Electric conductivity of hadronic matter

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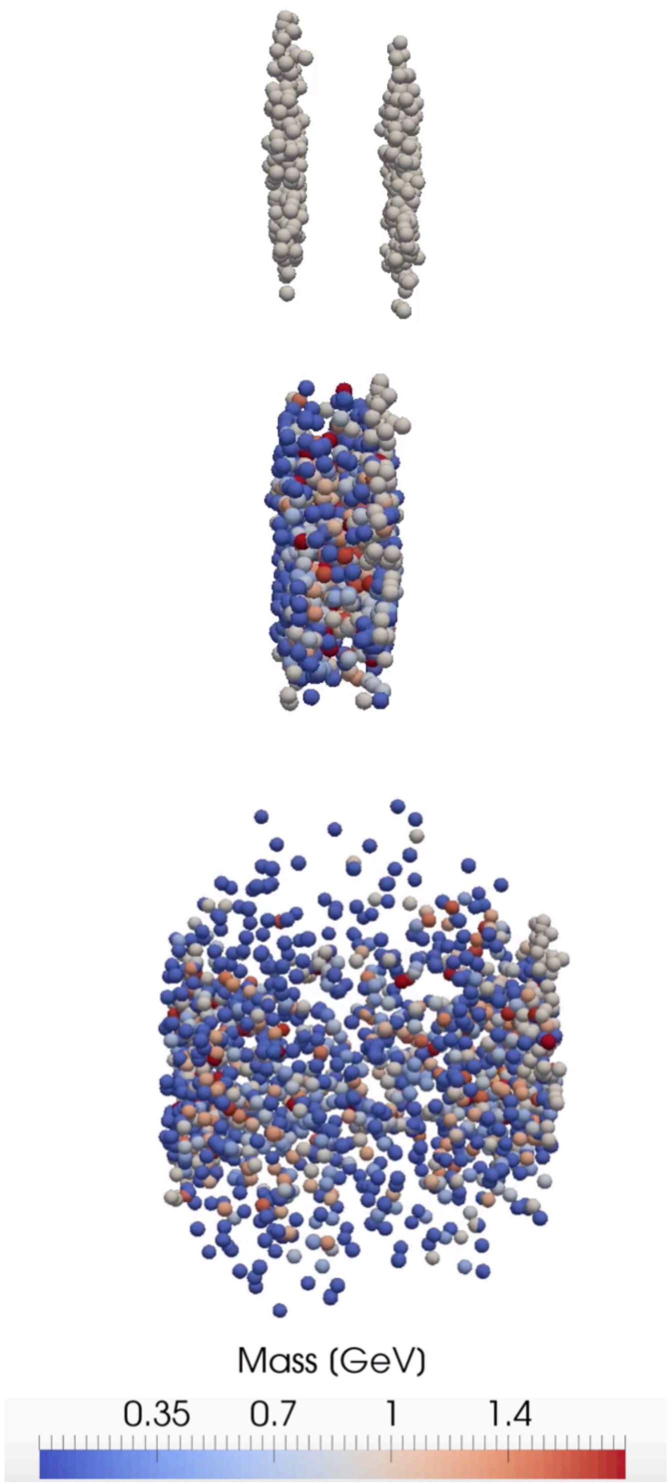
Transport meeting 24.01.19

See also:
arXiv:1810.12527

CRC-TR 211
FIAS Frankfurt Institute for Advanced Studies
GOETHE UNIVERSITÄT FRANKFURT AM MAIN
HIC for FAIR Helmholtz International Center
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Introduction

- Studying heavy ion collisions in order to understand properties of QCD matter
- Transport models give unique insights on dynamical evolution of heavy ion collisions
- Characterize the interacting hadron gas by its linear transport coefficients



Pb-Pb collision with 17.3 GeV center-of-mass energy
(by J. Mohs)

Definition: electric conductivity σ_{el}

Ohm's law: $$\vec{J}_Q(t, \vec{x}) = \sigma_{el}\vec{E}(t, \vec{x})$$

- Electric current $\vec{J}_Q$
- Electric conductivity σ_{el}
- Electric field $\vec{E}$

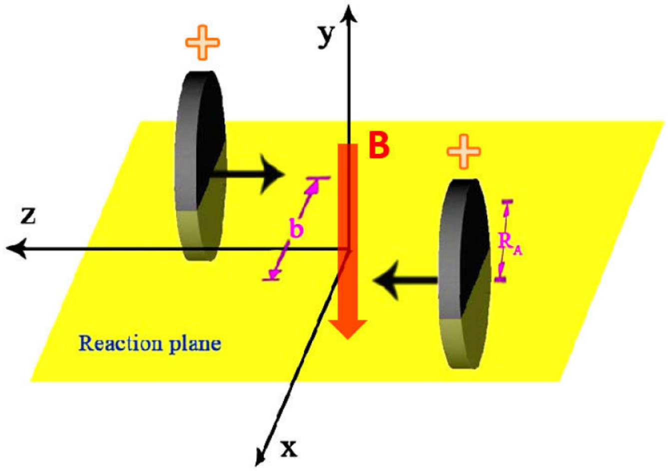
Conservation of *U(1)* electric charge leads to a transport of electric charge -> transport coefficient

Applications of σ_{el}

- High velocities of charged particle will lead to big electric and magnetic fields $\sim m_\pi^2$.
  -> Time evolution depends on σ_{el}

- Soft dilepton and photon rates are related to the electric conductivity

- Magneto-hydrodynamic description of the expanding plasma after the collision needs the values of transport coefficients



X-G Huang Rept. Prog. Phys. 79, 076302, [2016]

Model: SMASH

smash

- **S**imulating **M**any **A**ccelerated **S**trongly-interacting **H**adrons

- Hadronic transport approach, effectively solving Boltzmann equation

$$p^{\mu}\partial_{\mu}f_i(x,p) + m_i F^{\alpha}\partial^{p}_{\alpha}f_i(x,p) = C^{i}_{coll}$$

- Uses a geometric collision criterion

$$d_{coll} < \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- Good description of low energy heavy ion collisions and of late stages of high energy heavy ion collisions

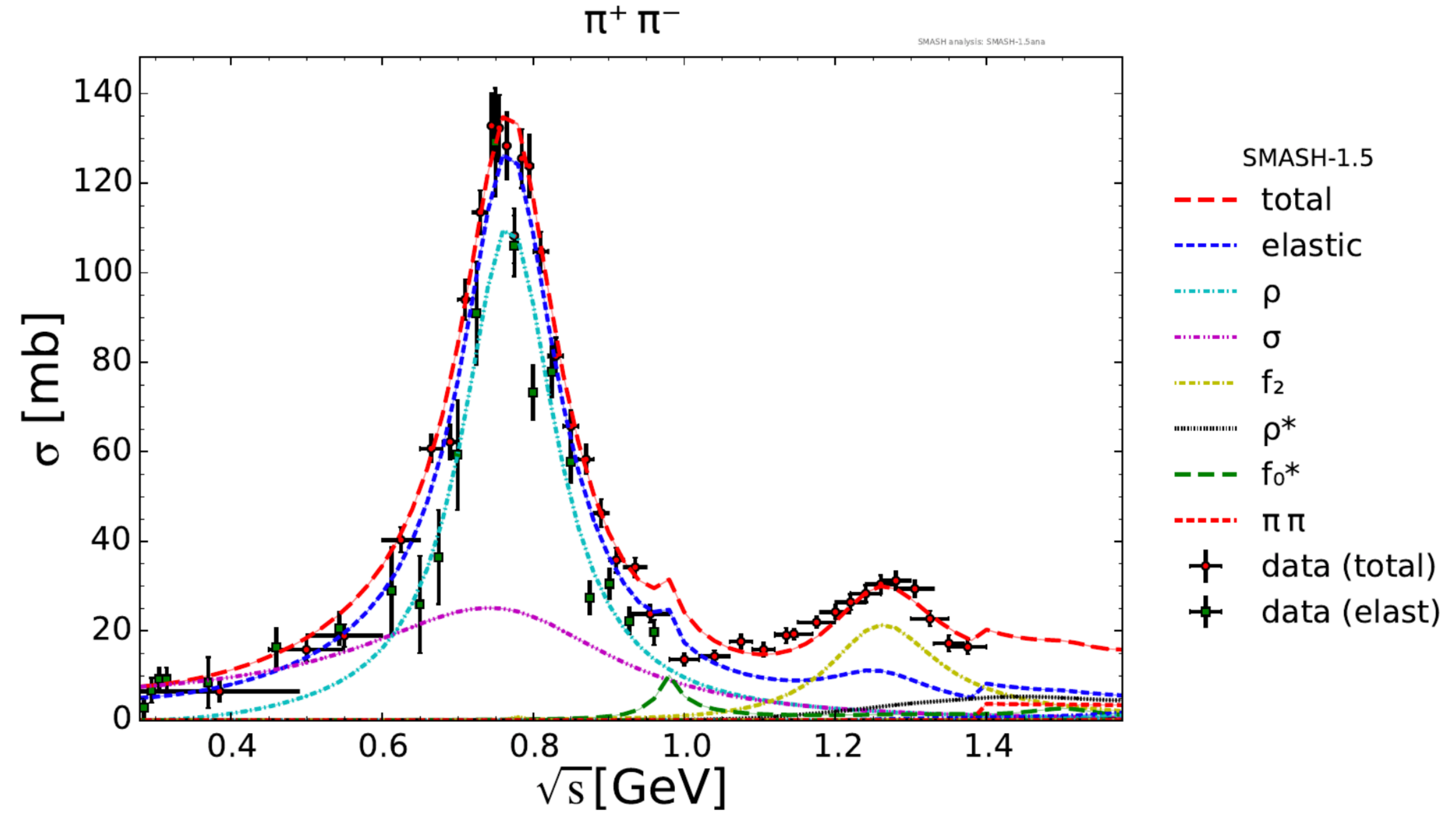
https://smash-transport.github.io

Weil et al PRC.94.054905 [2016]

Model: SMASH

Weil et al PRC.94.054905 [2016]

- SMASH contains all baryons and mesons with masses up to $\sim 2GeV$
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ collision modeled through resonance formation and decay
- Resonance properties obtained from experiment



Particle	Mass (MeV)	Decay width (MeV)
π	138	0
ρ	776	149
K	494	0
$K^\star$	892	50.8
N	938	0
Δ	1232	117

