Viscous Hydrodynamics in (3+1) Dimensions

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Outline

- General Introduction of the Problem
- Simple Tests
- Some (Very) Early Results

Problem Description

- Conservation of stress-energy
- Altered by effects of velocity gradients:

$$\partial^\mu T_{\mu\nu} = 0$$

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu - P g_{\mu\nu} + \Pi_{\mu\nu}$$

- In Navier-Stokes, these are explicitly functions of gradients, etc.

$$\Pi_{ij}^{(NS)} = -\eta[\partial_i n_j + \partial_j n_i - \delta_{ij}\partial_k n_k] - \zeta \partial_k n_k$$

Problem Description

- NS equations are famously both difficult and acausal.
- One solution is the Israel-Stewart theory.
  - Deviations from ideal decay toward Navier-Stokes values.

$$\partial_t(a_i/\alpha) = \frac{-(a_i - a_i^{(NS)})}{\tau_\eta \alpha} + \Omega_{ij}(a_j/\alpha)$$

$$\alpha^2 = \eta T s$$

Problem Description

- Using tracelessness of shear contribution there are five shear components, and one bulk component.

$$
\begin{aligned}
a_1 &= \Pi_{xx} - \Pi_{yy} \\
\sqrt{3}a_2 &= \Pi_{xx} + \Pi_{yy} - 2\Pi_{zz} \\
a_3 &= \Pi_{xy}, \quad a_4 = \Pi_{xz}, \quad a_5 = \Pi_{yz} \\
b &= (1/3)(\Pi_{xx} + \Pi_{yy} + \Pi_{zz})
\end{aligned}
$$

Basic Testing (non-viscous)

- Total entropy conservation.
- Central energy density evolution.
- Standard test solutions.

Exponential Gradient Test

- For initial condition: $\epsilon(x) = \epsilon_0 \cdot exp[-x/R]; v(x) = 0$
- Energy Conservation: $\partial_\mu T^{\mu 0} = 0$

$$2\epsilon(1+c_s^2)u\dot{u} + [u_0^2 + c_s^2(u_0^2-1)]\dot{\epsilon} = (1+c_s^2)u_0 u\frac{\epsilon}{R}$$

- Momentum Conservation: $\partial_\mu T^{\mu x} = 0$

$$\frac{(1+c_s^2)\epsilon}{u_0}[u^2+u_0^2]\dot{u} + (1+c_s^2)u_0 u\dot{\epsilon} = \left[u^2(1+c_s^2) - c_s^2\right]\frac{\epsilon}{R}$$

Exponential Test (u)

- This equation happens to be nicely separable for u:

$$\dot{u} = \frac{u_0 c_s^2}{R(1+c_s^2)(u_0^2 - c_s^2 u^2)}$$

- Which happens to be analytically integrable:

$$\frac{c_s^2}{R(1+c_s^2)} t = \frac{1}{2}\left[(1+c_s^2)sinh^{-1}(u) - (c_s^2-1)u\sqrt{u^2+1}\right]$$

$$\frac{c_s^2}{R(1+c_s^2)} t \approx u$$

- But not invertible.

Exponential Test (u)

