Coupled-charge transport: current status of fluid dynamic modeling

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The multi-component nature of nuclear matter

Traditionally:

Viewed as ‘blob’ of one type of matter (single component) with one velocity field

- usually ‘blob’ of energy with conserved particle number

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- usually ‘blob’ of energy with conserved particle number

In general:

Consists of multiple components with various properties with multiple velocity fields

- with multiple conserved quantities (e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry $\rightarrow$ coupled charge currents!
Hydrodynamics applied to heavy ion collisions

Hydro has been very successful in the **effective** description of the evolution of heavy-ion collisions ...

Molnár et al., PRC 90, 044904 (2014)
Hydro has been very successful in the **effective** description of the evolution of heavy-ion collisions ... but the theory **has to be extended** in order to explicitly account for effects and is dependent on the knowledge of the **underlying microscopic properties**.

Molnár et al., PRC 90, 044904 (2014)

Denicol et al., PRC 98, 034916 (2018)
Initial state with multiple conserved charges

- Energy density
- Net baryon number
- Net strangeness

Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

Plots not published!
Initial state with multiple conserved charges

Collaborations with T. Dore, O. Garcia-Montero, S. Schlicht (McDipper / KoMPoST) and H. Roch, N. Götz, H. Elfner (SMASH)
Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field $u^\mu$

Conservation of Energy and Momentum: $\partial_\mu T^{\mu \nu} = 0$

Conservation of charge: $\partial_\mu N_q^\mu = 0$
Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field $u^\mu$

Conservation of Energy and Momentum:
\[
\partial_\mu T^{\mu\nu} = 0
\]

\[
T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}
\]

$q$-th conserved charge (eg. B,Q,S)

Conservation of charge:
\[
\partial_\mu N^\mu_q = 0
\]

\[
N_q^\mu = \sum_i q_i N_i^\mu = \tau_q u^\mu + V_q^\mu
\]
Fluid dynamics of multi-component systems

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q-th conserved charge (eg. B,Q,S)

Conservation of charge:

$$\partial_\mu N_q^{\mu} = 0$$

$$N_q^{\mu} = \sum_i q_i N_i^{\mu} = \tau_q u^\mu + V_q^{\mu}$$

2nd-order (multi-component) hydro

$\Pi, \dot{V}_q^{\langle \mu \rangle}, \dot{\pi}^{\langle \mu\nu \rangle}$

Relaxation equations (Israel-Stewart-type causal theory)

$$\tau_\Pi \Pi + \Pi = S_\Pi$$

$$\sum_{q'} \tau_{qq'} \dot{V}_q^{\langle \mu \rangle} + V_q^{\mu} = S_q^{\mu}$$

$$\tau_{\pi} \dot{\pi}^{\langle \mu\nu \rangle} + \pi^{\mu\nu} = S_{\pi}^{\mu\nu}$$
Equations of motion with multiple conserved charges

2\textsuperscript{nd}-order (multi-component) hydro

\[ \dot{\Pi}, \dot{V}_q^{\langle \mu \rangle}, \dot{\pi}^{\langle \mu \nu \rangle} \]

\[ \tau_{\Pi} \dddot{\Pi} + \Pi = S_{\Pi} \]

\[ \sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle \mu \rangle} + V_q^{\mu} = S_q^{\mu} \]

\[ \tau_{\pi} \dddot{\pi}^{\langle \mu \nu \rangle} + \pi^{\mu \nu} = S_{\pi}^{\mu \nu} \]

Relaxation equations
(Israel-Stewart-type causal theory)
Equations of motion with multiple conserved charges

2nd-order (multi-component) hydro
\[ \dot{\Pi}, \dot{\mathcal{V}}^{(\mu)}, \dot{\pi}^{(\mu\nu)} \]

Relaxation equations (Israel-Stewart-type causal theory)
\[
\tau_{II} \ddot{\Pi} + \Pi = S_{II} \\
\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{(\mu)} + V_{q}^{\mu} = S_{q}^{\mu} \\
\tau_{\pi} \dot{\pi}^{(\mu\nu)} + \pi^{\mu\nu} = S_{\pi}^{\mu\nu}
\]

Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1st order term
2nd order terms: couples all currents to each other; depend on all gradients!
Explicit expressions for transport coefficients!