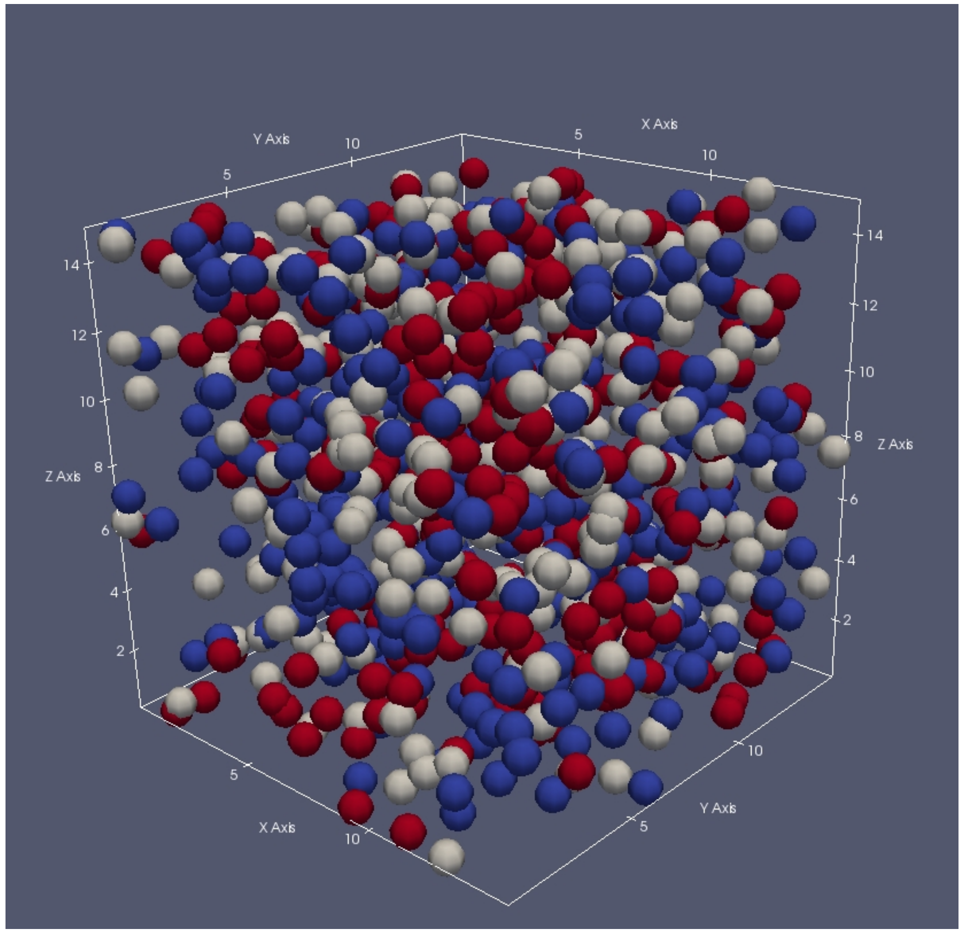
Box setup

- Box with periodic boundary conditions, simulating infinite matter

- Initial momenta distribution:

$$\frac{d^3N}{dp^3} \sim exp\left(-\sqrt{\vec{p}^2 + m^2}/T\right)$$

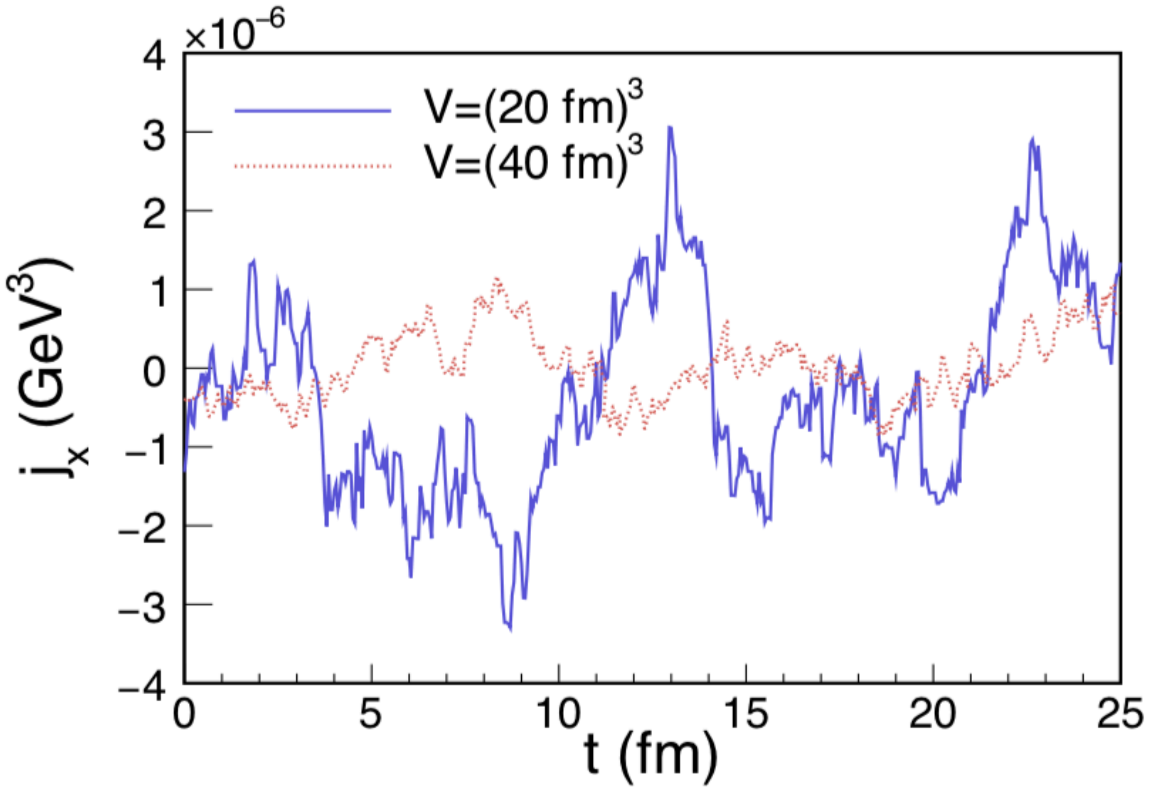
- One has to check for thermal/ chemical equilibrium



Box filled with π, $m_\pi = 0.138 GeV$ interacting via constant elastic cross section $\sigma_{tot} = 30 mb$

Electric current

For a set of particles: $j^i_Q(t) = \frac{1}{V}\sum_{k=1}^{N} \frac{q_k p^i_k}{p^0_k}|_t \sim \frac{1}{V}\int d^3x J^i_Q(t, \vec{x})$



Box filled with π, interacting via constant elastic cross section $\sigma_{tot} = 30mb$ at $T = 125MeV$

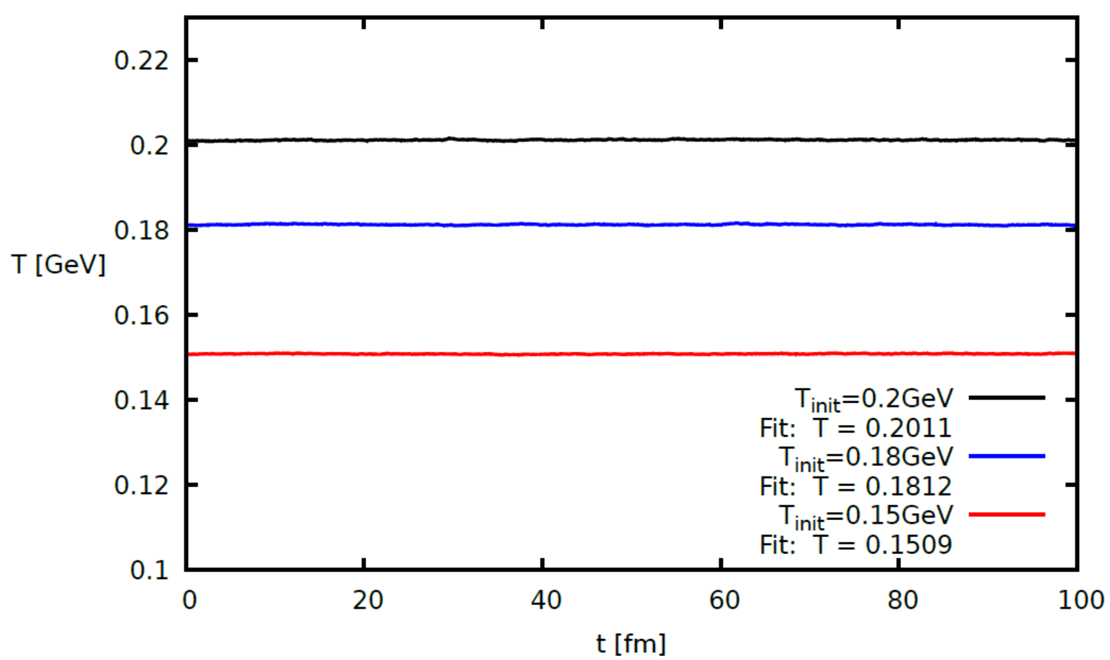
- Pions interacting as hard spheres
- Since no electric field is applied, the current fluctuates around zero
- Strength of fluctuation $\sim \frac{1}{\sqrt{V}}$

Calculation of the temperature

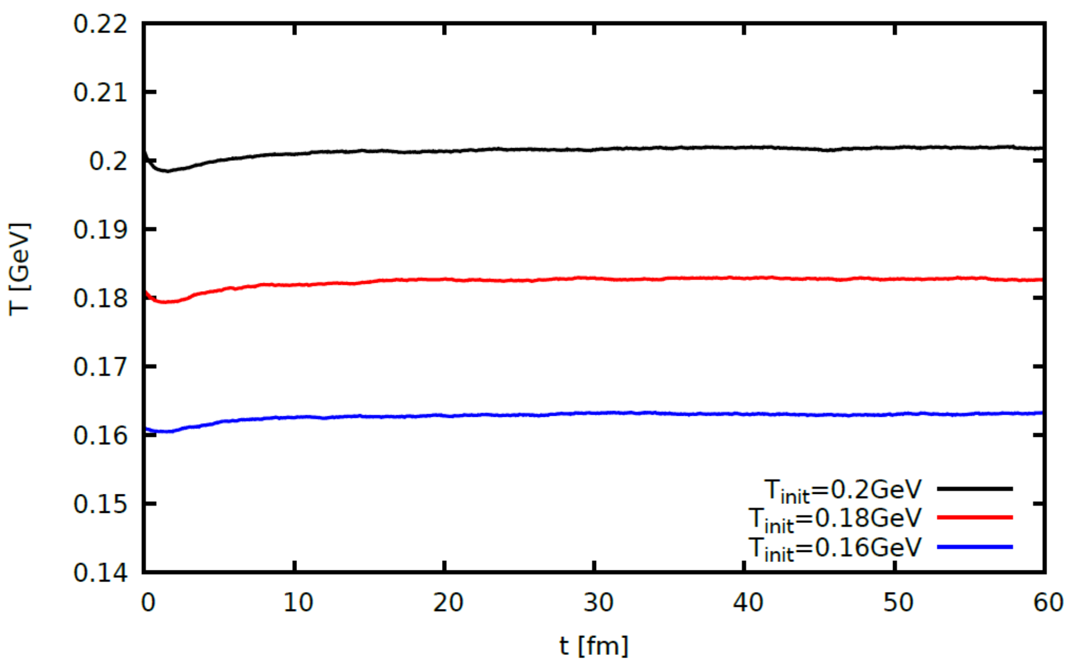
- Thermal equilibrium has to be reached in the box

- Calculate temperature by fitting the pion momenta spectrum

$$\frac{d^3N}{dp^3} \propto Ve^{-\sqrt{\vec{p}^2 + m^2}/T}$$



Box filled with π, $m_\pi = 0.138GeV$ interacting via constant elastic cross section $\sigma_{tot} = 30mb$



Box filled with π and ρ mesons interaction via $2 \leftrightarrow 1$ resonance formation/decay

Green-Kubo formalism

- Well-known formalism to extract transport coefficients
- Based on linear response theory
- Possible to extract transport properties by looking at fluctuations of currents around equilibrium

$$\sigma_{el} = \frac{V}{3T} \int_0^\infty \langle \overrightarrow{j_Q}(t) \overrightarrow{j_Q}(0) \rangle dt$$

Correlation function

- How does $\langle \vec{j}_Q(t)\vec{j}_Q(0)\rangle$ look like?

