Transport coefficients of the dense QGP along the chiral PT

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QGP in equilibrium: DQPM and PNJL

Transport coefficients at finite $T$ and $\mu_B$
1.) crossover (DQPM model)
2.) CEP and 1st order phase transition (PNJL model)
Motivation: QGP at finite baryon density

- Explore the QCD phase diagram at finite temperature and chemical potential through heavy-ion collisions
- Available information:
  - Experimental data at SPS, BES at RHIC
  - Lattice QCD calculations (only for vanishing $\mu_B$)

How to learn about degrees-of-freedom of QGP?

HIC simulations – transport models

Problem: Transport models need an input for the partonic phase: cross-sections, masses, ...

Solution: effective models

QGP in equilibrium: DQPM and PNJL
Properties of QGP: transport coefficients

Hydrodynamics

\[ T^{\mu\nu} = -P g^{\mu\nu} + w u^\mu u^\nu + \Delta T^{\mu\nu} \]

\[ J_B^\mu = n_B u^\mu + \Delta J_B^\mu \]

\[ \partial_\mu J_B^\mu = 0 \]

\[ \partial_\mu T^{\mu\nu} = 0 \]

Shear viscosity
Resistance to deformation

\[ \eta \nabla^{\langle \mu} u^{\nu \rangle} \]

Bulk viscosity
Resistance to expansion

\[ -\zeta \nabla u \]

Baryon/electric charge diffusion coefficients

\[ \kappa_B \nabla^\mu \frac{\mu_B}{T} \]

\[ \Delta T^{\mu\nu} = \eta \left( D^\mu u^\nu + D^\nu u^\mu + \frac{2}{3} \Delta^{\mu\nu} \partial_\rho u_\rho \right) - \zeta \Delta^{\mu\nu} \partial_\rho u_\rho \]

\[ \Delta J_B^\mu = \kappa_B D^\mu \left( \frac{\mu_B}{T} \right) \]

\[ D^\mu = \Delta^{\alpha\nu} \partial_\nu \]

\[ \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \]
Transport coefficients of QGP

Hydrodynamical model (macroscopic description)

\[ \Delta T^{\mu\nu} = \eta \left( D^{\mu} u^{\nu} + D^{\nu} u^{\mu} + \frac{2}{3} \Delta^{\mu\nu} \partial_\rho u^\rho \right) - \zeta \Delta^{\mu\nu} \partial_\rho u^\rho \]

input for hydro simulations

Shear viscosity to entropy density ratio is extremely small
QGP is the most ideal liquid!

Model predictions:

\[ \frac{\eta}{s} \]

Different models using the same EoS can have completely different transport coefficients!
Transport coefficients: approaches

- **Kubo formalism:** transport coefficients are expressed through correlation functions of stress-energy tensor

\[ \eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \left\langle \left[ S^{ij}(t, \mathbf{x}), S^{ij}(0, 0) \right] \right\rangle \theta(t) \]
\[ S^{ij} = T^{ij} - \delta^{ij} \mathcal{P} \]
\[ \zeta = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \left\langle \left[ \mathcal{P}(t, \mathbf{x}), \mathcal{P}(0, 0) \right] \right\rangle \theta(t) \]
\[ \mathcal{P} = -\frac{1}{3} T^i_i \]

R. Lang and W. Weise, EPJ. A 50, 63 (2014) (NJL model)
A. Harutyunyan et al, PRD 95, 114021, (2017)

**Kinetic theory:**

- **Relaxation time approximation (RTA):** consider relaxation time

\[ \frac{df_a^{eq}}{dt} = C_a = -\frac{f_a^{eq} \phi_a}{\tau_a} \]
\[ \tau_i(p, T, \mu_B) = \frac{1}{\Gamma_i(p, T, \mu_B)} \]

P. Chakraborty and J. I. Kapusta, PRC 83, 014906 (2011)

- **Chapman-Enskog:** expand the distribution in terms of the Knudsen number

J. A. Fotakis et al, PRD 101 (2020) 7, 076007 (HRG)

And more!

