Nuclear correlations, pre-clusters and light nuclei close to the QCD critical point

Juan M. Torres-Rincon
(Goethe University Frankfurt)

based on **arXiv:1910.08119**
also PRC 100 (2019) no.2, 024903
with E. Shuryak (Stony Brook Uni.)

Transport Meeting
Goethe University Frankfurt
Jan. 16, 2020

Outline

- **Motivation:** QCD critical point
- **Main idea:** Critical mode and *NN* interaction
- **Message:** Strongly-correlated systems
- **Results I:** Nuclear correlations close to the critical region
- **Results II:** Pre-clusters and light nuclei ratios
- **Comments:** Quantum effects at finite T for He-4
- **Summary**

QCD phase diagram and critical point

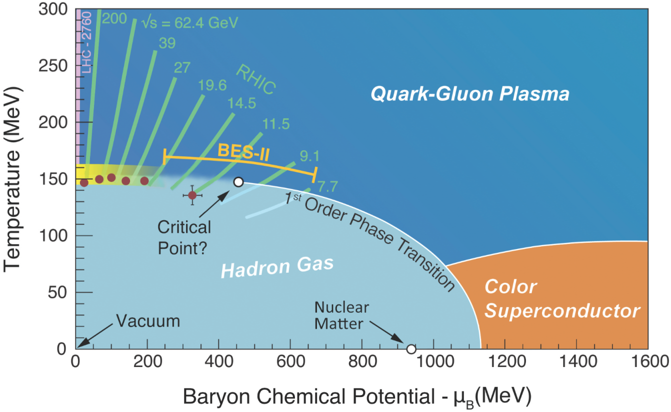
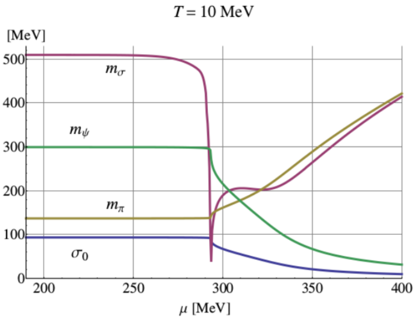


Image: S. Mukherjee (Brookhaven National Lab)

QCD critical mode

σ mass decreases close to the phase transition/critical point (correlation length ξ increases)



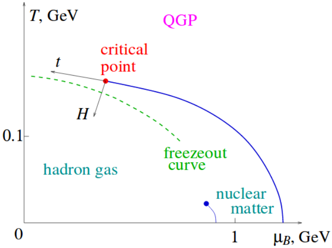
R.-A. Tripolt, Ph.D. Thesis, 2015
(quark-meson model with FRG approach)

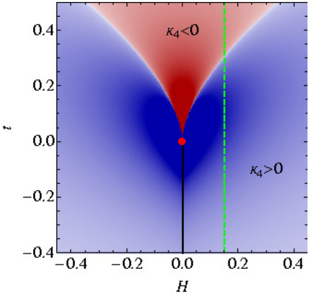
$$m_\sigma \sim \frac{1}{\xi} \sim \left(\frac{|T-T_c|}{T_c}\right)^\nu$$

(with ξ limited by finite lifetime effects)

Moments of the σ probability distribution

M. Stephanov, 2008 and 2011





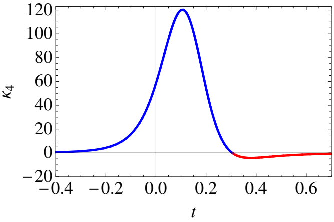
$$P[\sigma] \sim \exp\left(-\Omega/T\right)$$

$$\Omega = \int d^3x \, \frac{(\nabla\sigma)^2}{2} + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4$$

