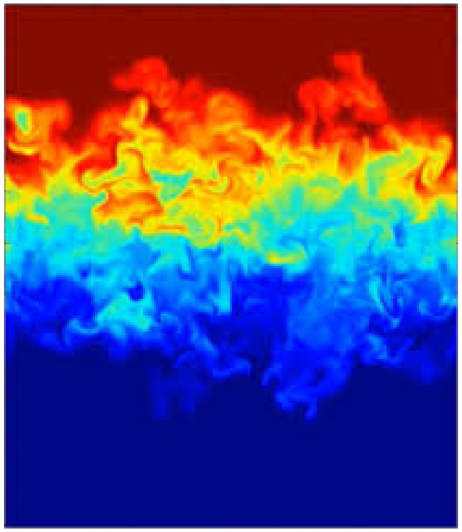
Andreas Schmitt

Mathematical Sciences and STAG Research Centre
University of Southampton
Southampton SO17 1BJ, United Kingdom

Hydrodynamical instabilities in two-component (super)fluids

without dissipation: A. Haber, A. Schmitt and S. Stetina, PRD 93, 025011 (2016)
with dissipation: N. Andersson, A. Schmitt, work in progress

- counterpropagating (super)fluids in neutron stars and the laboratory
- energetic and dynamical instabilities and effect of dissipation on them
- analogue of gravitational r-mode (instability) in two-fluid system

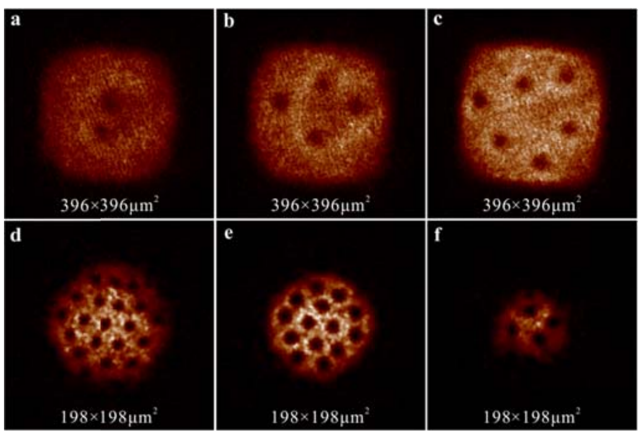


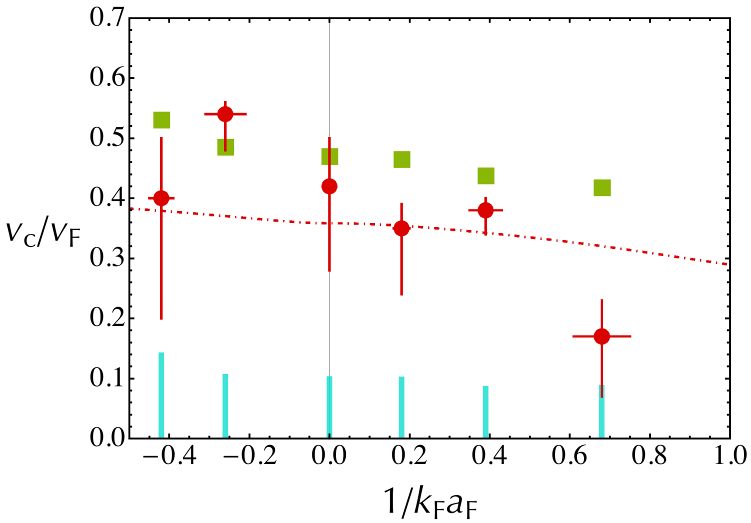
D. Livescu *et al.*, JPhCS, 318, 082007 (2011)

• Two-component superfluids in the laboratory

- Bose-Fermi gas mixtures
  ^{6}Li-^{7}Li superfluid I. Ferrier-Barbut *et al.*, Science 345, 1035 (2014)

- Simultaneous vortex lattices in ^{6}Li-^{41}K
  Yao, X.-c. *et al.*, PRL 117, 145301 (2016)





- Critical counterflow velocity in ^{6}Li-^{7}Li (comparing data to $v_{\rm two-stream} = v_{\rm L,1} + v_{\rm L,2}$)
  Delehaye, M. *et al.* PRL 115, 265303 (2015)

- ^{3}He-^{4}He mixtures: difficult to create experimentally
  J. Tuoriniemi, *et al.*, JLTP 129, 531 (2002)

- **Two-component (super)fluids in compact stars**
  transport in neutron star (recent review) A. Schmitt and P. Shternin, arXiv:1711.06520

- neutron superfluid/proton superconductor
  M. A. Alpar, S. A. Langer and J. A. Sauls, Astrophys. J. 282, 533 (1984)
  A. Haber and A. Schmitt, PRD 95, 116016 (2017)
  nucleon-hyperon (multi-fluid): M.E. Gusakov, E.M. Kantor, P. Haensel, PRC 79, 055806 (2009)

- neutron superfluid in ion lattice
  - two-stream instability as trigger for collective vortex unpinning
    $\rightarrow$ pulsar glitches N. Andersson, G.L. Comer, R. Prix, PRL 90, 091101 (2003)
  - Landau and dynamical instabilities of BEC in optical lattice
    B. Wu and Q. Niu, PRA 64, 061603 (2001)

- CFL-K^0 quark matter
  P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)
  D. B. Kaplan and S. Reddy, PRD 65, 054042 (2002)

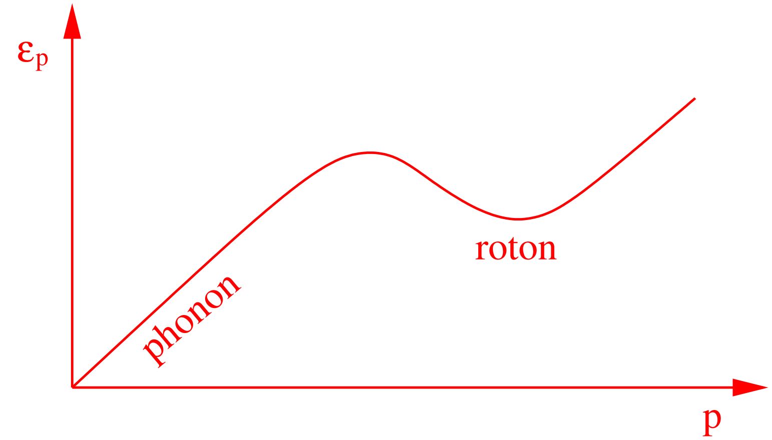
• Superfluid at $T > 0$ is a two-fluid system

London, Tisza (1938); Landau (1941)

relativistic: Khalatnikov, Lebedev (1982); Carter (1989)

from field theory: M. G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)

- "superfluid component": condensate, carries no entropy
- "normal component": excitations (Goldstone mode), carries entropy



- hydrodynamic equations ⇒ two sound modes

1st sound	2nd sound
in-phase oscillation (primarily) density wave	out-of-phase oscillation (primarily) entropy wave

(two-fluid picture also explains thermomechanical effect, "viscosity paradox", etc.)

- **$U(1) \times U(1)$ superfluid: setup**
  A. Haber, A. Schmitt, S. Stetina, PRD 93, 025011 (2016)

Lagrangian: $\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_I$

$$\mathcal{L}_i = \partial_\mu \varphi_i \partial^\mu \varphi_i^* - m_i^2 |\varphi_i|^2 - \lambda_i |\varphi_i|^4$$

entrainment coupling: $\mathcal{L}_I = -g(\varphi_1 \varphi_2 \partial_\mu \varphi_1^* \partial^\mu \varphi_2^* + \text{c.c.})$

(non-entrainment coupling: $\mathcal{L}_I = -h|\varphi_1|^2|\varphi_2|^2$)

- introduce chemical potentials $\partial_0 \to \partial_0 - i\mu_i$
- work at $T = 0$
- condensates $\langle \varphi_i \rangle = \rho_i e^{i\psi_i}$

- $U(1) \times U(1)$ **superfluid: setup**
  A. Haber, A. Schmitt, S. Stetina, PRD 93, 025011 (2016)

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$$\text{entrainment coupling: } \mathcal{L}_I = -g(\varphi_1 \varphi_2 \partial_\mu \varphi_1^* \partial^\mu \varphi_2^* + \text{c.c.})$$

- superfluid four-velocities $v_i^\mu = \partial^\mu \psi_i / p_i$
  (p_i chemical potentials in fluid rest frame)