**Holographic models:** AdS/CFT correspondence

Relaxation Time Approximation

- Boltzmann equation
  \[ \frac{df_a^{eq}}{dt} = C_a = \frac{f_a^{eq} \phi_a}{\tau_a} \]
  \[ f_a = f_a^{eq} (1 + \phi_a) \]
  RTA: system equilibrates within the relax time \( \tau \),
  Express collisional Integral via \( \tau \) and \( f_a \)

- Relaxation times:
  \[ \frac{1 + d_a f_a^{eq}}{\tau_a(E_a^*)} = \sum_{bcd} \frac{1}{1 + \delta_{ab}} \int d\Gamma_b^* d\Gamma_c^* d\Gamma_d^* W(a,b|c,d) f_b^{eq} (1 + d_c f_c^{eq}) (1 + d_d f_d^{eq}) + (cd), (bc) \]

\[ T^{\mu\nu} = -P g^{\mu\nu} + w u^\mu u^\nu + \Delta T^{\mu\nu} \]
\[ J_B^\mu = n_B u^\mu + \Delta J_B^\mu \]

Energy-momentum tensor and baryon diffusion current can be expressed using \( f_a \):

\[ T^{\mu\nu}(f_a, m_{q,g}), J_B^\mu(f_a, m_{q,g}) \]

\[ \Delta T^{\mu\nu} = \eta \left( D^{\mu} u^\nu + D^{\nu} u^\mu + \frac{2}{3} \Delta^{\mu\nu} \partial_\rho u^\rho \right) - \zeta \Delta^{\mu\nu} \partial_\rho u^\rho \]
\[ \Delta J_B^\mu = \lambda \left( \frac{n_B T}{w} \right)^2 D^\mu \left( \frac{\mu_B}{T} \right) \]

Obtain the transport coefficients using conservation laws, and \( f_a \):

\[ \begin{cases} 
  \partial_\mu J_B^\mu = 0 \\
  \partial_\mu T^{\mu\nu} = 0 
\end{cases} \]

\[ \eta_{RTA}(T, \mu_B) = \frac{1}{15 T} \sum_{i=q, \bar{q}, g} \int \frac{d^3 p}{(2\pi)^3} \frac{p^4}{E_i^2} \tau_i(p, T, \mu_B) d_i(1 \pm f_i) f_i \]

Relaxation time and scattering rate

\[ \delta f_i = f_i^{(0)} \phi_i \text{(while } \phi_j = \phi_c = \phi_d = 0) \].

\[
\frac{\partial f_i}{\partial t} + v_i \cdot \nabla f_i = \sum_{jcd} \frac{1}{1 + \delta_{cd}} \int \frac{d^3p_j d^3p_c d^3p_d}{(2\pi)^9} W(i, j|c, d)(f_c f_d - f_i f_j)
\]

\[
= \sum_{jcd} \frac{1}{1 + \delta_{cd}} \int \frac{d^3p_j d^3p_c d^3p_d}{(2\pi)^9} W(i, j|c, d)(-\delta f_i f_j^{(0)}) = -\Gamma_i(p, T, \mu) \delta f_i,
\]

1) \( \tau_i(p, T, \mu_B) = \frac{1}{\Gamma_i(p, T, \mu_B)} \)

2) \( \tau_i(T, \mu_B) = \frac{1}{2\gamma_i(T, \mu_B)} \),

\[ \Gamma_i^{on}(p_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q, \bar{q}, g} \int \frac{d^3p_j}{(2\pi)^3 2E_j} d_j f_j(E_j, T, \mu_q) \]

\[
\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4)
\]

\[ |\tilde{\mathcal{M}}|^2(p_i, p_j, p_3, p_4) (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4) \]
Dynamical QuasiParticle Model (DQPM)

