Pseudogauge fixing in interacting QFTs

Based on 2407.14345

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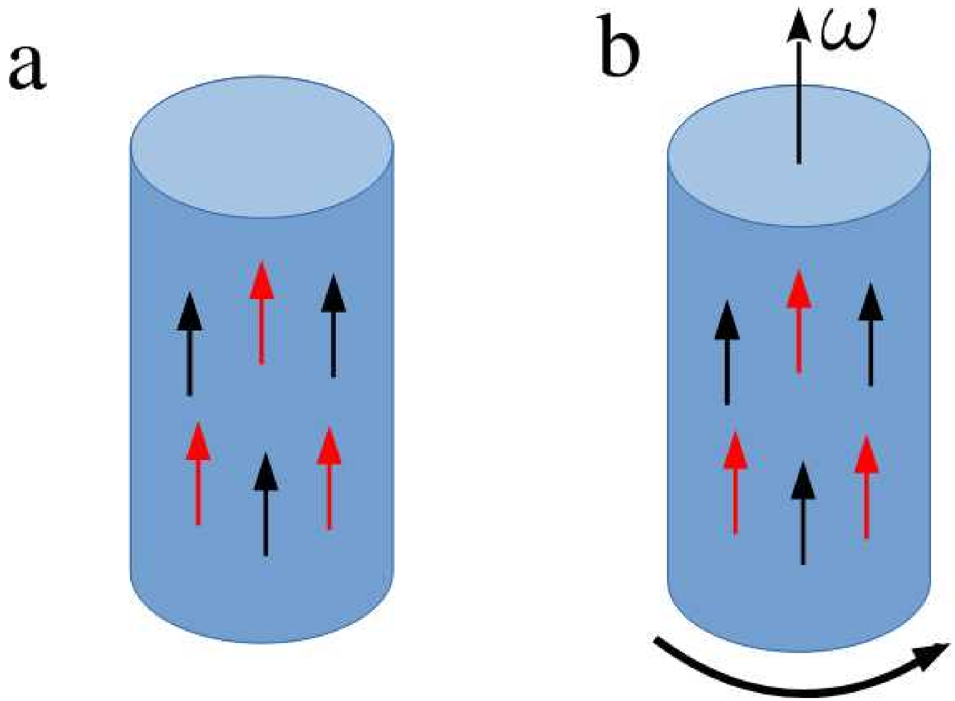
Motivations

We observe spin polarization in heavy ion collisions, and we know hydrodynamics works well for QGP physics. Maybe we should include the spin tensor there?



Becattini, Florkowski, Speranza Phys.Lett.B 789 (2019) 419-425

The spin tensor is related to the total angular momentum, it could allow polarization to happen without any fluid flow.
Spin hydro:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu S^{\mu,\rho\sigma} = T^{\sigma\rho} - T^{\rho\sigma}$$

Tremendous development, see permutations of: Rischke, Weickgenannt, Wagner, Florkowski and co-authors.

The definition of energy momentum tensor and spin tensor is not unique.

Theoretical ambiguity!

$$\begin{aligned}\widehat{T}'^{\mu\nu} =&\widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}\right)\\ \widehat{\mathcal{S}}'^{\lambda,\mu\nu} =&\widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} + \nabla_\rho \widehat{Z}^{\mu\nu,\lambda\rho}\end{aligned}$$

The new operators describe the same global charges (four-momentum and angular momentum boost operators).

$$\widehat{\mathcal{J}}^{\lambda,\mu\nu} = x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} + \widehat{\mathcal{S}}^{\lambda,\mu\nu}$$

$$\nabla_\mu \widehat{T}'^{\mu\nu} = \nabla_\mu \widehat{T}^{\mu\nu} = 0 \qquad \nabla_\lambda \widehat{\mathcal{J}'}^{\lambda,\mu\nu} = \nabla_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = 0$$

$$\widehat{P'}^{\mu} = \widehat{P}^\mu = \int \mathrm{d}\Sigma_\rho \widehat{T^{(\prime)}}^{\mu\nu} \qquad \widehat{J'}^{\mu\nu} = \widehat{J}^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \widehat{\mathcal{J}^{(\prime)}}^{\lambda,\mu\nu}$$

From the QFT standpoint, this is a problem in systems out-of-equilibrium

Kubo formula:

