Covariant transport, dissipation and flow

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Outline

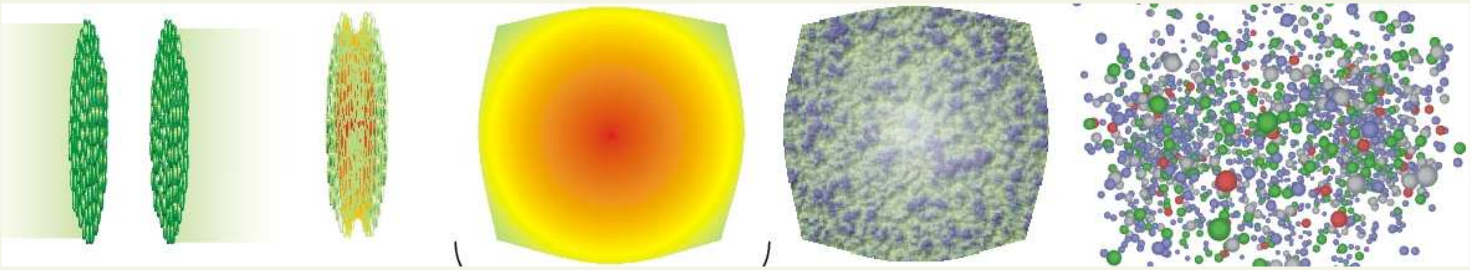
- **I. Covariant transport theory**

- **II. Thermalization at RHIC**

- **III. Applicability of viscous hydrodynamics**

- **IV. Open questions**

Dynamical frameworks



$\sim 10^{-14}$ **m** **= 10 fermis**

<—— $\sim 10^{-23}$ **sec = 10 yocto(!)-secs** ——>

- **hydrodynamics** Mueller, Israel, Stewart, ... Csernai, Stöcker, Rischke, Shuryak, Teaney, Heinz, Kolb, Huovinnen, Hirano, Muronga, Romatschke, DM, Kodama, Kodei, ...

  ideal (Euler) hydro, dissipative (viscous) hydro

- **covariant transport** Grad, Israel, de Groot, Leeuwen, ... Elze, Gyulassy, Heinz, Pang, Aichlen, Zhang, Vance, DM, Csizmadia, Pratt, Cheng, Greiner, Xu, ...

  parton cascade, hadron cascade

- **classical field theory** Venugopalan, McLerran, Rischke, Krasnitz, Nara, Lappi, Gelis, Arnold, Lenagan, Dumitru, Strickland, Romatschke, ...

  classical Yang-Mills (color glass)

I. Covariant transport theory

simple covariant nonequilibrium theory that can thermalize

local scattering rate

$$\frac{dN_{sc}}{d^3x\,dt} = n_{target} \cdot j_{beam} \cdot \sigma = n_{target}(\vec{x},t)\, n_{beam}(\vec{x},t)\, v_{rel}\, \sigma$$

consider also momenta: $n(\vec{x},t) \to f(\vec{x},\vec{p},t) \equiv dN/d^3x d^3p$

$$\frac{\partial f_b(\vec{x},\vec{p},t)}{\partial t} = -\int f_t(\vec{x},\vec{p}_1,t)\, f_b(\vec{x},\vec{p},t)\; v_{rel}(\vec{p},\vec{p}_1) \frac{d\sigma(p,p_1 \to p',p'_1)}{d^3p'\, d^3p'_1}\; d^3p' d^3p'_1 d^3p_1$$
$$+ \int f_t(\vec{x},\vec{p}',t)\, f_b(\vec{x},\vec{p}'_1,t)\; v_{rel}(\vec{p}',\vec{p}'_1) \frac{d\sigma(p',p'_1 \to p,p_1)}{d^3p\, d^3p_1}\; d^3p_1 d^3p'_1 d^3p'$$

and free streaming: $f(\vec{x},\vec{p},t) = f(\vec{x}+\vec{v}\Delta t, \vec{p}, t+\Delta t)$

$\Rightarrow$ **Boltzmann eq:** $p^\mu \partial_\mu f(x,\vec{p}) = C[f](x,\vec{p})$

An example - quarks & gluons

Elementary processes: elastic $2 \to 2$ processes + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

Equation for $f^i(x, \vec{p})$: $\quad i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$$
\begin{aligned}
p_1^\mu \partial_\mu \tilde{f}^i(x, \vec{p}_1) = & \frac{\pi^4}{2} \sum_{jkl} \int_2\!\!\int_3\!\!\int_4 \left(\tilde{f}^k_3 \tilde{f}^l_4 - \tilde{f}^i_1 \tilde{f}^j_2\right) \left|\bar{\mathcal{M}}^{i+j\to k+l}_{12\to 34}\right|^2 \delta^4(12-34) \quad \swarrow 2 \to 2 \\
& + \frac{\pi^4}{12} \int_2\!\!\int_3\!\!\int_4\!\!\int_5 \left(\frac{\tilde{f}^i_3 \tilde{f}^i_4 \tilde{f}^i_5}{g_i} - \tilde{f}^i_1 \tilde{f}^i_2\right) \left|\bar{\mathcal{M}}^{i+i\to i+i+i}_{12\to 345}\right|^2 \delta^4(12-345) \quad \swarrow 2 \leftrightarrow 3 \\
& + \frac{\pi^4}{8} \int_2\!\!\int_3\!\!\int_4\!\!\int_5 \left(\tilde{f}^i_4 \tilde{f}^i_5 - \frac{\tilde{f}^i_1 \tilde{f}^i_2 \tilde{f}^i_3}{g_i}\right) \left|\bar{\mathcal{M}}^{i+i\to i+i+i}_{45\to 123}\right|^2 \delta^4(123-45) \quad \swarrow 3 \leftrightarrow 2 \\
& + \tilde{S}^i(x, \vec{p}_1) \quad \leftarrow \textbf{initial conditions}
\end{aligned}
$$

with shorthands:

$$
\tilde{f}^q_i \equiv (2\pi)^3 f_q(x, \vec{p}_i), \qquad \int_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \qquad \delta^4(p_1 + p_2 - p_3 - p_4) \equiv \delta^4(12-34)
$$

Nonlinear 6+1D transport eqn: solvable numerically

$$p^\mu \partial_\mu f_i(\vec{x},\vec{p},t) = \overbrace{S_i(\vec{x},\vec{p},t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x},\vec{p},t)}^{2\to 2\ (\text{ZPC, GCP},\ldots)} + \overbrace{C_i^{inel.}[f](\vec{x},\vec{p},t)}^{2\leftrightarrow 3\ (\text{BAMS, MPC})} + \ldots$$

handful of covariant/causal codes: ZPC, MPC, Bjorken-τ, BAMS, ...

cascade algorithm: Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, ...

- rates implemented through a geometric "closest approach" prescription

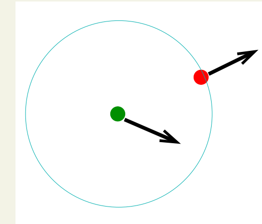
grid algorithm: Greiner, Xu ...

- rates estimated, in each timestep, based on test particles in the cell

correct (causal and covariant) numerical results require many test particles, and fine grids

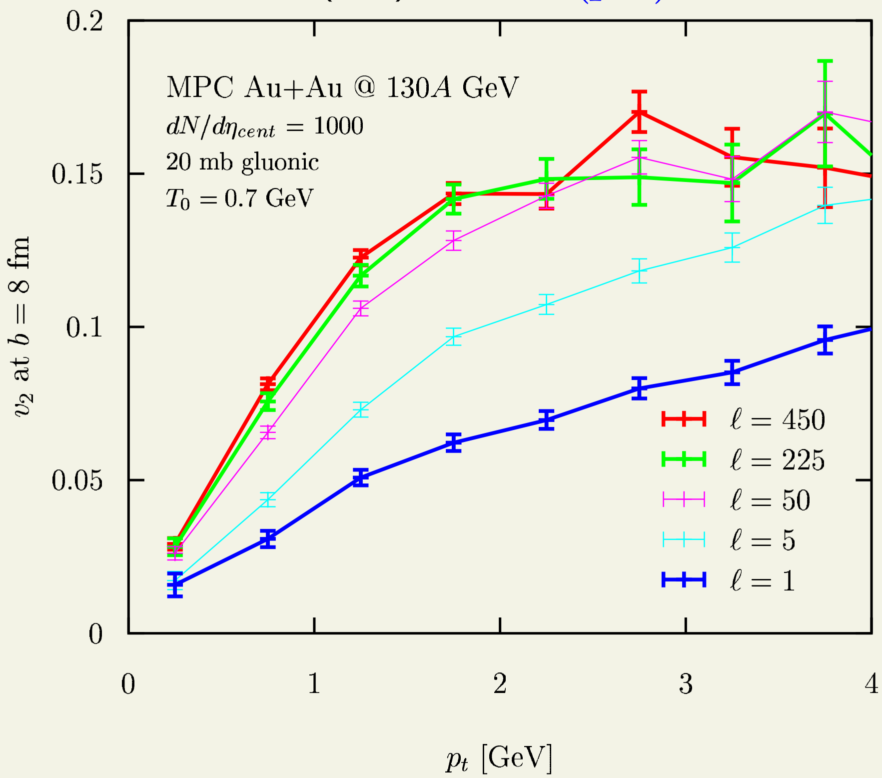
Nonlocal cascade artifacts

Naive $2 \to 2$ cascade nonlocal - action at distance $d < \sqrt{\frac{\sigma}{\pi}}$

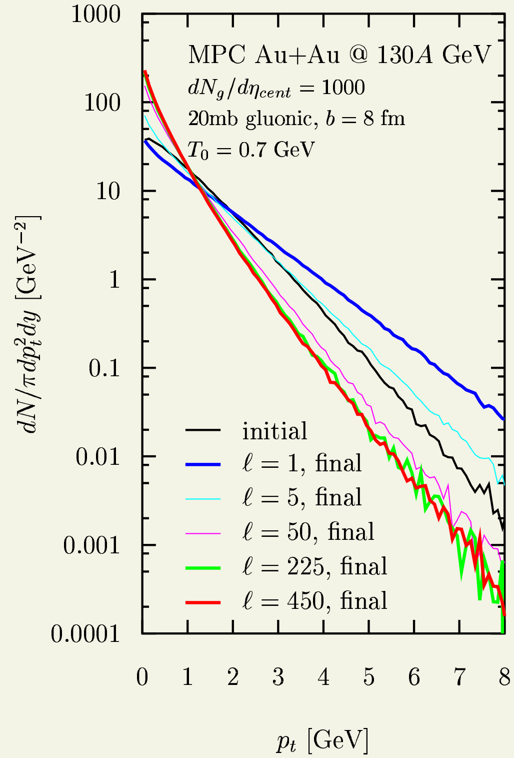


subdivision: rescale $f \to f \cdot \ell$, $\sigma \to \sigma/\ell$ $\Rightarrow$ $d \propto \ell^{-1/2}$ local as $\ell \to \infty$

DM & Gyulassy ('02): $v_2(p_T)$



spectra



at RHIC: need subdivision $\ell \sim 200$ to eliminate large artifacts

$\to$ computational challenge - CPU time scales as $\ell^{\sim 3/2}$ per run $\to$ barely fits on PC

