(not only) Transport effects due to conformal anomaly

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A motivation:

**Fermions and axial anomaly**

**Massless Dirac fermions**

Covariant formulation

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi \]

\[ \not{D} = \gamma^\mu D_\mu \]

\[ D_\mu = \partial_\mu + ieA_\mu \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

\[ \tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \]

**Semimetals (solid state):**

\[ \bar{\psi} \left[ i \gamma^0 \hbar \frac{\partial}{\partial t} + v_F \gamma (i \hbar \nabla - eA) \right] \psi \]

**Currents**

Vector\[ j^V_\mu = \bar{\psi} \gamma^\mu \psi, \]

Axial\[ j^A_\mu = \bar{\psi} \gamma^5 \gamma^\mu \psi \]

**Axial anomaly**

\[ \partial_\mu J^A_\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \]

**Transport:**

\[ j_A = \frac{\mu_V}{2\pi^2} eB, \quad j_V = \frac{\mu_A}{2\pi^2} eB \]

Chiral separation effect  Chiral magnetic effect  AVV diagram

Other anomalies / diagrams / anomalous transport effects?
Massless Dirac fermions

A generic system in particle physics, cosmology, solid state …

Covariant formulation (quantum field theory)

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i\gamma^\mu \partial_\mu \psi \]

Dirac semimetals (solid state):

\[ \bar{\psi} \left[ i\gamma^0 \hbar \frac{\partial}{\partial t} + v_F \gamma^\mu (i\hbar \nabla - eA) \right] \psi \]

Weyl semimetal
(non-degenerated bands)

- TaAs
- NbAs
- NbP
- TaP

Dirac semimetal
(doubly degenerated bands)

- ZrTe$_5$
- Na$_3$Bi,
- Cd$_3$As$_2$

Effective low energy description around band crossings in 3D crystals.
Classical symmetries

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \slashed{D} \psi \]

**Vector**

\[ \psi \rightarrow e^{i \omega V} \psi \]

Vector current is classically conserved

\[ j^\mu_V = \bar{\psi} \gamma^\mu \psi \quad \partial_\mu j^\mu_V = 0 \]

local/gauge symmetry

**Axial**

\[ \psi \rightarrow e^{i \omega 5 \gamma^5} \psi \]

Axial current is classically conserved

\[ j^\mu_A = \bar{\psi} \gamma^5 \gamma^\mu \psi \quad \partial_\mu j^\mu_A = 0 \]

global symmetry (no axial gauge field)

**Conformal**

\[ x \rightarrow \lambda^{-1} x, \quad A_\mu \rightarrow \lambda A_\mu, \quad \psi \rightarrow \lambda^{3/2} \psi \]

Dilatation current is classically conserved

\[ j^\mu_D = T^{\mu\nu} x_\nu \quad \partial_\mu j^\mu_D \equiv T^\mu_\mu \equiv 0 \]

\[ (T^\mu_\mu)_{\text{cl}} \equiv 0 \]

global scale transformations

**Energy-Momentum tensor**

\[ T^{\mu\nu} = -F^{\mu\alpha} F^\alpha_\nu + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \]

\[ + \frac{i}{2} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - \eta^{\mu\nu} \bar{\psi} i \slashed{D} \psi \]
Zoo of anomalies
(three out of six triangular vertices)

Axial

\[ \partial \mu j_A^\mu = \frac{e^2}{16\pi^2} F_{\mu \nu} \tilde{F}^{\mu \nu} \]

\[ \tilde{F}^{\mu \nu} = \frac{1}{2} \epsilon^{\mu \nu \alpha \beta} F_{\alpha \beta} \]

Mixed (axial-gravitational anomaly)

\[ \partial \mu j_A^\mu = -\frac{1}{384\pi^2} R_{\mu \nu \alpha \beta} \tilde{R}^{\mu \nu \alpha \beta} \]

\[ \tilde{R}^{\mu \nu \alpha \beta} = \frac{1}{2} \epsilon^{\mu \nu \gamma \lambda} R_{\gamma \lambda \alpha \beta} \]

Conformal

\[ \partial \mu j_D^\mu = T_\alpha^\alpha \]

\[ \langle T_{\mu \nu} \rangle = \frac{\beta(e)}{2e} F_{\mu \nu} F^{\mu \nu} \]

Currents

Vector \[ j_V^\mu = \bar{\psi} \gamma^\mu \psi \]

Axial \[ j_A^\mu = \bar{\psi} \gamma^5 \gamma^\mu \psi \]

Dilatation \[ j_D^\mu = T^{\mu \nu} x_\nu \]

Full list: AVV, ATT, TVV, TAA, AAA, TTT (not counting torsion!)

