

VISCOUS HYDRODYNAMIC PREDICTIONS FOR THE LHC

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Flow and Dissipation in Ultrarelativistic Heavy Ion Collisions
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GOAL

To make a prediction of the measured (integrated) elliptic flow for 5.5 TeV Pb+Pb collisions by:

- 1 taking best fit viscous hydrodynamic simulations from RHIC,
- 2 making appropriate modifications for LHC collisions,
- 3 running viscous hydrodynamic simulations to calculate v_2 .

(arXiv:0901.4588)

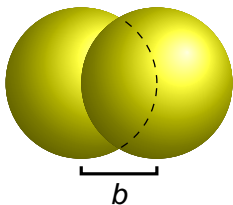
OUTLINE

- 1 INTRODUCTION/SETUP
 - Elliptic Flow
 - Hydrodynamic Equations
 - Viscous Hydrodynamic Simulations
- 2 RHIC ANALYSIS
- 3 LHC PREDICTIONS

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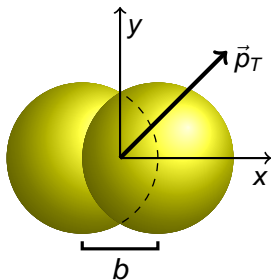
ELLIPTIC FLOW



ELLIPTIC FLOW

Single-particle momentum spectra:

$$\frac{dN}{dY d^2p_T}(\vec{p}, b)$$



IDEAL (RELATIVISTIC) HYDRODYNAMIC EQUATIONS

- Assume isotropic energy-momentum tensor:

$$T^{\mu\nu} = T_0^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$\Rightarrow T_{0_{rest}}^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Conservation equations:

$$\partial_\mu T^{\mu\nu} = 0$$

- Equation of State:

$$p = p(\epsilon)$$

- An additional relation for each additional conserved current (assumed unimportant for the following)

VISCOUS HYDRODYNAMICS

- Add dissipative (viscous) effects—derivative expansion:

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

- To first order (Navier-Stokes):

$$\Pi^{\mu\nu} = \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha$$

- Acausal signal propagation \Rightarrow instabilities \Rightarrow difficult to solve numerically.
- Can be fixed by adding second-order term(s).

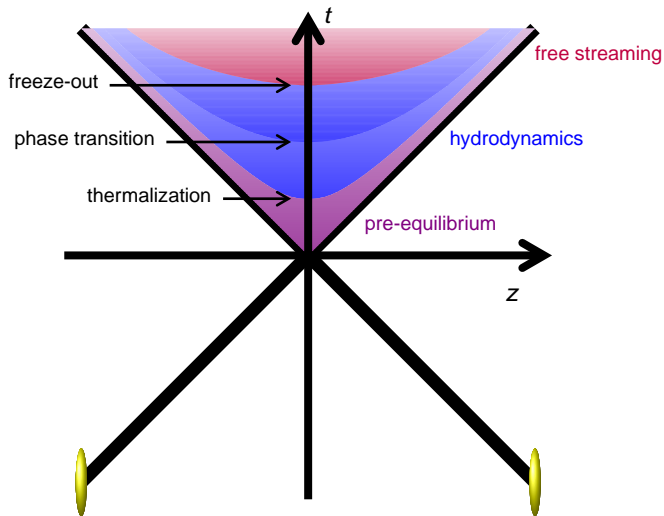
CAUSAL RELATIVISTIC VISCOUS HYDRODYNAMICS