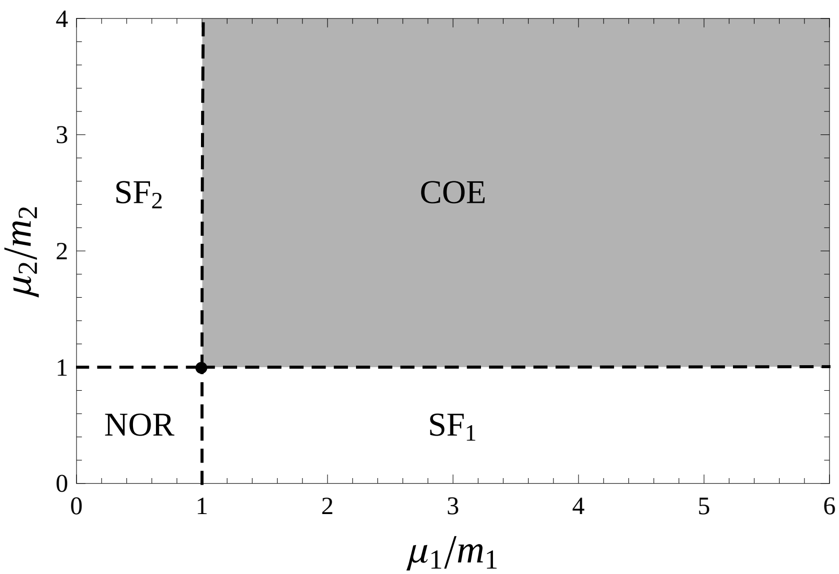
- conserved currents with entrainment

$$j_1^\mu = \rho_1^2 \left(\partial^\mu \psi_1 + \frac{g}{2} \rho_2^2 \partial^\mu \psi_2 \right)$$

$$j_2^\mu = \rho_2^2 \left(\partial^\mu \psi_2 + \frac{g}{2} \rho_1^2 \partial^\mu \psi_1 \right)$$

• Thermodynamics without superflow

$g = 0$



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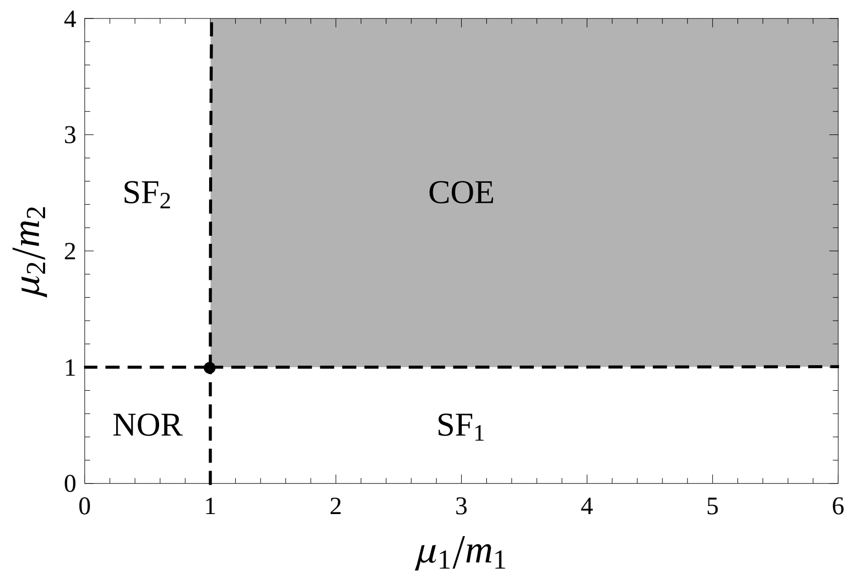
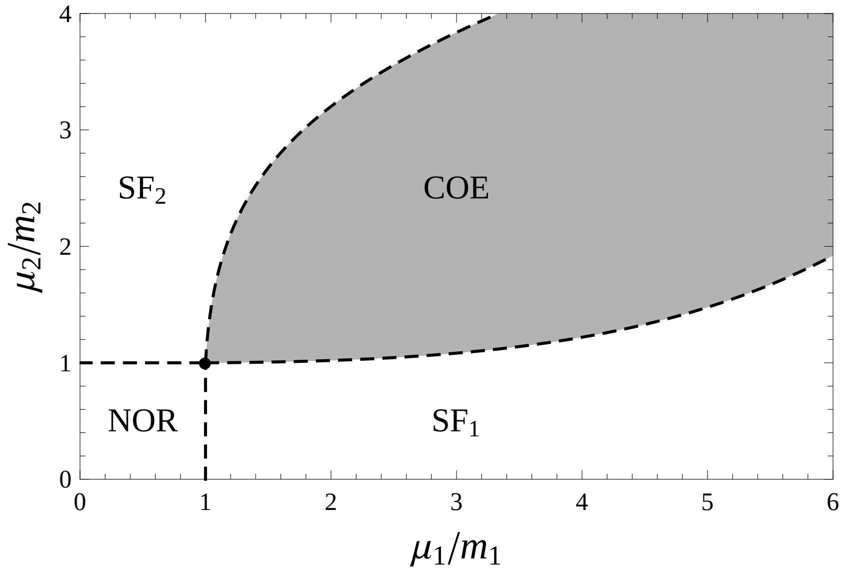
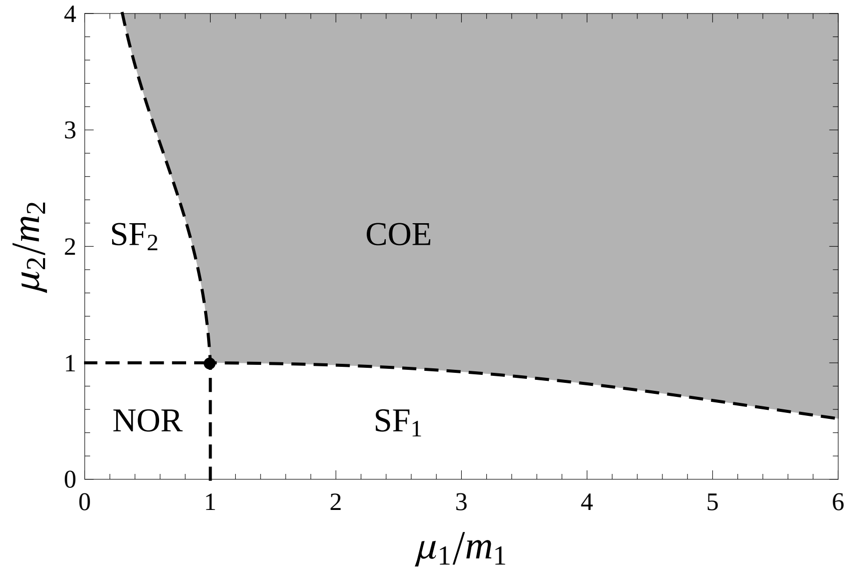


- COE: both superfluids coexist, $U(1) \times U(1) \to 1$
- SF_1, SF_2: only one superfluid, $U(1) \times U(1) \to U(1)$ or $U(1)$
- NOR: normal phase, no superfluid, $U(1) \times U(1)$ intact

• Thermodynamics without superflow

$g < 0$ $\qquad\qquad$ $g > 0$





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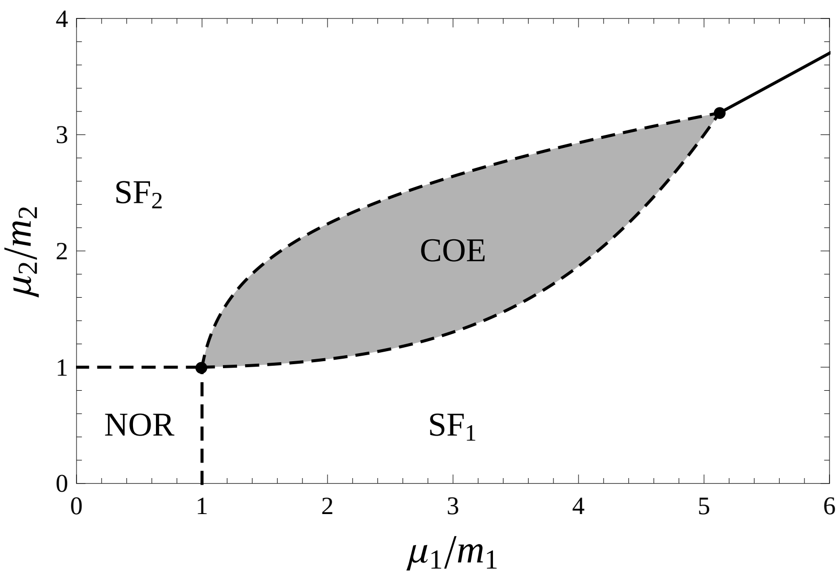