Energy-Momentum tensor

\[ T^{\mu \nu} = -F^{\mu \alpha} F_\alpha^\nu + \frac{1}{4} \eta^{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} \]

\[ + \frac{i}{2} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - \eta^{\mu \nu} \bar{\psi} i \slashed{D} \psi \]
Conformal anomaly and the beta function

Massless Dirac fermions

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \slashed{D} \psi \]

are (classically) invariant under the global (scale) transformations:

\[ x \rightarrow \lambda^{-1} x, \quad A_\mu \rightarrow \lambda A_\mu, \quad \psi \rightarrow \lambda^{3/2} \psi \]

The quantum theory generates an intrinsic scale due to a renormalization (in this particular case) of the electric charge:

\[ \beta(e) = \frac{d e(\mu)}{d \ln \mu} \]

In QED (for one Dirac fermion)

\[ \beta_{\text{QED}}^{1\text{-loop}} = \frac{e^3}{12\pi^2} \]

→ conformal/scale symmetry is broken at the quantum level

S. Odaka et al. (VENUS collaboration/KEK), PRL 81, 2428(1998)
Quantum anomaly $\rightarrow$ anomalous transport

**Axial anomaly**

$$\partial_{\mu} j_{A}^{\mu} = \frac{e^{2}}{16\pi^{2}} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

**Transport**

$$j_{A} = \frac{\mu_{V}}{2\pi^{2}} eB, \quad j_{V} = \frac{\mu_{A}}{2\pi^{2}} eB$$

Chiral separation and chiral magnetic effects

**Mixed axial-gravitational anomaly**

$$\partial_{\mu} j_{A}^{\mu} = -\frac{1}{384\pi^{2}} R_{\mu\nu\alpha\beta} \tilde{R}^{\mu\nu\alpha\beta}$$

**Transport**

$$j_{V} = \frac{\mu_{V}\mu_{A}}{\pi^{2}} \Omega, \quad j_{A} = \left( \frac{T^{2}}{6} + \frac{\mu_{V}^{2} + \mu_{A}^{2}}{2\pi^{2}} \right) \Omega$$

Thermal contribution to chiral vortical effects

**Conformal anomaly**

$$\partial_{\mu} j_{D}^{\mu} = T_{\alpha}^{\alpha}$$

$$\langle T_{\mu}^{\mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$$

**Transport?**

not topological ... not one-loop exact ... not interesting?
Conformal anomaly \rightarrow
Scale Electric Effect (SEE) and Scale Magnetic Effect (SME)

Oversimplified picture

Gravitational background: Weyl-transformed flat space

\[ g_{\mu \nu}(x) = e^{2\tau(x)} \eta_{\mu \nu} \]

The conformal anomaly leads to scale electromagnetic effects:

\[ j^\mu \equiv \langle j^\mu \rangle = -\frac{2\beta(e)}{e} F^{\mu \nu} \partial_\nu \tau \]

"Generation of an electric current in a background of electromagnetic and gravitational fields"

The matter is absent. This is a quantum effect in the vacuum.