Fotakis, Molnár, Niemi, Greiner, Rischke
PRD 106, 036009 (2022)
Equations of motion with multiple conserved charges

\[ \tau_{I} \dot{I} + I = S_{I} \]
\[ \sum_{q'} \tau_{qq'} \dot{V}_{q'}^{(\mu)} + V_{q}^{\mu} = S_{q}^{\mu} \]
\[ \tau_{\pi} \dot{\pi}^{(\mu \nu)} + \pi^{\mu \nu} = S_{\pi}^{\mu \nu} \]

2\textsuperscript{nd}-order (multi-component) hydro
\[ \dot{I}, \dot{V}_{q}^{(\mu)}, \dot{\pi}^{(\mu \nu)} \]

Relaxation equations
(Israel-Stewart-type causal theory)

\[ S_{q}^{\mu} = \sum_{q'} \kappa_{qq'} \nabla^{\mu} \alpha_{q'} - \sum_{q'} \tau_{qq'} V_{q',\nu} \omega^{\nu \mu} - \sum_{q'} \delta_{VV}^{(q,q')} V_{q'} \theta - \sum_{q'} \lambda_{VV}^{(q,q')} V_{q',\nu} \sigma^{\mu \nu} \]

\[ - \ell_{\Pi}^{(q)} \nabla^{\mu} \Pi + \ell_{\Pi}^{(q)} \Delta^{\mu \nu} \nabla^{\lambda} \rho^{\lambda \nu} + \tau_{\Pi \Pi}^{(q)} \Pi \dot{u}^{\mu} - \tau_{\Pi \nu}^{(q)} \pi^{\mu \nu} \dot{u}_{\nu} + \sum_{q'} \lambda_{VV}^{(q,q')} \Pi \nabla^{\mu} \alpha_{q'} - \sum_{q'} \lambda_{V\pi}^{(q,q')} \pi^{\mu \nu} \nabla_{\nu} \alpha_{q'} \]

Diffusion coefficients: extensively studied!

Fotakis, Molnár, Niemi, Greiner, Rischke
PRD 106, 036009 (2022)

Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1\textsuperscript{st} order term
2\textsuperscript{nd} order terms: couples all currents to each other; depend on all gradients!
Explicit expressions for transport coefficients!

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)
Computation of transport coefficients
(Example: diffusion coefficients)

\[ k_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} (C^{-1})_{ji,0n} q_j \left( q_i' J_{n+1,1}^{(i)} - \frac{n q'}{\epsilon + P_0} J_{n+2,1}^{(i)} \right) \]

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al., PRD 104, 034014 (2021)
Hammelmann et al., arXiv:2307.15606 (2023)

Insights in chemical composition of nuclear matter?
Upcoming publication!
(Fotakis, Lohr, Greiner)

Upcoming project with T. Dore and S. Schlichting
Single-component vs. Multi-component system

What is with the second-order terms?

\[ S^\mu_q = (...) + \ell^{(q)}_{\nu \pi} \Delta^{\mu \nu} \nabla \pi^\lambda_{\nu} + (...). \]
Single-component vs. Multi-component system

What is with the second-order terms?

\[ S^\mu_q = (...) + \ell^{(q)}_\nu \nabla^\mu \nabla^\lambda \pi^\lambda_\nu + (...) \]

**Ultrarelativistic**, classical system with hard-sphere interactions:

Used in simulations of heavy-ion collisions!

\[
\tau_n \dot{V}_q^{(\mu)} + V_q^{\mu} = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q, \nu} \omega^{\nu \mu} - \tau_n V_q^{\mu} \theta - \frac{3\tau_n}{5} V_{q, \nu} \sigma^{\mu \nu} + \frac{\tau_n}{20T} \Delta^{\mu \nu} \nabla^\lambda \pi^\lambda_\nu - \frac{\tau_n}{20T} \pi^{\mu \nu} \dot{\nu} - \frac{\tau_n}{20T} \pi^{\mu \nu} \nabla^\nu \alpha_q
\]
Single-component vs. Multi-component system

What is with the second-order terms?