- Since Ohm's law is inconsistent with causality it is modified to:

$$J^i_Q(t,\vec{x}) = \int_V d^3\vec{x}' \int_{t_0}^{t} dt' \Sigma^{ij}(t-t';\vec{x}-\vec{x}')E'^i(t',\vec{x}') + \Xi^i_Q(t,\vec{x})$$

where Σ^{ij} is the memory kernel of the electric conductivity and $\Xi^i_Q(t,\vec{x})$ a fluctuation term

- With an exponential Ansatz for the memory kernel:

$$\Sigma^{ij}(t-t';\vec{x}-\vec{x}') = \frac{\sigma_{el}\delta^{ij}}{\tau_Q}\delta^{(3)}(\vec{x}-\vec{x}')exp\left(\frac{|t-t'|}{\tau_Q}\right) \quad ,$$

It follows with the fluctuation dissipation theorem that the correlation function of the spatial average of the current-current correlator has an exponential decay

$$\langle j^i_Q(t) j^j_Q(t')\rangle = \frac{\sigma_{el}\delta^{ij}T}{\tau_Q V}exp\left(-\frac{|t-t'|}{\tau_Q}\right)$$

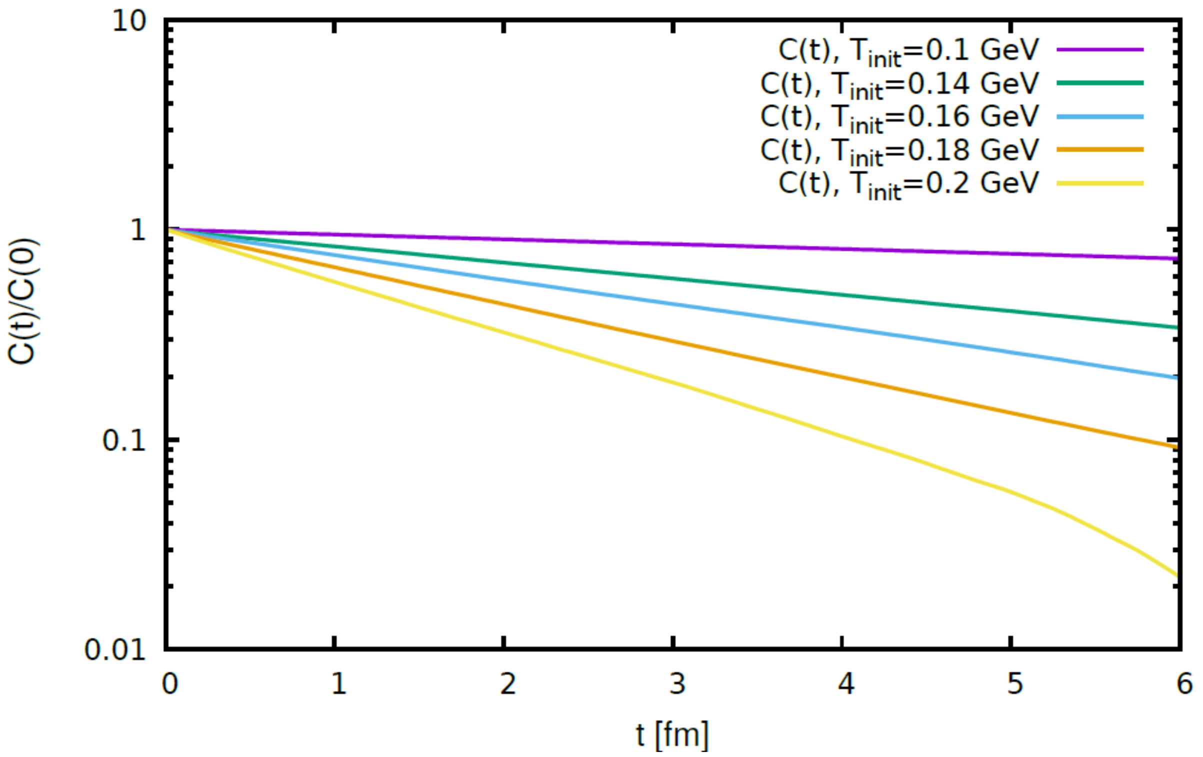
-> Ansatz: $\langle \vec{j}_Q(t)\vec{j}_Q(0)\rangle = C(0)e^{-t/\tau_Q}$

Murase, Hirano
arXiv:1304.3243 [nucl-th]

Correlation function

Correlation function: $C(t) \equiv \langle j^i_Q(t) j^i_Q(0) \rangle = \frac{1}{K-t} \sum_{s=0}^{K-t} j^i_Q(s\Delta t) j^i_Q(s\Delta t + t)$

Where K is the total number of time steps



Correlation function of a box filled with pions interacting with a constant cross section $\sigma_{tot} = 30mb$ for different initialization temperatures.

- Correlation function follows an exponential decay -> extract values of τ_Q and $C(0)$ by fitting an exponential function

$$C(t) = C(0)e^{-\frac{t}{\tau_Q}}$$

$$\rightarrow \quad \sigma_{el} = \frac{C(0)V\tau_Q}{T}$$

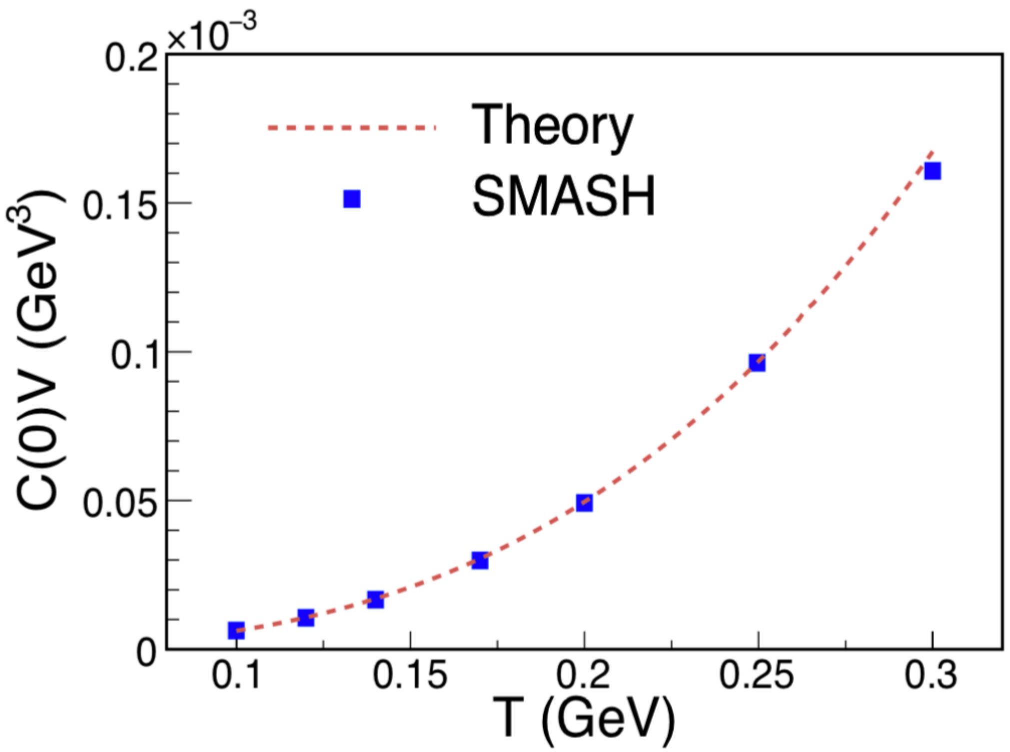
- Error grows for later times -> find cut-off criterion

Correlation function at t=0

- Can be computed analytically
- Well defined in thermal equilibrium
- Good cross check if calculation is done correctly

$$C(0)V = \sum_{a=1}^{N_s} \frac{g_a q_a^2 e^2}{6\pi^2} I(m_a, T, \mu_Q)$$

$$I(m_a, T, \mu_Q) = \int_0^\infty dp \frac{p^4}{m^2 + p^2} e^{-\frac{\sqrt{m_a^2 + p^2} - q_a e \mu_Q}{T}}$$

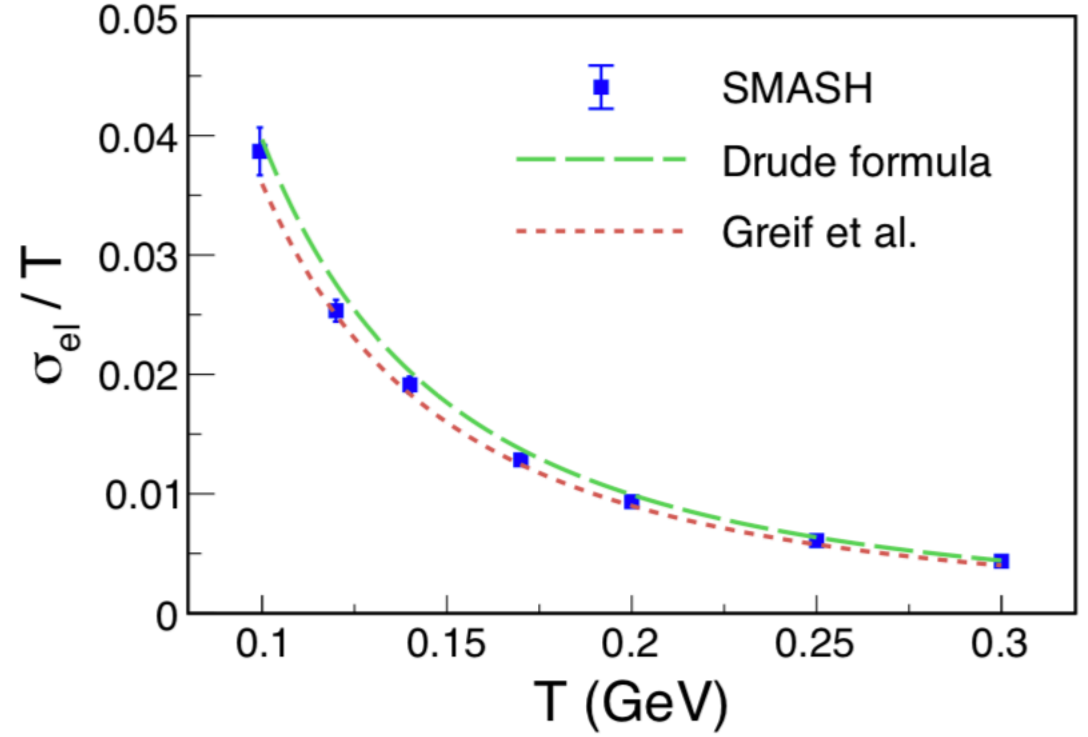


For a massless gas: $C(0) = \frac{2(qe)^2 n}{3V}$

$C(0)V$ of massless gas with three species $q_a = \pm 1, 0$ constant cross section $\sigma_{tot} = 30mb$