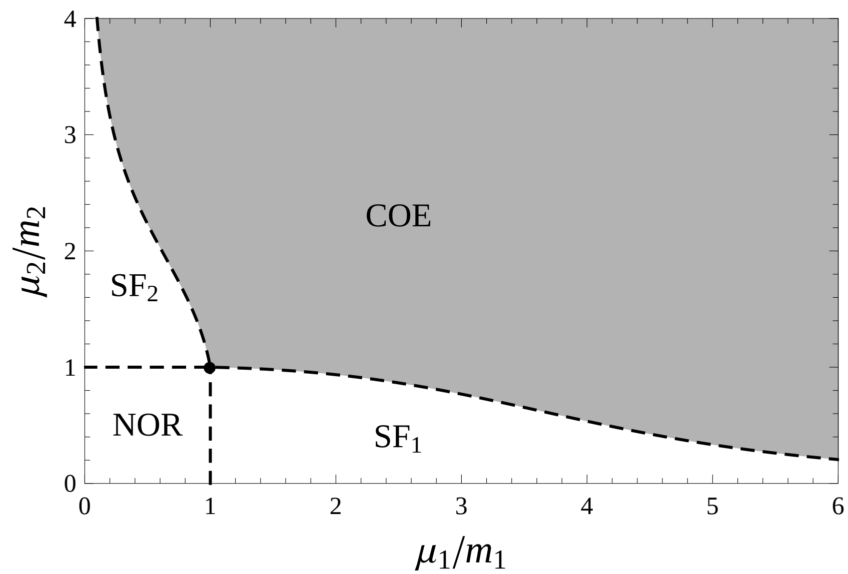

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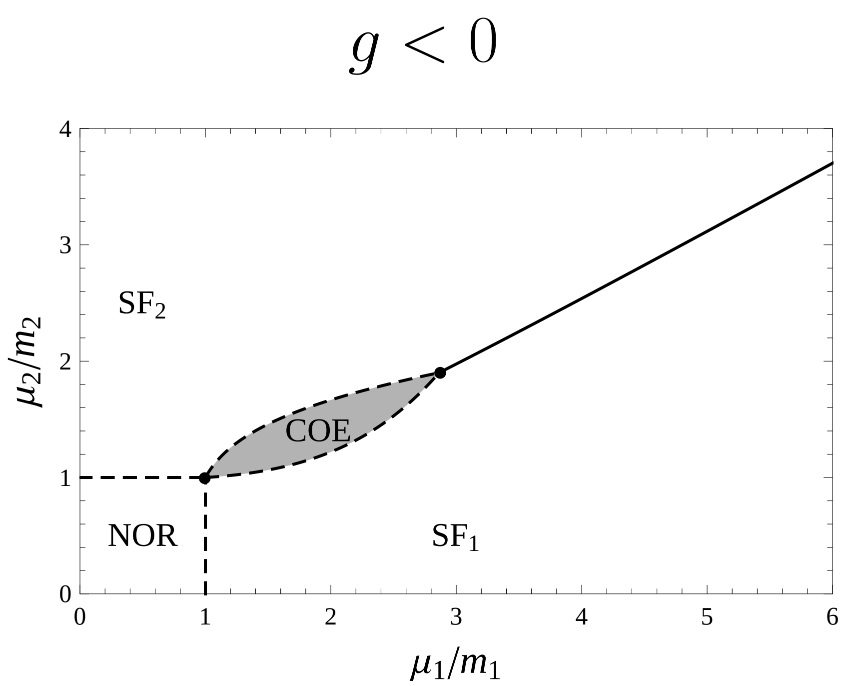

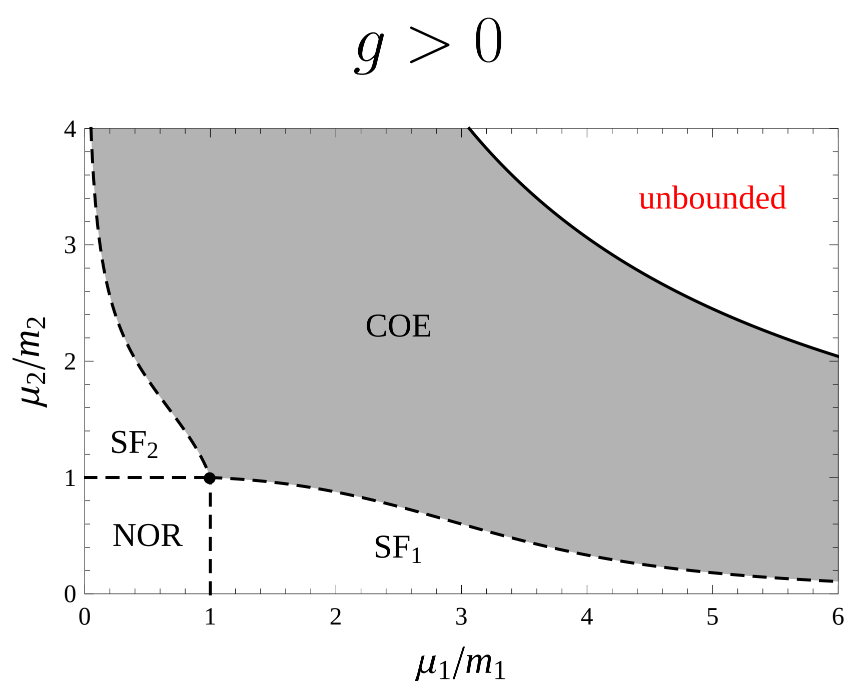

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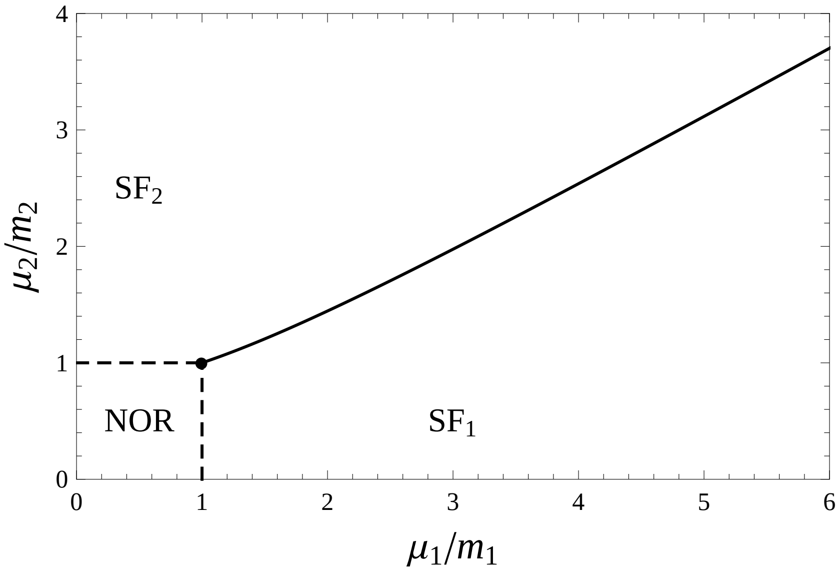

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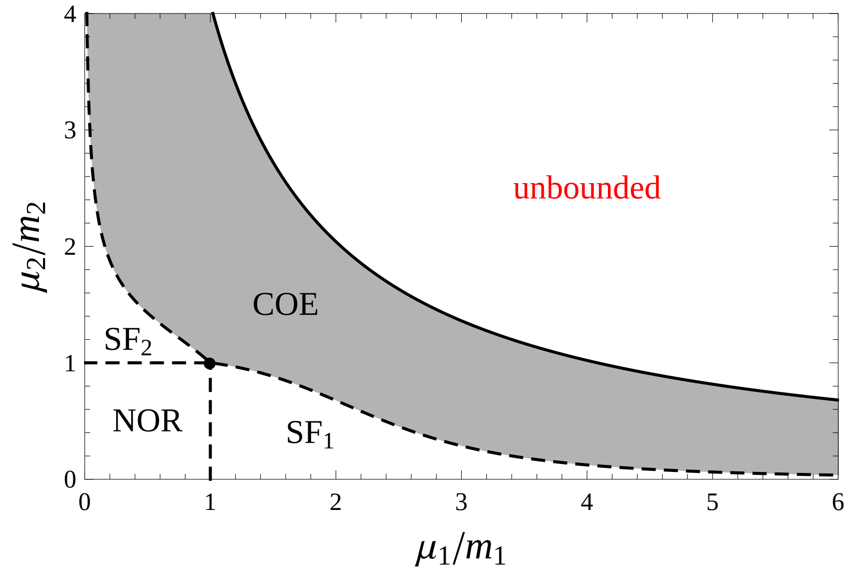