- To start, set bulk viscosity to zero.
- \Rightarrow most general form for a conformal fluid in flat space to second order [BRSSS]:

$$\begin{aligned} \Pi^{\mu\nu} = & \eta \nabla^{\langle\mu} u^{\nu\rangle} - \tau_{\pi} \left[\Delta_{\alpha}^{\mu} \Delta_{\beta}^{\nu} \mathcal{D} \Pi^{\alpha\beta} + \frac{4}{3} \Pi^{\mu\nu} (\nabla_{\alpha} u^{\alpha}) \right] \\ & - \frac{\lambda_1}{2\eta^2} \Pi^{\langle\mu}{}_{\lambda} \Pi^{\nu\rangle\lambda} + \frac{\lambda_2}{2\eta} \Pi^{\langle\mu}{}_{\lambda} \omega^{\nu\rangle\lambda} - \frac{\lambda_3}{2} \omega^{\langle\mu}{}_{\lambda} \omega^{\nu\rangle\lambda} \end{aligned}$$

- (Simulations insensitive to second-order transport coefficients \Rightarrow can isolate effect of shear viscosity η .)

ANATOMY OF A HEAVY ION COLLISION



THE REST OF OUR HYDRO MODEL

- “Realistic” QCD equation of state (Laine and Schröder '06)
 - similar to more recent lattice QCD-derived equations of state
 - no treatment of chemical non-equilibrium
- Glauber and Color Glass Condensate (fKLN) initial conditions
 - optical model (no fluctuations)
 - free parameters T_0, τ_0
- Cooper-Frye freeze out prescription (with resonance feeddown)
 - free parameter T_f

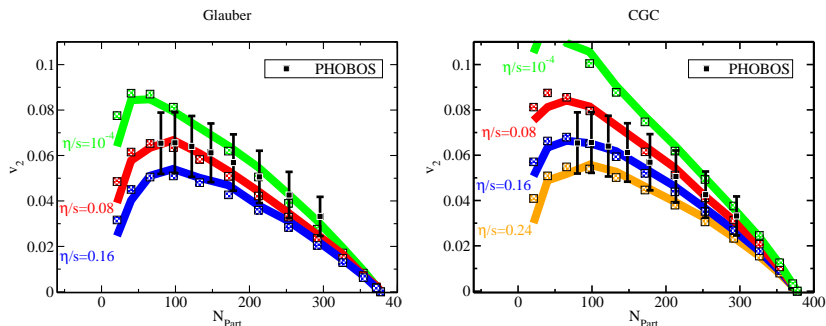
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ANALYSIS PROCEDURE/FREE PARAMETERS

THE PROCEDURE USED IS AS FOLLOWS:

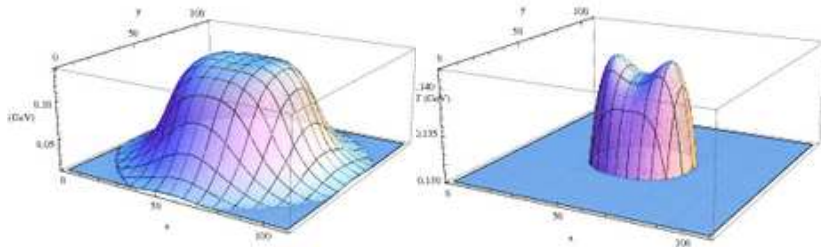
- 1 Choose model for initial conditions (Glauber or CGC)
- 2 Choose value of η/s to study (set to a constant throughout the evolution)
- 3 Use multiplicity data to fix the energy density normalization (T_0) and thermalization time (τ_0)
- 4 Use $\langle p_T \rangle$ data to fix the freezeout temperature (T_F)
- 5 With all the parameters now fixed, compare v_2 to RHIC data