\[ S_{q}^{\mu} = (...) + \ell_{\pi}^{(q)} \Delta^{\mu\nu} \nabla_{\lambda} \pi_{\nu}^{\lambda} + (...) \]

**Ultrarelativistic**, classical system with hard-sphere interactions:

Denicol et al., PRD 85, 114047 (2012)

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>( \tau_n[\lambda_{\text{mip}}] )</th>
<th>( \delta_{mn}[\tau_n] )</th>
<th>( \lambda_{nn}[\tau_n] )</th>
<th>( \lambda_{nx}[\tau_n] )</th>
<th>( \ell_{mz}[\tau_n] )</th>
<th>( \tau_{nx}[\tau_n] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/(16( \sigma ))</td>
<td>9/4</td>
<td>1</td>
<td>3/5</td>
<td>( \beta_0/20 )</td>
<td>( \beta_0/20 )</td>
<td>( \beta_0/80 )</td>
</tr>
</tbody>
</table>

Used in simulations of heavy-ion collisions!

Second-order transport coefficients not consistent with assumed system

\[ \tau_n \dot{V}_{q}^{(\mu)} + V^{\mu}_{q} = \sum_{q'} \kappa_{qq'} \nabla^{\mu} \alpha_{q'} - V_{q,\nu} \omega^{\nu \mu} - \tau_n V_{q}^{\mu} \theta - \frac{3\tau_n}{5} V_{q,\nu} \sigma^{\mu \nu} + \frac{\tau_n}{20T} \Delta^{\mu \nu} \nabla \lambda \pi_{\nu}^{\lambda} - \frac{\tau_n}{20T} \pi^{\mu \nu} \dot{u}_{\nu} - \frac{\tau_n}{20T} \pi^{\mu \nu} \nabla_{\nu} \alpha_{q} \]

Consistency is important in charge transport!

Use multi-component expressions.

→ generation of unphysical charge currents
Single-component vs. Multi-component system

What is with the second-order terms?

\[ S^\mu_q = (...) + \ell^{(q)}_V \Delta^{\mu\nu} \nabla \pi^\lambda \nu + (...) \]

**Ultrarelativistic**, classical system with hard-sphere interactions:

\[ \ell^{(q')}_V = \frac{9}{80 \sigma_{\text{tot}}} P c_{q'} \text{ single} \rightarrow \frac{\beta}{20} \tau_q \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\kappa)</td>
<td>(\frac{3}{16}\sigma)</td>
</tr>
<tr>
<td>(\tau_n)</td>
<td>(\frac{9}{4})</td>
</tr>
</tbody>
</table>

Used in simulations of heavy-ion collisions!

\[ \tau_n \dot{V}_q^{(\mu)} + V_q^{\mu} = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q',\nu} \omega^{\nu\mu} - \tau_n V_q^{\mu} \theta - \frac{3 \tau_n}{5} V_{q',\nu} \sigma^{\mu\nu} + \frac{\tau_n}{20T} \Delta^{\mu\nu} \nabla \pi^\lambda \nu - \frac{\tau_n}{20T} \pi^{\mu\nu} \hat{u}_\nu \]

Second-order transport coefficients not consistent with assumed system

\[ \rightarrow \text{generation of unphysical charge currents} \]

Consistency is important in charge transport! Use multi-component expressions.
The journey to a software package treating coupled-charged transport

- (3+1)D-hydro simulation
- multiple-conserved charges
- implemented multi-component theory
- user-defined EoS, transport coefficients, theories and and geometries
- maybe adaptive mesh in future

See recent talk by Jakob Lohr
The journey to a software package treating coupled-charged transport

HYDRA
- (3+1)D-hydro simulation
- multiple-conserved charges
- implemented multi-component theory
- user-defined EoS, transport coefficients, theories and geometries
- maybe adaptive mesh in future

See recent talk by Jakob Lohr

TraCoLinR
(TRAnsport COefficients from LINear Response theory)
- Calculates all first- and second-order transport coefficients from the linearized Boltzmann equation
- Multi-component systems with isotropic (in-)elastic, s-dependent cross sections
- thermal masses and cross sections possible (e.g. DQPM)
- quantum corrections are planned