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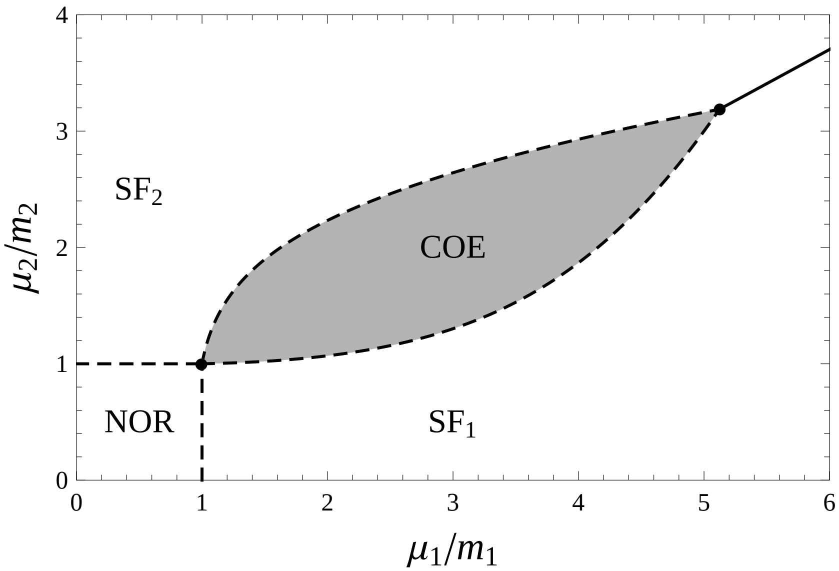
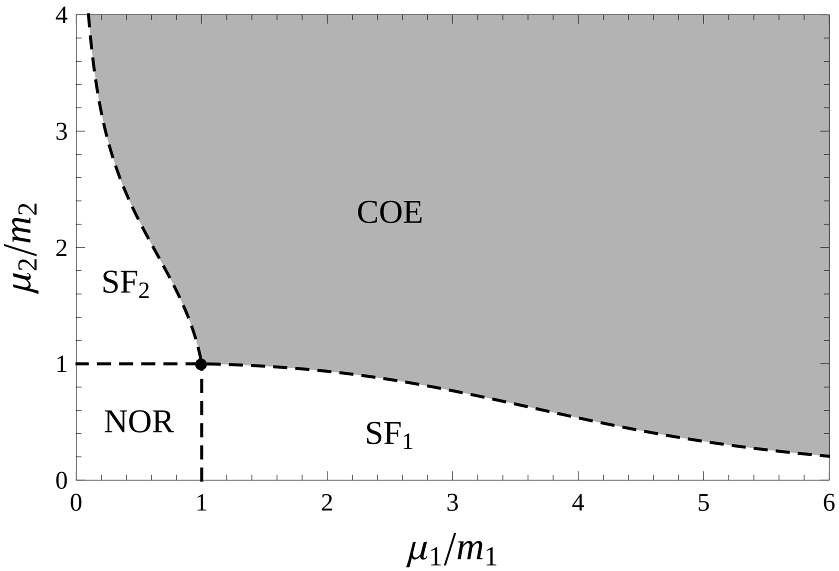
• Thermodynamics with (homogeneous) superflow

- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2

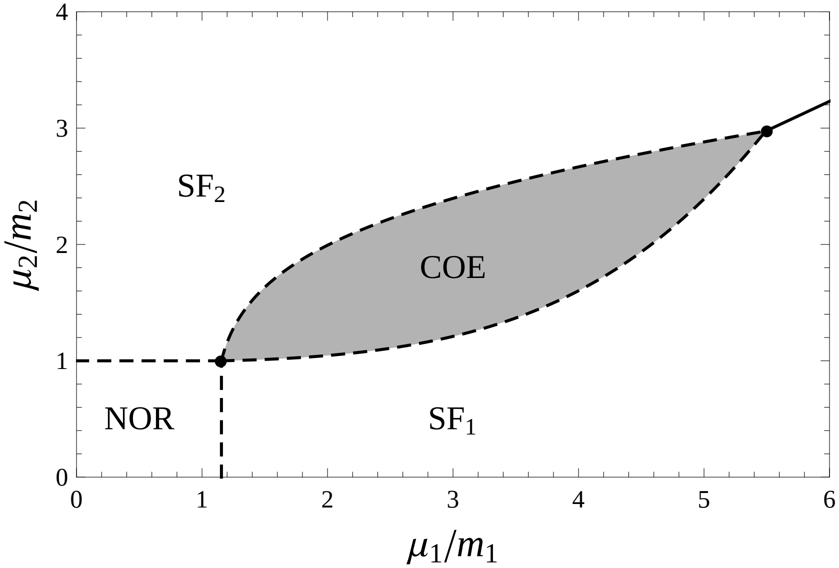


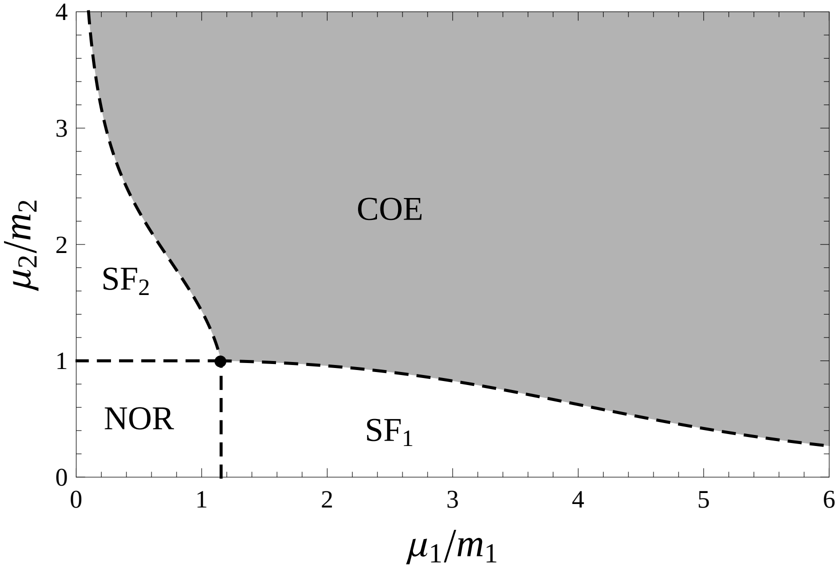


$$v_1 = 0$$

• Thermodynamics with (homogeneous) superflow

- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2

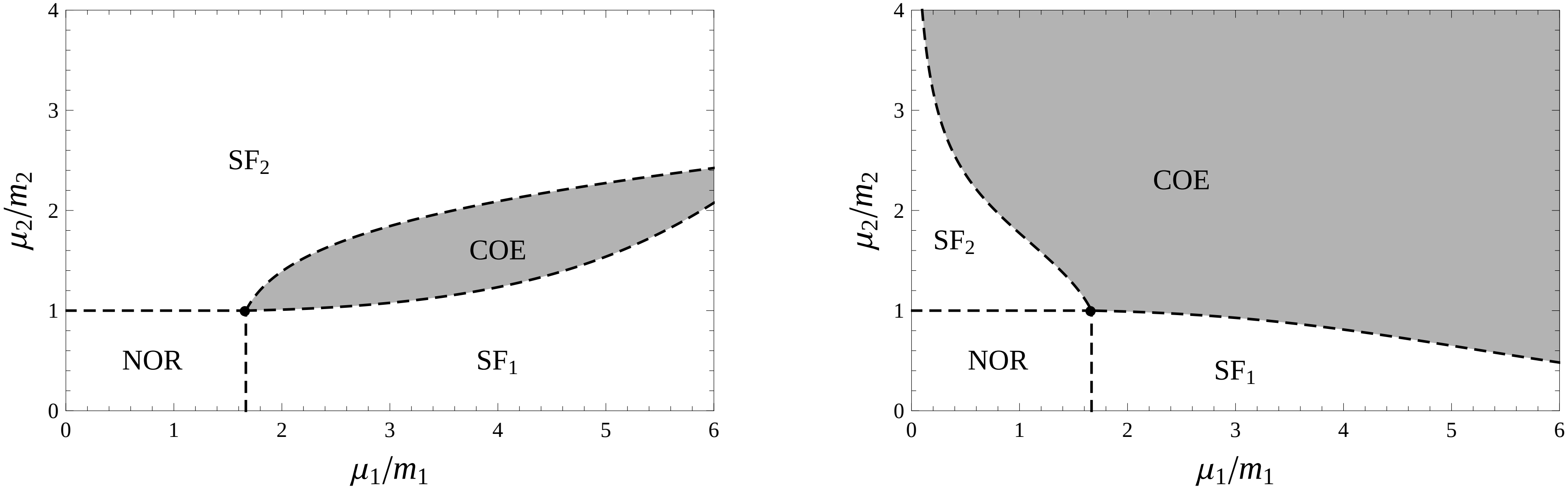




$$v_1 = 0.5$$

• Thermodynamics with (homogeneous) superflow

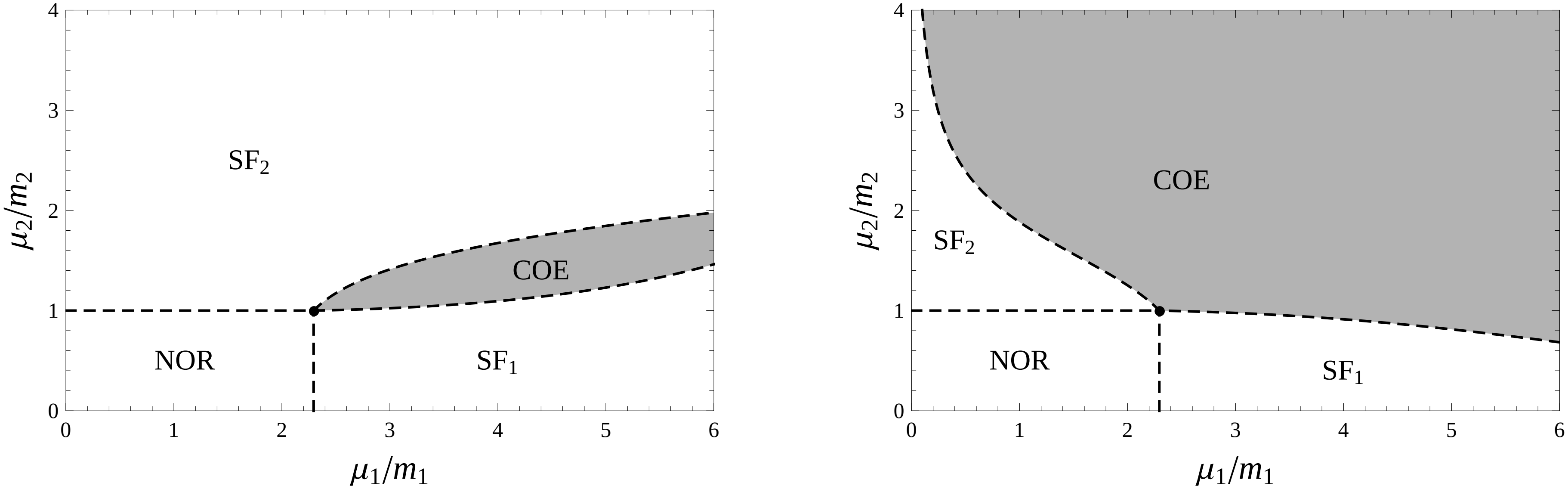
- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2



$$v_1 = 0.8$$

• Thermodynamics with (homogeneous) superflow

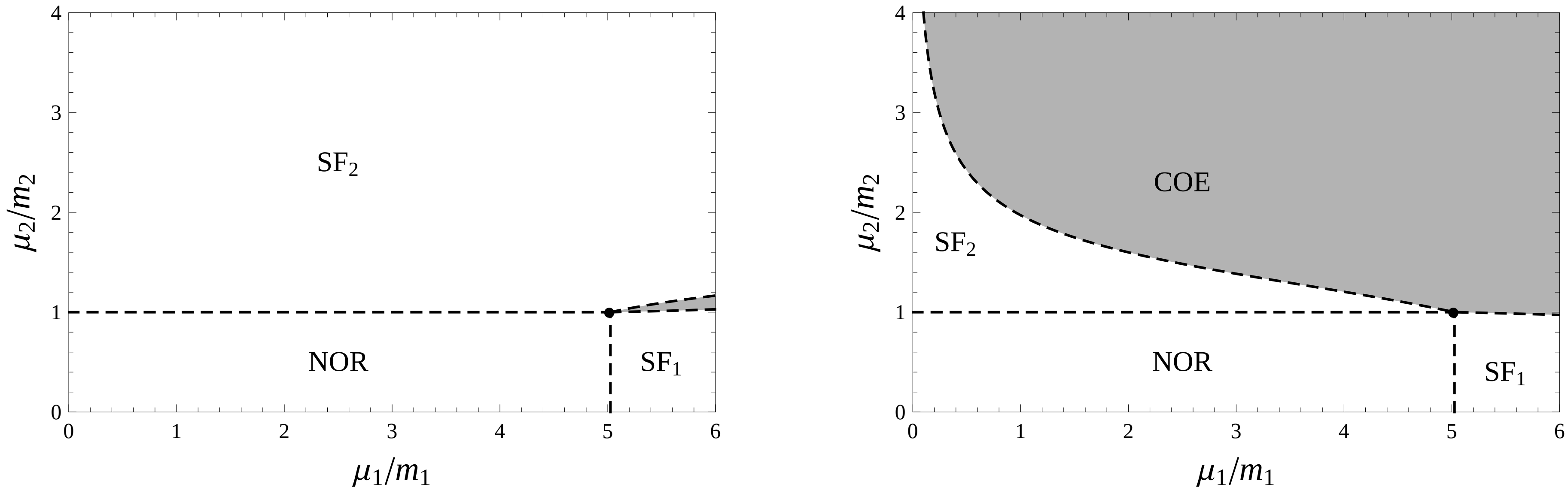
- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2



$$v_1 = 0.9$$

• Thermodynamics with (homogeneous) superflow

- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2



$$v_1 = 0.98$$

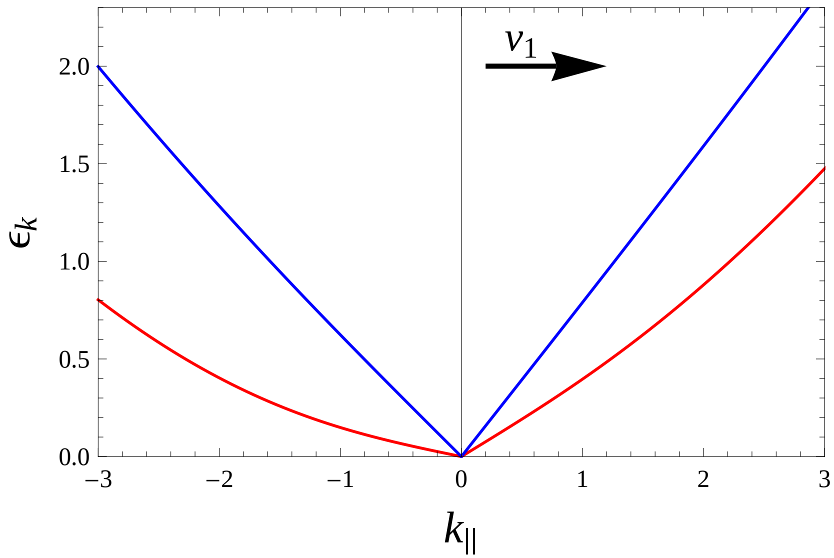
• Excitations and sound modes

- excitations = poles of (tree-level) propagator

- 2 Goldstone modes

  $\epsilon_{i,k} = c_i(\theta)k + d_i(\theta)k^3 + \ldots$

  (+ 2 massive modes)



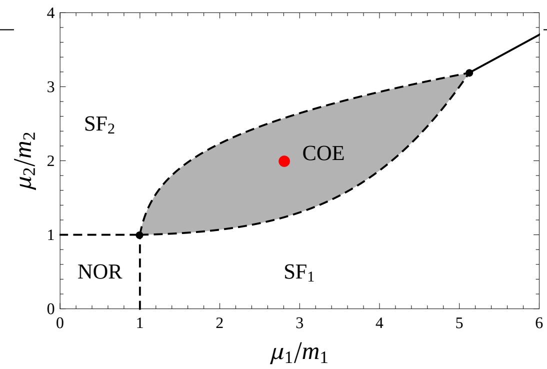
- alternatively: wave equations from (linearized) hydro