Becattini, Tinti, Phys.Rev.D 87 (2013) 2, 025029

$$\eta = \lim_{q\to 0} \frac{\mathrm{d}}{\mathrm{d}\omega} \mathrm{Im}\left\{ i \int \mathrm{d}^4 x e^{i\omega t - qx} \langle \left[\widehat{T}^{12}(x), \widehat{T}^{12}(0) \right] \rangle \theta(t) \right\}$$

Local equilibrium density operator:

$$\widehat{\rho} = \frac{1}{Z} \exp\left(- \int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu} \beta_\nu + n_\mu \mathfrak{S}_{\rho\sigma} \widehat{\mathcal{S}}^{\mu,\rho\sigma} \right)$$

Expectation values depend on this choice!

Results

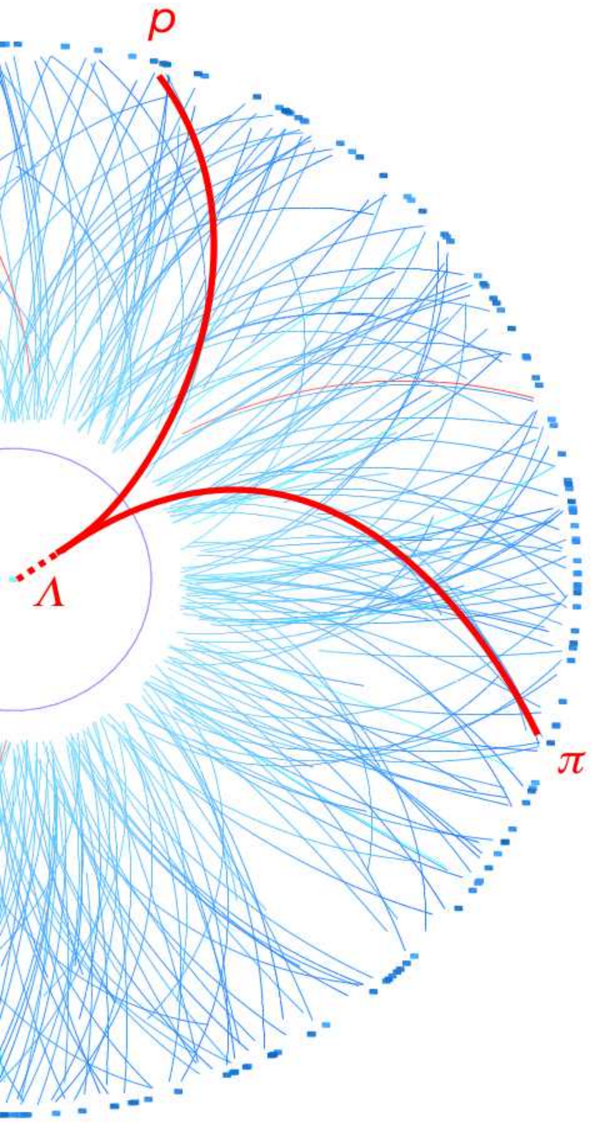
Interacting theories may make the choice! Consider the theory

$$\mathcal{L} = \psi \left(\frac{i}{2} \overleftrightarrow{\not{D}} - m \right) \psi + G_A \left(\bar{\psi} \gamma_\mu \gamma^5 \psi \right) \left(\bar{\psi} \gamma^\mu \gamma^5 \psi \right)$$

One can show that, in the mean field approximation, the following relation between partition functions hold

$$Z|_{\mathcal{L}+\text{rotation}} \equiv Z|_{\text{LEDO+canonical PG}}$$

So the interactions can be translated into a theory of ideal spin hydrodynamic.



Outline

- **Short theoretical review of the non equilibrium density operator formalism**
- **Path integral formulation**
- **Results and conclusions**

Density operator

Given a space-time foliation in hypersurfaces $\Sigma(\tau)$, the entropy $S = -\mathrm{Tr}\left(\widehat{\rho}\log\widehat{\rho}\right)$ is maximized constraining the mean values of the operators to be the actual ones *(van Weert, Annals of Physics 1982)*.