Fine print: 1) natural in a linear-response theory
2) very unnatural (would-be-wrong) in general relativity
→ to be refined (and made less simple) later

M.Ch., PRL 117, 141601 (2016)
Scale electric effect (SEE)

**Time-dependent background:** \( \tau = \tau(t) \)

**Scale Electric Effect:**

\[
\langle j(t, x) \rangle_{\text{scale}} = \sigma(t) E(t, x) \quad \text{for } \nabla \tau = 0
\]

**Metric:**

\[
g_{\mu\nu}(x) = e^{2\tau(x)} \eta_{\mu\nu}
\]

**Conformal conductivity:**

\[
\sigma(t, x) = -\frac{2\beta(e)}{e} \frac{\partial \tau(t, x)}{\partial t}
\]

**Negative conductivity in an expanding space-time!**

Indepedently obtained in the de Sitter spacetime (a version of the Schwinger effect, both for fermions and bosons)


Scale magnetic effect (SME)

Space-dependent background: \[ \mathcal{T} = \mathcal{T}(\mathcal{X}) \]

Scale Magnetic Effect:
\[ \langle j(t, \mathcal{X}) \rangle_{\text{scale}} = F(\mathcal{X}) \times B(t, \mathcal{X}) \quad \text{for} \quad \partial_t \mathcal{T} = 0 \]

Gravitational deformation vector:
\[ F(t, \mathcal{X}) = \frac{2\beta(e)}{e} \nabla \mathcal{T}(t, \mathcal{X}) \]

Distantly similar to Hall effects (no electric field, though).
Formal derivation of scale electromagnetic effects

Conformal anomaly in curved space-time in electromagnetic field background

\[
\langle T^\mu_\mu \rangle \equiv -\frac{2}{\sqrt{-g}} g^{\mu\nu} \frac{\delta S_{\text{anom}}}{\delta g_{\mu\nu}} \\
= -\frac{1}{4} \left[ bC^2 + b' \left( E - \frac{2}{3} \Box R \right) + cF_{\mu\nu}F^{\mu\nu} \right]
\]


\[ C^2 = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} \]

\[ \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3} \]

Weyl tensor squared

\[ E = *R_{\mu\nu\alpha\beta} *R^{\mu\nu\alpha\beta} \]

\[ \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \]

\[ *R_{\mu\nu\alpha\beta} = \frac{1}{2} \epsilon_{\mu\nu\mu'\nu'} R^{\mu'\nu'}_{\alpha\beta} \]

Euler (topological) density

Couplings

\[ b = \frac{1}{320\pi^2}, \quad b' = -\frac{11}{5670\pi^2}, \quad c = -\frac{e^2}{24\pi^2} \]

comes from beta function
Formal derivation of scale electromagnetic effects

Conformal anomaly in curved space-time in electromagnetic field background

\[
\langle T_{\mu}^{\mu} \rangle \equiv -\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta S_{\text{anom}}}{\delta g_{\mu\nu}} = -\frac{1}{4} \left[ bC^2 + b' \left( E - \frac{2}{3} \Box R \right) + cF_{\mu\nu}F^{\mu\nu} \right]
\]


Anomalous action:

\[
S_{\text{anom}}[g, A] = \frac{1}{8} \int d^4 x \sqrt{-g(x)} \int d^4 y \sqrt{-g(y)} \cdot H(x) \Delta_4^{-1}(x, y) \left[ 2bC^2(y) + b' H(y) + 2cF_{\mu\nu}(y)F^{\mu\nu}(y) \right]
\]


Nonlocality: Green’s function for 4th order conformal differential operator

\[
\Delta_4 = \nabla_\mu \left( \nabla^\mu \nabla^\nu + 2R^{\mu\nu} - \frac{2}{3} R g^{\mu\nu} \right) \nabla_\nu
\]

S. Paneitz, MIT preprint (1983)
Formal derivation of scale electromagnetic effects

Anomalous electric current from anomalous conformal action

$$J^\mu(x) = -\frac{1}{\sqrt{-g(x)}} \frac{\delta S_{\text{anom}}}{\delta A_\mu(x)}$$

variation with respect to the gauge field

$$= -\frac{1}{\sqrt{-g(x)}} \frac{\partial}{\partial x^\nu} \left[ \sqrt{-g(x)} F^{\mu\nu}(x) \right]$$

electromagnetic field strength

$$\cdot \int d^4y \sqrt{-g(y)} \Delta_4^{-1}(x,y) \left( E(y) - \frac{2}{3} \Box R(y) \right)$$