$$\kappa_2 = \langle \sigma_0^2 \rangle$$

$$\kappa_4 = \langle \sigma_0^4 \rangle - 3\langle \sigma_0^2 \rangle^2$$

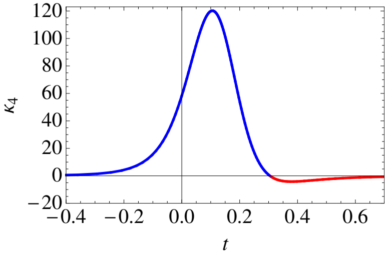
$$Kurtosis = \kappa_4/\kappa_2^2$$

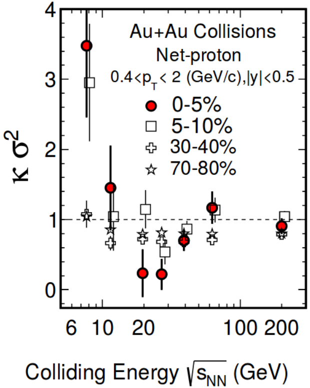


Critical mode couples to baryons

M. Stephanov, 2011

$$\mathcal{L}_{eff} = g\sigma p\bar{p}$$





$$C_2 = \langle \delta N^2_{p-\bar{p}} \rangle$$

$$C_4 = \langle \delta N^4_{p-\bar{p}} \rangle - 3\langle \delta N^2_{p-\bar{p}} \rangle^2$$

$$\kappa\sigma^2 = C_4/C_2$$

STAR Coll., 2015
preliminary

M. Stephanov, 2011

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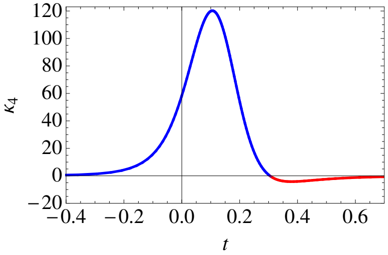

$$C_2 = \langle \delta N^2_{p-\bar{p}} \rangle$$

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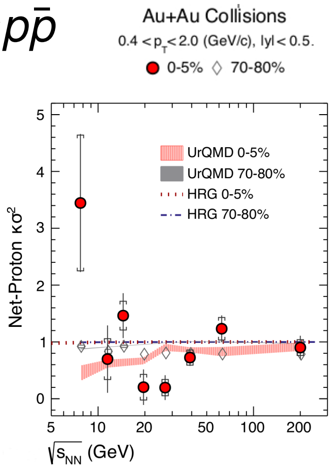
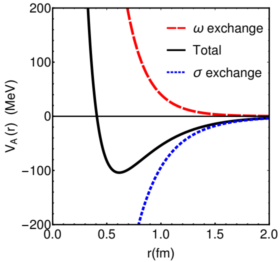
$$\kappa \sigma^2 = C_4 / C_2$$



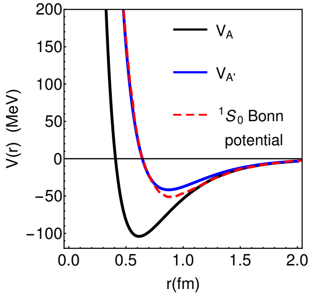
STAR Coll., 2001.02852

Simple-as-possible (but not simpler) model for *NN* interaction due to **Serot-Walecka (1984)**



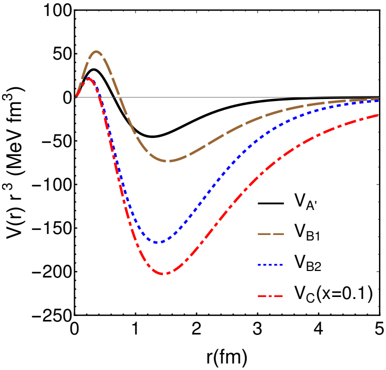
- Large cancellation between attraction and repulsion to produce bound nuclear matter
- Any small imbalance would strongly modify the potential!

$$V_A(r) = -\frac{\alpha_\sigma}{r} e^{-m_\sigma r} + \frac{\alpha_\omega}{r} e^{-m_\omega r}$$



$V_{A'}(r)$ has extra repulsion to match Bonn potential (Machleidt, 2000)

- Close to T_c a very light σ enhances the attraction
- ***NN* potential should be affected by the presence of the QCD critical point!**
- We consider more and more attractive potentials:



- V_A: Serot-Walecka with MF parameters
- $V_{A'}$: extra repulsion $\alpha_\omega \to 1.4\alpha_\omega$
- V_{B1}: $V_{A'}$ with $m_\sigma^2 \to m_\sigma^2/2$, $\alpha_\sigma \to \alpha_\sigma/2$
- V_{B2}: $V_{A'}$ with $m_\sigma^2 \to m_\sigma^2/2$
- V_C: very light critical mode $V_C(x) = (1-x)V_{B2} + xV_{A'}(m_\sigma^2 \to m_\sigma^2/6)$