$$n_\mu \langle \widehat{T}_B^{\mu\nu} \rangle = n_\mu \mathrm{Tr}\left(\widehat{\rho}\widehat{T}_B^{\mu\nu}\right) = n_\mu T_B^{\mu\nu}{}_{\mathrm{true}}$$

$$\widehat{\mathcal{J}}^{\lambda,\mu\nu} = x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} + \widehat{\mathcal{S}}^{\lambda,\mu\nu}$$

$$n_\mu \langle \widehat{S}^{\mu,\nu\rho} \rangle = n_\mu \mathrm{Tr}\left(\widehat{\rho}\widehat{S}^{\mu,\nu\rho}\right) = n_\mu S_{\mathrm{true}}^{\mu,\nu\rho}$$

“$\mathbf{n}$” is the normal vector to the hypersurface $\Sigma(\tau)$. The density operator reads:

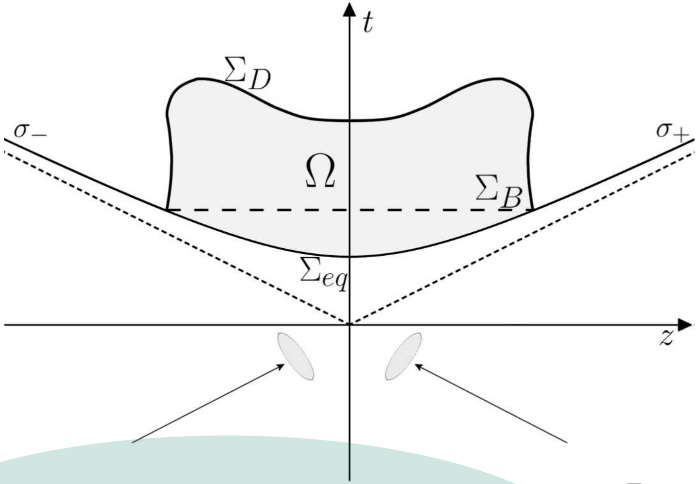
$$\widehat{\rho} = \frac{1}{Z}\exp\left(-\int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu}\beta_\nu - n_\mu \widehat{S}^{\mu,\nu\rho}\mathfrak{S}_{\nu\rho}\right)$$

$$Z = \mathrm{Tr}\left[\exp\left(-\int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu}\beta_\nu - n_\mu \widehat{S}^{\mu,\nu\rho}\mathfrak{S}_{\nu\rho}\right)\right]$$

β is the four temperature and $\mathfrak{S}$ is the spin potential: $\beta^\mu = \frac{u^\mu}{T}$

Warning: The true density operator should be constant in the Heisenberg representation!

$$\widehat{\rho}_{true} = \frac{1}{Z}\exp\left(-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu}\beta_\nu - n_\mu \widehat{S}^{\mu,\nu\rho}\mathfrak{S}_{\nu\rho}\right)$$



We use Gauss theorem to connect with present time.

$$\widehat{\rho}_{true} = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - n_\mu \widehat{S}^{\mu,\nu\rho}\mathfrak{S}_{\nu\rho}\right) + \int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu - \nabla_\mu\left(\widehat{S}^{\mu,\nu\rho}\mathfrak{S}_{\nu\rho}\right)\right)\right]$$

Local equilibrium **Dissipation**

Entropy production

The local thermodynamic equilibrium has vanishing entropy production

Zubarev, Prozorkevich, & Smolyanskii, 1979; Becattini, Buzzegoli, & Grossi, Particles, 2019, 1902.01089; Becattini, Daher, Sheng Phys.Lett.B 850 (2024)

$$S = -\mathrm{Tr}\,(\widehat{\rho}\,\log\widehat{\rho}) = \int_\Sigma \mathrm{d}\Sigma_\mu s^\mu$$

$$\nabla_\mu s^\mu = \left(T_S^{\mu\nu} - T_{S,LE}^{\mu\nu}\right)\partial_\mu\beta_\nu - \left(j^\mu - j^\mu_{LE}\right)\partial_\mu\zeta + \left(T_A^{\mu\nu} - T_{A,LE}^{\mu\nu}\right)(\varpi - \mathfrak{S})_{\mu\nu} \\ - \frac{1}{2}(\mathcal{S}^{\mu,\nu\rho} - \mathcal{S}^{\mu,\nu\rho}_{LE})\partial_\mu\mathfrak{S}_{\nu\rho}$$

Deviations of the actual values of the conserved current from the local equilibrium value are responsible for entropy production.

The local equilibrium operator describes an ideal fluid.