space-time curvature enters here

Green’s function for 4th order Paneitz conformal differential operator

Weak curvature, linear order:

$$J^\mu(x) = + \frac{e^2}{6\pi^2} F^{\mu\nu}(x) \partial_\nu \varphi(x)$$

Minkowski

small perturbation

Effective scale factor:

$$\varphi(x) = \frac{1}{6} \int d^4y \Box_{x,y}^{-1} \left[ \partial_\alpha \partial_\beta h^{\alpha\beta}(y) - \eta_{\alpha\beta} \Box h^{\alpha\beta}(y) \right]$$

for $$g_{\mu\nu}(x) = e^{2\tau(x)} \eta_{\mu\nu}$$

$$J^\mu = \frac{2\beta(e)}{e} F^{\mu\nu} \partial_\nu \tau$$

A. Cortijo, M. Vozmediano, M.Ch., PRL 120, 206601 (2018)
Tolman-Ehrenfest law

Temperature gradient vs. gravitational field?

In a static background gravitational field, the temperature of a system in a thermal equilibrium is not constant (Tolman-Ehrenfest):

\[ T(\mathbf{x}) \sqrt{g_{00}(\mathbf{x})} = T_0 \]

A temperature gradient that drives system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

\[ \frac{1}{T} \nabla T = -\frac{1}{c^2} \nabla \Phi \]

\[ g_{00} = 1 + \frac{2\Phi}{c^2} \]

Thermal equilibrium

— Consider a closed system divided arbitrarily into two subsystems

— Thermal equilibrium happens when the total entropy reaches its maximum

\[ S = S_1 + S_2 \]
\[ dS_1 + dS_2 = 0 \]

—in the absence of gravitational field, the temperature is constant

— Assume that we have no gravitational field

— If the quantity of heat leaves the first subsystem, it always enters the second subsystem:

\[ dE_1 = -dE \quad \Rightarrow \quad dE_2 = dE \]
\[ \frac{dS_1}{dE_1} = \frac{dS_2}{dE_2} \quad \Rightarrow \quad T_1 = T_2 \]

definition of temperature \[ T = \frac{dS}{dE} \]

In the absence of gravitational field, the temperature is constant
How to understand the Tolman-Ehrenfest law?

— In a static gravitational field $\Phi$, the heat quantity $dE$ possesses an inertial mass

$$dm = \frac{dE}{c^2}$$

— the equivalence between inertial and gravitational masses: a quantity of heat has a weight

— When heat leaves the first subsystem, $dE_1 = -dE$

it enters the second subsystem, and performs work against the gravity (heat = mass):

$$dE_2 = dE + (\Phi_2 - \Phi_1)dm = dE_2(1 + \Delta\Phi/c^2)$$

— Entropy maximum

$$dS_1 + dS_2 = 0$$

— Local temperature

$$T_2 = T_1(1 + \Delta\Phi/c^2)$$

Tolman-Ehrenfest law

$$T(x) = T_0/\sqrt{g_{00}(x)}$$

The SME in Dirac and Weyl semimetals

PHYSICAL REVIEW LETTERS 120, 206601 (2018)

Generation of a Nernst Current from the Conformal Anomaly in Dirac and Weyl Semimetals

M. N. Chernodub,\(^1\,^2\) Alberto Cortijo,\(^3\) and María A. H. Vozmediano\(^3\)

1.) A temperature gradient that drives system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

\[
\frac{1}{T} \nabla T = -\frac{1}{c^2} \nabla \Phi \\
g_{00} = 1 + \frac{2\Phi}{c^2} \\
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
\]

flat metric + small perturbation

2.) In a magnetic-field background in a curved space-time, the conformal anomaly generates an electric current via the scale magnetic effect.

→ Conformal anomaly generates thermoelectric transport!