Proceedings of RIKEN BNL Research Center Workshop
Volume 1

Open Standards for Cascade Models for RHIC

June 23 - 27, 1997

~~**Model = Code**~~

Model = Equations

Code = Tool to solve eqns

Organizer
Miklos Gyulassy + Yang Pang & T.D. Lee

code repository @ http://karman.physics.columbia.edu/OSCAR

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Useful approximations to transport

- **Fokker-Planck - limit of many small-angle scatterings ($\Delta p \ll p$)**

$$p^\mu \partial_\mu f = \frac{\partial}{\partial p^\mu}(A^\mu(x,p)f) + \frac{\partial^2}{\partial p^\mu \partial p^\nu}(B^{\mu\nu}(x,p)f)$$

- lends itself to stochastic (Langevin) treatment

$$\frac{d\vec{p}}{dt} = -\eta_D\,\vec{p} + \vec{\xi}(t)\ , \quad \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')\ , \quad \eta_D = \frac{\kappa}{2TM} = \frac{T}{DM}$$

- drag η_D, noise κ and spatial diffusion $D \equiv \langle \Delta\vec{x}^{\ 2}(t)\rangle/6t$ related

- breaks down at high p_T

- **relaxation time approximation**

$$p^\mu \partial_\mu f = -\frac{f - f_{eq}}{\tau}$$

- especially useful in case of high symmetry, and with ansatz $\tau(x)$

Connection to hydro

energy-momentum tensor: $T^{\mu\nu}(x) = \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(x, \vec{p})$

charge current: $N_c^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu c_i f_i(x, \vec{p})$

entropy current: $S^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu f_i(x, \vec{p}) \left\{1 - \ln[f_i(x, \vec{p}) h^3]\right\}$

conservation laws: $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu N_c^\mu = 0$

dissipation: $\partial_\mu S^\mu \geq 0 \quad \Rightarrow$ **entropy production, equilibration**

mean free path:

$$\lambda \equiv \frac{1}{\text{cross section} \times \text{density}}$$

$$\begin{cases} \lambda \to 0 & - \text{hydrodynamics} \\ \lambda \to \infty & - \text{free streaming} \end{cases}$$

transport opacity:

$$\chi \equiv \langle n_{coll} \rangle \underbrace{\langle \sin^2 \theta_{CM} \rangle} \quad \sim \text{\# of collisions} \times \text{deflection weight}$$

$\searrow \sigma^{-1} \int d\Omega \frac{d\sigma}{d\Omega} \sin^2\theta \equiv \sigma_{tr}/\sigma \ \rightarrow$ **2/3 for isotropic**

e.g., for $n_B = 0$:

- **Ideal fluid approximation $\lambda \to 0$: local equilibrium $f = (2\pi)^{-3} \exp[p_\mu u^\mu(x)/T(x)]$**

$$T^{\mu\nu}_{id} = (e+p) u^\mu u^\nu - p\, g^{\mu\nu}$$

$$\partial_\mu S^\mu = 0 \quad \Rightarrow \text{entropy conserved}$$

- **Navier-Stokes approx. $\lambda \ll \textit{length scales}$: near local equilibrium, $f = f_{eq} + \delta f$**

$\eta \propto T/\sigma$

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{id} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \zeta \Delta^{\mu\nu}\nabla_\alpha u^\alpha \quad \rightarrow \text{shear \& bulk viscosity}$$

$$\partial_\mu S^\mu_{NS} = \tfrac{\eta}{2T}(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha)^2 + \tfrac{\zeta}{T}(\nabla_\mu u^\mu)^2 \quad \rightarrow \text{entropy production}$$

- **Israel-Stewart hydro: near local equil, $\delta f_{IS} = C_{\alpha\beta} p^\alpha p^\beta$ - $C_{\alpha\beta}$ linear in $\pi^{\mu\nu}$, Π**

$$T^{\mu\nu}_{IS} = T^{\mu\nu}_{id} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu} \quad \rightarrow \text{shear stress \& bulk pressure}$$

$\tau_\pi \propto \lambda_{tr}$, $\eta \propto T/\sigma$

$$D\pi^{\mu\nu} = -\tfrac{1}{\tau_\pi}\left(\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu) D u_\lambda - \tfrac{1}{2}\pi^{\mu\nu}\left(\nabla_\lambda u^\lambda + D \ln \tfrac{\beta_2}{T}\right) - 2\pi_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda}$$

$$D\Pi = -\tfrac{1}{\tau_\Pi}\left(\Pi - \Pi_{NS}\right) - \tfrac{1}{2}\Pi\left(\nabla_\mu u^\mu + D \ln \tfrac{\beta_0}{T}\right)$$

$$\partial_\mu S^\mu_{IS} = \tfrac{\eta}{2T}\pi^{\mu\nu}\pi_{\mu\nu} + \tfrac{\zeta}{T}\Pi^2 \quad \rightarrow \text{entropy production}$$

II. Thermalization at RHIC

- **yields and spectra of particles look thermal**

  $T_{chem} \sim 165$ MeV, $T_{kin} \sim 110$ MeV, $\langle v_T \rangle \sim 0.5c$

- **large azimuthal anisotropy ("elliptic flow")**

- **strong suppression of high-momentum probes, even heavy quarks**

- **remarkable success of hydrodynamics**

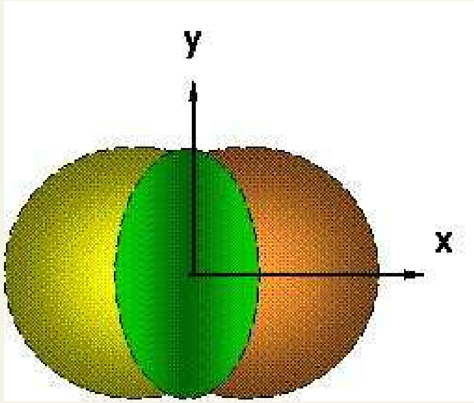
looks like matter does thermalize to a large degree

- do we understand how it thermalizes?

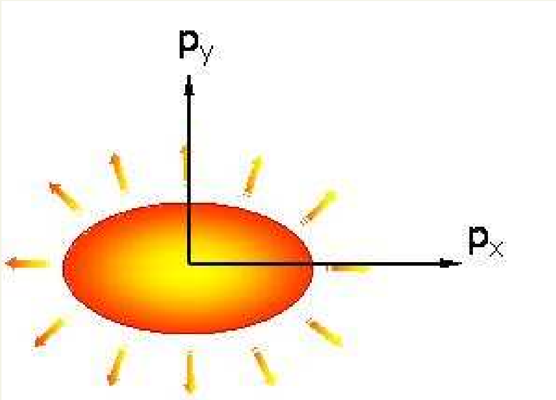
Elliptic flow (v_2)