Global equilibrium

The global equilibrium is achieved if there is no dependence on the hypersurface:

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$$

$$\zeta = \text{const.}$$

$$\mathfrak{S} = \varpi = \text{const.}$$

$$\beta^\mu = b^\mu + \varpi^{\mu\nu} x_\nu$$

ϖ Constant thermal vorticity

The statistical operator in this case reads:

$$\widehat{\rho}_{GE} = \frac{1}{Z} \exp\left(-\widehat{P}^\mu b_\mu + \frac{\varpi_{\mu\nu}}{2} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right)$$

P and J are the generators of the Poincaré group.
Q conserved charge.

ϖ describes rotation and acceleration

$$\varpi^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} w_\alpha u_\beta + \alpha^\mu u^\nu - \alpha^\nu u^\mu \qquad w^\mu = \frac{\omega^\mu}{T} \qquad \alpha^\mu = \frac{A^\mu}{T}$$

Polarization in local equilibrium

Local equilibrium (Belinfante pseudogauge) $\widehat{\rho}_{LE} = \frac{1}{Z_{LE}} \exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}\beta_\nu\right]$ $\beta^\mu = \frac{u^\mu}{T}$

Hydrodynamic approximation: gradients are small, **linear response theory**

$$S^\mu(k) = \frac{1}{2} \frac{\int_{\Sigma_D} \mathrm{d}\Sigma \cdot k \ \mathrm{tr}\left(\gamma^\mu \gamma_5 W_+(x,k)\right)}{\int_{\Sigma_D} \mathrm{d}\Sigma \cdot k \ \mathrm{tr}\left(W_+(x,k)\right)}$$

$$W(x,k)_{ab} = -\frac{1}{(2\pi)^4} \int \mathrm{d}^4 y e^{-ik\cdot y} \langle : \bar{\Psi}_b(x+y/2)\Psi_a(x-y/2) : \rangle$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z} \exp\left[-\beta(x)\cdot\widehat{P} + \partial_\nu \beta_\mu(x) \int \mathrm{d}\Sigma_\alpha(y) \widehat{T}^{\alpha\mu}(y-x)^\nu\right]$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z_\beta} e^{-\beta(x)\cdot\widehat{P}} + \frac{1}{Z_\beta} \partial_\alpha \beta_\nu \int \mathrm{d}\Sigma_\mu \int_0^1 \mathrm{d}z \ e^{-(1+z)\beta(x)\cdot\widehat{P}} \widehat{T}^{\mu\nu} e^{z\beta(x)\cdot\widehat{P}} (y-x)^\alpha$$

Corrections to the spin operator (Pauli-Lubanski vector): $\langle\widehat{O}\rangle_\beta = \frac{1}{Z}\mathrm{Tr}\left(e^{-\beta(x)\cdot\widehat{P}}\widehat{O}\right)$

$$\langle\widehat{S}^\mu(p)\rangle_{LE} = \langle\widehat{S}^\mu(p)\rangle_\beta + \partial_\nu\beta_\mu(x)\int \mathrm{d}\Sigma_\alpha(y)\ (y-x)^\nu\langle\widehat{S}^\mu(p)\widehat{T}^{\alpha\nu}(y)\rangle_\beta$$

In the end we get:

$$S^\mu_B(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma\frac{\int \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)[\varpi_{\nu\rho} + 2\hat{t}_\nu\xi_{\lambda\rho}\frac{p^\lambda}{\varepsilon}]}{\int \mathrm{d}\Sigma\cdot p\, n_F}$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$

But what happens in other pseudogauges?

Repeat the procedure with your favorite spin operator and with a spin potential:
Buzzegoli, Phys.Rev.C 105 (2022) 4, 044907

$$\widehat{\rho} = \frac{1}{Z} \exp\left(- \int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu} \beta_\nu + n_\mu \mathfrak{S}_{\rho\sigma} \widehat{\mathcal{S}}^{\mu,\rho\sigma} \right)$$

Canonical:

$$S^\mu_C = S^\mu_B + \frac{\epsilon^{\lambda\rho\sigma\tau} \hat{t}_\lambda (k^\mu k_\tau - \eta^\mu_\tau m^2)}{8m\varepsilon_k} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot k \, n_F \,(1 - n_F)\,(\varpi_{\rho\sigma} - \mathfrak{S}_{\rho\sigma})}{\int_\Sigma \mathrm{d}\Sigma \cdot k \, n_F}$$

HW, GLW:

$$S^\mu_{GLW,HW} = -\frac{1}{8m} \epsilon^{\mu\rho\sigma\tau} k_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot k \, n_F \,(1 - n_F)\, \mathfrak{S}_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot k \, n_F}$$

All pseudogauges agree in global equilibrium, when $\varpi = \mathfrak{S}$, as they should.