DQPM: consider the effects of the nonperturbative nature of the strongly interacting quark-gluon plasma (sQGP) constituents (vs. pQCD models)

The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:

\[
\rho_j(\omega, p) = \frac{\gamma_j}{E_j} \left( \frac{1}{(\omega - \bar{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \bar{E}_j)^2 + \gamma_j^2} \right)
\]

\[
\equiv \frac{4\omega\gamma_j}{(\omega^2 - p^2 - M_j^2)^2 + 4\gamma_j^2\omega^2}
\]

Dynamical QuasiParticle Model (DQPM)

- Resummed properties of the quasiparticles are specified by scalar complex self-energies:
  
<table>
<thead>
<tr>
<th>Gluon Propagator: $\Delta^{-1} = P^2 - \Pi$ &amp; Quark Propagator: $S_q^{-1} = P^2 - \Sigma_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluon Self-Energy: $\Pi = M_g^2 - i2\gamma_g\omega$ &amp; Quark Self-Energy: $\Sigma_q = M_q^2 - i2\gamma_q\omega$</td>
</tr>
</tbody>
</table>

- Re $\Pi, \Sigma_q$ : Thermal Mass $(M_g, M_q)$ & Im $\Pi, \Sigma_q$ : Interaction Width $(\gamma_g, \gamma_q)$

- Modeling of the quark/gluon masses and widths (inspired by HTL calculations)

DQPM $g^2$: fixed within s(lQCD) at $\mu_B=0$

- **Input:** entropy density as a $f(T, \mu_B = 0)$
  
  \[ g^2(s/s_{SB}) = d ((s/s_{SB})^e - 1)^f \]

  \[ s^{QCD} = 19/9\pi^2T^3 \]

  \[ s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice} \]

- **Scaling hypothesis at finite $\mu_B \approx 3\mu_q$**
  
  \[ g^2(T/T_c, \mu_B) = g^2 \left( \frac{T^*}{T_c(\mu_B)} \right), \mu_B = 0 \]

  **Output:**
  
  \[ T^* = \sqrt{T^2 + \mu_q^2/\pi^2} \]

- **Input:** lattice EoS $\mu_B = 0$ (dots)
  
  **Output:** DQPM EoS $\mu_B > 0$
Transport coefficients: specific shear viscosity

$$\eta_{RTA}(T, \mu_B) = \frac{1}{15T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \frac{p_i^4}{E_i^2} \tau_i(p, T, \mu_B) d_i(1 \pm f_i)f_i$$

Main contribution comes from light quarks and anti-quarks

Relaxation times

Introduction

Transport coefficients

DQPM

PNJL
Transport coefficients: increasing with $\mu_B$

Specific bulk viscosity

Electric conductivity
Polyakov Nambu Jona-Lasinio (PNJL) model

- **Effective lagrangian with the same symmetries for the quark dof as QCD**

\[
{\mathcal{L}}_{\text{PNJL}} = \sum_i \bar{\psi}_i (iD - m_0 + \mu_i \gamma_0) \psi_i \\
+ G \sum_a \sum_{ijkl} \left[ (\bar{\psi}_i \gamma_5 \tau_{ij}^a \psi_j) (\bar{\psi}_k \gamma_5 \tau_{kl}^a \psi_l) + (\bar{\psi}_i \tau_{ij}^a \psi_j) (\bar{\psi}_k \tau_{kl}^a \psi_l) \right] \\
- K \det_{ij} \left[ \bar{\psi}_i (-\gamma_5) \psi_j \right] - K \det_{ij} \left[ \bar{\psi}_i (+\gamma_5) \psi_j \right] \\
- {\mathcal{U}}(T; \Phi, \bar{\Phi})
\]

Polyakov potential fitted to the YM

5 parameters fixed by vacuum values K,π masses, η-η’ mass splitting, π decay constant, Chiral condensate