Numerical study: Molecular Dynamics + Langevin

NN potential in a classical nonrelativistic Molecular Dynamics scheme

$$\begin{cases} \dfrac{d\vec{x}_i}{dt} = \dfrac{\vec{p}_i}{m_N} \\ \dfrac{d\vec{p}_i}{dt} = -\sum_{j\neq i} \dfrac{\partial V(|\vec{x}_i - \vec{x}_j|)}{\partial \vec{x}_i} - \lambda \vec{p}_i + \vec{\xi}_i \end{cases}$$

with Langevin dynamics,

$$\langle \vec{\xi}_i(t) \rangle = 0$$
$$\langle \xi_i^a(t) \xi_j^b(t') \rangle = 2T\lambda m_N \delta^{ab} \delta_{ij} \delta(t-t')$$

where $a, b = 1, 2, 3$ and

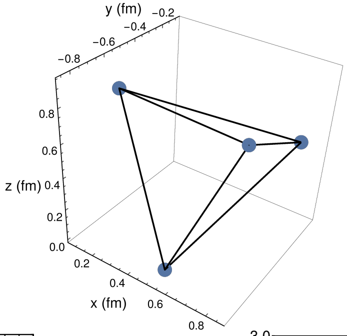
$$\lambda = T/(m_N D_B)$$

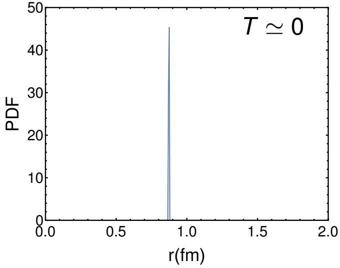
with D_B: baryon diffusion coefficient

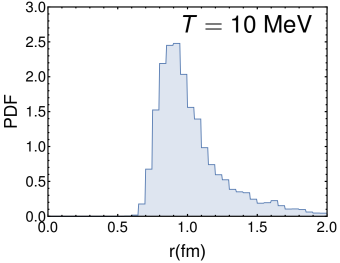
- Quantum effects neglected at high temperature T (see later)

Small clusters, $N = 4$

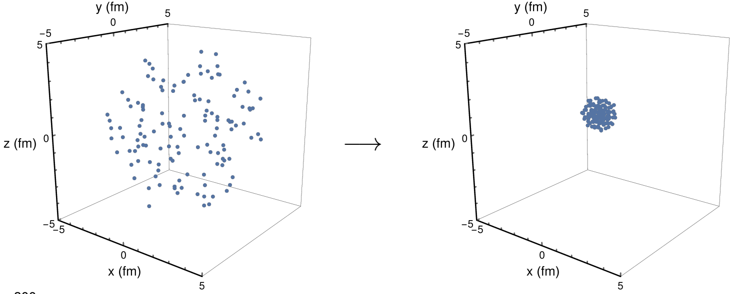
$V_{A'}$ potential
(no modifications yet)

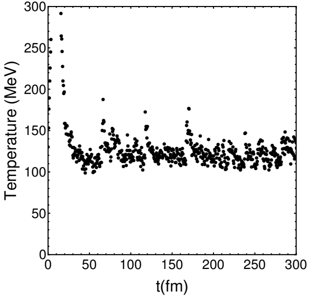






Big clusters, $N = 128$



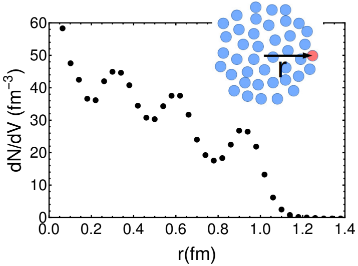


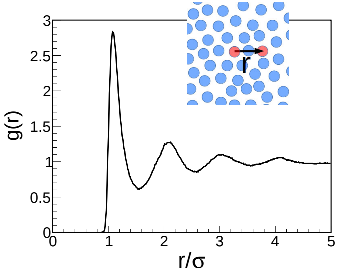
$T = 120$ MeV

At large N the potential energy always wins over entropy: **clustering effect**.

This is an illustrative example: unreachable time scales for HICs!