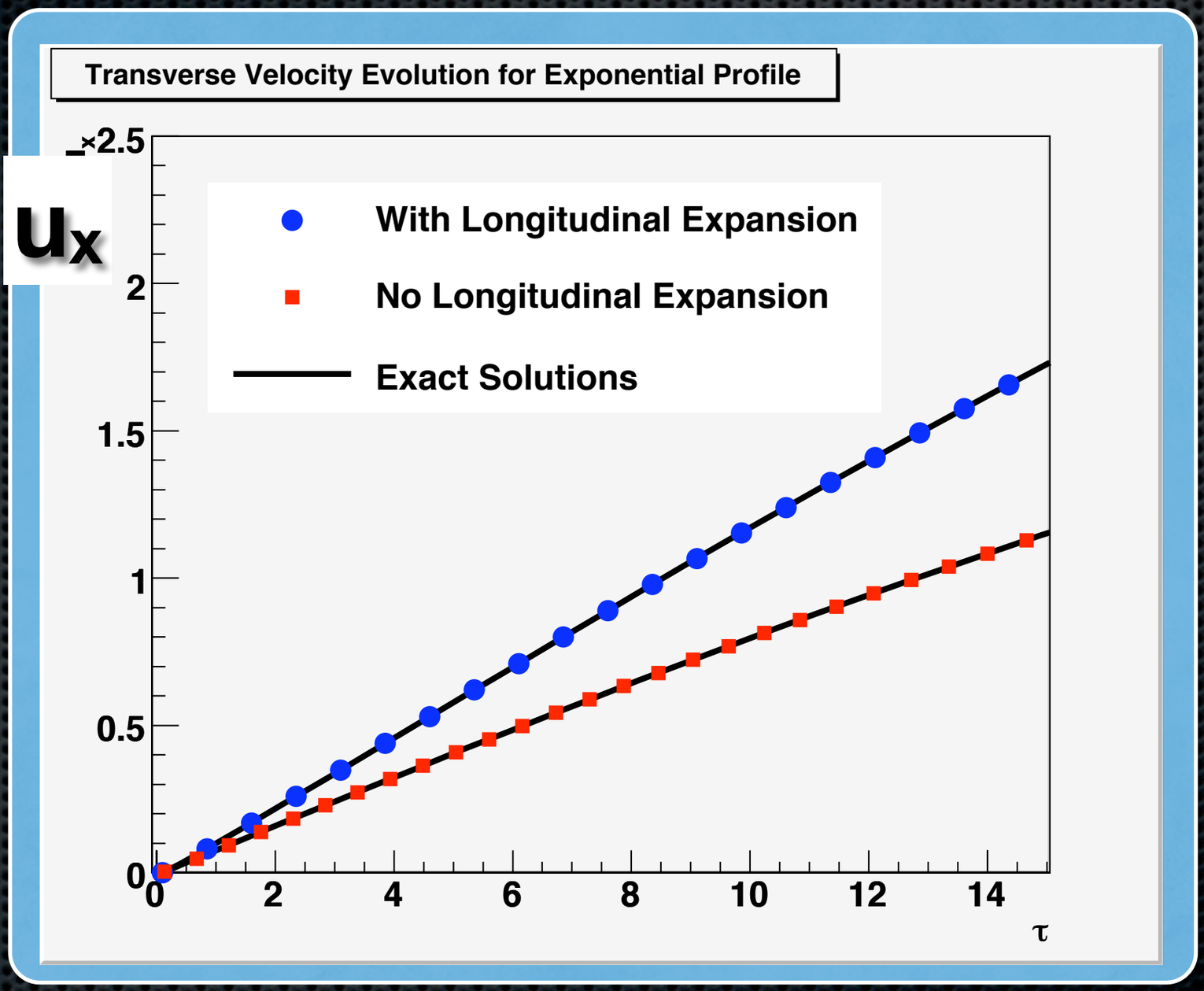
- The acceleration in Cartesian coordinates:

$$\dot{u} = \frac{u_0 c_s^2}{R(1+c_s^2)(u_0^2 - c_s^2 u^2)}$$

- This is modified in the presence of a longitudinal expansion.
- Then the transverse acceleration is:

$$\dot{u} = \frac{u_0 c_s^2}{u_0^2 - c_s^2 u^2} \cdot \left[\frac{u_0 u}{\tau} + \frac{1}{R(1+c_s^2)} \right]$$

Exponential Test (u)



Exponential Test (ε)

- We could have instead solved conservation equations for energy.
- With no Bjorken expansion:

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{uu_0(1-c_s^2)}{R(u^2(1-c_s^2)+1)}$$