**Anomalous electric current:**

\[
J^\mu(x) = + \frac{e^2}{6\pi^2} F^{\mu\nu}(x) \partial_\nu \varphi(x)
\]

with the effective scale factor:

\[
\varphi(x) = \frac{1}{6} \int d^4 y \Box^{-1}_{x,y} \left[ \partial_\alpha \partial_\beta h^{\alpha\beta}(y) - \eta_{\alpha\beta} \Box h^{\alpha\beta}(y) \right]
\]

(it will become a local quantity)
Nernst-Ettingshausen Effect in Weyl semimetals

**Giant Nernst Effect due to SME**

\[ J = \frac{e^2 v_F}{18\pi^2 T\hbar} B \times \nabla T \]

The electric current is proportional to the beta function (conformal anomaly!)

**Longitudinal anomalous transport in Weyl semimetals:**

\[ S_{11} \equiv \frac{E_1}{B_3 \nabla_1 T} = \frac{\rho_{12} L_{21}^{12}}{B_3} \sim \frac{v_F}{9|b|T} \]

**Estimations for an undoped Weyl semimetal** (\( v_F \sim 10^5 \text{ m/s}, \ T \sim 10 \text{ K}, \ |2b| \sim 0.3 \text{ Å}^{-1} \))

\[ S_{11}/T \sim 0.6 \mu\text{V/T K}^{-2} \]

**Accessible experimentally!**

works via the anomalous Hall current

\[ J = \frac{e^2}{2\pi^2 \hbar} b \times E \]

A. Cortijo, M. A. H. Vozmediano, M.Ch., PRL 120, 206601 (2018)
Remarks on the Scale Magnetic Effect (SME)

- Appears due to conformal anomaly
  \[ \langle T^\mu_\mu \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu} \]

- Bulk phenomenon, works at zero chemical potential

- Leads to the Nernst effect in Dirac/Weyl semimetals
  \[ J = \frac{e^2 v_F}{18\pi^2 T\hbar} B \times \nabla T \]

- Is not related to axial or axial-gravitational anomalies

- Strength is given by the beta function
  \[ \beta(e) = \frac{d e(\mu)}{d \ln \mu} \]

- Universal: works both in fermionic and bosonic systems
Purely gravitational part of conformal anomaly

Conformal anomaly in curved space-time in electromagnetic field background

\[ \langle T^\mu_\mu \rangle \equiv -\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta S_{\text{anom}}}{\delta g_{\mu\nu}} = -\frac{1}{4} \left[ bC^2 + b' \left( E - \frac{2}{3} \Box R \right) + c F_{\mu\nu} F^{\mu\nu} \right] \]

Anomalous action:

\[ S_{\text{anom}}[g, A] = \frac{1}{8} \int d^4 x \sqrt{-g(x)} \int d^4 y \sqrt{-g(y)} H(x) \Delta_{A}^{-1}(x, y) \left[ 2bC^2(y) + b'H(y) + 2c F_{\mu\nu}(y) F^{\mu\nu}(y) \right] \]

 gravitational part

 electromagnetic part

 pure gravity

 couplings

 comes from beta function

\[ \beta(e) = \frac{dc(\mu)}{d \ln \mu} \]

TTT vertex

TVV vertex
Anomalous TTT vertex

A temperature gradient that drives a system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

\[
\frac{1}{T} \nabla T = -\frac{1}{c^2} \nabla \Phi \quad g_{00} = 1 + \frac{2\Phi}{c^2} \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
\]

Expectation value of the energy-momentum tensor:

\[
\langle T^\mu_{\nu_1}(x_1) \rangle = \frac{1}{8} \int dx_2 \, dx_3 \langle T^\mu_{\nu_1}(x_1) T^\mu_{\nu_2}(x_2) T^\mu_{\nu_3}(x_3) \rangle \, h_{\mu_2\nu_2}(x_2) \, h_{\mu_3\nu_3}(x_3)
\]

Inspired by Tolman–Ehrenfest effect:

\[
T(x) = T_0 \sqrt{g_{00}(x)}
\]

\[
\Phi(x) \equiv \frac{h_{00}(x)}{2} = -\frac{1}{2} \left( \frac{T^2(x)}{T_0^2} - 1 \right)
\]

Contribution of the anomalous TTT correlator to the energy-momentum tensor.
Anomalous TTT vertex