Why does the result change?

The spin operator is PG independent, as it depends on the total angular momentum

$$\widehat{S}^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\widehat{J}_{\nu\rho}p_\sigma$$

But the local equilibrium density operator is PG-dependent. Expectation values depend on the choice of densities

$$Tr\left(\rho_{LE}[\widehat{T}^{\mu\nu},\widehat{\mathcal{S}}^{\mu\nu\rho}]\widehat{O}\right) \neq Tr\left(\rho_{LE}[\widehat{T}'^{\mu\nu},\widehat{\mathcal{S}}'^{\mu\nu\rho}]\widehat{O}\right)$$

Path Integral formulation of the local equilibrium density operator

Developed by M. Hongo in Annals Phys. 383 (2017) 1-32

The local values of four-temperature induce a thermal metric background, where one can write the path integral.

Our goal is to write the path integral for the partition function of the free Dirac field with local equilibrium density operator in the canonical pseudogauge

The Dirac Lagrangian in a generic background metric, with spin connection and vielbein

$$\mathcal{L} = \bar{\psi}\left(i\frac{\overleftrightarrow{\not{D}}}{2} - m\right)\psi \qquad \gamma^\mu = e^\mu_a \gamma^a \qquad \overrightarrow{D}_\mu = \overrightarrow{\nabla}_\mu - i\omega_\mu{}^{ab}\Sigma_{ab}$$

$$e_{a\mu}e^\mu{}_b = \eta_{ab} \qquad e_\mu{}^a e_{a\nu} = g_{\mu\nu} \qquad \omega_\mu{}^a{}_b = e_\nu{}^a e^\lambda{}_b \Gamma^\nu_{\mu\lambda} - e^\lambda{}_b \partial_\mu e_\lambda{}^a$$

The canonical energy momentum and spin tensor

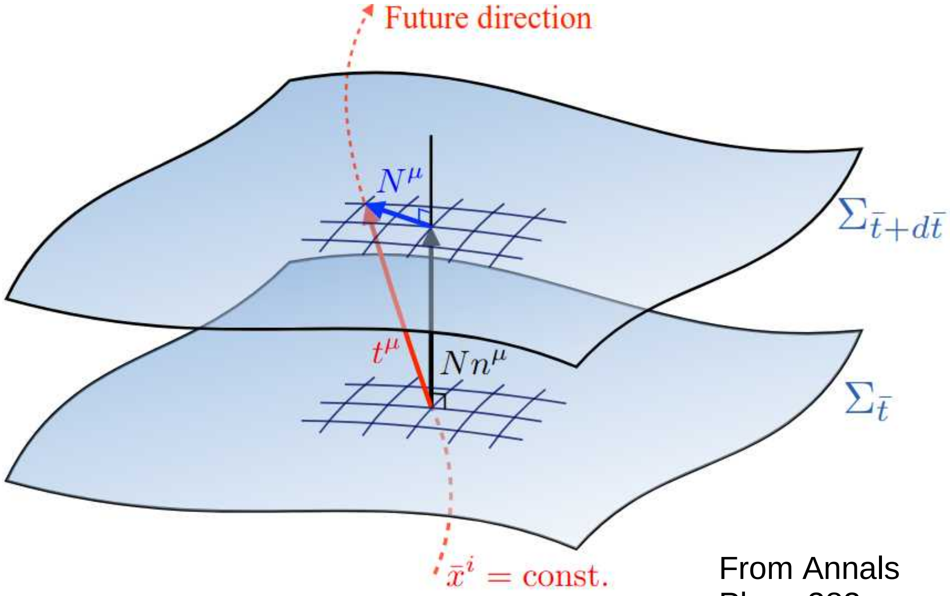
$$\widehat{T}^\mu_\nu = \bar{\psi} i\gamma^\mu \frac{\overleftrightarrow{D}_\nu}{2}\psi - g^\mu_\nu \mathcal{L} \qquad \widehat{\mathcal{S}}^{\mu,\nu\rho}_C = \frac{1}{2}\bar{\psi}\{\gamma^\mu, \Sigma^{\nu\rho}\}\psi = \epsilon^{\mu\nu\rho\sigma} j^5_\sigma$$