Test Case: Massless gas

Massless gas with three species
$q_a = +1, 0, -1$
interacting via constant cross section
$\sigma_{el} = 30mb$



- Drude Formula (classical kinetic theory)

$$\sigma_{el}^{Drude} = \frac{1}{2}\frac{2e^2}{3\sigma T} = \frac{3.9686 \cdot 10^{-4}}{T}(GeV^2)$$

Greif et al. Phys. Rev. D90, 094014 (2014)

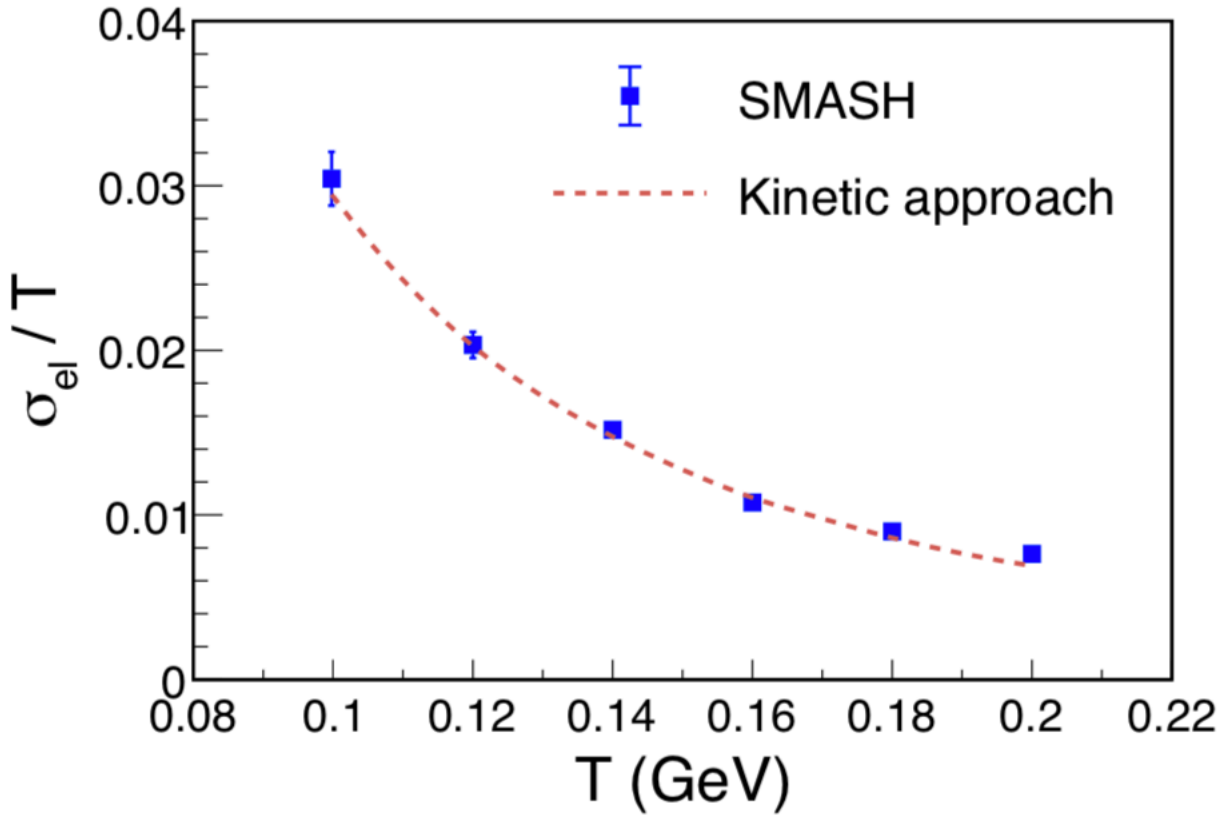
- Kinetic theory

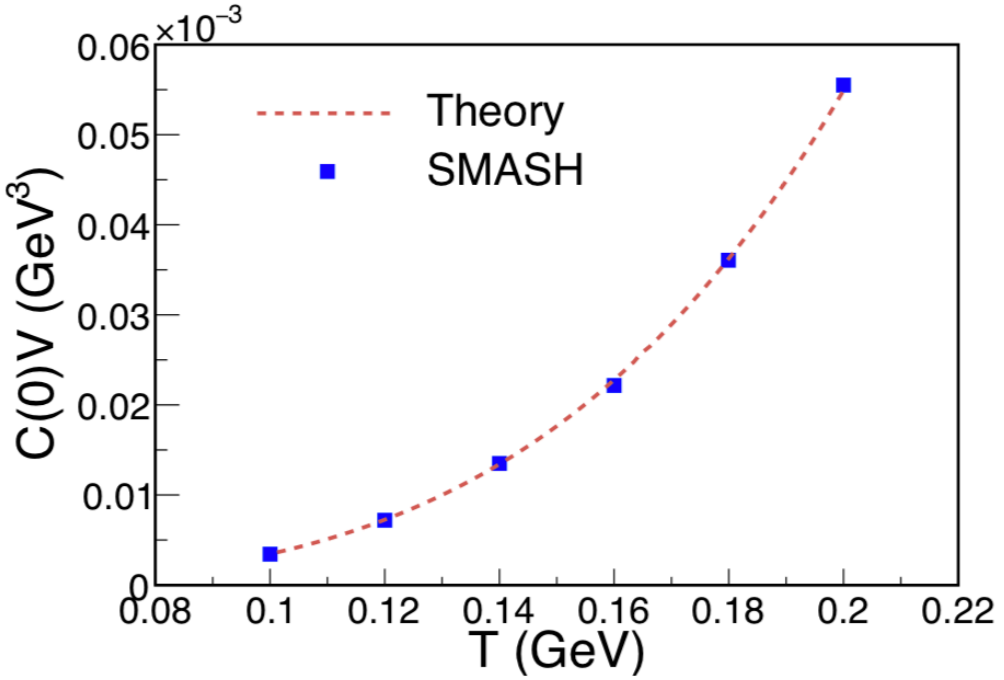
$$\sigma_{el}^{Drude} = \frac{2/3}{3/13}\frac{3}{30}\frac{0.000832737}{T} = \frac{3.5967 \cdot 10^{-4}}{T}(GeV^2)$$

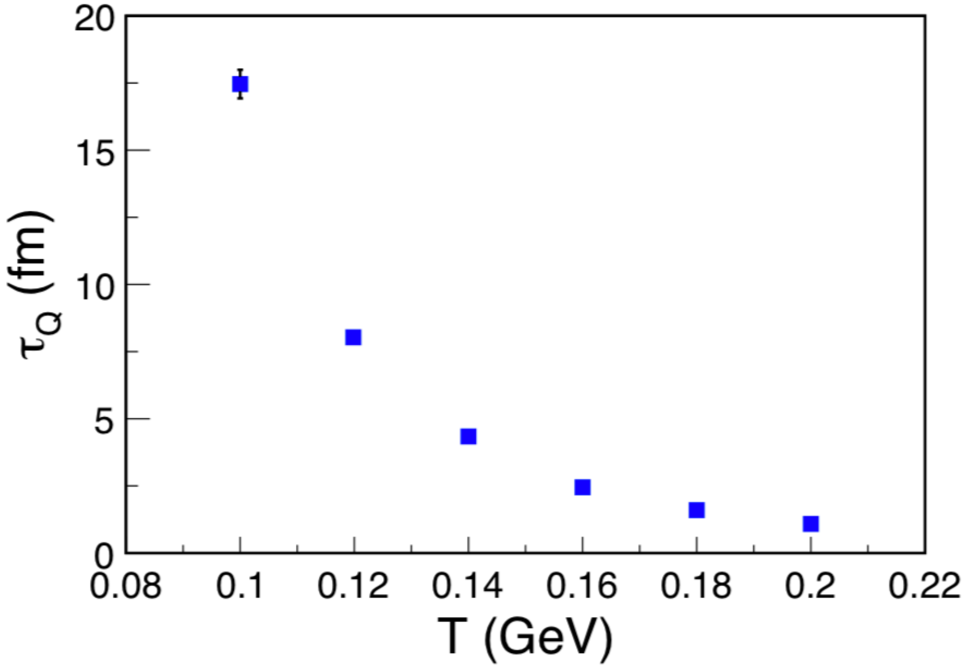
Greif et al. Phys. Rev. D93, 096012 (2016)

Test case: Hadron mixture

Three types of massive hadrons (π, K, N) interacting with a constant cross section $\sigma_{el} = 30mb$