- **1st order PT at high \(\mu_B\)**

(sudden change of q and meson masses)

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D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203
Quark masses NJL and PNJL

- Gap equation + minimization of the grand potential → Chiral masses \((M_L, M_S)\)

\[
m_i = m_{0i} - 4G\langle \bar{\psi}_i \psi_i \rangle + 2K\langle \bar{\psi}_j \psi_j \rangle \langle \bar{\psi}_k \psi_k \rangle
\]

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**Chiral masses (NJL)**

- in PNJL transition is steeper than in NJL

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R. Marty et al. PRC 88 (2013) 4 045204

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**Chiral masses (PNJL)**

- \(\mu = 0 \text{ MeV}\)
- \(\mu = 0 \text{ GeV}\)

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**Introduction**  |  **Transport coefficients**  |  **DQPM**  |  **PNJL**
Mesons in PNJL

- The meson pole mass and the width can be obtained by

\[ 1 - 2G_{\text{eff}} \Pi(p_0 = M_{\text{meson}} - i\Gamma_{\text{meson}}/2, p = 0) = 0 \]

At T=0 good agreement with the physical masses
After T>T_{\text{Mott}} mesons become unstable
Polyakov Nambu Jona-Lasinio (PNJL) model: EOS

- PNJL allow for predictions for finite $T$ and $\mu_B$: D. Fuseau, T. Steinernert, J. Aichelin
  PRC 101 (2020) 6 065203

- Parameters fixed, EoS at $\mu_B = 0$:
  HotQCD Phys. Rev. D90 (2014) 094503
  Comparison with lQCD (HotQCD)

- EoS at high $\mu_B$:
  pQCD: A. Kurkela, A. Vuorinen, PRL 117 (2016) 4 042501

- CEP: $(T, \mu_B) = (110,960) \text{ MeV}, \mu_B/T = 8.73$

- 1$^{st}$ order PT at high $\mu_B$ (sudden change of $q$ and meson masses)
Relaxation time: PNJL

\[ 1) \tau_i(p, T, \mu_B) = \frac{1}{\Gamma_i(p, T, \mu_B)} \]

- on-shell scattering (interaction) rates

\[ \Gamma_i^{on}(p_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q,g} \int \frac{d^3p_j}{(2\pi)^32E_j} \int \frac{d^3p_3}{(2\pi)^32E_3} \int \frac{d^3p_4}{(2\pi)^32E_4} \left( 1 \pm f_3 \right) \left( 1 \pm f_4 \right) \]

\[ |M|^2(p_i, p_j, p_3, p_4) = \frac{1}{(2\pi)^4} \delta^{(4)}(p_i + p_j - p_3 - p_4) \]

4 point interaction -> meson exchange

Effective interaction in RPA

\[ \tau = \frac{-ig_{\pi qq}^2}{k^2 - m^2_\pi} \]

\[ \mathcal{D} = \frac{2ig_m}{1 - 2g_m \Pi^\pm_{jj}(k_0, \vec{k})} \]

Introduction   Transport coefficients   DQPM   PNJL
Relaxation time: PNJL

1) \( \tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)} \)

- on-shell scattering (interaction) rates

\[
\Gamma_{i}^{\text{on}}(\mathbf{p}_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3} \frac{\mathcal{D}_j(E_j, T, \mu_q)}{2E_j} \\
\int \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} (1 + f_3)(1 + f_4) \\
|\tilde{M}|^2(p_i, p_j, p_3, p_4) (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4)
\]