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



$$\longrightarrow \quad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

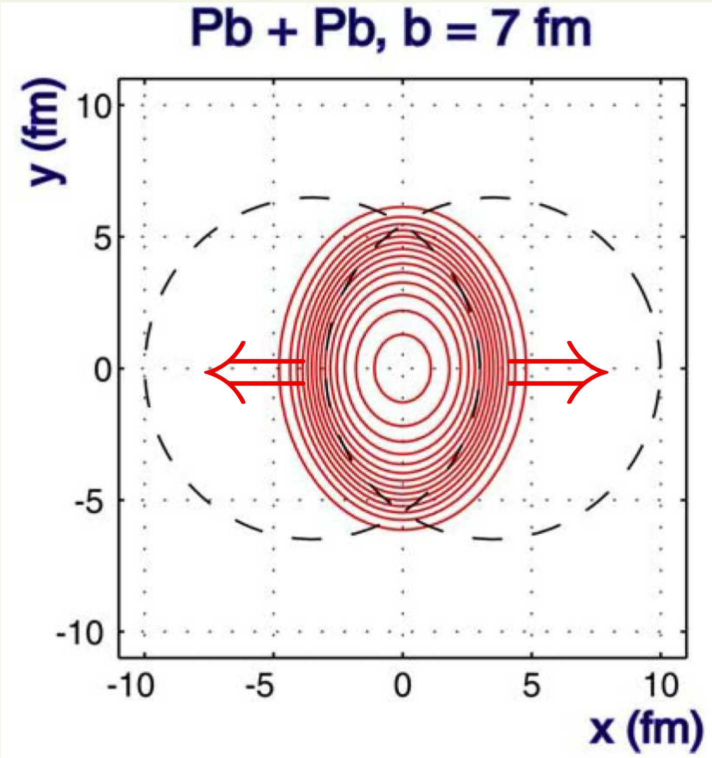


- measures strength of interactions
- self-quenching, develops at early times

What v_2 measures

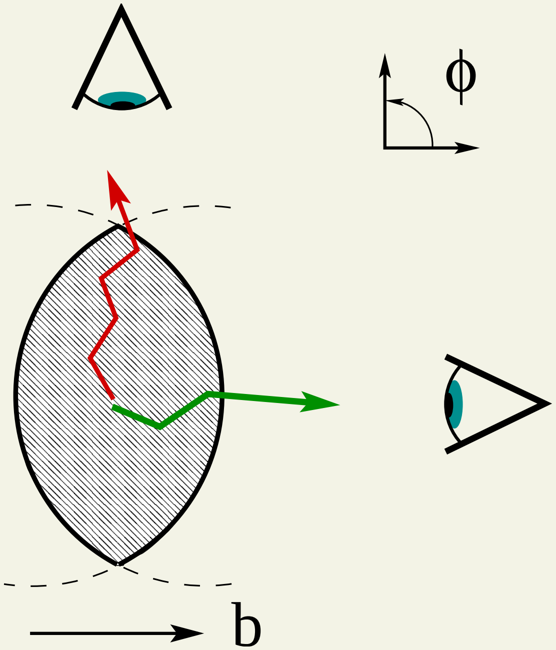
macroscopically: pressure gradients

$$\Delta\vec{F}/\Delta V = -\vec{\nabla}p$$



⇒ **larger acceleration in impact parameter direction**

microscopically: transport opacity

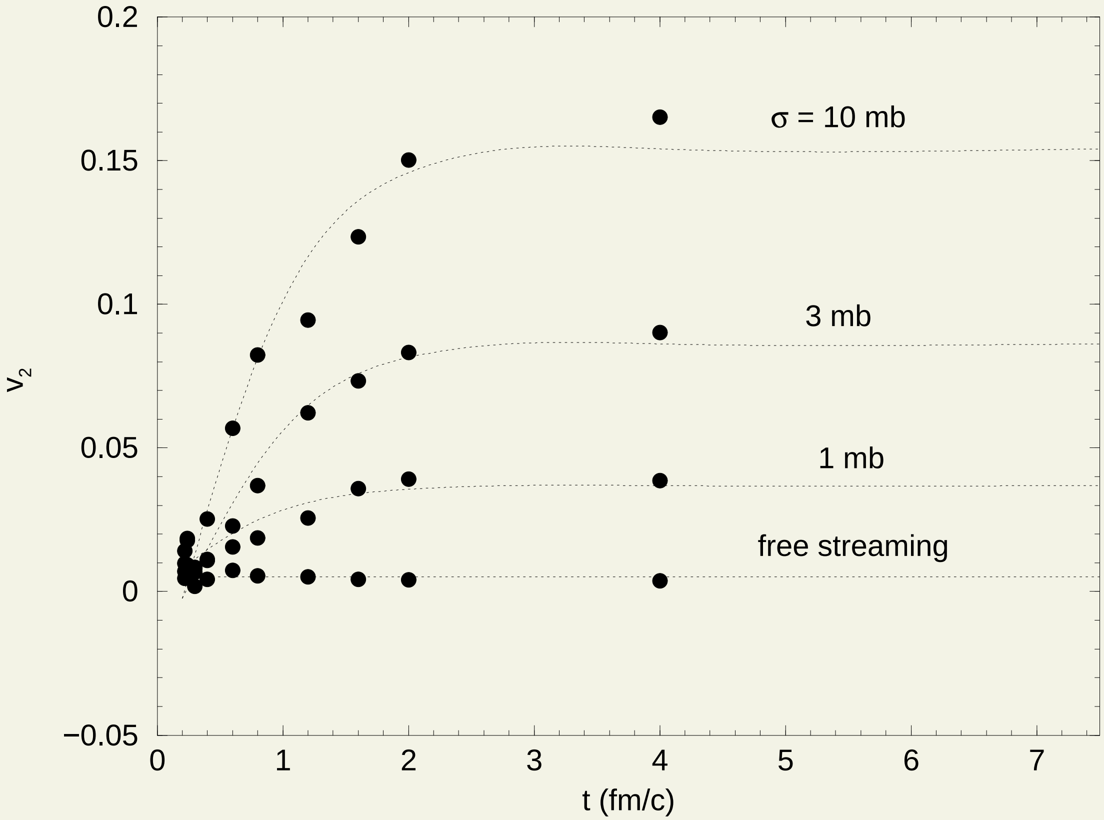


variation in pathlength

⇒ **momentum anisotropy** v_2

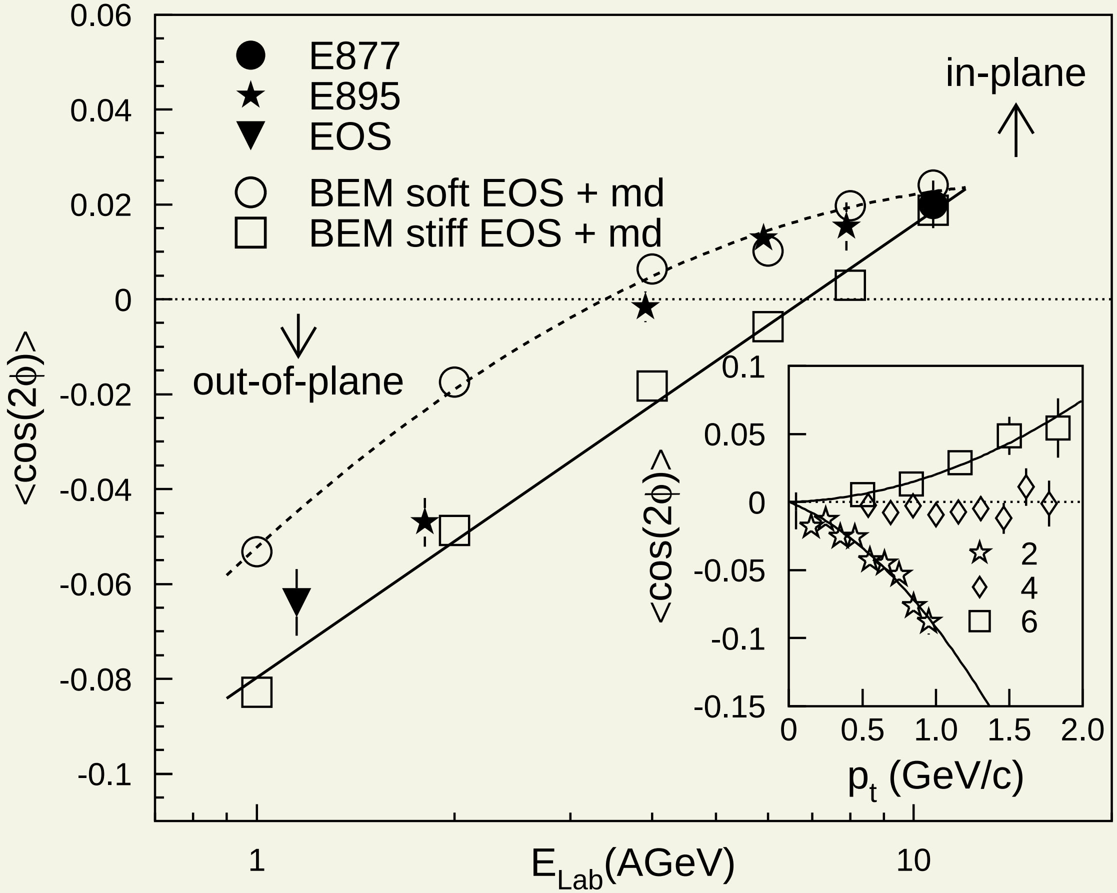
v_2 builds up early

Zhang, Gyulassy & Ko ('99): $2 \to 2$ **transport ZPC**



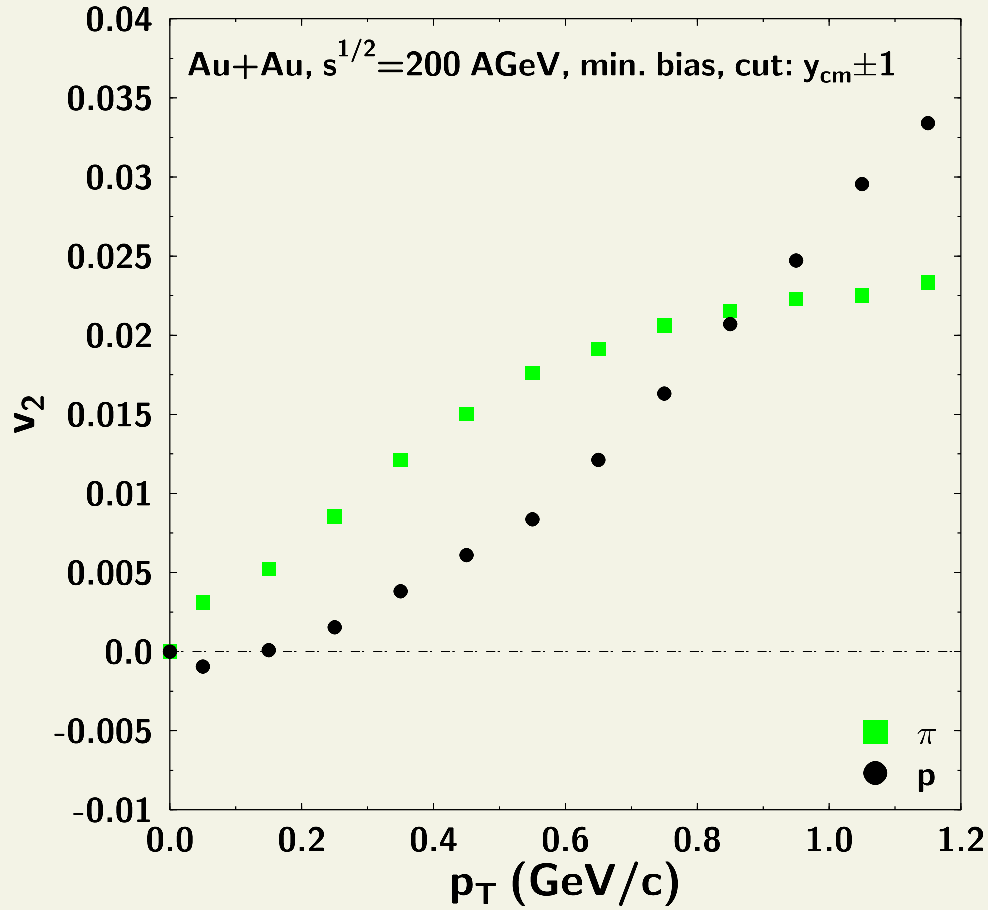
sharp cylinder $R = 5$ fm, $\tau_0 = 0.2$ fm$/c$, $b = 7.5$ fm, $dN^{cent}/dy = 300$

E895, PRL83 ('99): v_2 **at the AGS - squeeze-out** $(v_2 < 0)$ **vs. in-plane** $(v_2 > 0)$



passage time $t \sim 2R/\gamma$ **relevant**

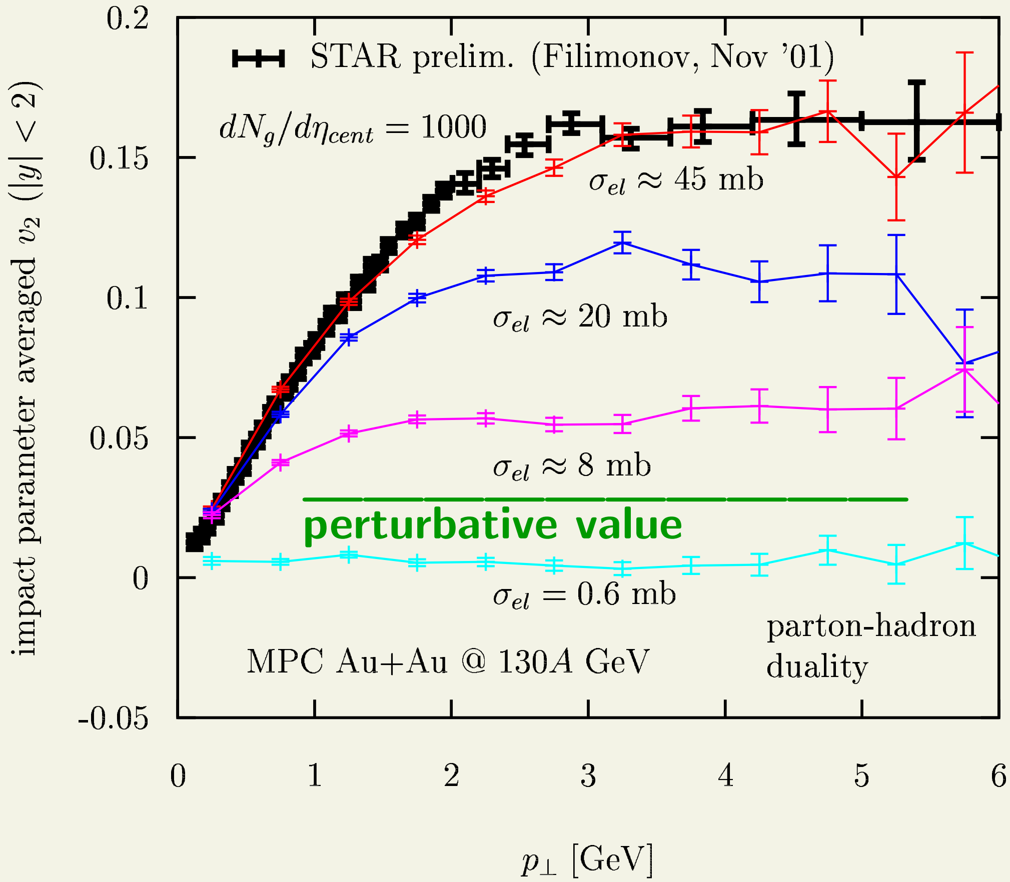
Bleicher & Stoecker, PLB526 ('02): **hadronic string model UrQMD**



too small!