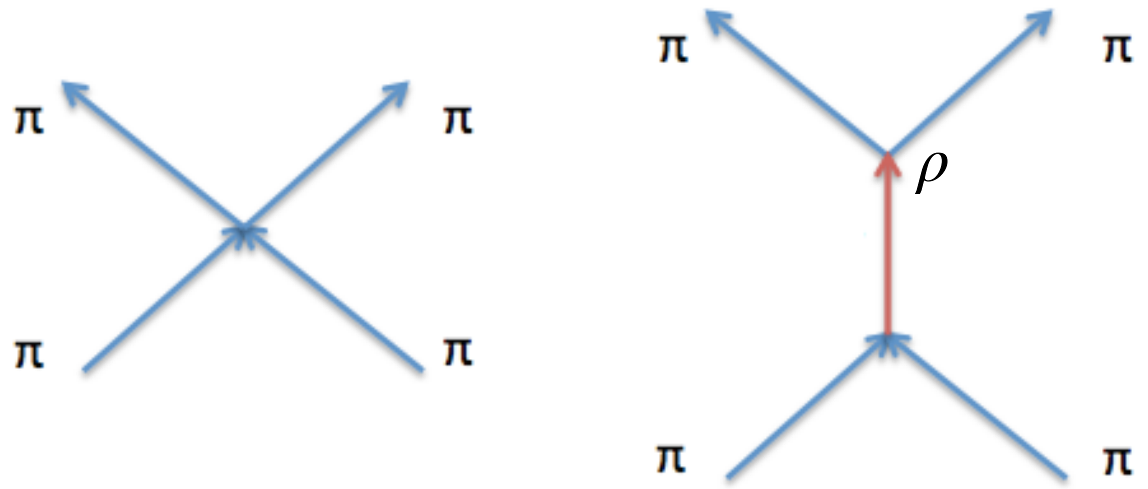




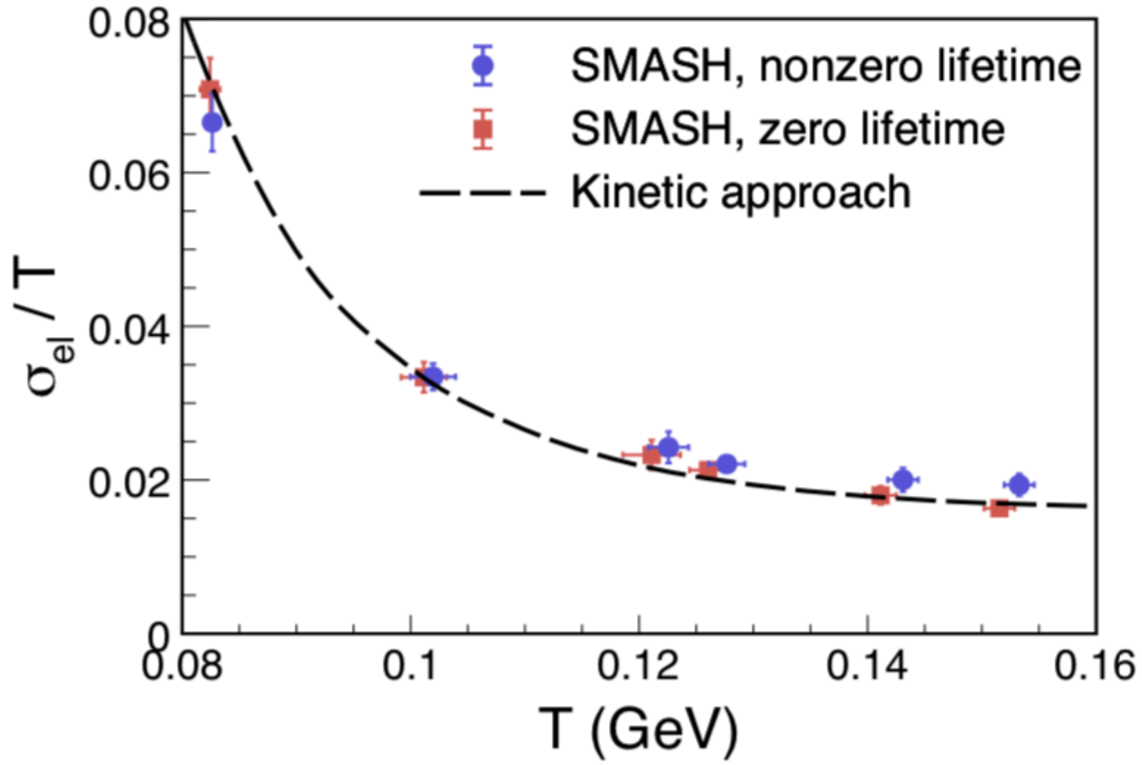
Greif et al.: Phys. Rev. D **96**, 059902

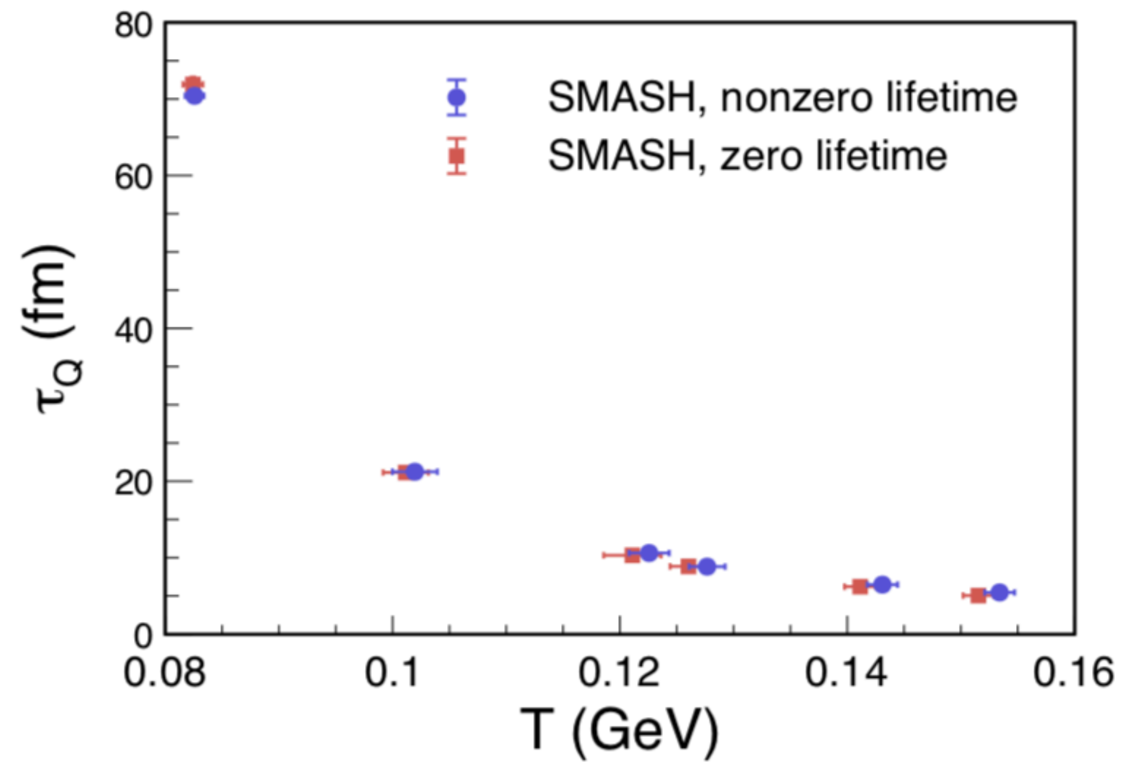
Test case: $\pi\rho$ - gas

- Pions interacting via energy dependent elastic cross section by forming a ρ meson.
- Analytic calculation only models the interaction with an energy dependent cross section, ρ does not propagate.
- How is the transport coefficient affected by resonance lifetime?



Test case: $\pi\rho$ - gas



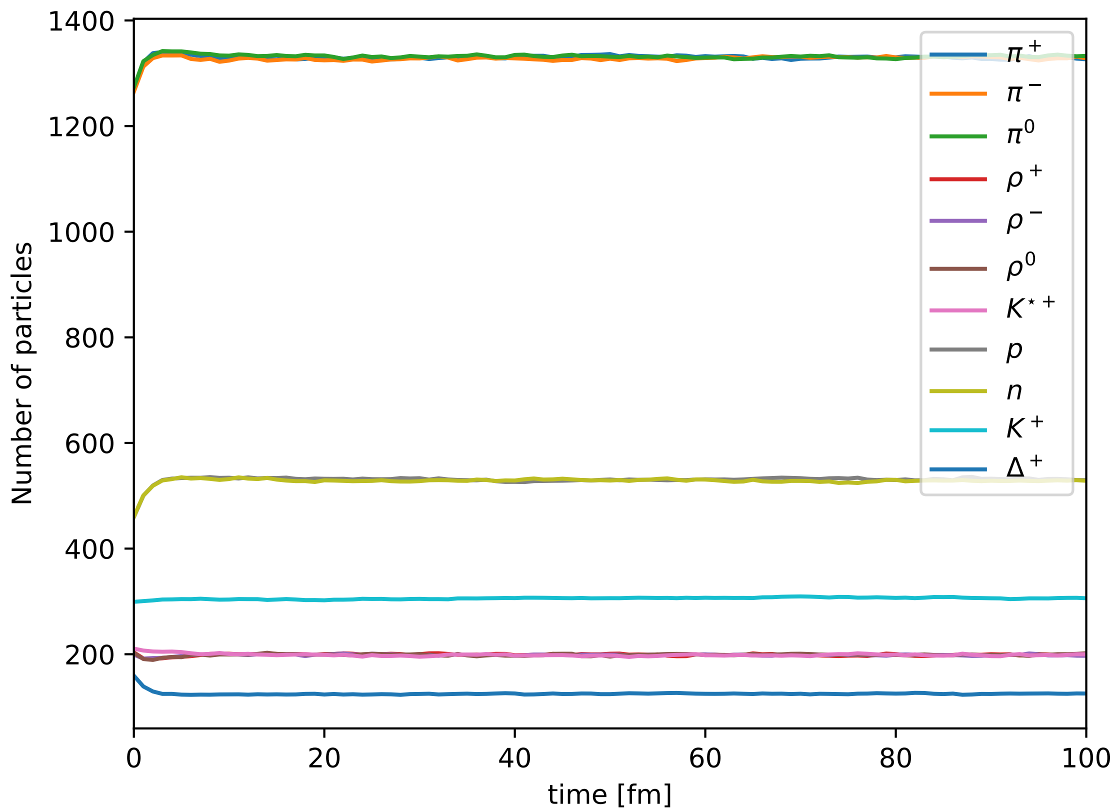


- Excellent agreement between analytic calculation and SMASH
- Resonance lifetime does not affect significantly the relaxation time τ_Q
- Difference in the electric conductivity comes from $C(0)$

Greif et al. :
Phys. Rev. D **93**, 096012 (2016)