The density operator is

$$\widehat{\rho} = \frac{1}{\mathcal{Z}} \exp\left\{-\int_\Sigma \mathrm{d}\Sigma\, n_\mu \left[\widehat{T}^{\mu\nu}_C \beta_\nu - \frac{1}{2}\mathfrak{S}_{\rho\sigma}\widehat{\mathcal{S}}^{\mu,\rho\sigma}_{\mathrm{C}}\right]\right\}$$

It is convenient to use ADM coordinates, which one defines starting from a foliation

$$\Sigma \equiv (\bar{t} = \text{const.}) \quad n_\mu = N\partial_\mu \bar{t} \quad \partial_{\bar{t}} x^\mu = N n^\mu + N^\mu$$



From Annals Phys. 383 (2017) 1-32

$$g_{\mu\nu} = \begin{pmatrix} N^2 + N_{\bar{i}} N^{\bar{i}} & N_{\bar{i}} \\ N_{\bar{i}} & \gamma_{\bar{i}\bar{j}} \end{pmatrix}$$

In the coordinate system $(\bar{t}, \bar{\boldsymbol{x}})$

$$\widehat{\rho} = \frac{1}{\mathcal{Z}} \exp\left[-\int_\Sigma \mathrm{d}^3 x \, \sqrt{\gamma} N \left(\widehat{T}^{0\nu} \beta_\nu + \frac{\mathfrak{S}_{\mu\nu}}{2} \widehat{\mathcal{S}}^{0,\mu\nu} \right) \right]$$

To compute the path integral one proceeds as usual. I ignore the spin part for now

The value of β is constant, and equal to its value on the hypersurface.

$$\widehat{R} = \exp\left\{-\frac{B}{B}\int_{\bar{t}} \mathrm{d}^3x \sqrt{\gamma} N\ \widehat{T}_C^{0\nu}\beta_\nu\right\} = \exp\left\{\frac{B}{B}\int \mathrm{d}^3x \widehat{O}\right\}$$

$$\mathrm{Tr}\widehat{R} = \lim_{N\to\infty}\sum_a\prod_i \mathrm{d}\pi_i \mathrm{d}\psi_i \langle\psi_a|\pi_i\rangle\langle\pi_i|\exp\left\{\frac{\delta B}{B}\int \mathrm{d}^3x(\widehat{O})\right\}|\psi_i\rangle\langle\psi_i|\ldots|\psi_a\rangle$$

The operators get evaluated on the fields, then we need to use the identity

$$\langle\phi_i|\pi_j\rangle = \exp\left(i\int \mathrm{d}^3x\phi_i\pi_j\right)$$

Eventually, in ADM imaginary time, the path integral is computed on the action:

$$S = \int_0^B \mathrm{d}\tau \mathrm{d}^3 x \sqrt{\gamma} N B^{-1} \beta^0 \left[-\bar{\psi}\gamma^0 \frac{\overleftrightarrow{\nabla}_0}{2\beta^0 B^{-1}} \psi + \bar{\psi} i \gamma^i \frac{\overleftrightarrow{D}_i}{2} \psi - m\bar{\psi}\psi - \bar{\psi} i \gamma^0 \frac{\overleftrightarrow{D}_i}{2} \psi \frac{\beta^i}{\beta^0} \right]$$

Introducing Eucledian gamma matrices, and making the vielbein explicit:

$$\gamma_E^i = -i\gamma^i$$

$$S = \int \mathrm{d}\tau \mathrm{d}^3 x \sqrt{\gamma} N B^{-1} \beta^0 \left\{ -\bar{\psi}\gamma_E^a \left[\frac{e_a^0}{B^{-1}\beta^0} \overleftrightarrow{\nabla}_0 + \left(e_a^i + i e_a^0 \frac{\beta^i}{\beta^0} \right) \overleftrightarrow{D}_i \right] \psi - m\bar{\psi}\psi \right\}$$