Modified distribution function: Polyakov loop contributions

\[
f_q \rightarrow f_q^\Phi(\mathbf{p}, T, \mu) = \frac{(\Phi + 2\Phi e^{-(E_p - \mu)/T})e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T}}{1 + 3(\Phi + 2\Phi e^{-(E_p - \mu)/T})e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T}}
\]

\[
f_{\bar{q}} \rightarrow f_{\bar{q}}^\Phi(\mathbf{p}, T, \mu) = \frac{(\Phi + 2\Phi e^{-(E_p + \mu)/T})e^{-(E_p + \mu)/T} + e^{-3(E_p + \mu)/T}}{1 + 3(\Phi + 2\Phi e^{-(E_p + \mu)/T})e^{-(E_p + \mu)/T} + e^{-3(E_p + \mu)/T}}
\]
Relaxation time: increases with $\mu_B$

\[ \tau_i^{-1}(T, \mu_q) = \frac{1}{n_i(T, \mu_q)} \int \frac{d^3p_i}{(2\pi)^3} d_q f_i^{(0)} \tau_i^{-1}(p_i, T, \mu_q) \]

- on-shell scattering (interaction) rates

\[ \Gamma_{i}^{\text{on}}(p_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q}} \int \frac{d^3p_j}{(2\pi)^3 2E_j} d_j f_j(E_j, T, \mu_q) \]

\[ \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4) |\tilde{M}|^2(p_i, p_j, p_3, p_4) (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4) \]

\[ \tau_i(p, T, \mu_B) = \frac{1}{\Gamma_i(p, T, \mu_B)} \]

\[ N_i=3 \text{ PNJL } \tau_q(\bar{w}_{ji}) \]

$\mu_B = 3\mu_q$

\[ \mu_q = 0 \]

PNJL $N_i=3$: $\tau_q(\bar{w}_{ji})$

NJL $N_i=3$: $\tau_q(\bar{\sigma}_{ji})$
Specific shear viscosity at high $\mu_B$

$$\eta^{\text{RTA}}(T, \mu_B) = \frac{1}{15T} \sum_{i=q,\bar{q},g} \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E_i^2} \tau_i(p, T, \mu_B) d_q f_i^\phi$$

$f_i^\phi = \frac{\phi e^{-(E_i+\mu)/T} + 2\phi e^{-2(E_i+\mu)/T} + e^{-3(E_i+\mu)/T}}{1 + \phi e^{-(E_i+\mu)/T} + \phi e^{-2(E_i+\mu)/T} + e^{-3(E_i+\mu)/T}}$

In agreement with Nf=2 NJL results C. Sasaki et al, NPA 832 (2010)

Electric conductivity at high $\mu_B$

$$\sigma_0^{\text{RTA}}(T, \mu_B) = \frac{e^2}{3T} \sum_{i=q,\bar{q}} q_i^2 \int \frac{d^3p}{(2\pi)^3} \frac{P^2_i}{E_i^2} \tau_i(p, T, \mu_B) d_q f_i^\phi$$

with Polyakov loops

$$(f_i^\phi)^2 = \frac{\phi e^{-(E_i+\mu_i)/T} + 2\phi e^{-2(E_i+\mu_i)/T} + e^{-3(E_i+\mu_i)/T}}{1 + 3\phi e^{-(E_i+\mu_i)/T} + 3\phi e^{-2(E_i+\mu_i)/T} + e^{-3(E_i+\mu_i)/T}}$$
Transport coefficients at finite $T$ and $\mu_B$ have been found using the $(T, \mu_B)$-dependent cross sections in the DQPM and PNJL models.

At $\mu_B = 0$ good agreement with the Bayesian analysis estimations and lQCD estimations of QGP transport coefficients.

At large values of $\mu_B$ (1.2 GeV in this work) presence of the 1st order phase transition changes $T$ dependence of transport coefficients drastically and a discontinuity can be seen approaching the $T_c$.

Outlook:

- More precise EoS large $\mu_B$
- Possible 1st order phase transition at large $\mu_B$ in DQPM, comparison w PNJL model
Transport coefficients at finite $T$ and $\mu_B$ have been found using the $(T, \mu_B)$-dependent cross sections in the DQPM and PNJL models.

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Thank you for your attention!