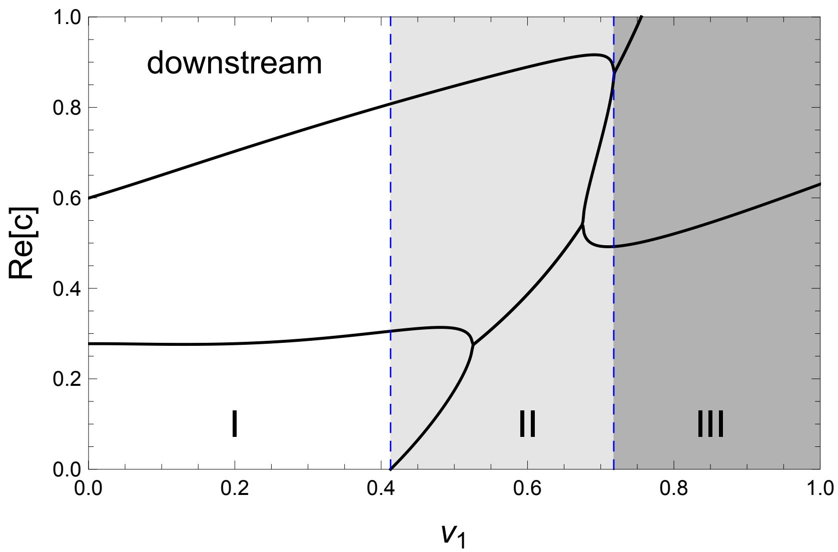
$$\partial_\mu j_1^\mu = 0 \, , \qquad \partial_\mu j_2^\mu = 0 \, , \qquad \partial_\mu T^{\mu\nu} = 0$$

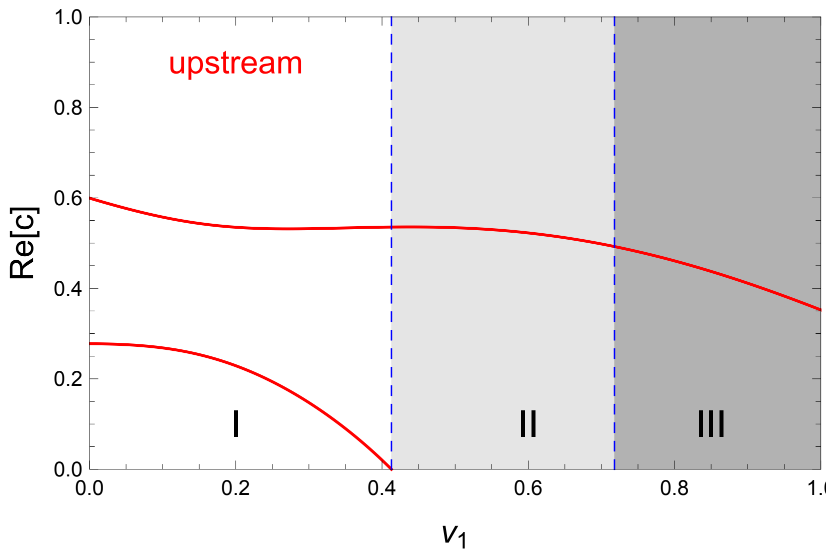
- 2 "first sounds" with sound velocities $c_i(\theta)$

  ($T > 0$: speeds of first and second sound in general different from Goldstone mode!)

• Instabilities with superflow

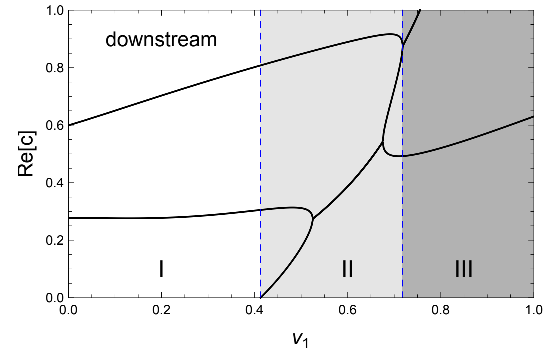


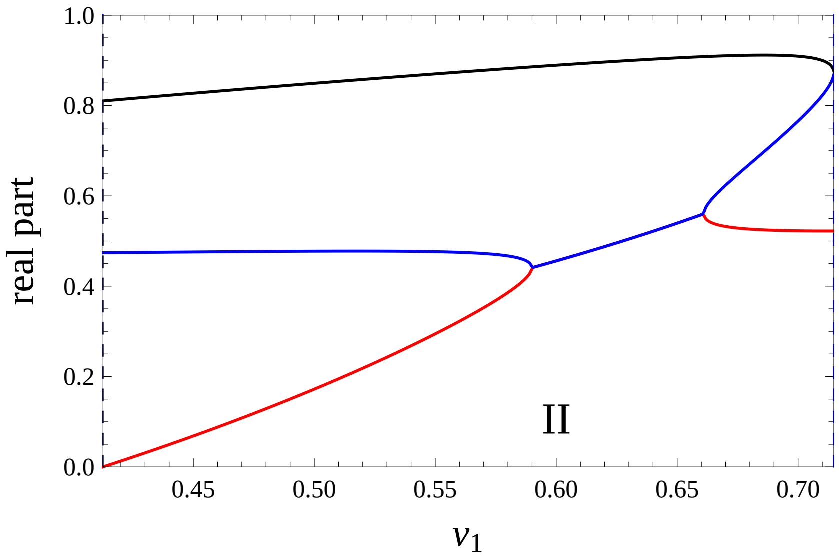


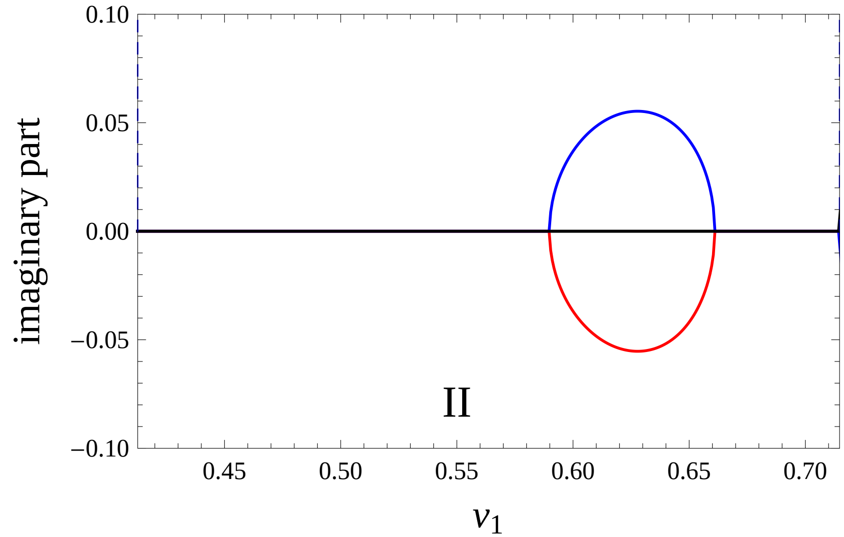


- region I: stable
- region III: SF_2 preferred

- **Two-stream instability**







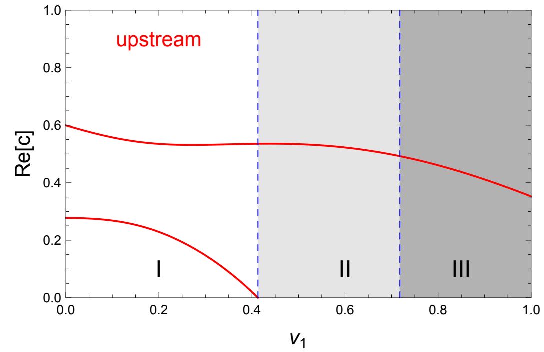
- complex sound speeds → one mode damped, one mode explodes

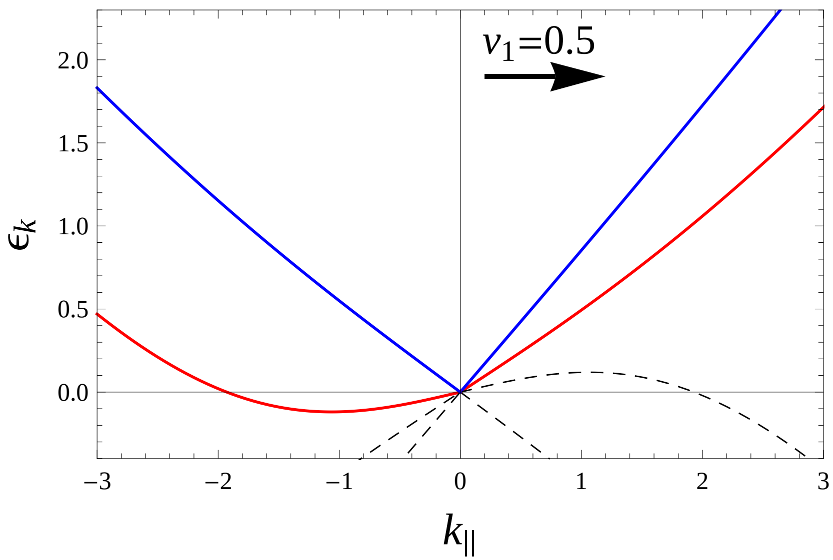
  plasma physics: O. Buneman, Phys.Rev. 115, 503 (1959); D.T. Farley, PRL 10, 279 (1963)

  general two-fluid system: L. Samuelsson *et al.* Gen. Rel. Grav. 42, 413 (2010)

  atomic gases: M. Abad, A. Recati, S. Stringari, F. Chevy, EPJD 69, 126 (2015)