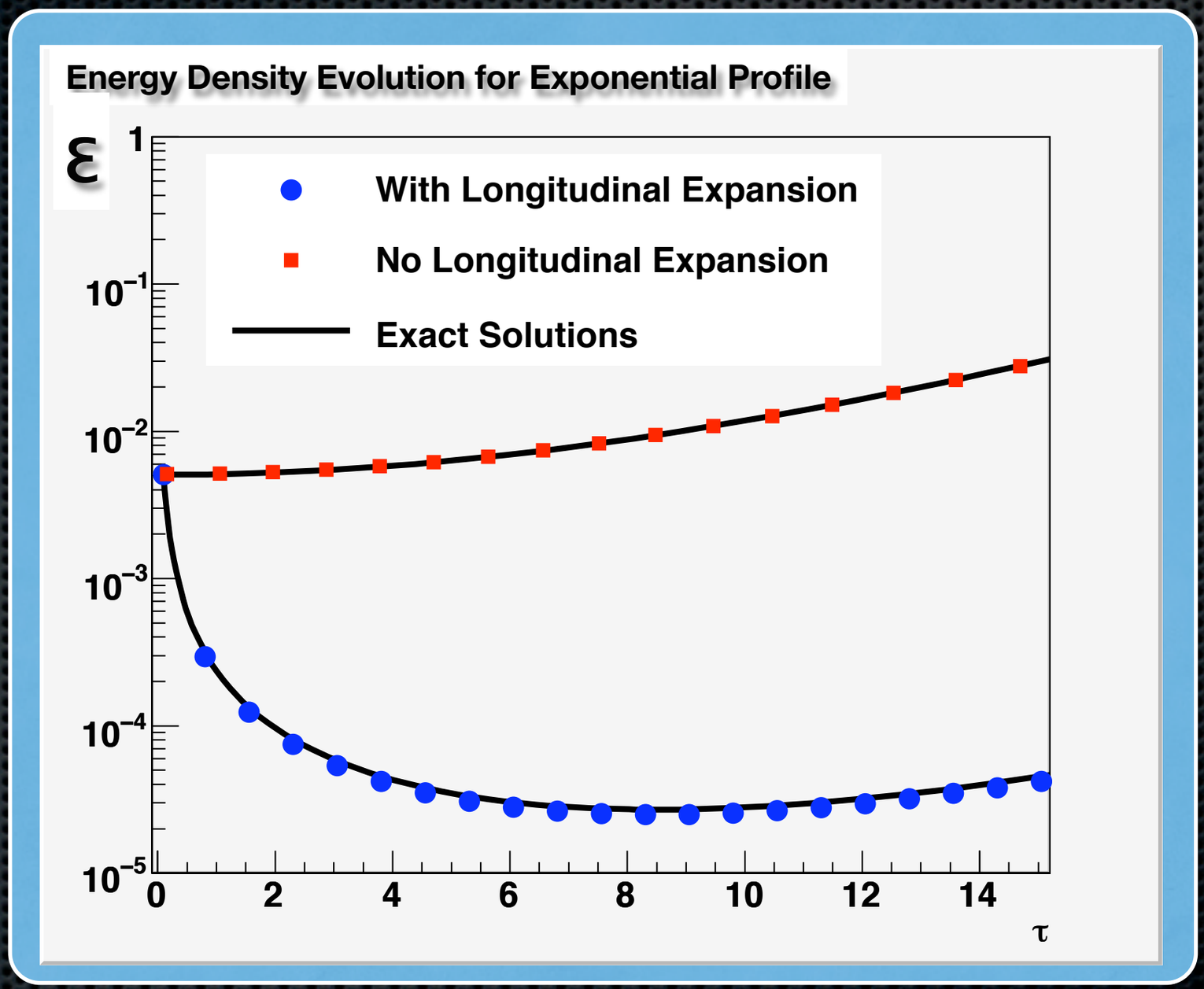
- Which may be integrated:

$$\epsilon = \epsilon_0 \cdot exp\left[\frac{1-c_s^4}{2c_s^2}u^2\right]$$

- Or with Bjorken expansion:

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{u_0}{u^2(1-c_s^2)+1} \cdot \left[\frac{(1-c_s^2)u}{R} - \frac{(1+c_s^2)u_0}{\tau}\right]$$