Strong interactions at RHIC

DM & Gyulassy, NPA 697 ('02): **$v_2(p_T, \chi)$ from $2 \to 2$ transport MPC**



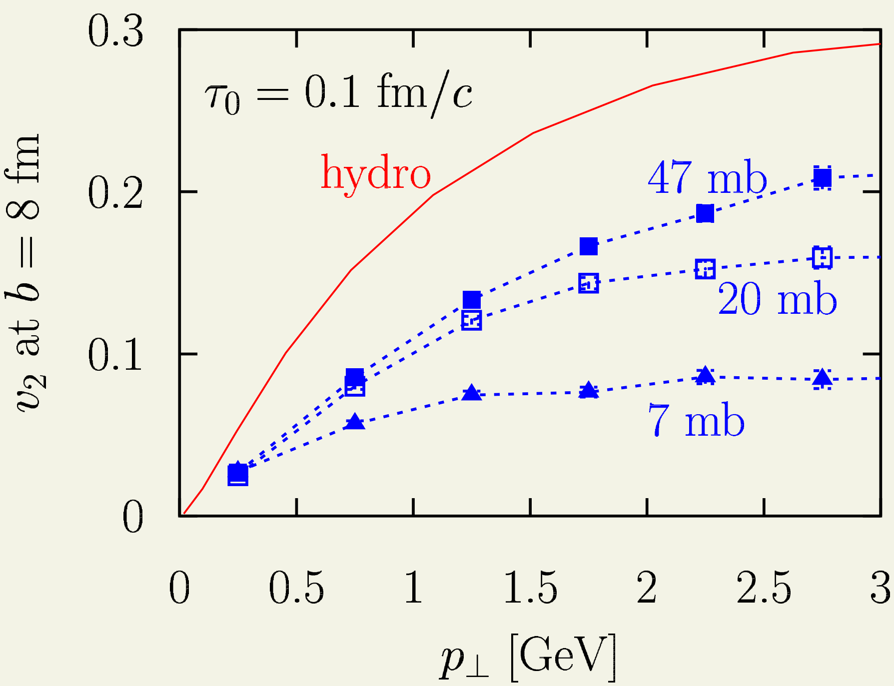
need $15\times$ perturbative opacities - $\sigma_{el} \times dN_g/d\eta \approx 45$ mb $\times 1000 \Rightarrow$ sQGP?

(saturated gluon $\frac{dN^{cent}}{d\eta} = 1000$, $T_{eff} \approx 0.7$ GeV, $\tau_0 = 0.1$ fm, 1 parton $\to$ 1 π hadronization)

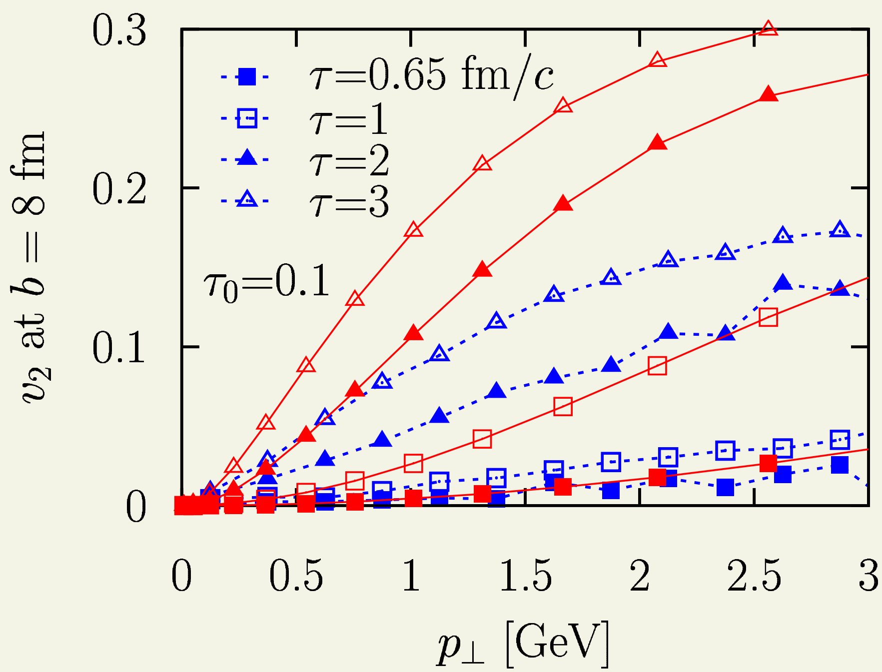
Not ideal fluid(!)

dissipation reduces v_2 by $30 - 50$% even for $\sigma_{gg\to gg} \sim 50$ mb

DM & Huovinen, PRL94 ('05): **final $v_2(p_T)$**



$v_2(\tau, p_T)$

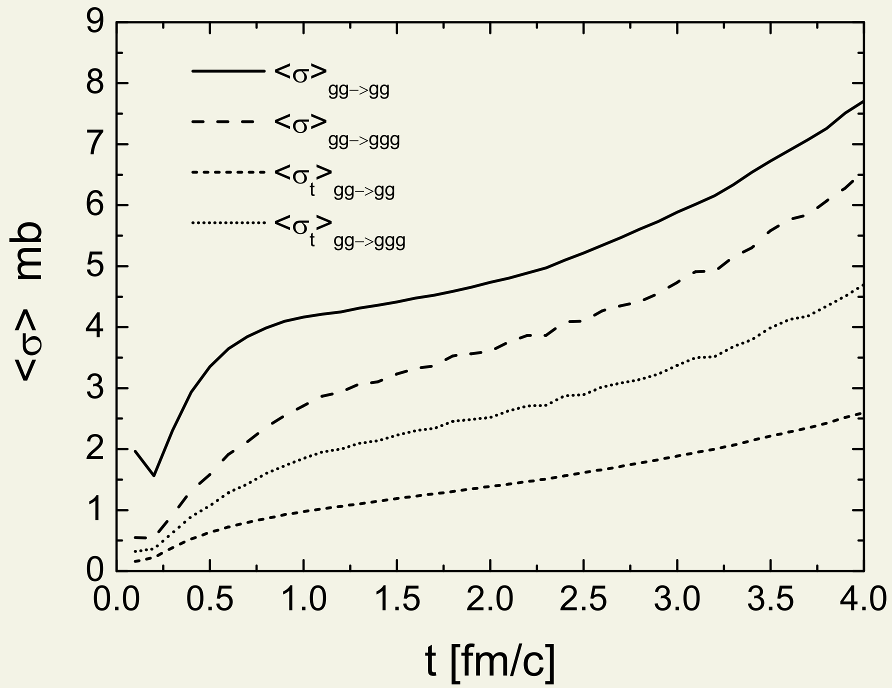


→ **opaque, highly collective system, but still dissipative**

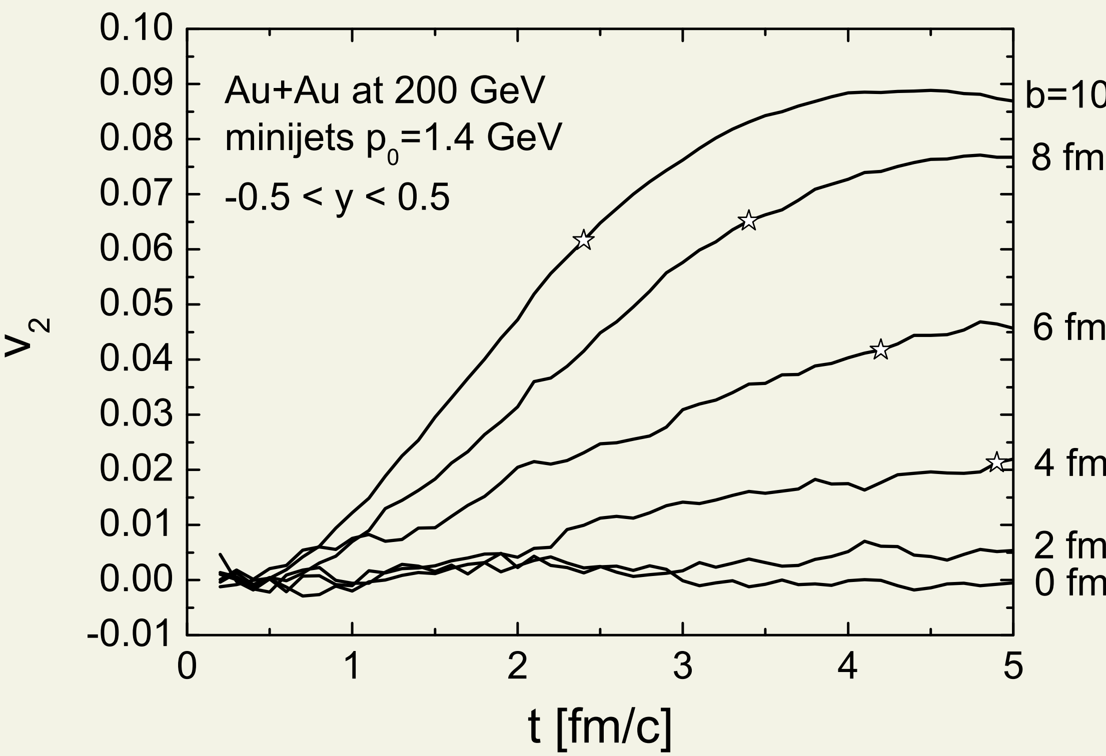
Radiative transport, $3 \leftrightarrow 2$

radiative $ggg \leftrightarrow gg$ **allows for** **large-angle scattering** **and can provide the needed rate enhancement** Baier, Mueller, Son... ('01), Greiner, Xu... ('05)

Greiner & Xu '04:



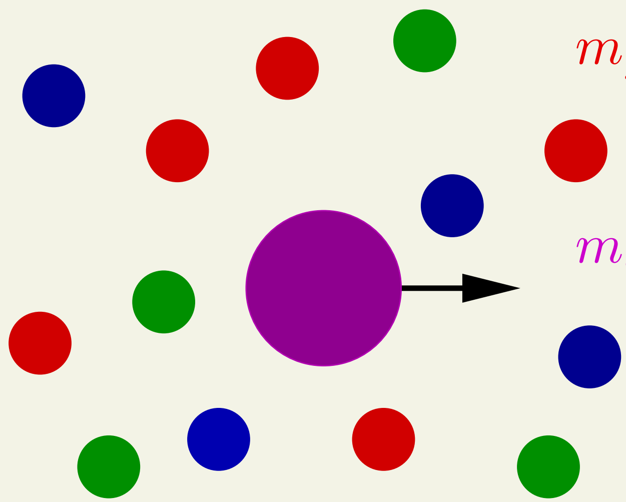
El, Xu & Greiner, NPA 785 ('07):



$\Rightarrow$ **alternative to the strongly-coupled nonperturbative plasma paradigm**