• Landau's critical velocity





- negative energies in Goldstone dispersion $\epsilon_k(\vec{v}) < 0$
- Landau's original argument

$$\epsilon_k - \vec{k} \cdot \vec{v} < 0$$

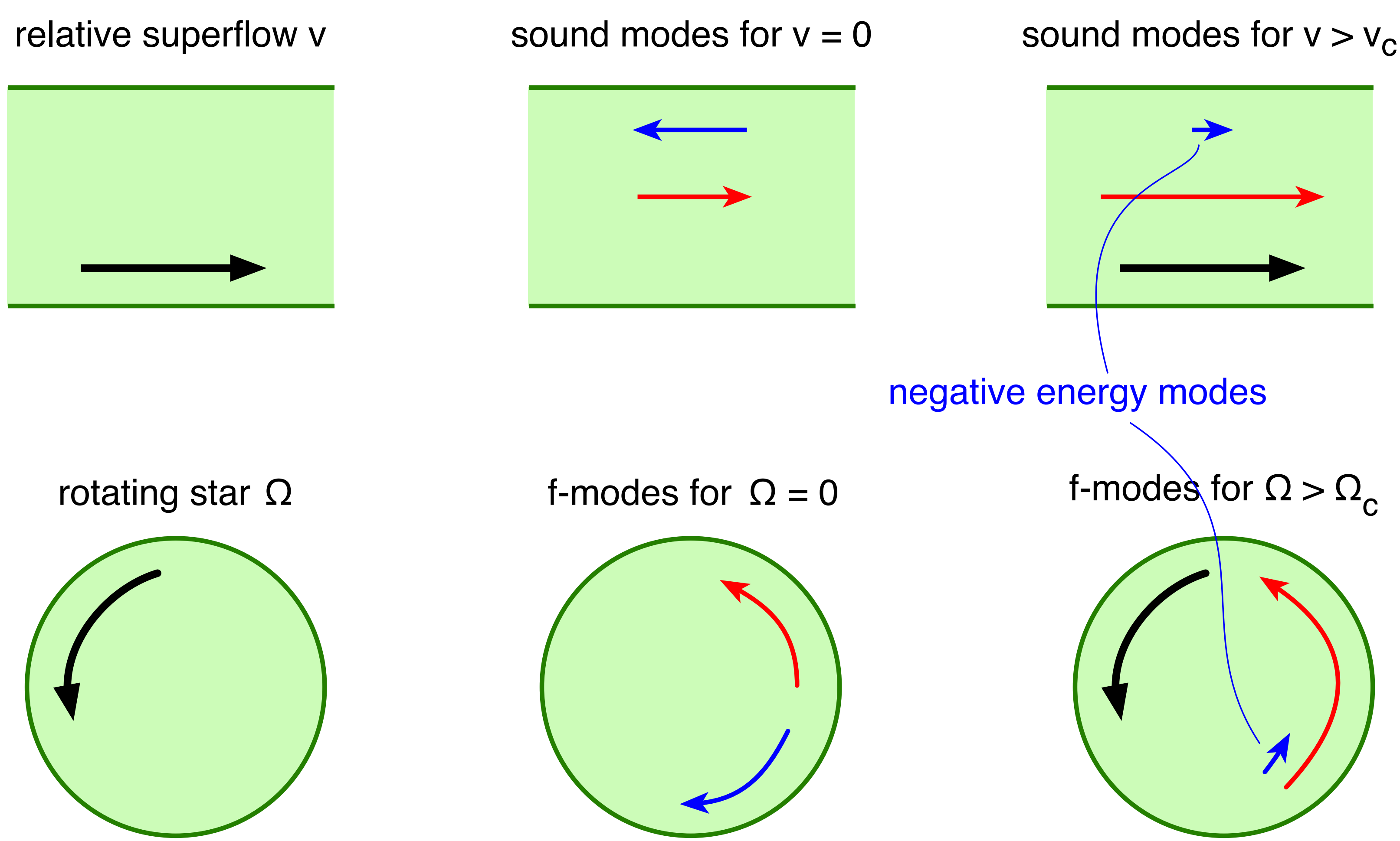
(for a single fluid)

• Two qualitatively different instabilities

“energetic instability” (Landau)
vs.
“dynamical instability” (two-stream)

- dynamical instability always occurs ”after” energetic instability
  A. Haber, A. Schmitt and S. Stetina, PRD 93, 025011 (2016)
- does energetic instability suggest new (inhomogeneous) ground state?
- if (at least) one of the fluids is a normal fluid:
  what is the effect of dissipation on these instabilities?
  → next part of talk

• Energetic instability: analogy to star pulsations



(note difference to r-modes, which only exist in rotating star, and have $\Omega_c = 0$)

- **General picture**

(I) fluid (star, superfluid, ...) with
propagating modes (sound modes, f-modes, ...)
+
(II) second rest frame
(non-rotating frame, second fluid, walls of a capillary, ...)

- if relative (angular) velocity between (I) and (II) is sufficiently large to flip direction of propagating mode $\rightarrow$ energetic ("secular") instability

- negative energy mode can become exponentially growing mode if (angular) momentum is exchanged (gravitational waves, dissipation, interaction with the walls of the capillary...)

• Dynamical instability from dissipation: setup (p. 1/2)

N. Andersson, A. Schmitt, work in progress

- consider two-fluid hydrodynamics with one fluid being dissipative

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal}(v_1, v_2) + T^{\mu\nu}_{\rm diss}(v_1)$$

- ideal two-fluid stress-energy tensor

$$T^{\mu\nu}_{\rm ideal} = j_1^\mu p_1^\nu + j_2^\mu p_2^\nu - g^{\mu\nu} P$$

with pressure $P(p_1^2, p_2^2, p_1 \cdot p_2)$

- four-currents

$$j_1^\mu = \mathcal{B}_1 p_1^\mu + \mathcal{A} p_2^\mu$$

$$j_2^\mu = \mathcal{A} p_1^\mu + \mathcal{B}_2 p_2^\mu$$

- conjugate momentum p^μ such that

$$j^\mu = \frac{\partial P}{\partial p_\mu}$$

and fluid velocity $v^\mu = p^\mu / p$

- entrainment coupling $\mathcal{A}$

$$\mathcal{A} = \frac{\partial P}{\partial (p_1 \cdot p_2)^2}, \quad \mathcal{B}_i = 2\frac{\partial P}{\partial p_i^2}$$

- density coupling through $\frac{\partial^2 P}{\partial p_1^2 \partial p_2^2}$

• Dynamical instability from dissipation: setup (p. 2/2)

N. Andersson, A. Schmitt, work in progress

- consider two-fluid hydrodynamics with one fluid being dissipative

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal}(v_1, v_2) + T^{\mu\nu}_{\rm diss}(v_1)$$

- dissipative terms (first order)

$$T^{\mu\nu}_{\rm diss}(v) = \eta\Delta^{\mu\gamma}\Delta^{\nu\delta}\left(\partial_\delta v_\gamma + \partial_\gamma v_\delta - \frac{2}{3}g_{\gamma\delta}\partial\cdot v\right) + \zeta\Delta^{\mu\nu}\partial\cdot v$$

$$+\kappa(\Delta^{\mu\gamma}v^\nu + \Delta^{\nu\gamma}v^\mu)[\partial_\gamma T - T(v\cdot\partial)v_\gamma]$$

with $\Delta^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$, shear viscosity η, bulk viscosity ζ, heat conductivity κ

- not unlike a superfluid: condensate (non-diss.) and entropy fluid (diss.)
- neglect additional dissipative coefficients due to two-fluid nature (e.g., ζ_1, ζ_2, ζ_3 in superfluid)
- equation of state unspecified; most of following results generic

• Computing sound modes

- conservation equations $\partial_\mu j_1^\mu = \partial_\mu j_2^\mu = \partial_\mu T^{\mu\nu} = 0$
- linearize in harmonic fluctuations $\delta\vec{v}\, e^{i(\omega t - \vec{k}\cdot\vec{x})}$, ...
- superfluid: constraint on fluctuations, $\omega\delta(\mu\vec{v}) = \vec{k}\delta\mu$ (since both μ and $\vec{v}$ are related to phase of condensate)
- compute sound modes

$$\omega(k) = ck + i\Gamma k^2 + \ldots$$

  with sound speed c, sound attenuation Γ
- dynamical instability for $\mathrm{Im}\,[c] < 0$ or $\mathrm{Re}\,[\Gamma] < 0$
- without dissipation: equivalent to above results from poles of propagator