Hadron gas

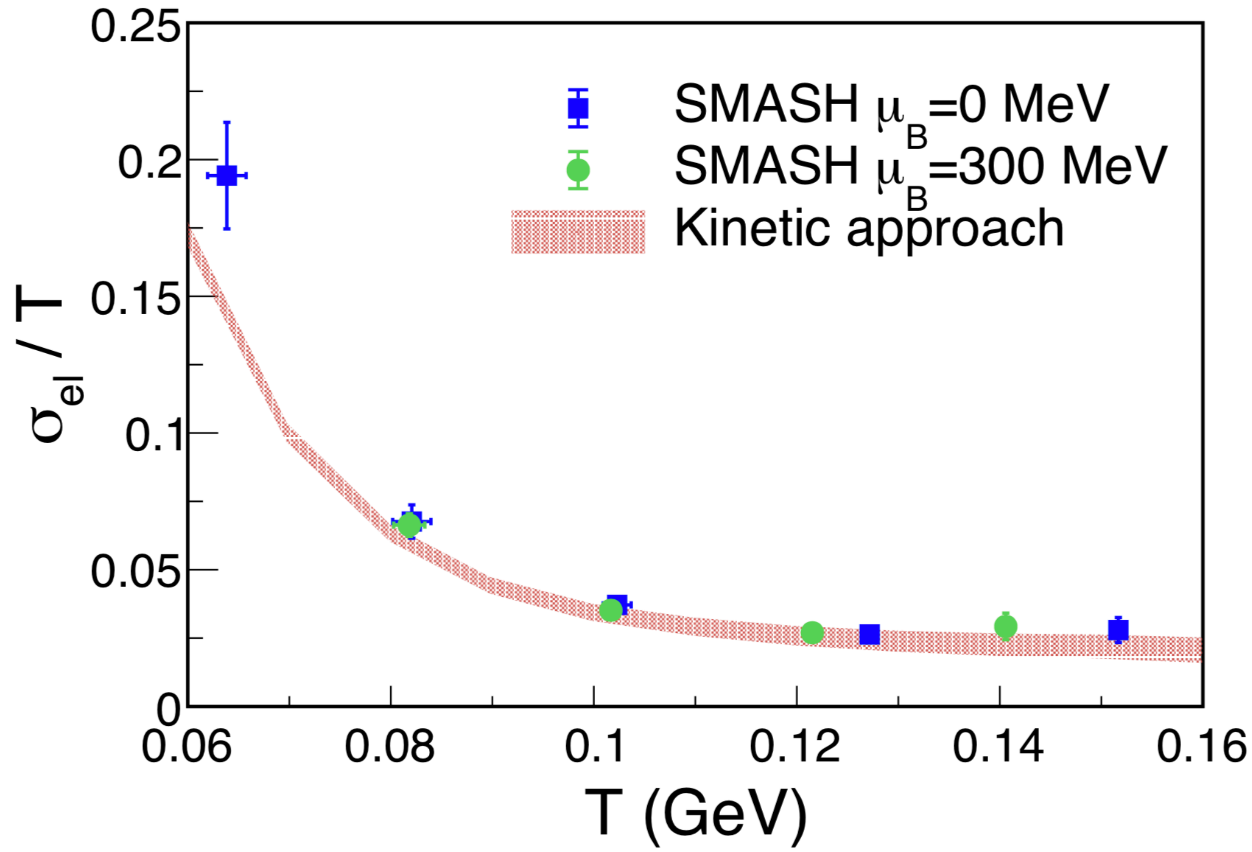
- Relatively simple hadron gas model with pions, kaons and nucleons, which represents a physical hadron gas below T_c.
- Both thermal and chemical equilibrium has to be reached for Green-Kubo method to be valid



	π	K	N
π	ρ	$K^\star$	Δ
K		0	0
N			0

Multiplicities of individual species in the hadron gas at $T = 140 MeV$ and $\mu_B = 300 MeV$

Hadron gas



- Slightly different result since the analytic calculation includes constant cross section + zero resonance lifetime

- Similar to the shear viscosity, baryochemical potential does not affect the electric conductivity as long as $\mu_B << m_N$

Greif et al.: Phys. Rev. D **96**, 059902 Rose et al. Phys. Rev. C **97**, 055204

Shear viscosity

- Linear transport coefficient which measures how momentum is transferred in transverse direction

- Same methodology as for the electric conductivity with different current

- Shear viscosity is calculated with

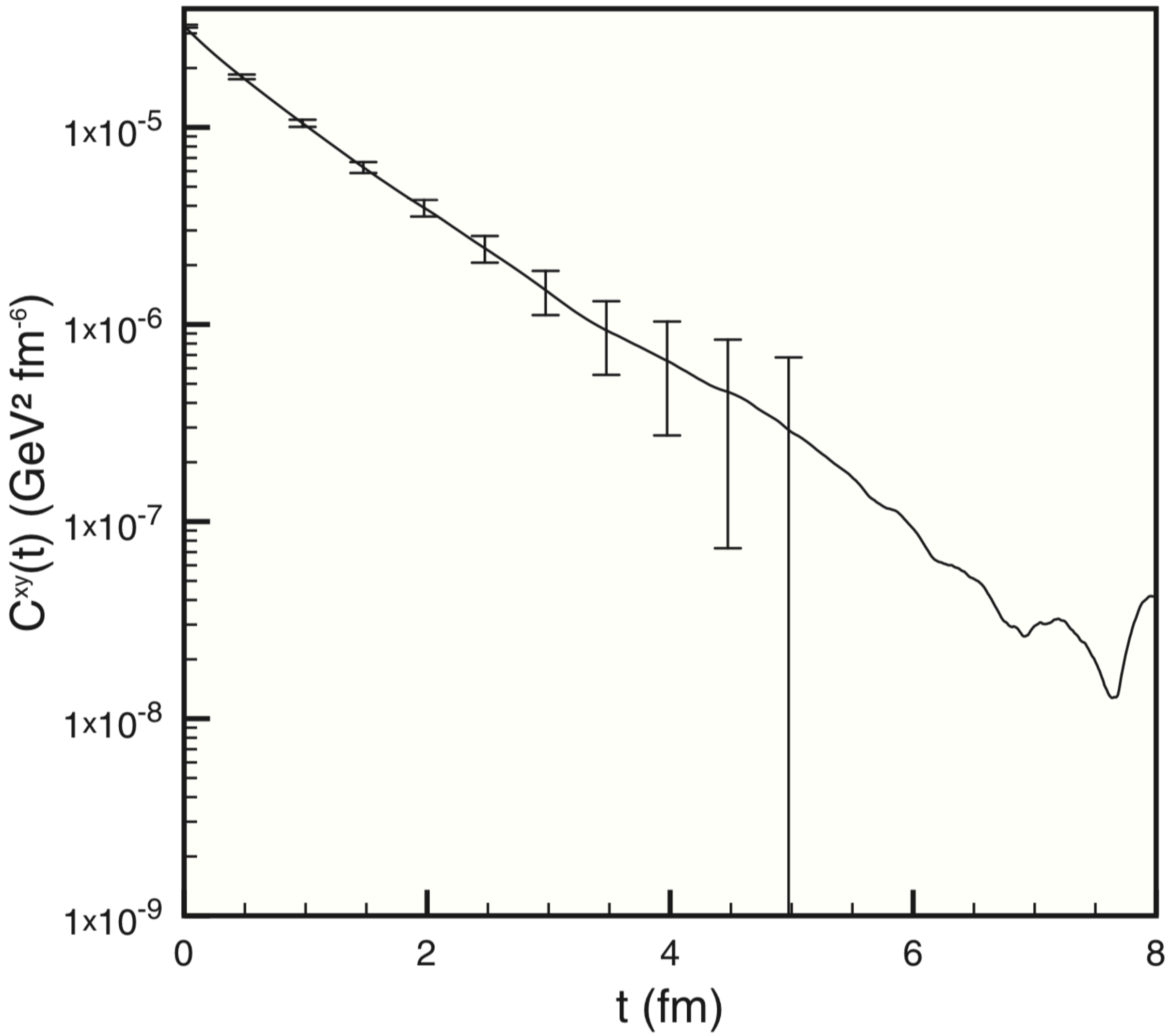
$$\eta = \frac{V}{T}\int_0^\infty \langle T^{xy}(t)T^{xy}(0)\rangle$$

with the energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{V}\sum_i^{N_{part}} \frac{p_i^\mu p_i^{nu}}{p_i^0}$$

and by fitting the correlation function with

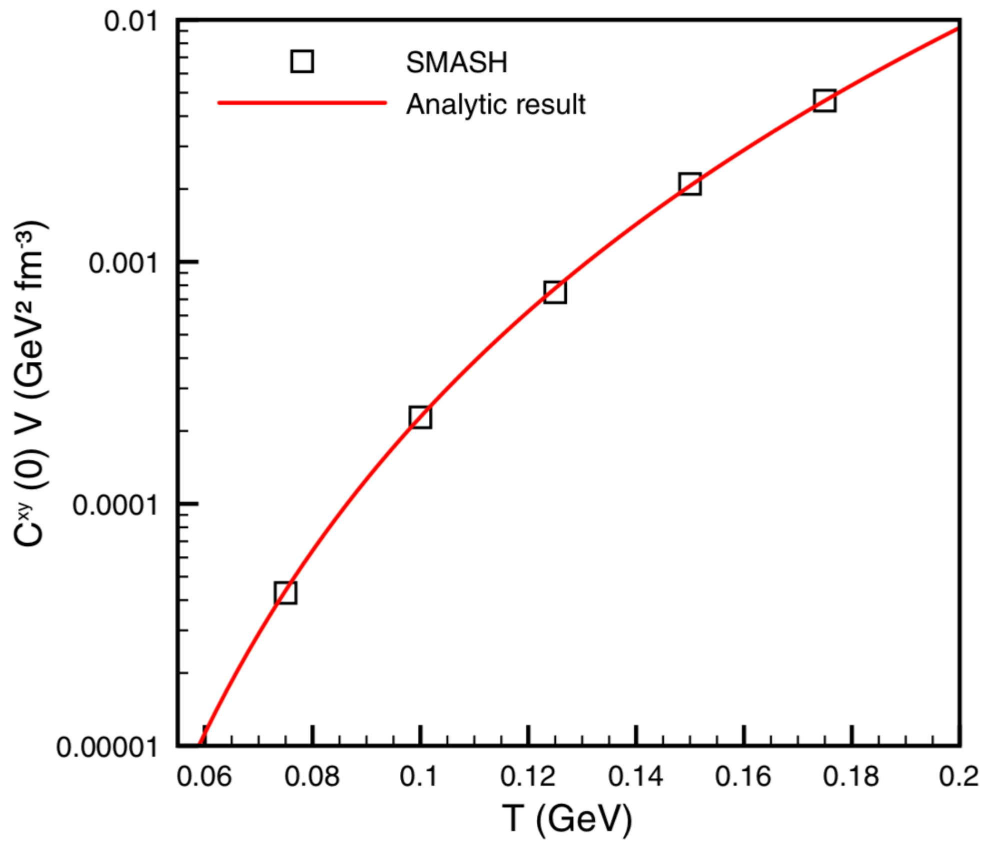
$$C^{xy} \equiv \langle T^{xy}(t)T^{xy}(0)\rangle = C^{xy}(0)e^{-t/\tau}$$