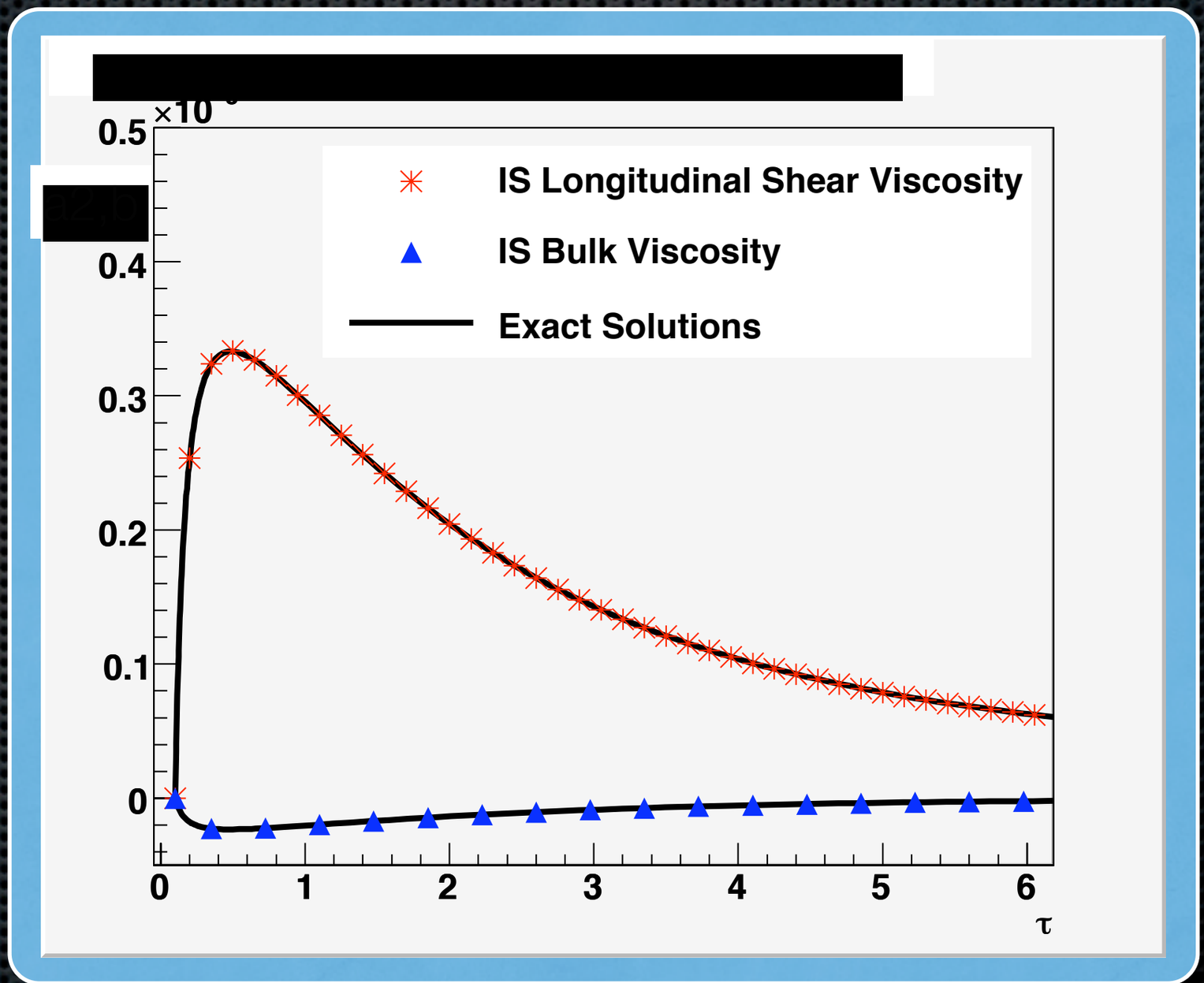
Energy Density Test



Exponential Test ($\pi_{\alpha\beta}$)