Now we perform the coordinate change

$$\tilde{e}_a^{\tilde{0}} = \frac{e_a^0}{B^{-1}\beta^0} \qquad \tilde{e}_a^{\tilde{i}} = e_a^i + i e_a^{\bar{0}} \frac{\beta^i}{\beta^{\bar{0}}}$$

$$\tilde{N} = N B^{-1} \beta^0 \qquad \tilde{N}^{\tilde{i}} = \beta^0 B^{-1} \left(N^i + \frac{\beta^i}{\beta^0} \right)$$

To get

$$S = -\int \mathrm{d}\tau \mathrm{d}^3 x \sqrt{\tilde{\gamma}} \tilde{N} \left\{ \bar{\psi} \gamma_E^a \left[\tilde{e}_a^0 \overleftrightarrow{\nabla}_0 + \tilde{e}_a^i \overleftrightarrow{D}_{\bar{i}} \right] \psi + m \bar{\psi} \psi \right\}$$

But one derivative is partial and the other is covariant: add and subtract the metric induced spin connection (thermal vorticity)

$$\omega_{\tilde{0}\tilde{i}\tilde{j}} = \frac{1}{2B} (\partial_{\tilde{i}} \beta_{\tilde{j}} - \partial_{\tilde{j}} \beta_{\tilde{i}}) = \frac{\varpi_{\tilde{i}\tilde{j}}}{B}$$

The final result is

$$Z = \int \mathcal{D}[\psi, \psi^\dagger] \exp[-S_E - S_\Theta]$$

$$S_E = \int d^4\widetilde{x}_E \sqrt{\widetilde{g_E}}\, \bar{\psi} \left(\frac{\overleftrightarrow{\not{D}}}{2} + m \right) \psi \qquad S_\Theta = \int d^4\widetilde{x}_E \sqrt{\widetilde{g_E}}\, \frac{1}{2} (\varpi - \mathfrak{S})_{\widetilde{\mu}\widetilde{\nu}} n_{\widetilde{\alpha}} S^{\widetilde{\alpha},\widetilde{\mu}\widetilde{\nu}}$$

The action in the path integral is modified by the spin tensor out of equilibrium!

The spin potential can be absorbed in a the spin-connection (torsion).

We can also write:

$$S_\Theta = -\int_0^B \mathrm{d}\tau \mathrm{d}^3\widetilde{\boldsymbol{x}} \sqrt{\widetilde{g}_E}\, j_5^{\widetilde{\mu}} f_{\widetilde{\mu}} \qquad f^{\widetilde{\sigma}} = \frac{1}{2} \epsilon^{\widetilde{\sigma}\widetilde{\mu}\widetilde{\nu}\widetilde{\rho}} (\varpi_{\widetilde{\mu}\widetilde{\nu}} - \mathfrak{S}_{\widetilde{\mu}\widetilde{\nu}}) n_{\widetilde{\rho}}$$

Interacting systems under rotation

Consider, instead, the NJL-like lagrangian in rotating coordinates

$$\mathcal{L} = \psi \left(\frac{i}{2} \overleftrightarrow{\not{D}} - m \right) \psi + G_A \left(\bar{\psi} \gamma_\mu \gamma^5 \psi \right) \left(\bar{\psi} \gamma^\mu \gamma^5 \psi \right)$$

The path integral is written in curved spacetime

$$S_E = \int \mathrm{d}\tau \mathrm{d}^3 x \sqrt{\widetilde{g_E}}\, \psi \left(\frac{i}{2} \overleftrightarrow{\not{D}} - m \right) \psi - G_A j_5^\mu j_{5\,\mu}$$

In other words, one can use the Hubbard-Stratonovich transformation, introducing an axial vector field:

$$\mathcal{Z}_{\mathcal{L}} = \int \mathcal{D}[q, q^{\dagger}]\mathcal{D}f\, \mathrm{e}^{-S_f[q,\,\bar{q},\,f_{\widetilde{\mu}};\,e_a^{\widetilde{\mu}}]}$$