Cross-check: heavy quarks

$\sim$ **"Brownian motion" in plasma**



$m_g, m_{u,d}, m_s \sim T$

$m_c \sim 1.2$ **GeV** $\gg T$

$v \sim \sqrt{T/m}$

$p \sim \sqrt{m \cdot T}$

$N_{coll} \sim p/\Delta q \sim \sqrt{m/T}$

charm quarks very heavy $\Rightarrow$ need more collisions to randomize

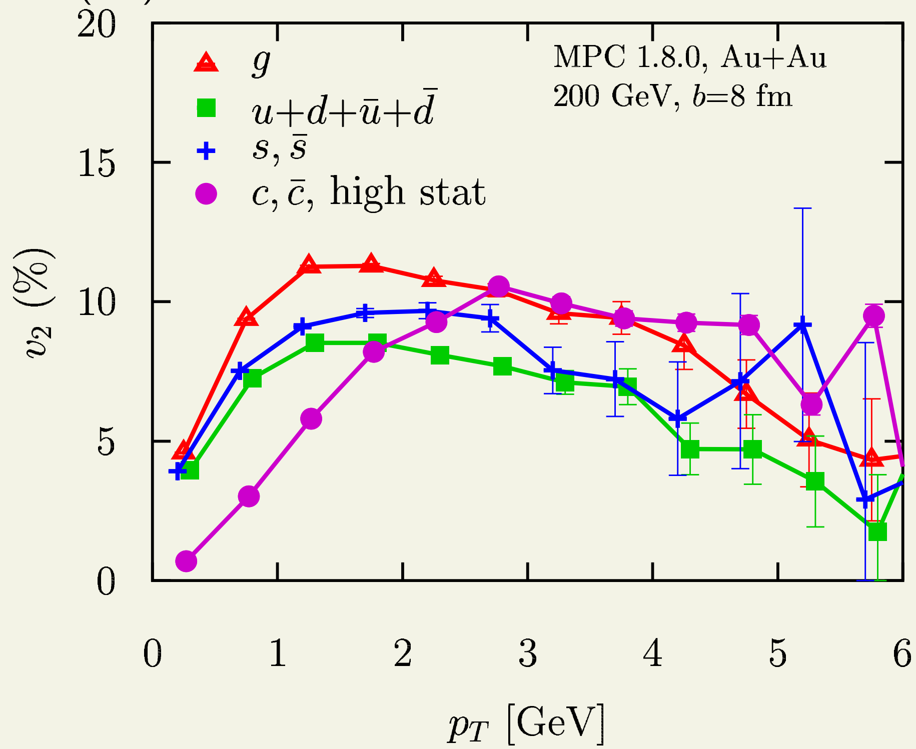
$\Rightarrow$ **at low momenta: expect reduced anisotropy** v_2

$\Rightarrow$ **at high momenta: mass difference should not matter as** $m/p \to 0$

Expect charm flow

parton transport **MPC 1.8.0** **vs** **ideal hydro + Langevin:**

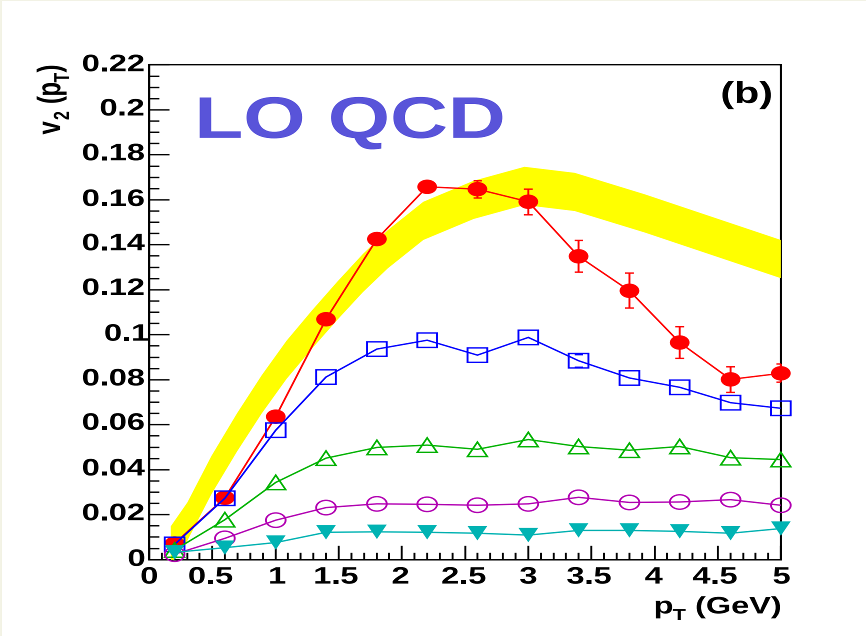
DM ('04):



elastic & inel. $2 \to 2$

$6\times$ perturbative opacities

Teaney & Moore ('05):



for $D \cdot (2\pi T) = 1.5, 3, 6, 12, 24$ **(top to bottom)**

qualitatively very similar results, except at high p_T

III. Applicability of viscous hydro

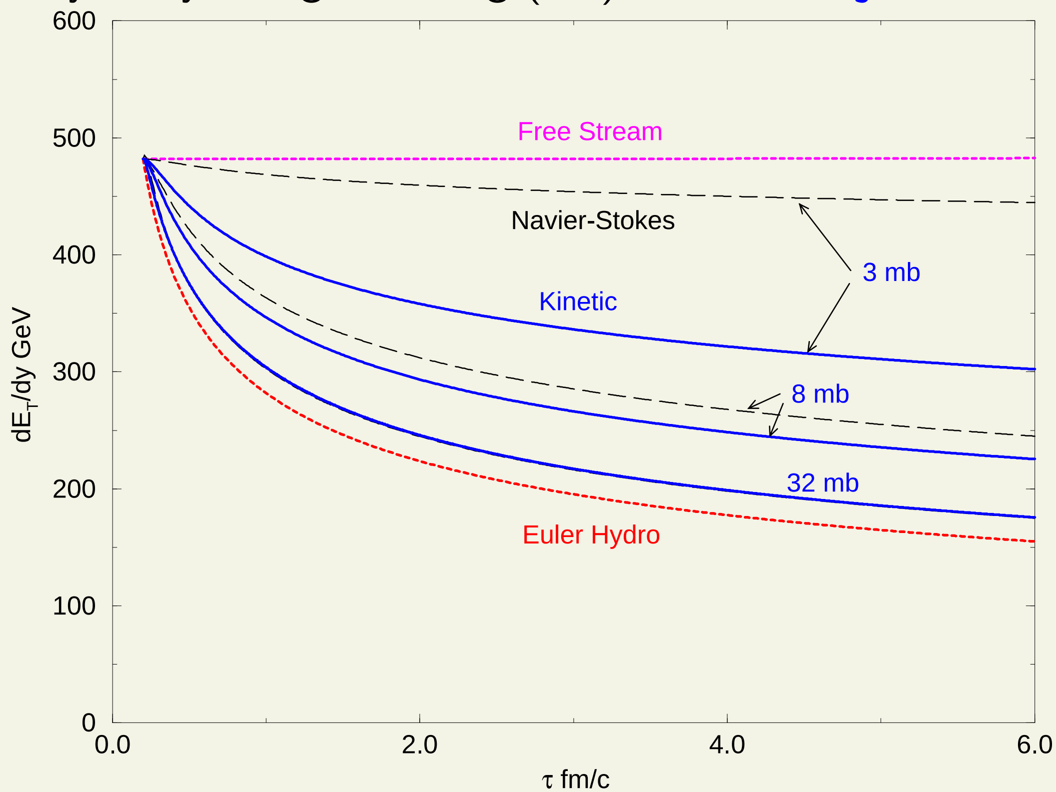
hydrodynamics is a truncation of some nonequilibrium theory

$\Rightarrow$ **use nonequilibrium transport to test validity**

An early test of Navier-Stokes

cooling due to $p\,dV$ work - ideal hydro vs Navier-Stokes vs transport

Gyulassy, Pang & Zhang ('97): **1+1D Bjorken scenario**



dissipation slows cooling

Second-order theories (hydro)

gradient expansion in kinetic theory Chapman-Enskog, ... de Groot, ... Moore and Yor k ('08)

$$f(x,p) = f_{eq}(x,p) + \lambda f_1(x,p) + \lambda^2 f_2(x,p) + ..., \qquad (\lambda \text{ counts derivatives})$$

$\rightarrow$ **1st order gives Navier-Stokes (acausal, unstable)**

moment expansion in kinetic theory Grad..., Israel, Stewart, ...

$$f(x,p) = f_{eq}(x,p)\,(1 + p_\alpha C^\alpha(x) + p_\alpha p_\beta C^{\alpha\beta}(x) + ...)$$

$\rightarrow$ **2nd order gives Israel-Stewart hydrodynamics**

generalized thermodynamics Israel, Stewart

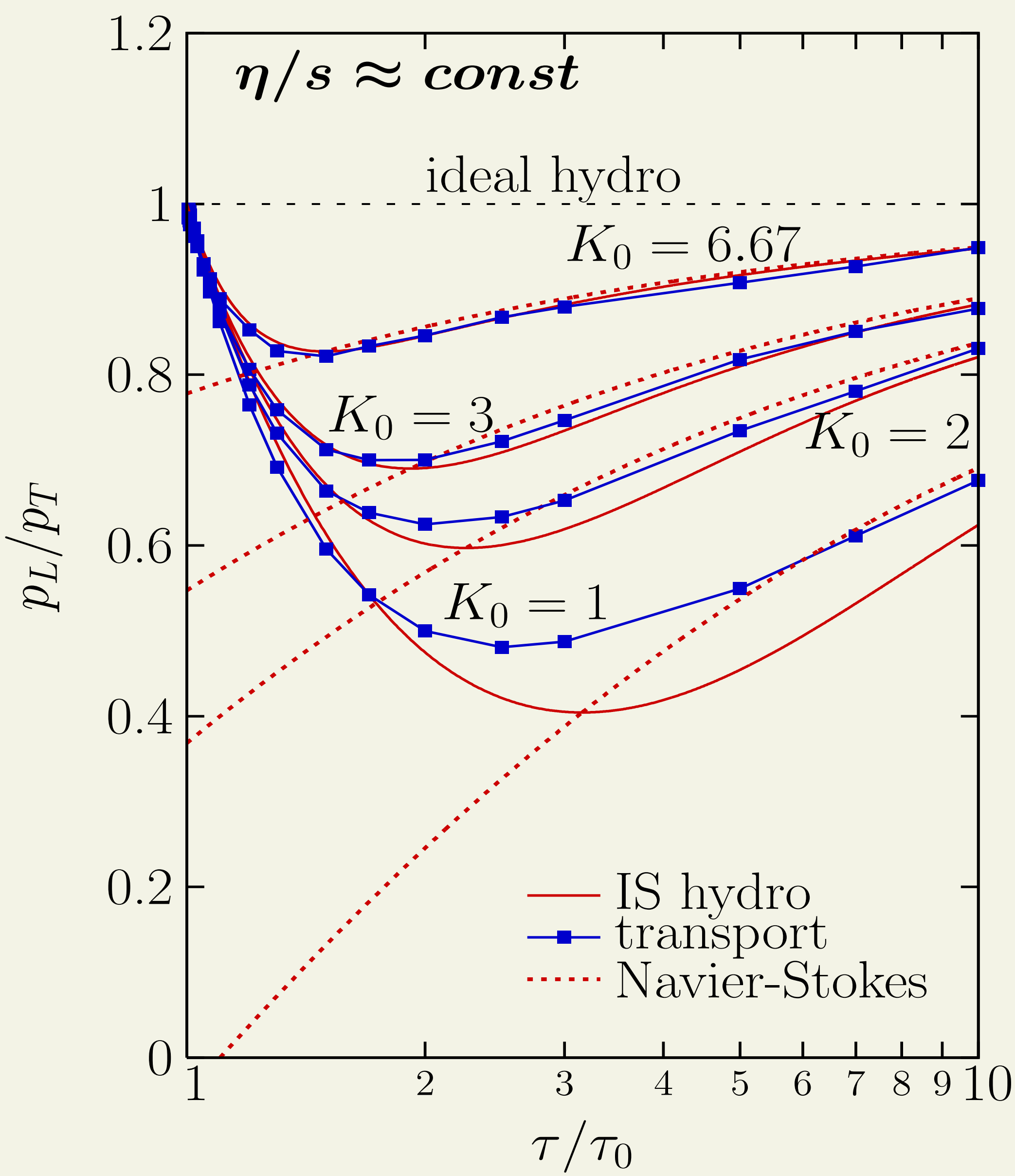
$$S^\mu = s_{eq} u^\mu + q^\mu \frac{\mu n}{T(e+p)} + all\ quadratic\ terms\ with\ \Pi, \pi_{\mu\nu}, q_\mu + ...$$

match most general equation of motion to correlators Baier, Romatschke et al...