Upcoming publication!
(Fotakis, Lohr, Wagner, Potesnov et al.)
Preliminary results from TraCoLinR

Pion-Kaon gas with resonant cross sections ($\rho$, $K^*$) with hard-sphere “background”

\[ S_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} + \ldots \]

\[ \sum_{q'} \tau_{qq'} \dot{V}_{q'}^\mu + \ldots = S_q^\mu \]
**Preliminary results from TraCoLinR**

**Pion-Kaon gas with resonant cross sections** (rho, K*) with hard-sphere “background”

\[
S_q^\mu = \ldots - \sum_{q'} \delta^{(q,q')}_{VV} V^{\mu \theta} + \ldots
\]

\[
S_q^\mu = \ldots - \sum_{q'} \lambda^{(q,q')}_{VV} V^{q',\nu \sigma_{\mu \nu}} + \ldots
\]
Conclusion

• Derived 2\textsuperscript{nd}-order relativistic fluid dynamic theory for \textit{multi-component systems} from the Boltzmann equation

• \textbf{Transport coefficients given explicitly} containing all information about particle interactions

• Mixed chemistry correlates particle flow \rightarrow \textit{coupled charge-transport}

• \textbf{Consistency} of equation of state, 1\textsuperscript{st}- and 2\textsuperscript{nd}-order transport coefficients \textbf{is important}!

• Implemented derived fluid dynamic theory in \textbf{(3+1)D-hydro code HYDRA}

• First preliminary calculations of \textbf{second-order coefficients} of massive mixture with more realistic cross sections \textit{with TraCoLinR}

• \textbf{We plan to publish both codes}
Backup
Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field \( u^\mu \)

Conservation of Energy and Momentum: \( \partial_\mu T^{\mu\nu} = 0 \)

\[
T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}
\]

10 + 4N_{\text{ch}} degrees of freedom, \( 4 + N_{\text{ch}} \) equations \( \rightarrow \) 6 + 3N_{\text{ch}} unknowns

Conservation of charge: \( \partial_\mu N_q^{\mu} = 0 \)

\[
N_q^{\mu} = \sum_i q_i N_i^{\mu} = n_q u^\mu + V_q^{\mu}
\]

What needs to be known:

- Equation of state \( P_0 = P_0(\epsilon, n_q), \quad T = T(\epsilon, n_q), \quad \alpha_q = \mu_q/T = \alpha_q(\epsilon, n_q) \)
- Equations of motion for dissipative fields \& transport coefficients \( \Pi, V_q^{\mu}, \pi^{\mu\nu} \)
- Initial state
- Final state: freeze-out and \( \delta f \)-correction
Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of **DNMR theory**: derivation from the Boltzmann equation with method of moments

Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009


Irreducible off-equilibrium moments obey Boltzmann eq.:

\[ k_i^\mu \partial_{\mu} f_{i,k} = C_i[f_i] \]

**Problem:** infinitely many coupled PDEs.

**Aim:** Truncate in a well-defined manner (“perturbation theory”)
Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of **DNMR theory**: derivation from the Boltzmann equation with method of moments

Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009


relativistic Boltzmann eq.

\[ \eta^\mu_{i} \partial_{\mu} f_{i,k} = C_{i}[f_{i}] \]

\[ 2^{nd\text{-order}} \text{(multi-component) hydro} \]

\[ \tilde{I}, \tilde{V}_{q}^{\langle \mu \rangle}, \tilde{\pi}^{\langle \mu \nu \rangle} \]

**Aim:** Truncate in a well-defined manner (“perturbation theory”)

“Order-of-magnitude approximation”:
relate off-equilibrium moments to the dissipative fields

\[ \rho_{i,n}^{\mu \nu} = \eta_{i,n} \pi_{\mu \nu} + \mathcal{O}(2) \]

**Counting scheme:**

Gradients in velocity, temperature etc. \( \sigma^{\mu \nu} \sim \mathcal{O}(1), \mathcal{O}(Kn) \)

Dissipative fields \( \pi^{\mu \nu} \sim \mathcal{O}(1), \mathcal{O}(Rn^{-1}) \)
Equation of state with multiple conserved charges

\[ P_0(T) \rightarrow P_0(T, \mu_B, \mu_Q, \mu_S) \]

