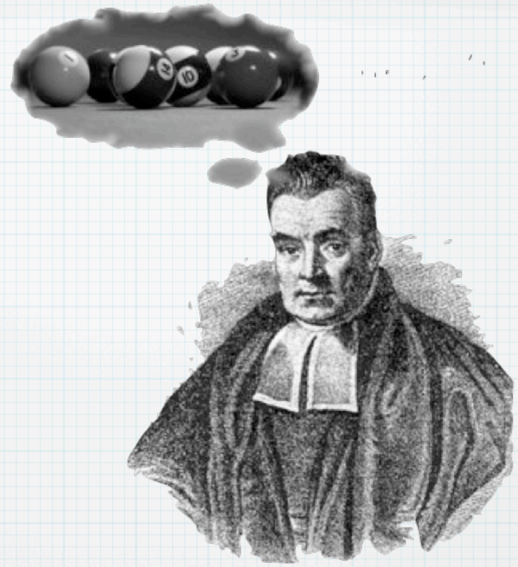
$$P(H \mid E) = \frac{P(E \mid H) P(H)}{P(E)}$$

- P(H) is probability (in absence of E) for parameter set H a.k.a. the "prior distribution"
- P(E|H) is probability of E given H, i.e,

$$P \sim \exp\left(-\sum \delta_i^2 / 2\sigma_i^2\right)$$

- P(E) is net probabiltiy of E, i.e., a normalization factor
- P(H|E) is probability of parameter set H given E

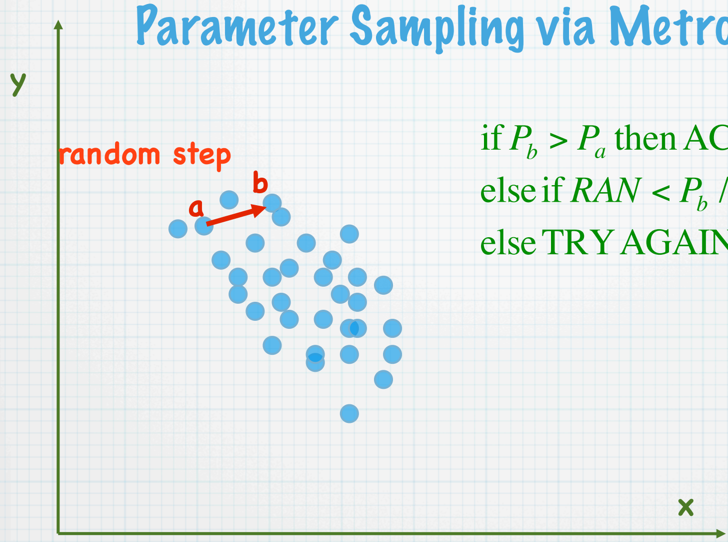
Bayes' Theorem



$$P(E \,\&\, H) = P(E \mid H) \cdot P(H) = P(H \mid E) \cdot P(E)$$

$$P(H \mid E) = \frac{P(E \mid H) \cdot P(H)}{P(E)}$$

Parameter Sampling via Metropolis



if $P_b > P_a$ then ACCEPT

else if $RAN < P_b / P_a$ ACCEPT

else TRY AGAIN

Can find disjoint regions

No problem with undeterminable parameters

Surrogate Models
(a.k.a. Emulators, Meta-Models)

Brute Force:

- Sampling requires millions of runs
- Each run requires 1 work-station day

Alternative:

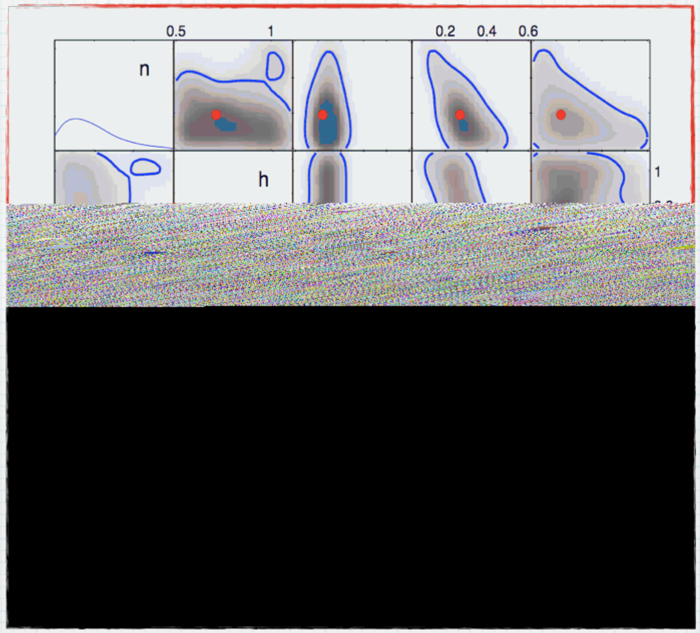
- Run 10^2 - 10^3 times at various points
- ``Interpolate'' to find values at all other points
- Competing "interpolation" schemes:
  - Gaussian fields
  - Multi-dimensional splines

An Emerging Science

Assessing Prospects for New Data

- Make Simulated Data
- Analyze Like Real Data
- Calculate Resolving Power of Subset

Approach taken from astrophysics.....

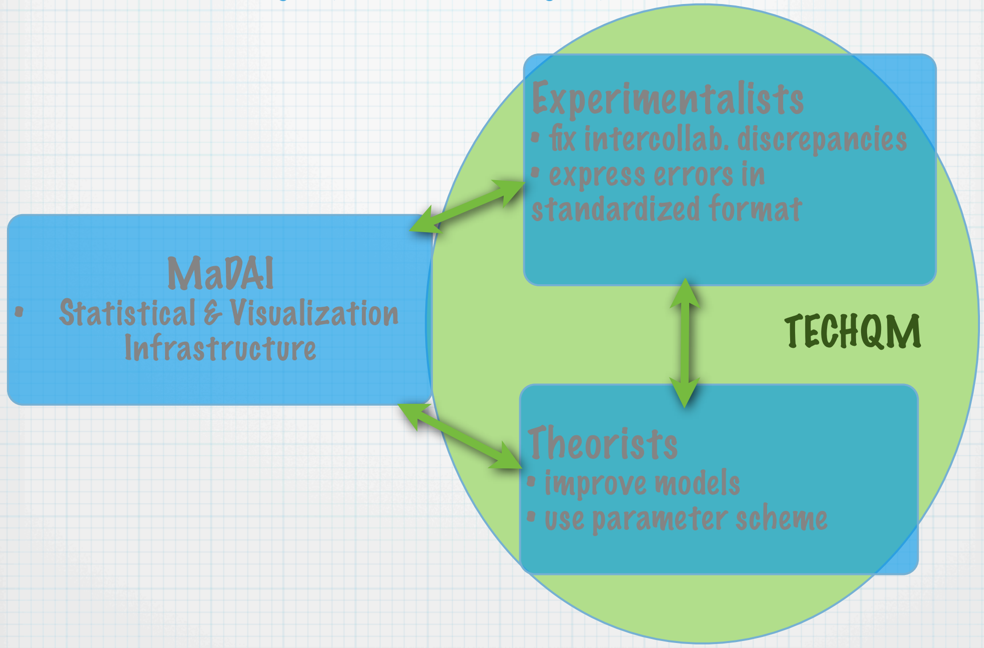


Cosmological parameters (Habib et al, astro-ph/0702348 1)

Can this work for RHIC?

- Must be amenable to parameterization
  - Model must contain basic truth
  - Not too many competing theories
  - First apply to soft observables (spectra, flow, HBT...)
  - Provide validated base for other calculations
- Must have well stated errors
  - Statistical & systematic for both theory and experiment
  - Cross correlated errors
  - May require re-expression of experimental results
  - Intimate theory/experimental discussions

Who Does What



Outcomes

- Rigorous Statement of Bulk Properties
- Validated Base for Calculating:
  Fluctuations, Jet Quenching, Heavy Flavors...