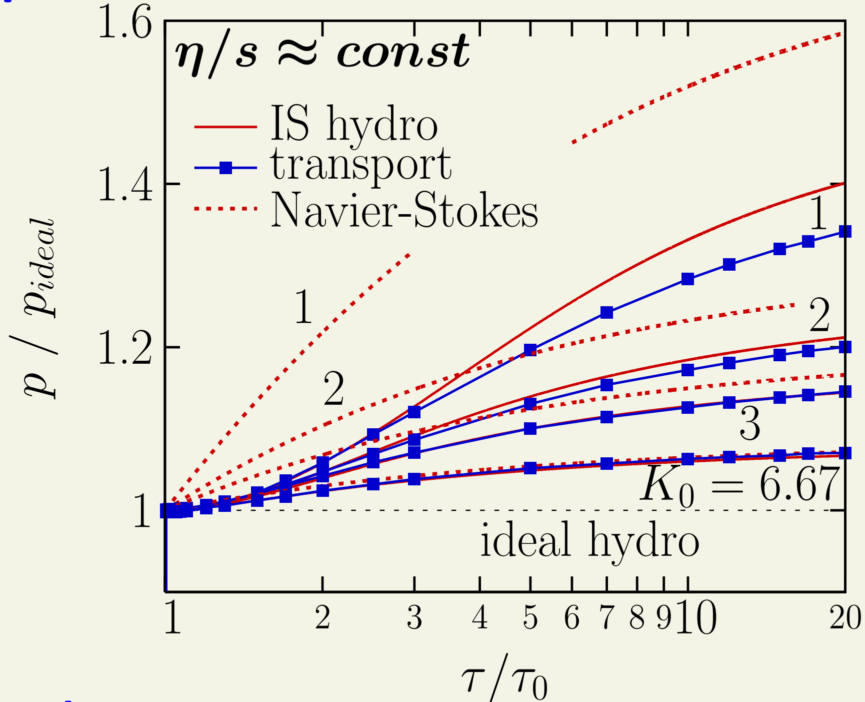
$\Rightarrow$ **basically all convert Navier-Stokes to relaxation equations**

hydro vs transport for $\eta/s \approx const$: Huovinen & DM ('08) $K \equiv \tau/\lambda_{MFP}$

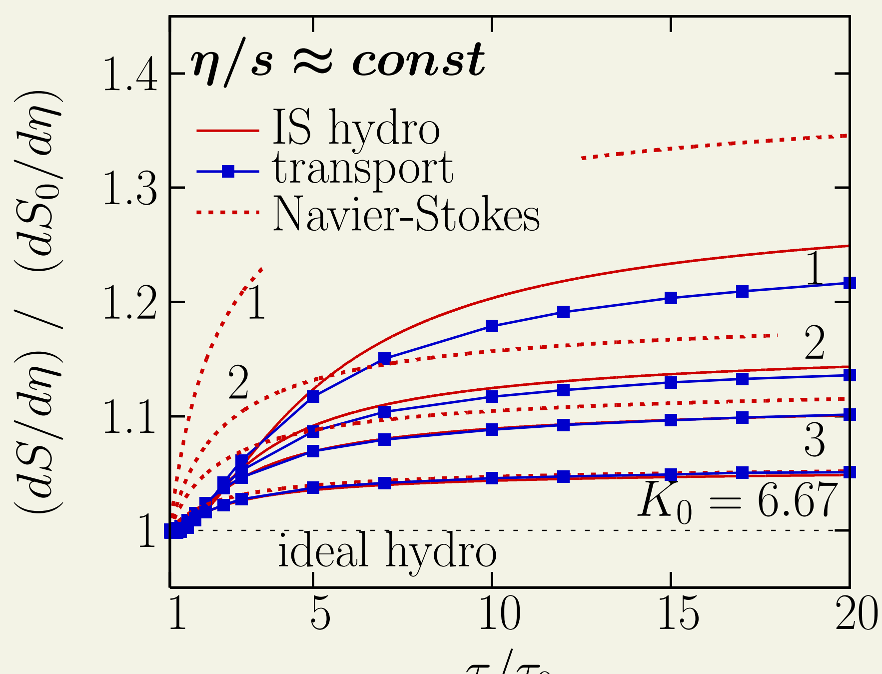
pressure anisotropy T_{zz}/T_{xx}



pressure



entropy



IS hydro 10% accurate when $K_0 \gtrsim 2$

Connection to viscosity

$$K_0 \approx \frac{T_0\tau_0}{5}\frac{s_0}{\eta_{s,0}} \approx 12.8 \times \left(\frac{T_0}{1\ \mathrm{GeV}}\right)\left(\frac{\tau_0}{1\ \mathrm{fm}}\right)\left(\frac{1/(4\pi)}{\eta_0/s_0}\right) \tag{1}$$

For typical RHIC hydro initconds $T_0\tau_0 \sim 1$, therefore

$$K_0 \gtrsim 2-3 \qquad \Rightarrow \qquad \frac{\eta}{s} \lesssim \frac{1-2}{4\pi} \tag{2}$$

I.e., shear viscosity cannot be many times more than the conjectured bound, for IS hydro to be applicable.

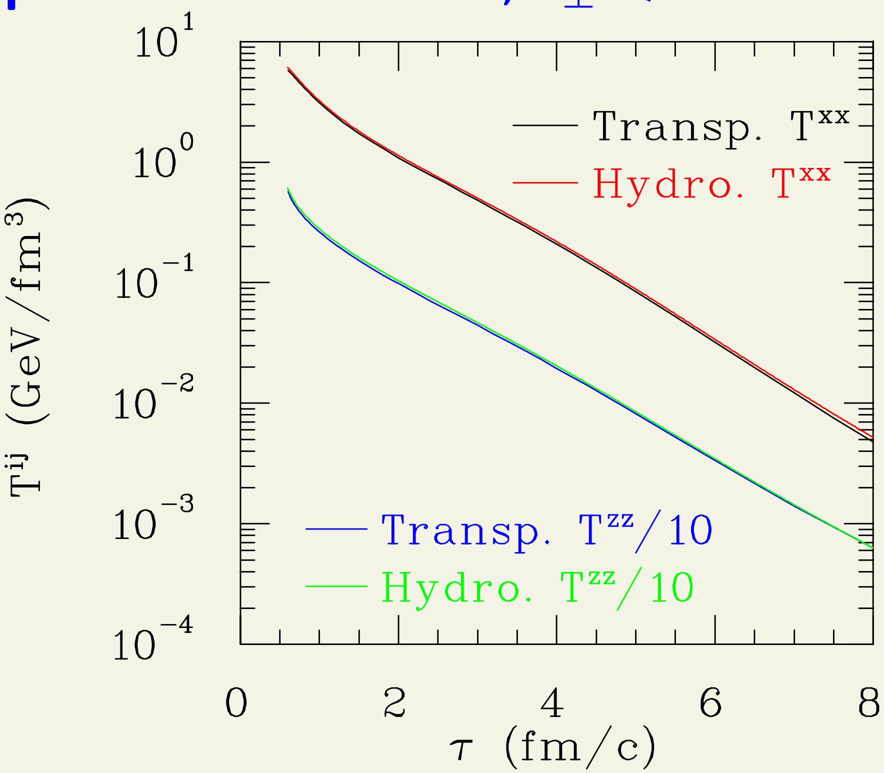
When IS hydro is accurate, dissipative corrections to pressure and entropy do not exceed 20% significantly (a necessary condition). This holds for a wide range $[0.476, 1.697]$ of initial pressure anisotropies.

Also works in 2+1D

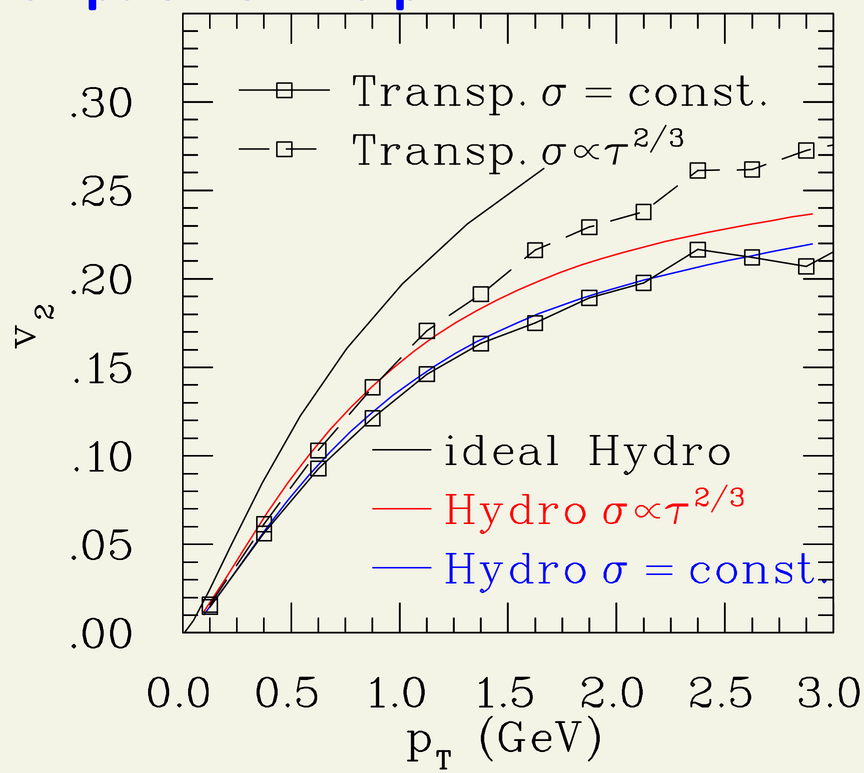
Au+Au at RHIC, $b = 8$ fm

Huovinen & DM ('08)