The expectation value of the energy-momentum tensor

\[
\langle T^{00}\rangle_{TTT} = \frac{4b}{9} \left[ 3(\partial_3^2 \Phi)^2 + 4(\partial_3 \Phi)(\partial_3^3 \Phi) + 2\Phi \partial_3^4 \Phi \right] \quad \text{other components are zero}
\]

\[
\langle T^{11}\rangle_{TTT} = \langle T^{22}\rangle_{TTT} = \frac{4b}{9} \left[ 2(\partial_3 \Phi)(\partial_3^3 \Phi) + \Phi \partial_3^4 \Phi \right].
\]

Pressure asymmetry:

\[
P_{\parallel} = \langle T^{33}\rangle
\]

\[
\delta P = P_{\parallel} - P_{\perp}
\]

Leading contribution:

comes from the gravitational part of the conformal anomaly

\[
\delta P = \frac{16b}{3} \hbar c \left( \frac{\nabla T}{T} \right)^2 \left( \frac{\nabla^2 T}{T} \right) \equiv \frac{\hbar c}{60\pi^2} \left( \frac{\nabla T}{T} \right)^2 \left( \frac{\nabla^2 T}{T} \right)
\]

Relative pressure asymmetry:

\[
\frac{\delta P}{P_{\text{th}}} = \frac{3}{7} \left( \frac{\nabla T}{\pi T^2} \right)^2 \left( \frac{\nabla^2 T}{\pi^2 T^3} \right)
\]

very small effect

\[
\delta P/P_{\text{th}} \sim 10^{-7}
\]

Dirac/Weyl semimetals & quark–gluon plasma
Conformal anomaly and transport effects at the edge

What is about the boundaries? Take massless QED (or any theory ...)

Electric current along the edge due to tangential magnetic field

\[ J^\mu = -\frac{2\beta_e}{e\hbar} F^{\mu\nu} n_\nu \]

spatial distance to the reflective boundary

Relation to the scale anomaly: C.-S. Chu and R.-X. Miao, JHEP 07, 005 (2018), PRL 121, 251602 (2018);

Scale Magnetic Effect at the Edge (SMEE):
Electric current along the edge due to tangential magnetic field

\[ j(x) = - f_n(x) n \times B \]

- Effect due to conformal anomaly
- No topology (Berry, Chern, etc)
- No matter at all (= quantum vacuum)

diverges at the boundary!
Scale Magnetic Effect at the Edge (SMEE)

A physical picture

Ingredients: vacuum, edge and magnetic field

Skipping orbits (like in the Hall effect, but now in the vacuum)

Absent: No Fermi surface, no temperature.
SMEE: numerical check
Generates the current at the boundary?

Scalar electrodynamics at a conformal point:

$$\mathcal{L}_{\text{SQED}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \left[ ( \partial_\mu - ie A_\mu ) \phi \right]^* ( \partial^\mu - ie A^\mu ) \phi$$

Massless one-component electrically-charged scalar field

Numerical Monte-Carlo simulations

We see the generated electric current!
We see the $1/x$ divergence of the current at the boundary.

We see the correct coefficient and recover the beta function!

$$\beta_{sQED}^{1\text{-loop}} = \frac{e^3}{48\pi^2}$$

(Notice that the beta function of the scalar QED is four times smaller than the beta function in the usual QED)
Scale magnetic effect at the edge and superconductivity

Scalar electrodynamics at a conformal point (massless scalar):

$$\mathcal{L}_{\text{S}Q\text{ED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[ \left( \partial_\mu - ie A_\mu \right) \phi \right]^* \left( \partial^\mu - ie A^\mu \right) \phi$$

Charged fields may condense!

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* D^\mu \varphi - V(\varphi)$$

What happens to the SEEE current in the superconducting phase?

V.A. Goy, A.V. Molochkov, M.Ch., in preparation
Scale magnetic effect at the edge and superconductivity

What happens to the SEEE boundary current in superconducting phase?

It becomes the Meissner current!