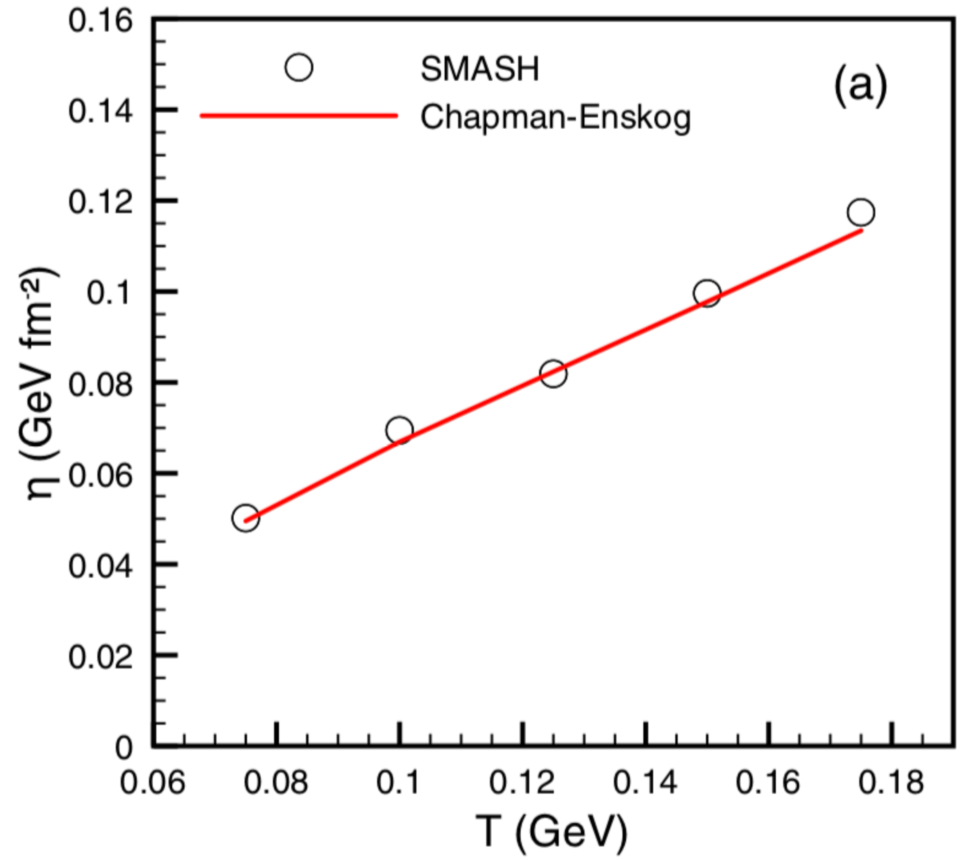


Rose et al. Phys. Rev. C **97**, 055204

Shear viscosity of a π - gas

Test case: simple π - gas interacting with a constant cross section

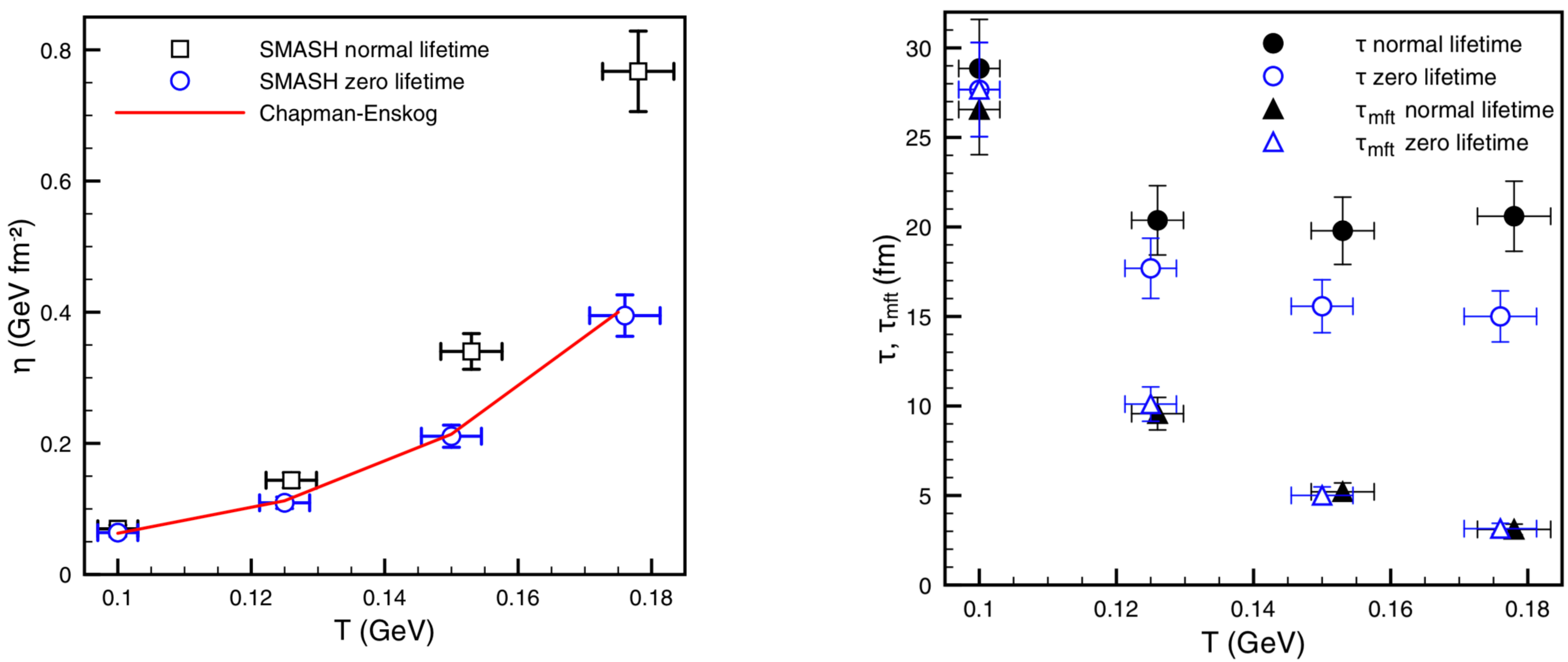




➡ Similarly to the electric conductivity there is a perfect agreement between SMASH and analytic calculations for the simple test cases!

Rose et al. Phys. Rev. C **97**, 055204

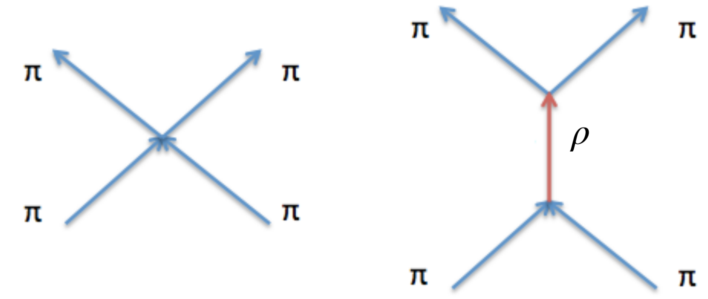
Shear viscosity of a $\pi\rho$ - gas

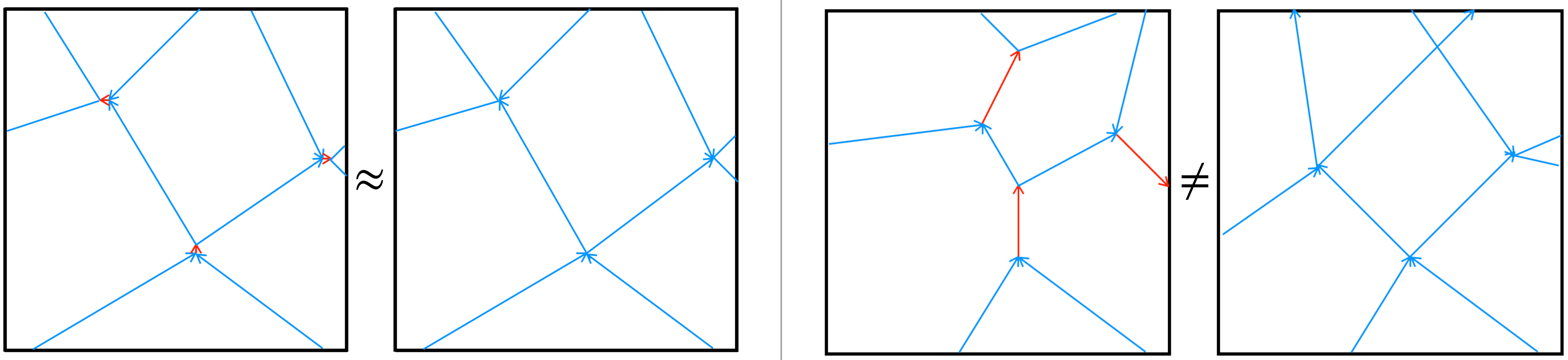


- Perfect agreement between SMASH with zero lifetime and analytic calculation
- Difference in τ between the zero and normal resonance lifetime cases

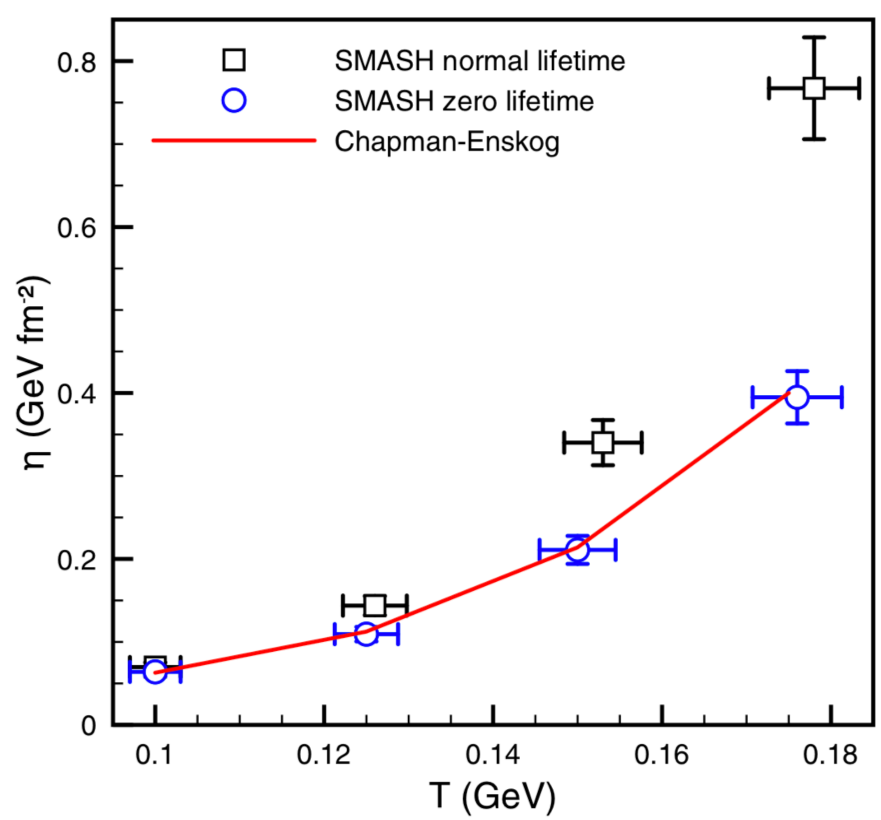
Rose et al. Phys. Rev. C **97**, 055204

Impact of lifetime of the resonance: Shear viscosity





Low temperature
$\tau_{mft} >> \tau_{res}$

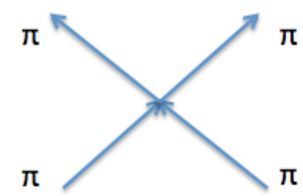


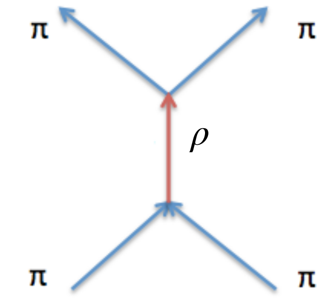
High temperature
$\tau_{mft} \sim \tau_{res}$