RHIC RESULTS: MOMENTUM INTEGRATED v_2 

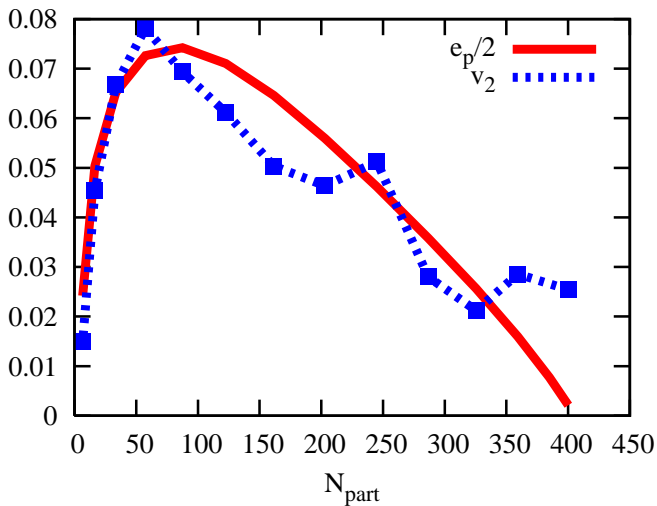
Note that squares follow curves:

$$\frac{1}{2} e_p \equiv \frac{1}{2} \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle} \simeq v_2$$

PROBLEMATIC FREEZE OUT



Temperature profile in the transverse plane just before complete freeze out of a Pb+Pb collision

v_2 VS. $e_p/2$ 

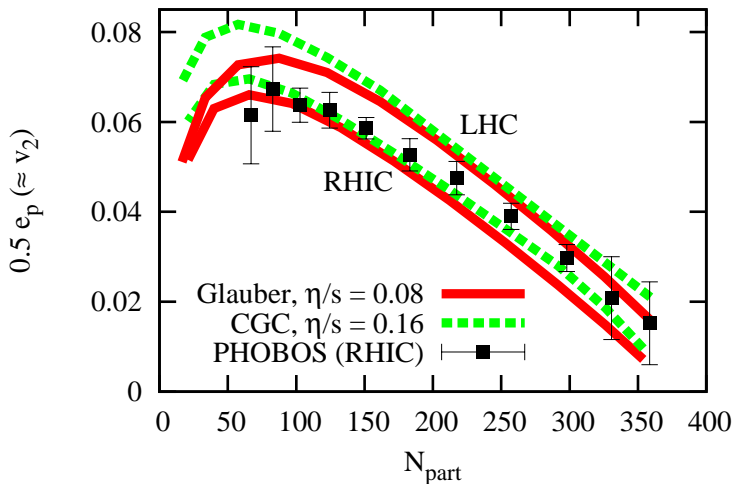
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HOW TO PREDICT RESULTS AT THE LHC

Using the knowledge gained from RHIC, we can make a prediction for Pb-Pb collisions at top LHC energies.

- 1 Assume $T_f, \frac{\eta}{s}, \tau_0$ do not change much \Rightarrow use best RHIC values for each initial condition (Glauber and CGC)
- 2 Choose T_0 to match predicted multiplicity $\frac{dN_{ch}}{dY} \approx 1800$.
(Can instead fix $\frac{dS}{dY} \simeq 7.85 \frac{dN_{ch}}{dY}$)
- 3 Make appropriate changes to the initial conditions (Pb instead of Au and increased collision energy)
- 4 $\Rightarrow v_2$ prediction!

MOMENTUM INTEGRATED v_2 AT RHIC AND LHC

SUMMARY/CONCLUSIONS

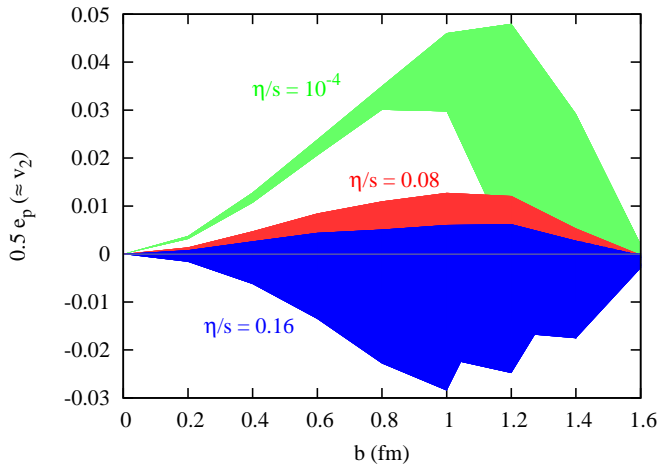
- Viscous hydrodynamic simulations were performed of top-energy heavy ion collisions at LHC
- Prediction: Integrated v_2 will be $\sim 10\%$ larger than measured at RHIC