- Strongly correlated system ($P/K \simeq \mathcal{O}(N) > 1$): beyond mean field





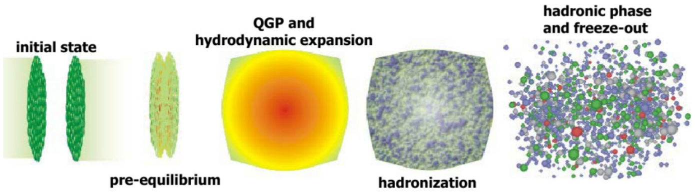
- Infinite systems: internal structure described by **pair correlation function** $g(r)$ e.g. liquid Argon ($N = 108$) via Lennard-Jones potential

- Message: Approaches based on Boltzmann assumptions would NOT capture these effects

Effects preventing clustering

- Expansion, radial collective flow
- Freeze-out temperatures $T \sim 150$ MeV
- Finite time effects (duration of hadronic phase)

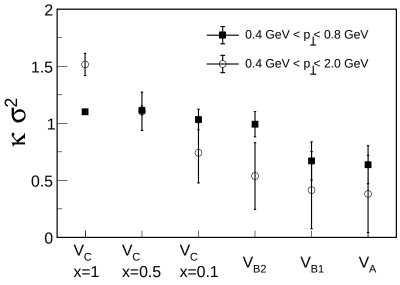
We need to address these for RHIC collisions at the Beam Energy Scan

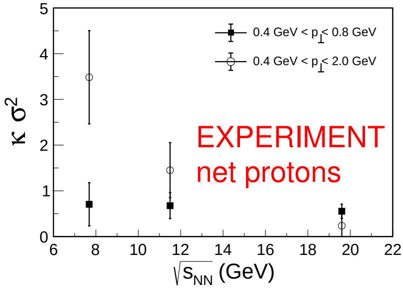


Focus on BES I at $\sqrt{s_{NN}} < 19.6$ GeV, as measured by STAR @ RHIC (STAR Collab. 2016 & 2017)

Few-body correlations should contribute to proton moments

Scaled kurtosis: $\kappa\sigma^2 = C_4/C_2$





Expected increase with enhanced attraction, esp. in the wider $p_\perp$ window.

Pre-clusters to light nuclei: Triton-proton/deuteron ratio

$$\frac{N_t N_p}{N_d^2} = g \qquad (g = 0.29)$$

- We assume that the **statistical thermal model** should give a good description

$$N = Vol \, \frac{(2S+1)}{2\pi^2} m^2 T \, K_2(m/T) \exp\left(\frac{B\mu_B + q\mu_q}{T}\right)$$

- Ratio considered before by Sun, Chen, Ko, Xu (2017) with a similar motivation (critical point) but a different perspective (coalescence)

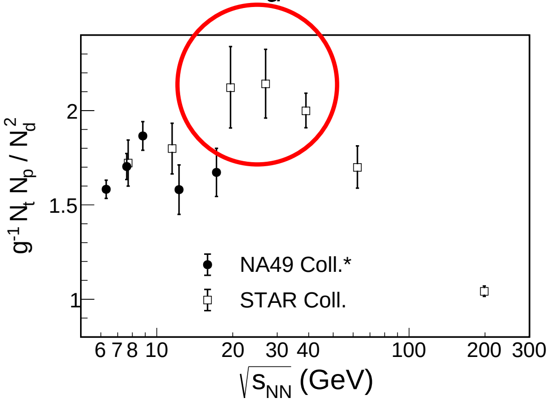
Triton-proton/deuteron ratio

$$g^{-1}\frac{N_t N_p}{N_d^2} \sim \left\langle e^{-\frac{V(x)}{T}} \right\rangle \qquad (g = 0.29)$$

$V(x)$ is the NN potential that binds nuclei,
V/T non negligible close to T_c

Important: The measured multiplicities also populated by feed down additions (especially proton). Important for statistical thermal fits.

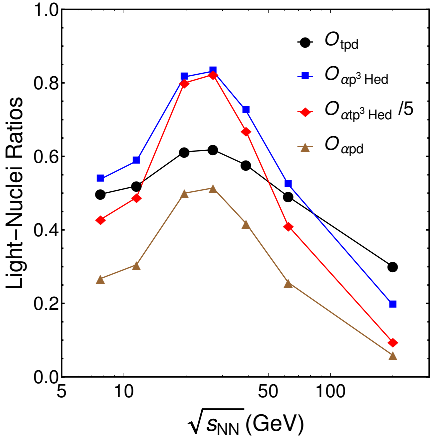
$$g^{-1}\frac{N_t N_p}{N_d^2} \sim \left\langle e^{-\frac{V(x)}{T}} \right\rangle \qquad (g = 0.29)$$



*Sun, Chen, Ko, Xu 2017, based on NA49 exp. data

STAR Collaboration, preliminary 0%-10% (QM2018, arXiv:1909.07028)

If clustering effects at T_c are the main source of the maximum...
explore ^{4}He at the same energies!