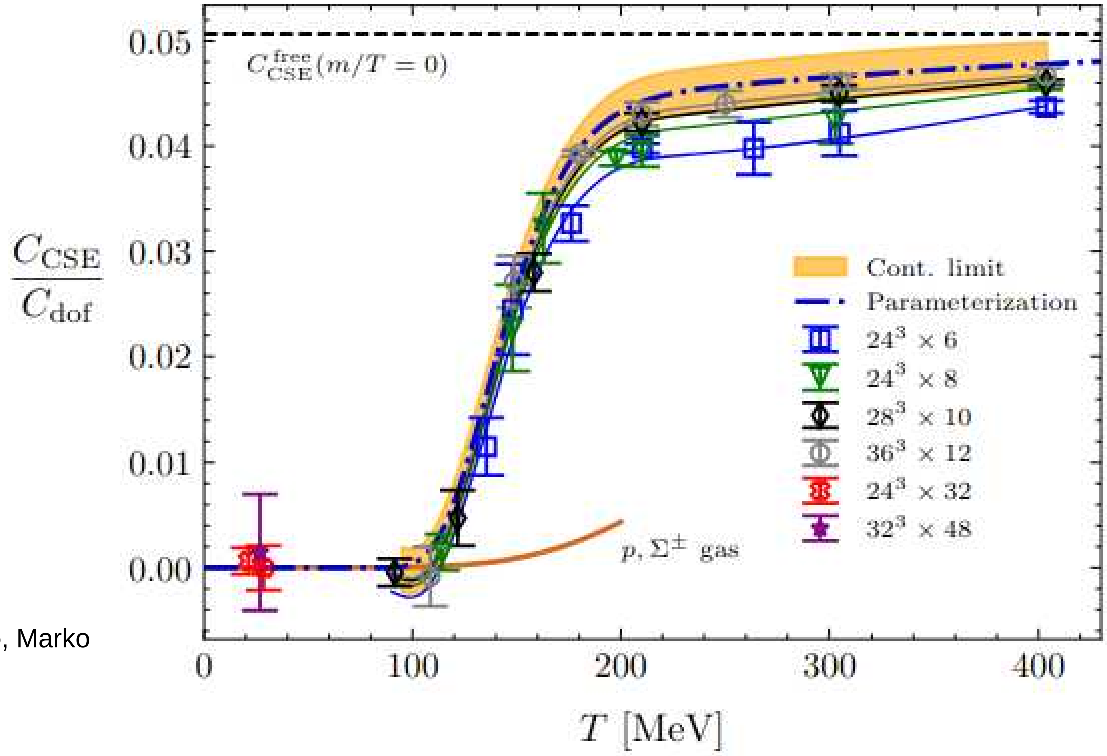
$$S_f = \int_0^B \mathrm{d}\tau \int \mathrm{d}^3\widetilde{\boldsymbol{x}}\,\sqrt{\widetilde{g}_E}\,\left[\mathcal{L}_0 + \frac{1}{4G_A} f_{\widetilde{\mu}} f^{\widetilde{\mu}} - j_5^{\widetilde{\mu}} f_{\widetilde{\mu}}\right]$$

In the mean field limit $f \to \langle f \rangle$, and the mean axial field is the difference between chemical potential and vorticity

$$\langle f^{\widetilde{\sigma}} \rangle = \frac{1}{2}\epsilon^{\widetilde{\sigma}\widetilde{\mu}\widetilde{\nu}\widetilde{\rho}}(\varpi_{\widetilde{\mu}\widetilde{\nu}} - \mathfrak{S}_{\widetilde{\mu}\widetilde{\nu}})n_{\widetilde{\rho}}$$

If there is rotation or magnetic field, an axial current is induced. In the chiral limit

$$\langle \boldsymbol{j_5} \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right) \boldsymbol{\omega} + \frac{\mu}{2\pi^2} \boldsymbol{B} + o(B, \omega, \mu)$$



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Some considerations

- The same effect is produced by magnetic field
- The f field has the same quantum numbers as the f_1 (rotation) and h_1 (magnetic field) mesons
- NJL-like interactions emerge from QCD as effective 4-quarks interaction. If the system is under rotation, the interaction considered here is probably sizeable.
- Our conclusions are model dependent. NJL model is anaolgous to a canonical spin tensor, but for different models it may be different.

Conclusions

Which pseudogauge should one use in heavy ion collisions? Can we make an objective choice at all?

- Interactions in underlying theories may choose for us

- NJL-like theory with axial current interactions at global equilibrium is equivalent to a theory of ideal idodynamics out-of-equilibrium with a spin potential

This duality should be studied beyond the mean field limit

THANK YOU FOR THE ATTENTION!