pressure in the core, $r_\perp < 1$ fm



elliptic flow vs pT



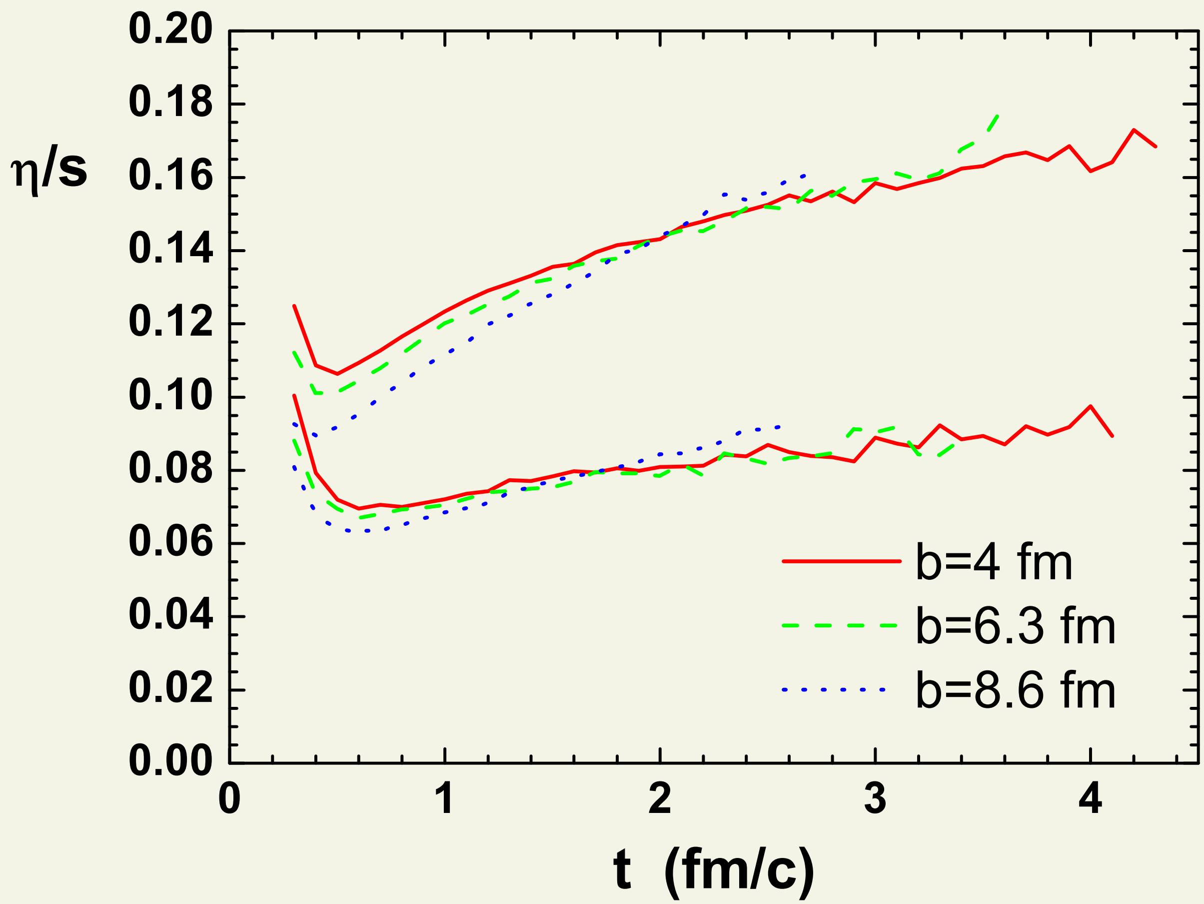
$\boldsymbol{\eta/s \approx 1/(4\pi)}$ $(\sigma \propto \tau^{2/3})$

- $\sigma =$ **const** $\sim 47mb$
- $\eta/s \approx 1/(4\pi)$, **i.e.,** $\sigma \propto \tau^{2/3}$

$\Rightarrow$ **if** $\eta/s \approx 1/(4\pi)$**, or** $\sigma \sim 47mb$**, IS hydro is a good approximation**

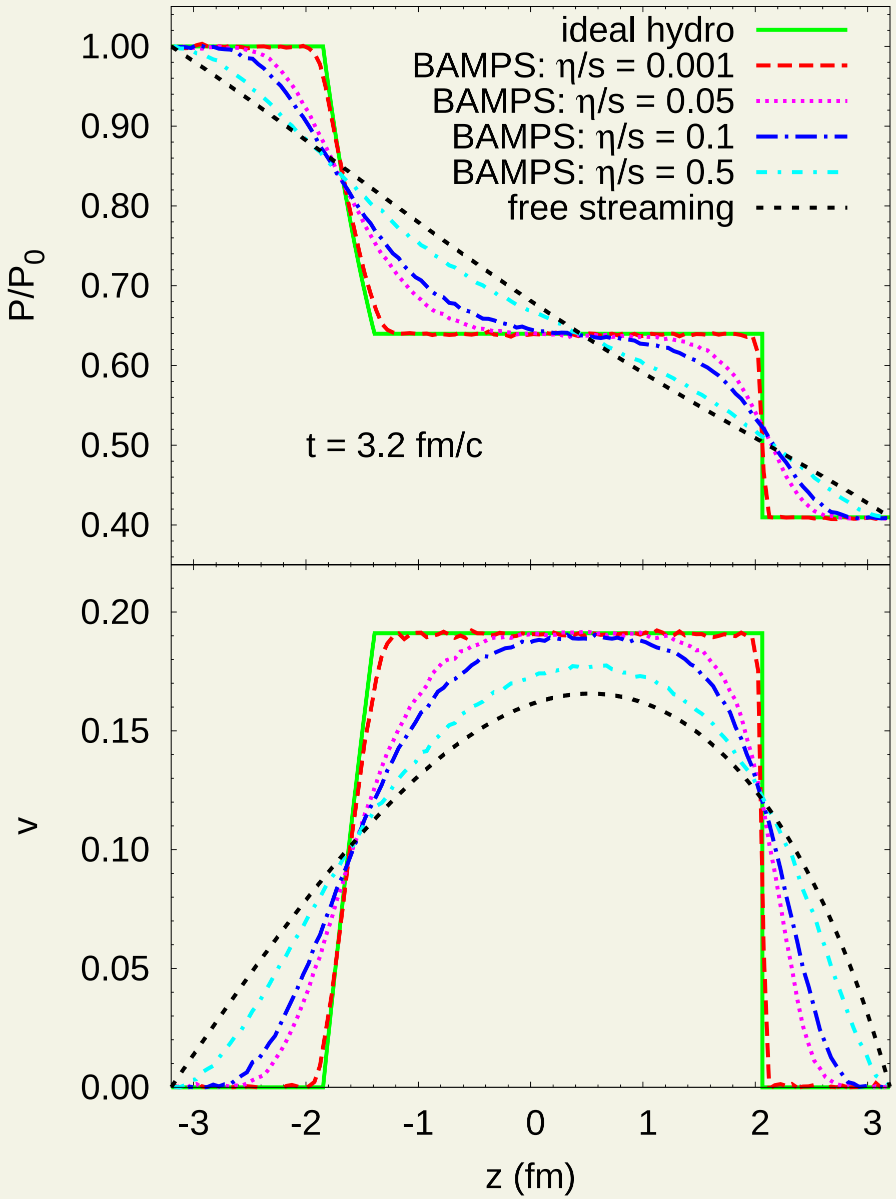
and η/s **is close to** $\sim 1/4\pi$ **at RHIC**

Xu, Greiner et al ('08): η/s **from radiative transport**



similar conclusions - mainly the viscosity matters, not the microscopics

Another test - shock in a box



shock propagation in 1+1D (Riemann problem)

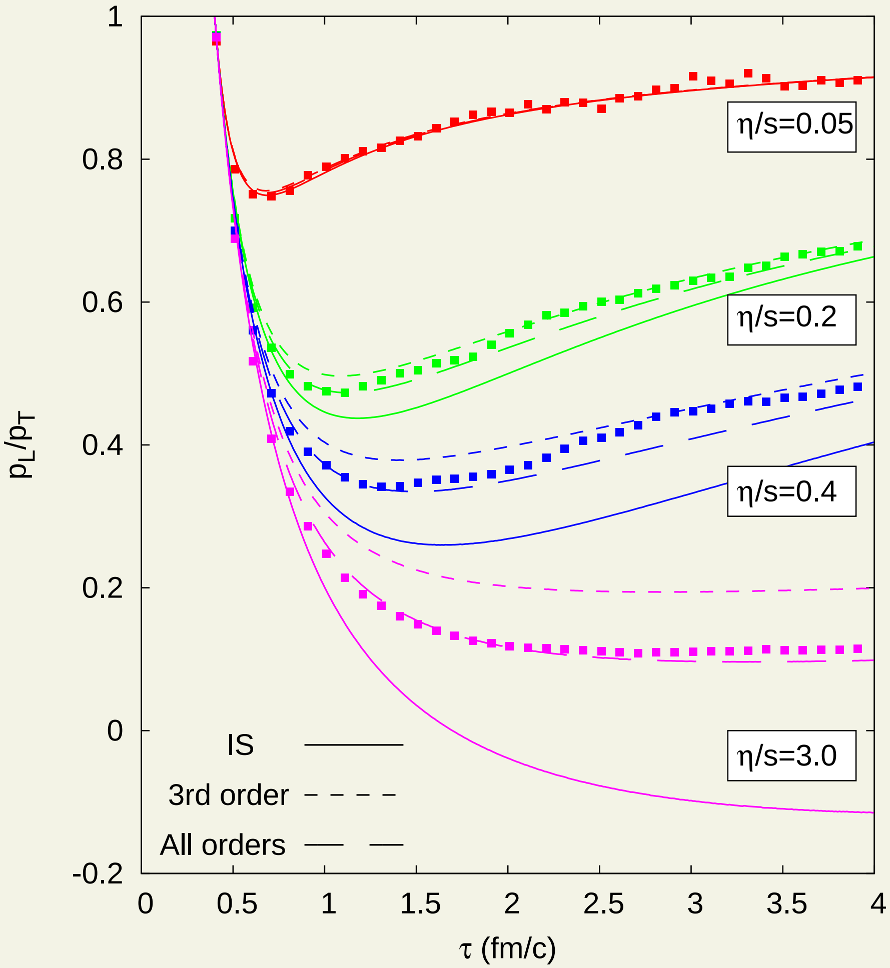
Bouras et al, arXiv:0902.1927:

transport converges to ideal hydro as $\eta/s \to 0$

Higher-order hydrodynamics

generalized entropy to 3rd order in shear stress

$$S^\mu = s_0\, u^\mu - \frac{\beta_2}{2T}\pi_{\alpha\beta}\pi^{\alpha\beta}u^\mu + \alpha\frac{\beta_2^2}{2T}\pi_{\alpha\beta}\pi^{\beta\sigma}\pi_\sigma{}^\alpha u^\mu$$