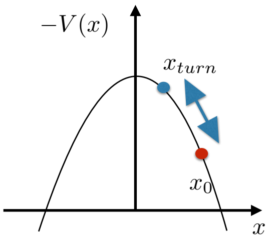


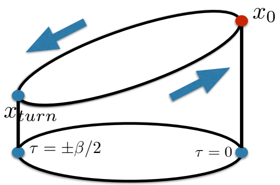
$$\mathcal{O}_{\alpha p^3\mathrm{He}d} = \frac{N_\alpha N_p}{N_{^3\mathrm{He}} N_d} \sim \langle e^{-\mathbf{2}V(x)/T} \rangle$$

$$\mathcal{O}_{\alpha t p^3\mathrm{He}d} = \frac{N_\alpha N_t N_p^2}{N_{^3\mathrm{He}} N_d^3} \sim \langle e^{-\mathbf{3}V(x)/T} \rangle$$

$$\mathcal{O}_{\alpha pd} = \frac{N_\alpha N_p^2}{N_d^3} \sim \langle e^{-\mathbf{3}V(x)/T} \rangle$$

The flucton is a semiclassical solution of the EoMs in Euclidean time with period $\beta = 1/T$ (Shuryak, 1988). Conceptually similar to the instanton.





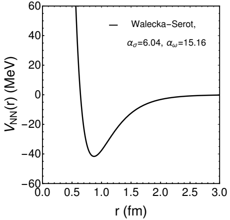
Unlike the instanton it is periodic $x(\beta) = x(0) = x_0$, and it does not require a double well. We applied to 2,3,4-body systems at finite temperature (E.Shuryak, J.M.T.-R., arxiv: 1910.08119).

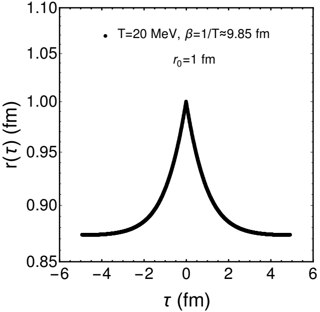
$$P(x_0) = \langle x_0 | e^{-\hat{H}\beta} | x_0 \rangle = \int_{x(0)=x_0}^{x(\beta)=x_0} \mathcal{D}x(\tau)\, e^{-S_E[x(\tau)]}$$

Flucton solution for 2 particles

Euclidean action

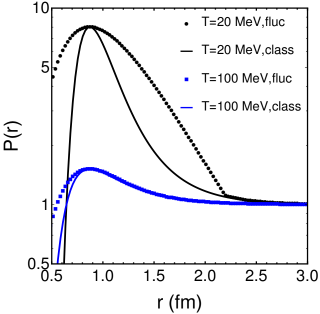
$$S_E = \int d\tau \left[\frac{m_N}{2} (\dot{x}_1^2 + \dot{x}_2^2) + V_{NN}(|\mathbf{x}_1 - \mathbf{x}_2|) \right]$$
$$= \int d\tau \left(\frac{m_N}{4} \dot{r}^2 + V_{NN}(r) \right) + C.M.$$



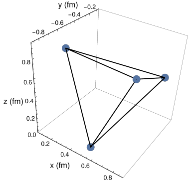


Flucton for Walecka potential

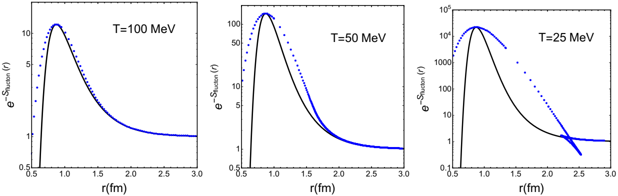
$$P(r_0) = e^{-S_E[r_{fluc}(\tau, r_0)]} \; ; \; P_{class}(r) = e^{-V(r)/T}$$



Flucton solution for ^{4}He



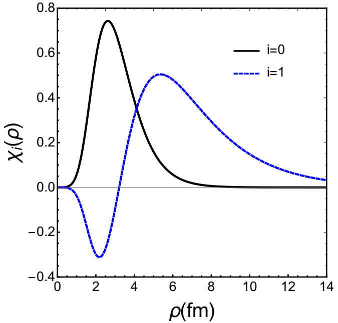
$$S_E = \int d\tau \left(\sum_{i=1}^{4} \frac{m_N}{2} \dot{x}_i^2 + \sum_{i,j\neq i} V_{NN}(r = |\mathbf{x}_i - \mathbf{x}_j|) \right)$$