τ_{mft} = mean free time
τ_{res} = resonance lifetime

Rose et al. Phys. Rev. C **97**, 055204

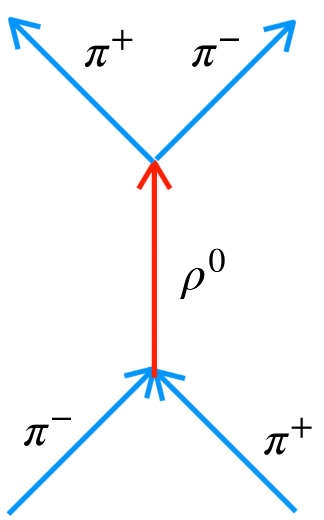
Impact of lifetime of the resonance: electric conductivity





- There are several processes in which the equilibration has already happened at the first interaction
- Since $m_\rho >> m_\pi$, the contribution of ρ mesons to current is small compared to that of pions
- Rate of formation/decays is not affected, when changing the resonance lifetime

Take for example the following reaction:



- Rho does not contribute to the electric current
- Flux already disappears in the first interaction
- value of resonance life time does not play a significant role

Conclusions and Outlook

- Conclusion
  - Computed the electric conductivity of a hadron gas using the Green-Kubo formalism
  - Found excellent agreement with analytic calculation
  - Resonance lifetimes have to be taken into account when comparing transport coefficients from analytic and numerical calculations
- Outlook
  - Add more species to the calculation
  - Go to higher values of μ_B
  - Compute diffusion coefficient matrix of conserved charges (S,Q,B) numerically within SMASH, as in [Greif et al. Phys. Rev. Lett. 120, 242301]

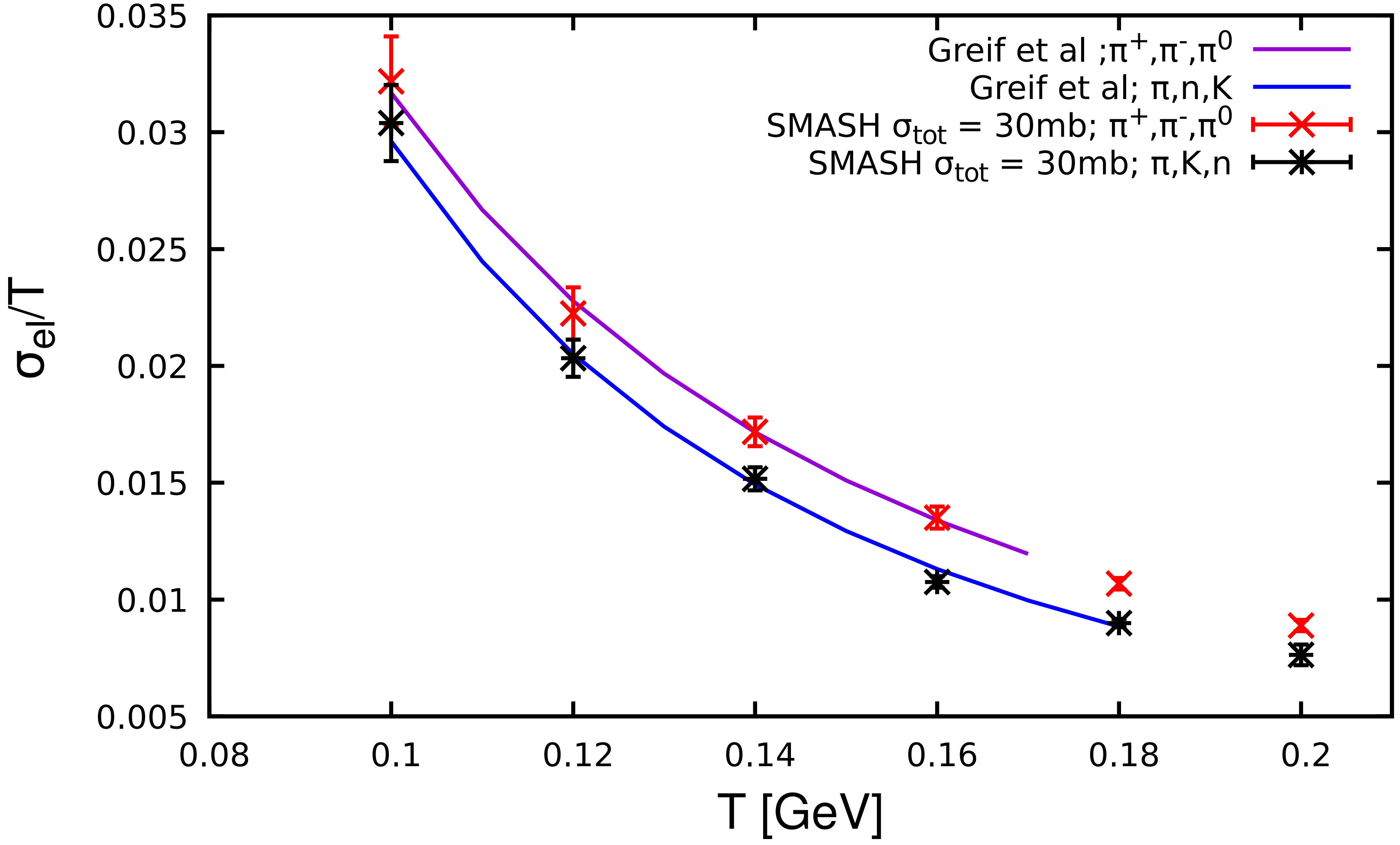
Backup slides

Hadron gas

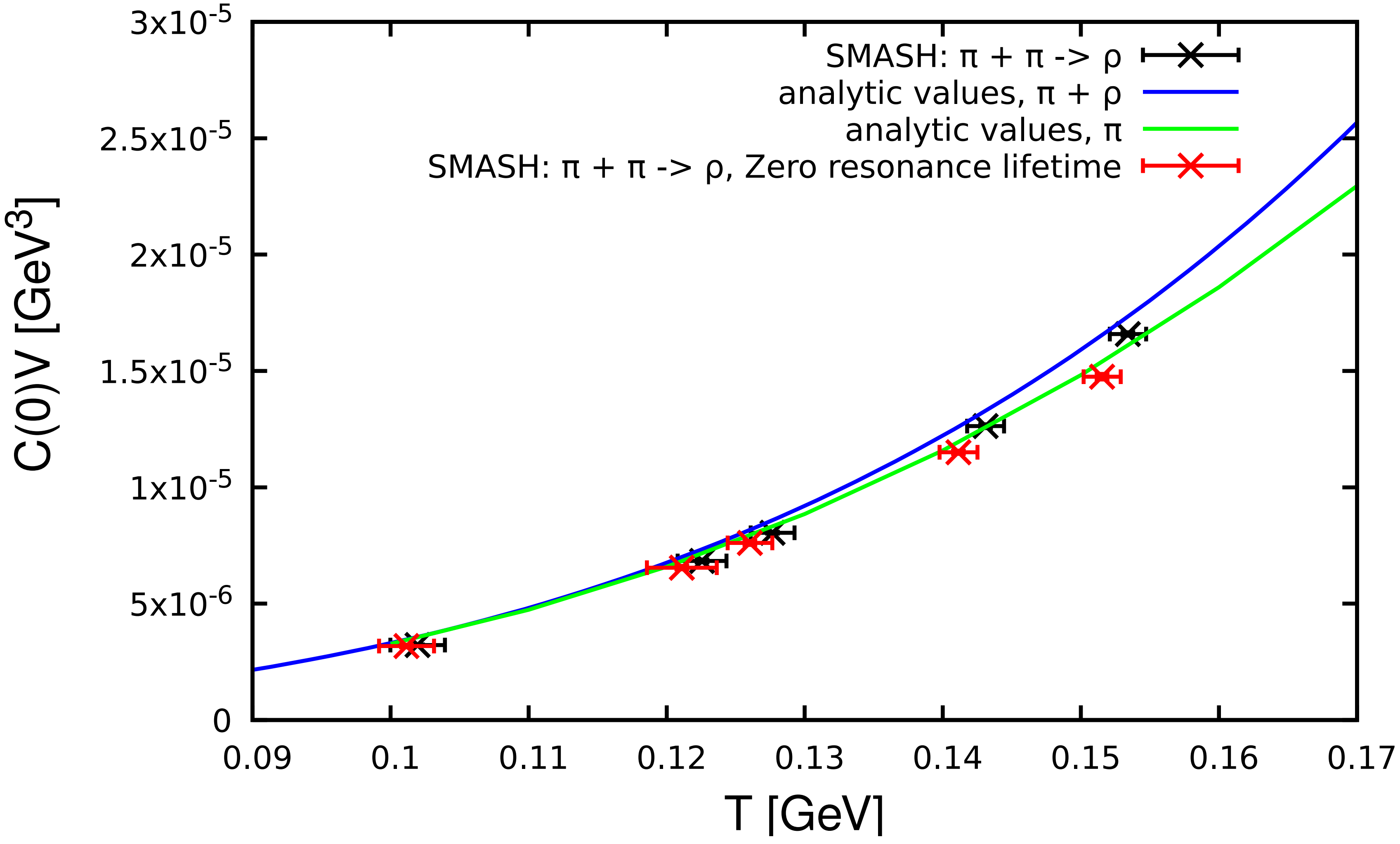
Interaction channels from Greif et al.: Phys. Rev. D **96**, 059902

	π^+	π^-	π^0	K^+	K^-	K^0	$\bar{K}^0$	p	n	$\bar{p}$	$\bar{n}$
π^+	10	ρ	ρ	10	10	$K^\star$	10	Δ	10	10	Δ
π^-		10	ρ	$K^\star$	10	10	$K^\star$	10	Δ	Δ	10
π^0			5	$K^\star$	10	$K^\star$	$K^\star$	Δ	Δ	Δ	Δ
K^+				10	10	10	50	6	10	20	10
K^-					10	50	10	20	10	6	10
K^0						10	50	6	6	20	20
$\bar{K}^0$							10	8	20	6	6
p								20	20	100	20
n									20	20	100
$\bar{p}$										10	10
$\bar{n}$											10

Electric conductivity



C(0) $\pi\rho$ - gas


SMASH: π + π -> ρ
analytic values, π + ρ
analytic values, π
SMASH: π + π -> ρ, Zero resonance lifetime
C(0)V [GeV3]
T [GeV]
3x10-5
2.5x10-5
2x10-5
1.5x10-5
1x10-5
5x10-6
0
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17