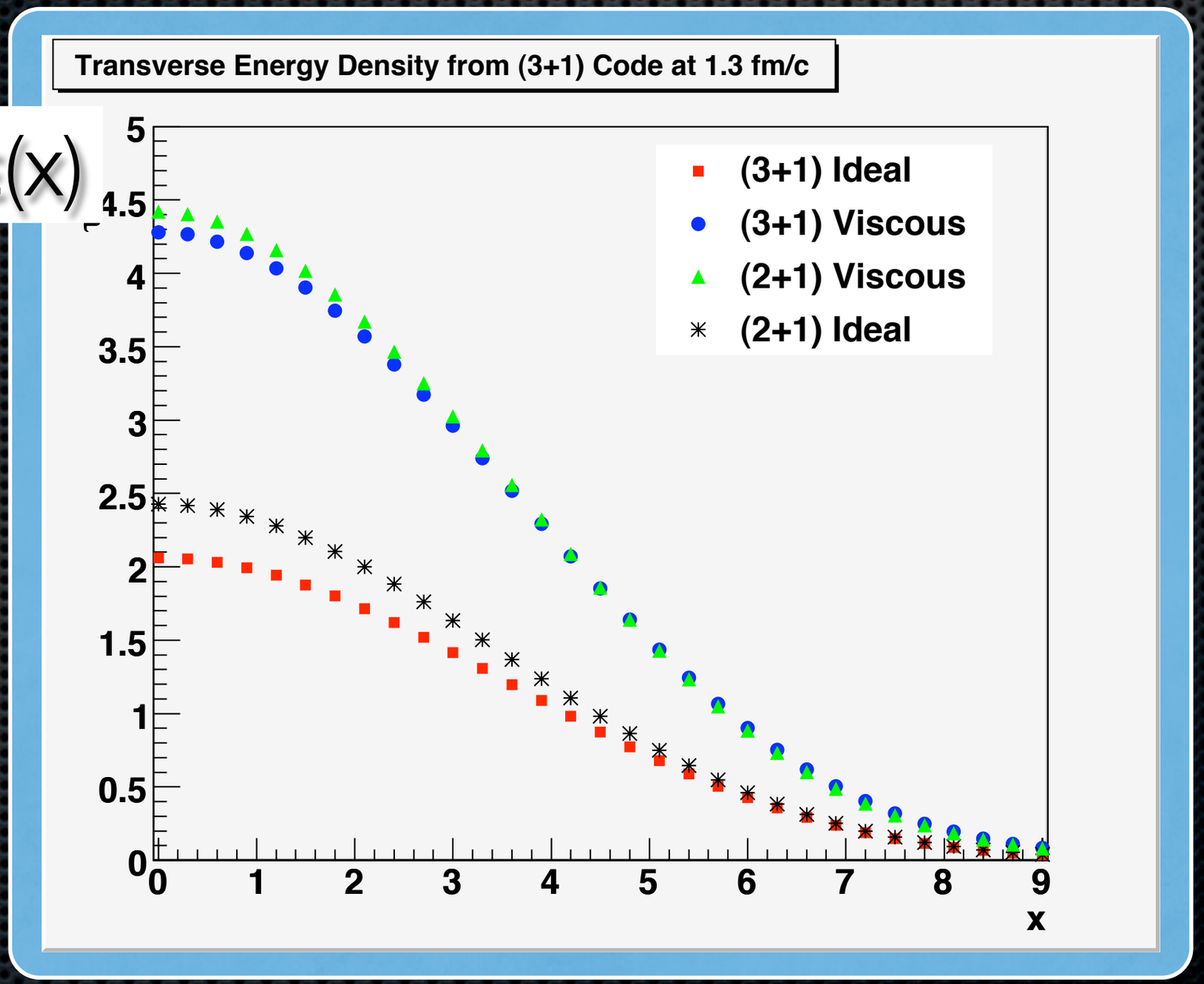
- Similar testing was done for several of the viscous deviations.
- For simplicity, un-physically set:

$$\eta = \epsilon/4\pi$$



Effects of (3+1) - Energy Density Profiles

$\tau^{4/3}\,\varepsilon(x)$



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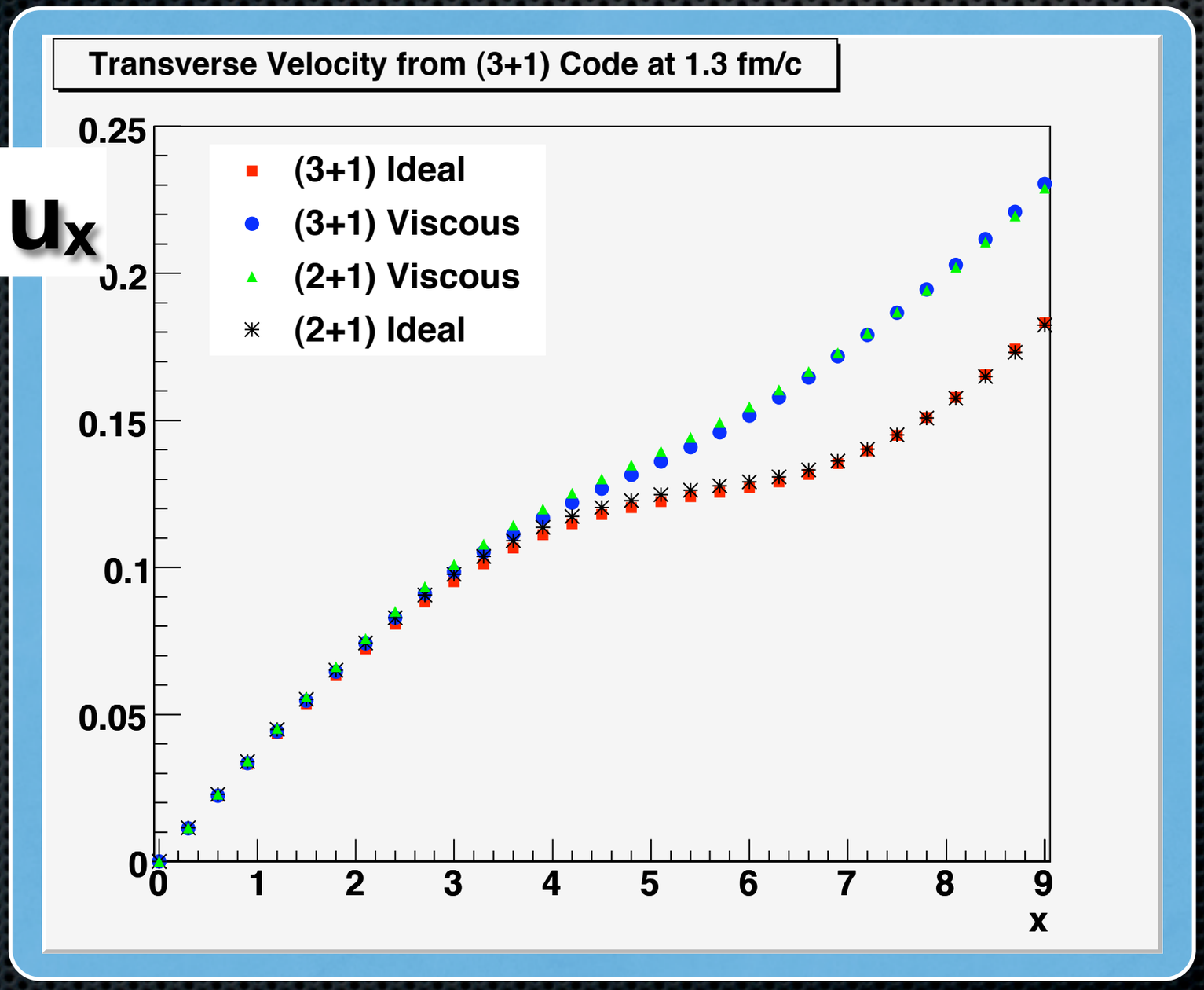
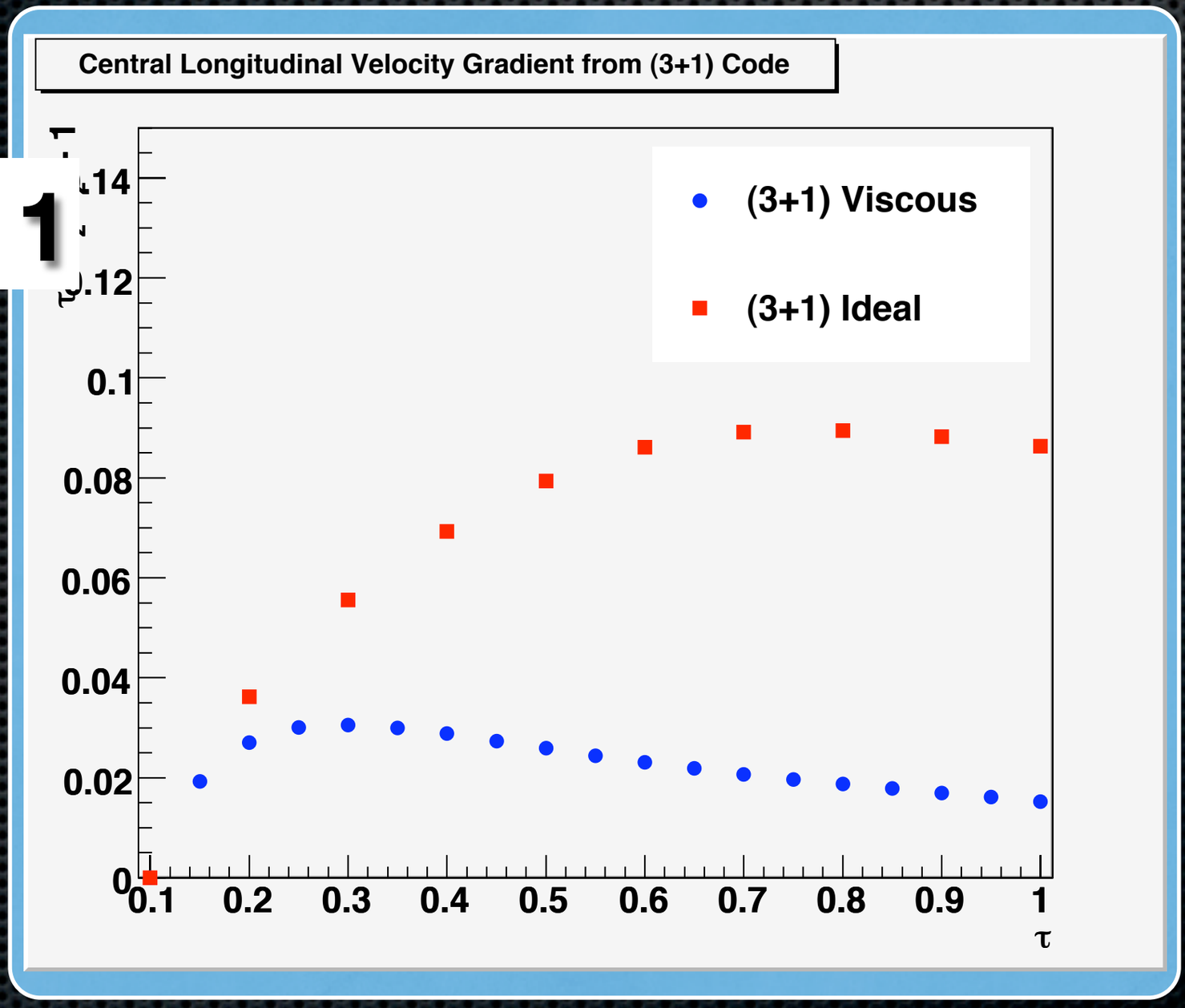