Solid: classical weight, $e^{-6V_{NN}(r)/T}$; Dots: fluction, $e^{-S_E[fluction]}$

Quantum effects important at low T; in general, when $V(r) \sim T$

One can try to solve the Schrödinger equation for ^{4}He.
Dimensionality reduction $\rightarrow$ K-harmonics (Badalyan, Simonov, 1966)



$$\frac{d^2\chi}{d\rho^2} - \frac{12}{\rho^2}\chi - \frac{2m_N}{\hbar^2}[W(\rho) + V_C(\rho) - E]\chi = 0$$

radial wave function: $\chi(\rho) = \psi(\rho)\rho^4$
hyperdistance: $\rho^2 = \frac{1}{4}\left[\sum_{i\neq j}(\mathbf{x}_i - \mathbf{x}_j)^2\right]$.
$W(\rho)$ contains NN interaction
$V_C(\rho)$ describes Coulomb repulsion

We reproduced the result for the ground state (Castilho Alcaras, Pimentel Escobar, 1974) and found an excited 0^+ state with $E_B = -5$ MeV.

E (MeV)	J^P	Γ (MeV)	decay modes, in %
20.21	0^+	0.50	$p = 100$

Excited states of Helium 4

^{4}He has many excited states (*www.nndc.bnl.gov/nudat2/*)

E (MeV)	J^P	Γ (MeV)	decay modes, in %
20.21	0^+	0.50	$p = 100$
21.01	0^-	0.84	$n = 24,\ p = 76$
21.84	2^-	2.01	$n = 37,\ p = 63$
23.33	2^-	5.01	$n = 47,\ p = 53$
23.64	1^-	6.20	$n = 45,\ p = 55$
24.25	1^-	6.10	$n = 47,\ p = 50,\ d = 3$
25.28	0^-	7.97	$n = 48,\ p = 52$
25.95	1^-	12.66	$n = 48,\ p = 52$
27.42	2^+	8.69	$n = 3,\ p = 3,\ d = 94$
28.31	1^+	9.89	$n = 47,\ p = 48,\ d = 5$
28.37	1^-	3.92	$n = 2,\ p = 2,\ d = 96$
28.39	2^-	8.75	$n = 0.2,\ p = 0.2,\ d = 99.6$
28.64	0^-	4.89	$d = 100$
28.67	2^+	3.78	$d = 100$
29.89	2^+	9.72	$n = 0.4,\ p = 0.4,\ d = 99.2$

- **Statistical thermal model** $\rightarrow$ all these states should be equally populated.
- They necessarily account for feed-down in t, d, p yields.
- Proposed nuclear ratios should include this feed-down in addition to the potential V_{NN} modifications.

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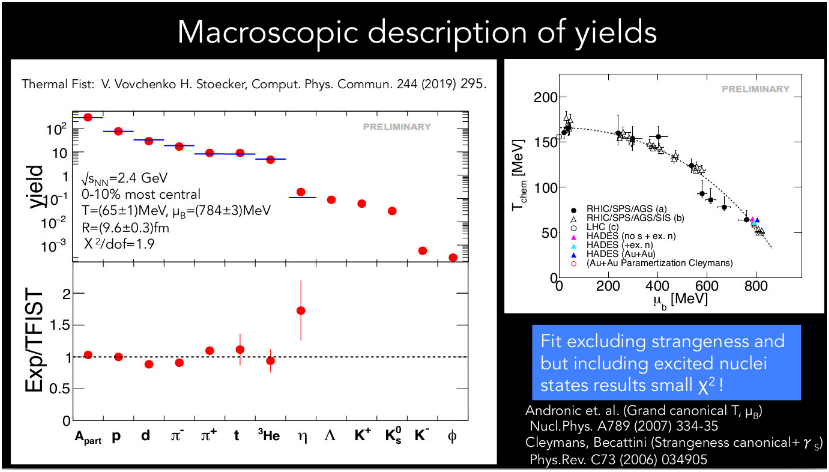
This is implemented by V. Vovchenko using Thermal-FIST: *10-40 % effect for RHIC/SPS and* $\mathcal{O}(1)$ *effect GSI/FAIR*. 3rd EMMI Workshop at Wroclaw Dec.2019