Noronha-Hostler et al., PRC 100, 064910 (2019)

Monnai et al., PRC 100, 024907 (2019)
Computation of transport coefficients
(Example: diffusion coefficients)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments
Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009

\[
\begin{align*}
C_{i,n-1}^{(\mu)} &= \int \frac{d^3k_i}{(2\pi)^3} \frac{E_{i,k}^{n-1} k_i^{(\mu)}}{C_i[f_i]} \\
&= - \sum_{m=0}^{\infty} \sum_{j} C_{i,j,mm}^{(1)} \rho_{j,m}^{\mu} + \text{non-linear terms}
\end{align*}
\]

Entries of „collision matrix“ (for diffusive moments)

\[
K_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left( C^{(1)} \right)_{ij,0n}^{-1} q_i \left( q_j' J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1} \right)
\]

Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al., PRD 104, 034014 (2021)
Core features:

- (3+1)D-hydro – optimized reduction to 2D and 1D
- (v)SHASTA solver
- Shear-stress and multiple conserved charges (2 charges)
- Ultrarelativistic, tabled and/or any user-defined equations of state
- DNMR theory, this theory, and/or any user-defined theory
- any (tabled, user-defined) transport coefficients
- Curve-linear geometry (so far Cartesian and Hyperbolic coordinates)
- state of the art unit and physical tests
- available in the CRC-TR211 collaboration soon (hopefully)
Yet another hydro code - „Hydra“

Core features:

- (3+1)D-hydro – optimized reduction to 2D and 1D
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Equation of State - details

- Hadronic system including lightest 19 species
  \[ \pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^\pm, \bar{\Sigma}^\pm \]

- Assume classical statistics and non-interacting limit

\[
P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{d^3p}{(2\pi)^3 E_{i,p}} \left( \frac{E_{i,p}^2 - m_i^2}{2} \right) g_i \exp\left(-\frac{E_{i,p}}{T} + \sum_q q_i \alpha_q \right)
\]

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density and net charge densities

\[
T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})
\]
Diffusion coefficient matrix - details

\[
\begin{pmatrix}
V_B^\mu \\
V_S^\mu
\end{pmatrix}
\sim
\begin{pmatrix}
\kappa_{BB} & \kappa_{BS} \\
\kappa_{SB} & \kappa_{SS}
\end{pmatrix}
\begin{pmatrix}
\nabla^\mu \alpha_B \\
\nabla^\mu \alpha_S
\end{pmatrix}
\]

- Matrix is symmetric

\[\kappa_{SB}\text{ is negative and has similar magnitude as }\kappa_{BB}\]

\[\Rightarrow \text{ significant coupling?}\]

- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD

- Tabulate coefficient matrix over \(T, \mu_B, \mu_S\)

\[\mu_Q = 0\]

Initial conditions - details

- $\tau_0 = 2 \text{ fm}/c$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density
Coupled charge-transport

Simplistic case study: no viscosity, diffusion only, no 2\textsuperscript{nd}-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation

\[ \Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{(\mu)} + V_q^\mu = \sum_{q'} K_{qq'} \nabla^\mu \left( \frac{\mu_{q'}}{T} \right) \]
Coupled charge-transport

Greif, Fotakis et al., PRL 120, 242301 (2018)
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\[ \Pi = 0, \quad \pi^{\mu\nu} = 0, \quad \tau_q \dot{V}^{(\mu)}_q + V^\mu_q = \sum_{q'} K_{qq'} \nabla^\mu \left( \frac{\mu_{q'}}{T} \right) \]
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Hydrodynamic (1+1)D-simulation

\[ \Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{(\mu)} + V_q^{\mu} = \sum_{q'} \kappa_{qq'} \nabla^\mu \left( \frac{\mu_{qq'}}{T} \right) \]
Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS

\[ \mu_S \equiv \mu_S(\epsilon, n_B, n_S) \]

\[ \nabla^\mu \alpha_S \sim \nabla^\mu n_B \]

Generation of domains of non-vanishing local net charge (here net strangeness)!

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)