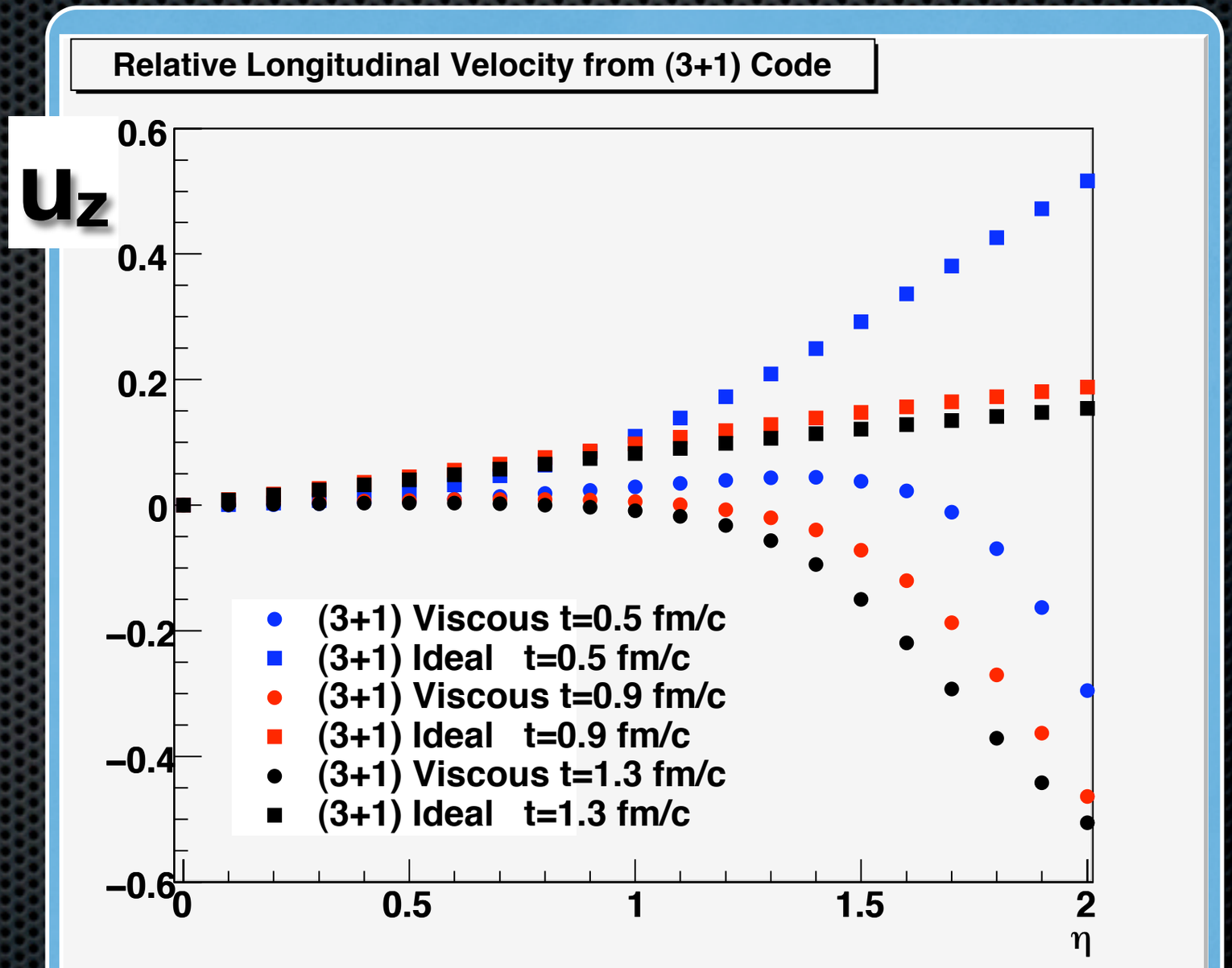
Effects of (3+1) - Velocity Gradient

$\tau du_z d_z - 1$



Effects of (3+1) - Longitudinal Flow

- Shear viscosity rapidly slows the longitudinal expansion.



Moving Forward

- Significant progress has been made toward a (3+1) dimensional Israel-Stewart code.
- Simple tests verify the general features of the evolution of the matter.
- Additional testing required.
- Longitudinal dynamics can affect central rapidity observables.