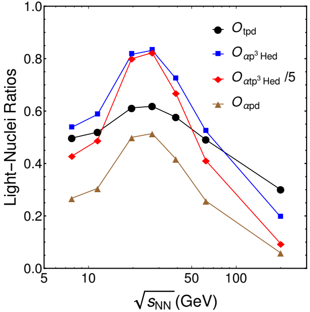
Talk by M. Lorenz at 3rd EMMI workshop at Wroclaw



$$\chi^2/dof = 6.7 \text{ (all hadrons)} \rightarrow 6.1 \text{ (+ excited nuclei)} \rightarrow 1.9 \text{ (- strangeness)}$$

Summary

- Close to T_c the critical mode becomes very light, $m_\sigma \propto (T - T_c)^\nu$
- Significant attractive and long-ranged *NN* potential near T_c
  Modifications from usual (cold) nuclear matter potential
- Increased correlations among nucleons (proton kurtosis...)
  Mean field/*Stosszahlansatz* **not enough** to capture the whole effect
- Possible formation of pre-nuclei (statistical correlations among nucleons)
- Potential production of light nuclei (t, ^{4}He) at "critical" $\sqrt{s_{NN}}$
  Important feed down from excited states of ^{4}He at these energies



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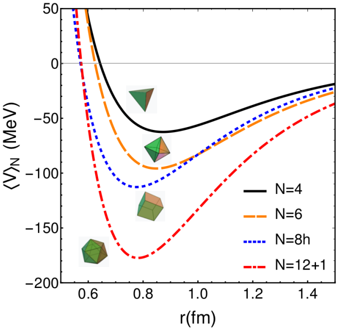
Few-body systems usually follow geometry arguments.

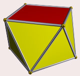


$V_{A'}$ potential

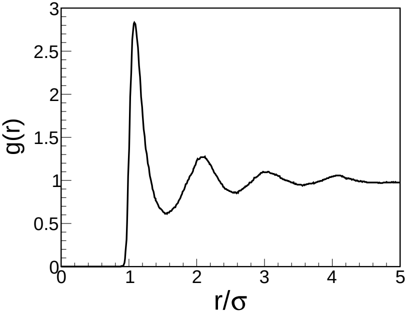
Curious fact

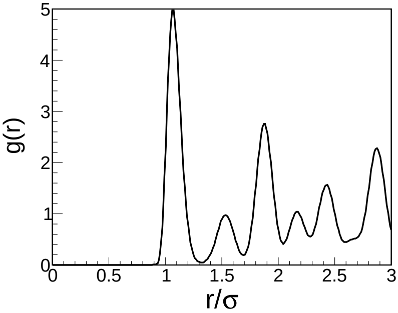
For $N = 8$ the cube is **not** the equilibrium configuration.

In a good approximation it is a **square antiprism**



Lennard-Jones potential, for N=108 Ar atoms, liquid vs solid

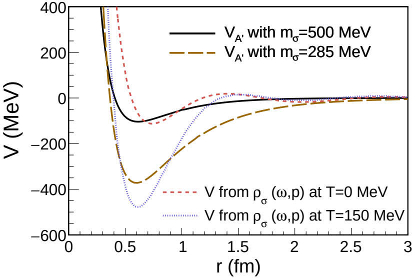




Boltzmann approximation assumes $g(r) = 1$ (dilute gas)
Correlations are important in our system!

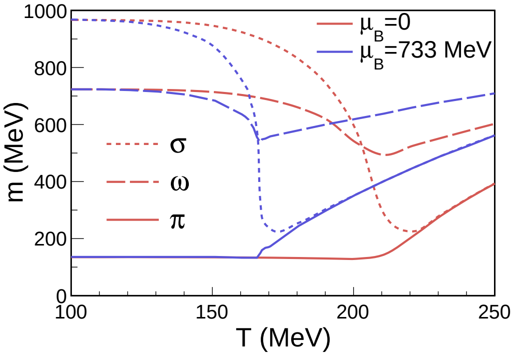
$$V_\sigma(\mathbf{r}) = g_\sigma^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} D_\sigma^R(p_0, \mathbf{p})$$

$$D_\sigma^R(p_0, \mathbf{p}) = -\int_{-\infty}^{\infty} d\omega \frac{\rho_\sigma(\omega, \mathbf{p})}{\omega - p_0 - i\epsilon}$$