Superconducting condensate

Vortex density

Total (integrated) electric current

Meissner: total current is large, but it vanishes at the boundary

SMEE: total current is small, but it diverges at the boundary
Scale electric effect at the edge: conformal screening

Screening of electrostatic field in metals:

\[ E(x) \sim E(0) e^{-x/\lambda} \]

Screening lengths:

\[ \lambda_D = \sqrt{\frac{\varepsilon_0 k_B T}{n e^2}}, \quad \lambda_{FT} = \sqrt{\frac{\varepsilon_0 \pi^2 \hbar^3}{m e^2 p_F}} \]

Fermi momentum

\[ p_F = (2\pi^2 n)^{1/3} \hbar \]

Density of carriers \( n \)

What if the medium is totally conformal and possess no dimensionful parameters?

For example, take a Dirac semimetal at particle-hole symmetric point.

– We have the mobile carriers (massless fermionic quasiparticles)

– Classically, there is no dimensionful scale.

– No classical scale → no screening?

No quantity to construct the screening length from!
Scale electric effect at the edge

\[ J^\mu = -\frac{2c\beta e}{\epsilon\hbar} F^{\mu\nu} n_\nu \]

\[ \rho = -\frac{2\beta e}{\epsilon\hbar} nE \]

the density of the electric charge accumulated at the boundary

Physics: the screening is due to the Schwinger effect (skipping orbits in time)

Works efficiently due to the absence of a mass gap

Generated by the conformal anomaly! (proportional to the beta function)

Mechanism in semimetals: creation of electron-hole pairs in the presence of a uniform electric field (the Zener effect)
Scale electric effect at the edge of a semimetal

Consider QED:

\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{a=1}^{N_f} \bar{\psi}_a i \gamma^\mu D_\mu \psi_a \]

Charge density due to conformal anomaly:

\[ \rho = -\frac{2\beta_e}{e\hbar} \frac{nE}{x} \]

Solve the Maxwell equation

\[ \partial_x E_x(x) = \frac{1}{\varepsilon_0} \rho(x) \]

At the boundary the conformal screening is polynomial:

\[ E_x(x) = \frac{C}{x^\nu} \quad \rho(x) = -\frac{C\varepsilon_0 \nu}{x^{1+\nu}} \quad \phi(x) = \phi_0 - \frac{C x^{1-\nu}}{1 - \nu} \]

Electric field  Charge density  Electrostatic potential

Conformal screening exponent:

\[ \nu = \frac{2\beta_e}{e\hbar\varepsilon_0} \]
Scale electric effect at the edge

Conformal exponent in a Dirac semimetal:

\[ \nu = \frac{e^2}{6\pi^2\hbar v_F \varepsilon \varepsilon_0} = \frac{2\beta_e}{e\hbar \varepsilon_0} \]

Particle density in a finite sample with two boundaries:

\[ \rho(x) = \frac{\Delta\phi}{L^2 \varepsilon_0 \nu h(\nu)} \left(1 - \frac{2x}{L}\right) \left[\frac{x}{L} \left(1 - \frac{x}{L}\right)\right]^{-1-\nu} \]

Direct measurement of the beta function. Indirect evidence of the Schwinger effect.

Accessible experimentally

- direct measurement of the beta function associated with the renormalization of the electric charge (never done in solid state)

- evidence of the elusive Schwinger effect (particle-antiparticle production by electric field)

Conformal exponent in a Dirac semimetal:

$$\nu = \frac{e^2}{6\pi^2 \hbar v_F \varepsilon \varepsilon_0}$$

In typical Dirac/Weyl materials \( v_F \sim 10^{-3}c \) and \( \varepsilon \sim 10 \)

→ large, experimentally accessible conformal exponent: \( \nu \sim 10^{-1} \)

Electrostatic screening potential vs. distance from the boundary of a Dirac material (at the Lifshitz point)
Summary

Conformal anomaly leads to a number of new transport effects:
— in the bulk (unbounded systems)
— at reflective boundaries (edges) of bounded systems

Currents are proportional to the beta function.
Accessible experimentally in Dirac and Weyl semimetals.

Scale electric effect:
— negative conductivity in expanding systems
— polynomial screening of electrostatic fields
— particle creation via the Schwinger effect

Scale magnetic effect:
— thermoelectric transport, Nernst effects
— edge currents in the absence of matter