PP COLLISIONS AT LHC

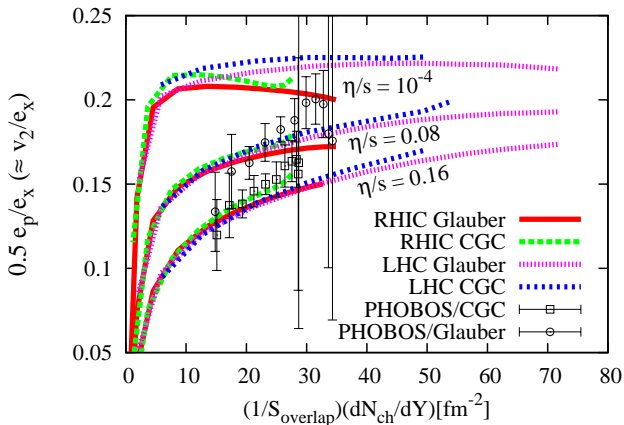
One can perform the same procedure for proton-proton collisions at LHC:

- Assume $dN_{ch}/dY \approx 6$ for "central" collisions
- Glauber initial conditions, using the charge density of the proton.

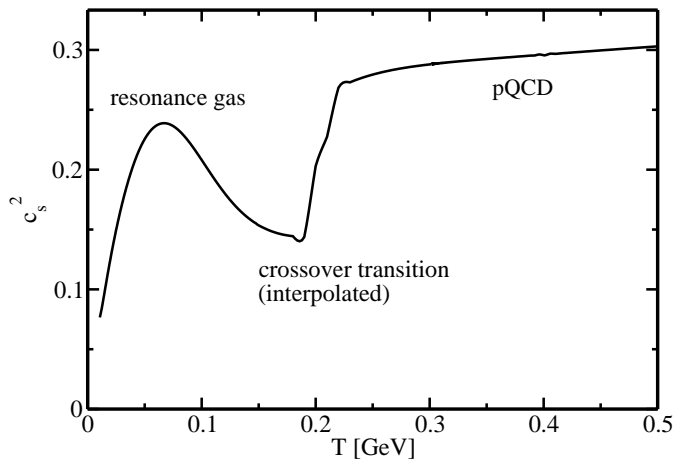
INTEGRATED ELLIPTIC FLOW



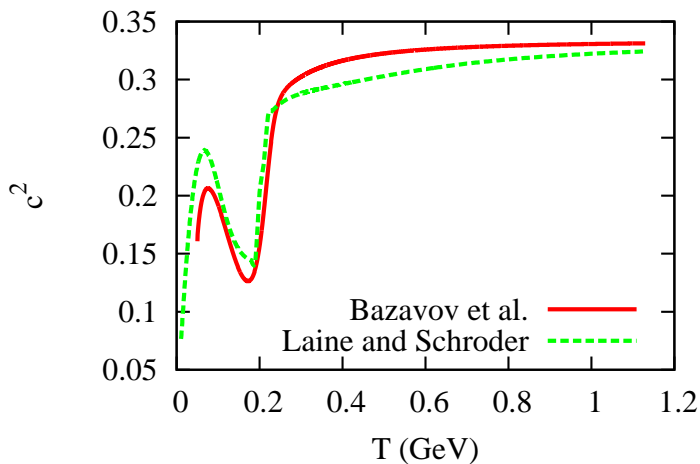
UNIVERSAL CURVES

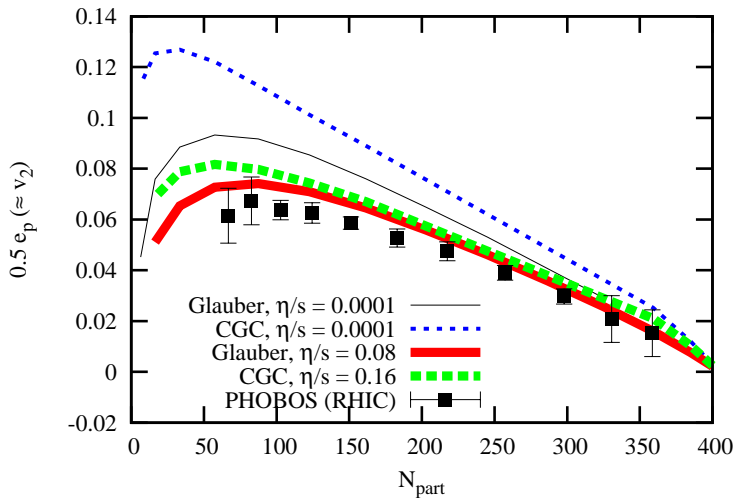


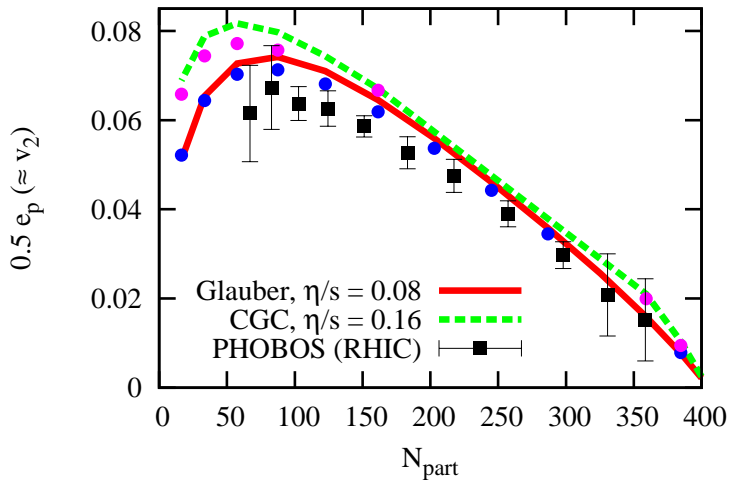
$$\left(S_{\text{overlap}} \equiv \pi \sqrt{\langle x^2 \rangle \langle y^2 \rangle} \right)$$



(Laine and Schröder)







PARAMETER VALUES

Beam	Initial cond.	$\frac{dN_{ch}}{dY}$	T_i [GeV]	\sqrt{s} [GeV]	τ_0 [fm/c]
Gold	Glauber	800	0.34	200	1
Gold	CGC	800	0.31	200	1
Lead	Glauber	1800	0.42	5500	1
Lead	CGC	1800	0.39	5500	1

TABLE: Central collision parameters used for the viscous hydrodynamics simulations ($T_f = 0.14$ GeV for all).

NOTATION

$$A_{\langle\mu} B_{\nu\rangle} \equiv \left(\Delta_{\mu}^{\alpha} \Delta_{\nu}^{\beta} + \Delta_{\nu}^{\alpha} \Delta_{\mu}^{\beta} - \frac{2}{3} \Delta^{\alpha\beta} \Delta_{\mu\nu} \right) A_{\alpha} B_{\beta}$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^{\mu} u^{\nu}$$

$$\nabla^{\mu} \equiv \Delta^{\mu\alpha} D_{\alpha}$$

$$D \equiv u_{\alpha} D^{\alpha}$$

$$\omega_{\mu\nu} \equiv \frac{1}{2} [\nabla_{\nu} u_{\mu} - \nabla_{\mu} u_{\nu}]$$

e.g. Navier Stokes term:

$$\nabla^{\langle\mu} u^{\nu\rangle} = \nabla^{\mu} u^{\nu} + \nabla^{\nu} u^{\mu} - \Delta^{\mu\nu} \nabla_{\alpha} u^{\alpha}$$