Spectral function from quark-meson model using FRG.
R.-A. Tripolt, Ph.D. Thesis 2015

σ and ω pole masses in PNJL model



JMT-R, 2018 ($N_f = 3$ Polyakov-Nambu-Jona–Lasinio model)

- 80 % σ mass reduction at T_c
- 25 % ω mass reduction at T_c

Caveat: σ is to be identified with $f_0(980)$

BES STAR freeze out

We try to mimic as much as possible experimental situation in BES I, as measured by STAR @ RHIC (STAR Collab. 2016 & 2017)

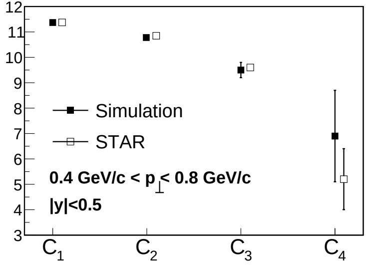
- Temperature $T \simeq 150$ MeV
- Densities: 1-2 n_0
- Finite time evolution: $t = 5$ fm
- Non-relativistic nucleon dynamics
- Fireball expansion: mapping of y and p_T distributions to experimental measured distributions
- Simulations: 32 nucleons, 10^5 events (similar to experiment for 5% most central events)
- Antinucleons: For $\sqrt{s_{NN}} < 19.6$ GeV they are suppressed, at least, a factor of 10 w.r.t. protons

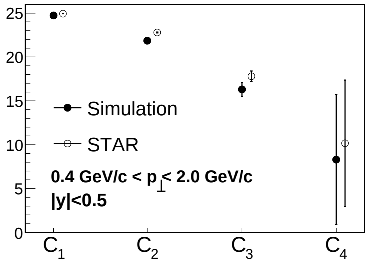
Note: It is a crude model and several effects not covered.
Understand as a first approximation to the physical situation.

Poisson distribution at $\sqrt{s_{NN}} = 19.6$ GeV $\leftrightarrow$ Noncritical potential $V_{A'}$

- $|y| < 0.5, \quad 0.4$ GeV/$c < p_\perp < 0.8$ GeV/c
- $|y| < 0.5, \quad 0.4$ GeV/$c < p_\perp < 2$ GeV/c

protons

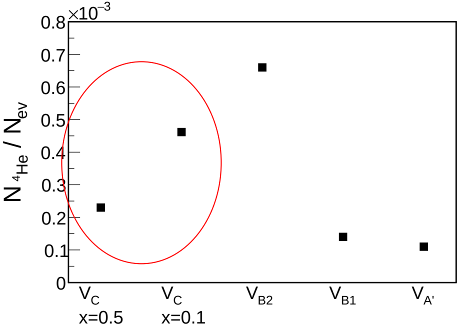




$$C_1 = \langle N_p \rangle \ , \quad C_2 = \langle \delta N_p^2 \rangle \ , \quad C_3 = \langle \delta N_p^3 \rangle \ , \quad C_4 = \langle \delta N_p^4 \rangle - 3 \langle \delta N_p^2 \rangle^2$$

Aggregation of few nucleons (**pre-clusters**) can be formed within few fm/c.
We search 4 isolated nucleons close in phase space in the same simulation

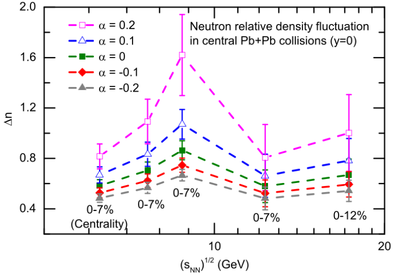
Nucleons belong to bigger clusters for these potentials

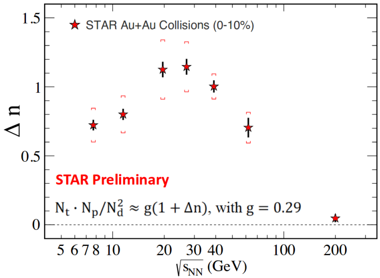


Close to T_c, we expect an **excess of light nuclei** over thermal expectations.

Neutron density fluctuation

$$\frac{N_t N_p}{N_d^2} = g(1 + \Delta n) \qquad (\alpha = 0)$$





Sun, Chen, Ko, Xu 2017, based on NA49 exp. data

STAR Collaboration (QM2018)