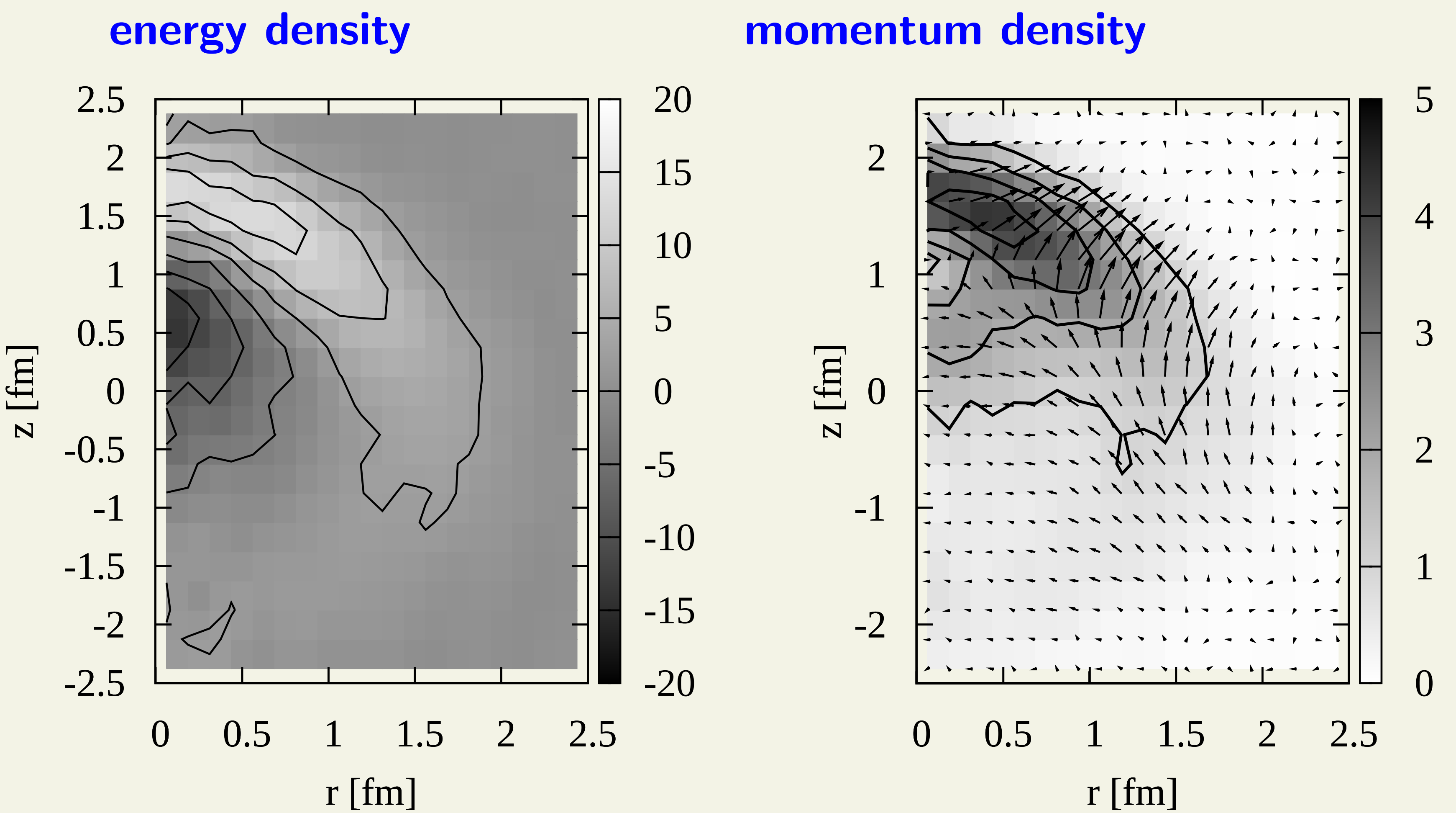
$2 \to 2$ transport vs higher-order IS hydro in 1+1D El et al ('09)

higher-order hydro equation approximates transport better

still a truncation, but can extend range of validity to $\eta/s \sim a\, few$

need 3+1D viscous hydro? - can use 3+1D transport

e.g., conical flow in static medium DM ('09)

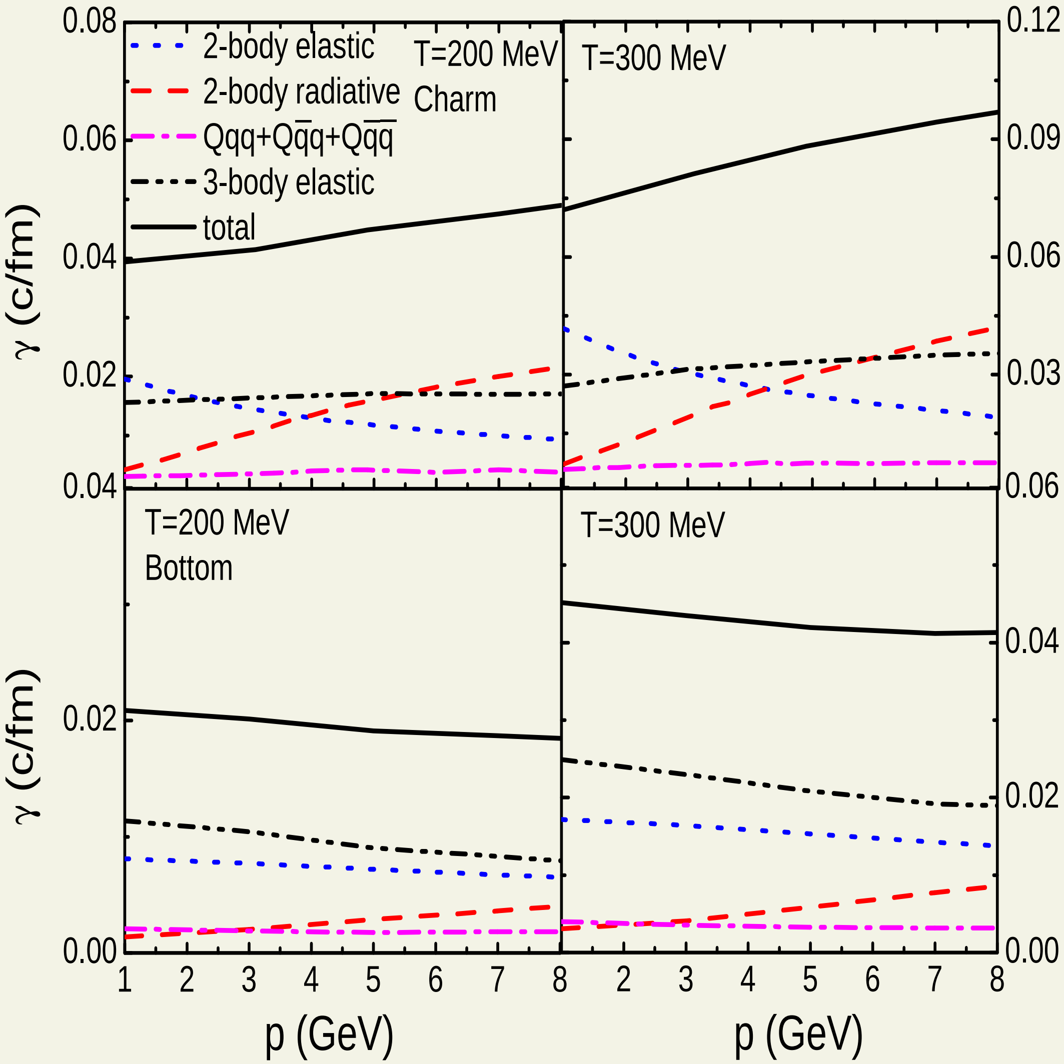


($dE/dx = 68$ GeV/fm, $\lambda_{MFP} = 0.125$ fm, $T = 0.385$ GeV, $v = 0.9$)

IV. Some open questions

1) If radiative $3 \leftrightarrow 2$ is crucial, how about $3 \to 3$, $4 \leftrightarrow n$, ...?

elastic $3 \to 3$ looks significant Liu & Ko ('06): **charm drag $\gamma \equiv \eta_D$**



if subsequent terms are relevant, what mechanism makes the combined effect of all terms to damping, viscosity, etc, FINITE (as it should)?

2) What is the role of classical fields?

Wong equation: color Vlasov-Boltzmann Wong, Heinz

$$p^\mu \left(\partial_\mu + g t_a F^a_{\mu\nu} \partial_{p_\nu} + g f_{abc} A^b_\mu t^c \partial_{t_a}\right) f = C[f]$$

$$[D_\mu, F^{\mu\nu}_a] = J^\nu_a = g \int p^\nu t_a \left(f_q - \bar{f}_q + f_g\right) dP dQ$$

thermal, perturbative plasma at $T \gg T_c$ would appear neutral - fast color rotations

$$\tau^{-1}_{color} \sim g^2 \ln(1/g) T \gg \tau^{-1}_{mom} \sim g^4 \ln(1/g) T$$ Selikhov '91
Gyulassy '92

However, anistropic particle distributions are rapidly isotropised ($\sim$ two-stream instability) Mrowczynski, Arnold, Lenagan, Romatschke, Strikland, Dumitru, ...

realistic 3+1D simulations (expansion) are still lacking...

3) Does $3 \leftrightarrow 2$ transport correctly describe transport properties in the perturbative regime?

at NLO, QCD shear viscosity Arnold, Moore, Jaffe ('03)

$$\eta \approx \frac{106.66}{g^4 \ln(2.414/g)}$$

determined by $1 \leftrightarrow 2$ and $2 \rightarrow 2$ processes.

Is binary $2 \rightarrow 2$ plus the $3 \leftrightarrow 2$ rate controlled by

$$|\mathcal{M}_{gg\to ggg}|^2 = \left(\frac{9g^4}{2}\frac{s^2}{(\mathbf{q}_T^2+\mu_D^2)^2}\right)\left(\frac{12g^2\mathbf{q}_T^2}{\mathbf{k}_T^2[(\mathbf{k}_T-\mathbf{q}_T)^2+\mu_D^2]}\right)\Theta(k_T\lambda_{MFP}-\mathrm{ch}y)$$

equivalent?

Also, how do results depend on the approximate LPM effect implementation?

4) What are the limitations of higher-order hydrodynamics?

Hydrodynamics, at ANY order is a truncation that assumes that the evolution of the energy-momentum tensor (or the generalized entropy current) only depends on the energy momentum tensor itself

$$DT^{\mu\nu} = F^{\mu\nu}(\{T_{\alpha\beta}\})$$

Whereas from covariant transport, higher moments of phase space density enter in the dynamics (BBGKY hierarchy).

Do we have any other nonequilibrum theory we can solve?

5) What is the consistent extension of causal dissipative hydrodynamics to mixtures?

Depending on microscopic rates, some species will be closer to or further away from equilibrium than others (e.g., supernova neutrino flux).

Summary

Covariant transport provides a simple nonequilibrium framework that is fully causal and stable.

Applications to heavy-ion collisions at RHIC indicate high opacities and very low shear viscosities $\eta/s \sim 0.1$. Radiative $3 \leftrightarrow 2$ processes are crucial to address the elliptic flow data. At such high opacities, significant charm quark elliptic flow is expected at RHIC.

Covariant transport can be utilized to establish the region of validity of hydrodynamics, both ideal, Navier-Stokes, and second- and higher-order dissipative formulations. Studies with $2 \to 2$ transport find that Israel-Stewart hydrodynamics is accurate if $\eta/s \lesssim (1-2)/(4\pi)$.

Several open questions remain - more work ahead.