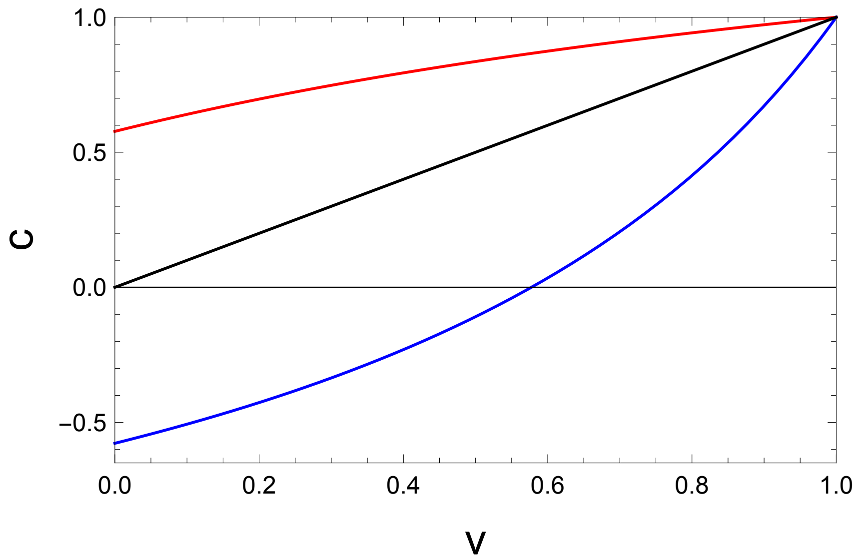
• Warm-up: single-fluid modes (page 1/2)

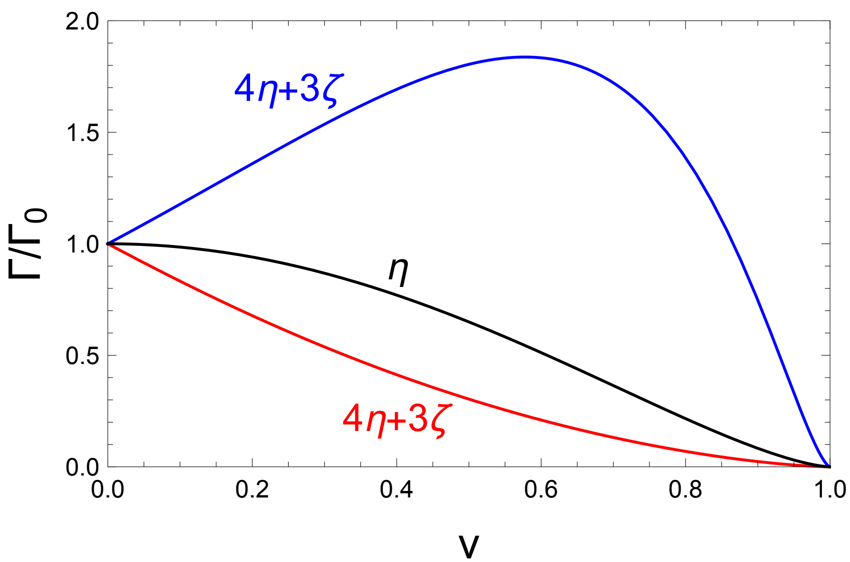
1	k	k^2	$T=0$	[1]
0	$+c$	$i\frac{4\eta+3\zeta}{6w}+i\kappa[\ldots]$	✓	L_4^+
0	$-c$	$i\frac{4\eta+3\zeta}{6w}+i\kappa[\ldots]$	✓	L_4^-
$-\frac{iw}{\gamma(\eta v^2+\kappa T)}$	$v\cos\theta\frac{\eta(2-v^2)+\kappa T}{\eta v^2+\kappa T}$	$-\frac{i\eta(1-v^2\cos^2\theta)}{\gamma w}$	✓	T_3
0	$v\cos\theta$	$\frac{i\eta(1-v^2\cos^2\theta)}{\gamma w}$	✓	T_4
$-\frac{iw}{\kappa T}$	0	$-i\frac{4\eta+3\zeta}{3w}-i\kappa[\ldots]$	–	L_2
0	$v\cos\theta$	$\frac{i\kappa n^2(1-v^2\cos^2\theta)}{\gamma T\left[n^2\frac{\partial s}{\partial T}+s^2\frac{\partial n}{\partial\mu}-ns\left(\frac{\partial n}{\partial T}+\frac{\partial s}{\partial\mu}\right)\right]}$	–	L_5

- blue modes only propagate for nonzero velocity v
- red modes show unphysical instabilities in first-order hydrodynamics
  [1] W. A. Hiscock and L. Lindblom, PRD 31, 725 (1985)

  $\rightarrow$ consider modes that are stable without counterflow:
  $T=0$ & work in rest frame of dissipative fluid

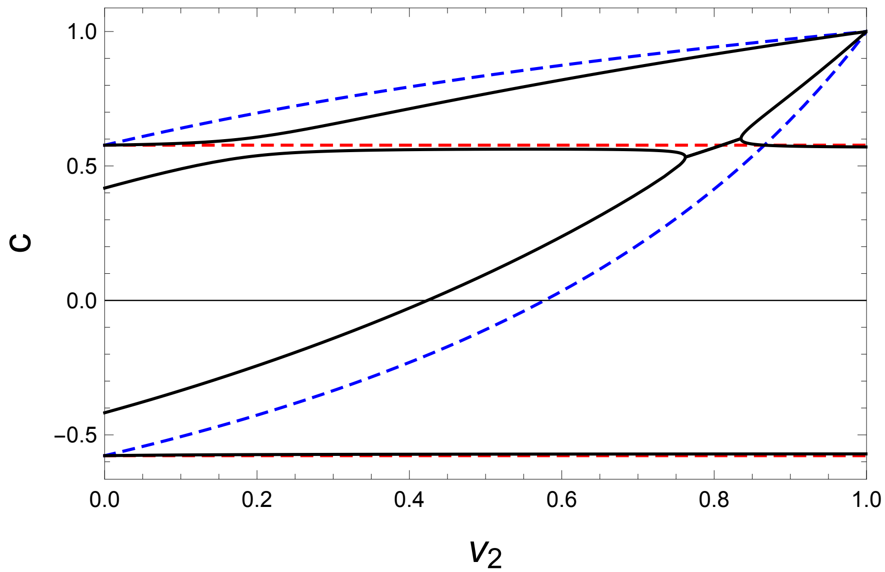
• Warm-up: single-fluid modes (page 2/2)

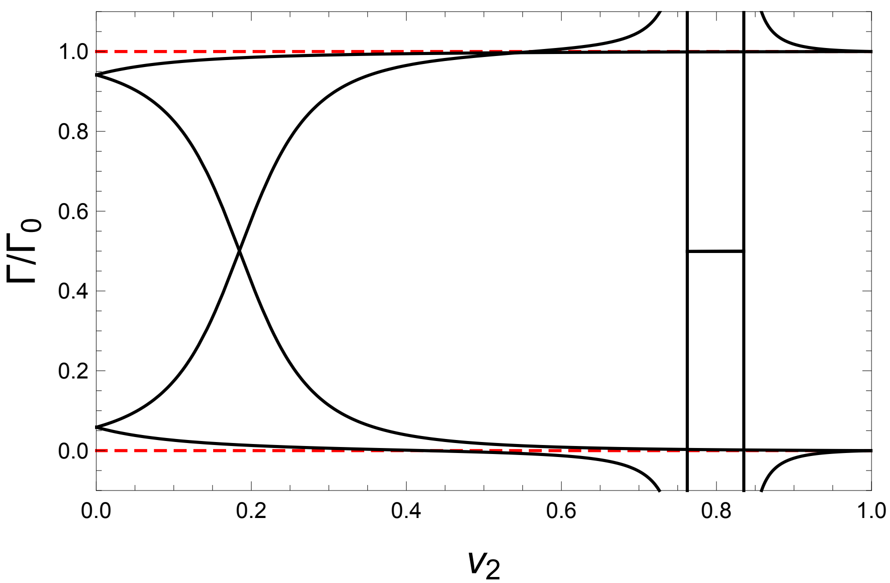




- upstream and downstream sound speeds for $c_0 = \frac{1}{\sqrt{3}}$
- upstream mode "flips over" at $v = c$
- transverse mode $c = v \cos\theta$
- all modes damped ($\Gamma > 0$) by bulk and shear viscosity (here $T = 0$)
- no dynamical instabilities

• Two-fluid modes

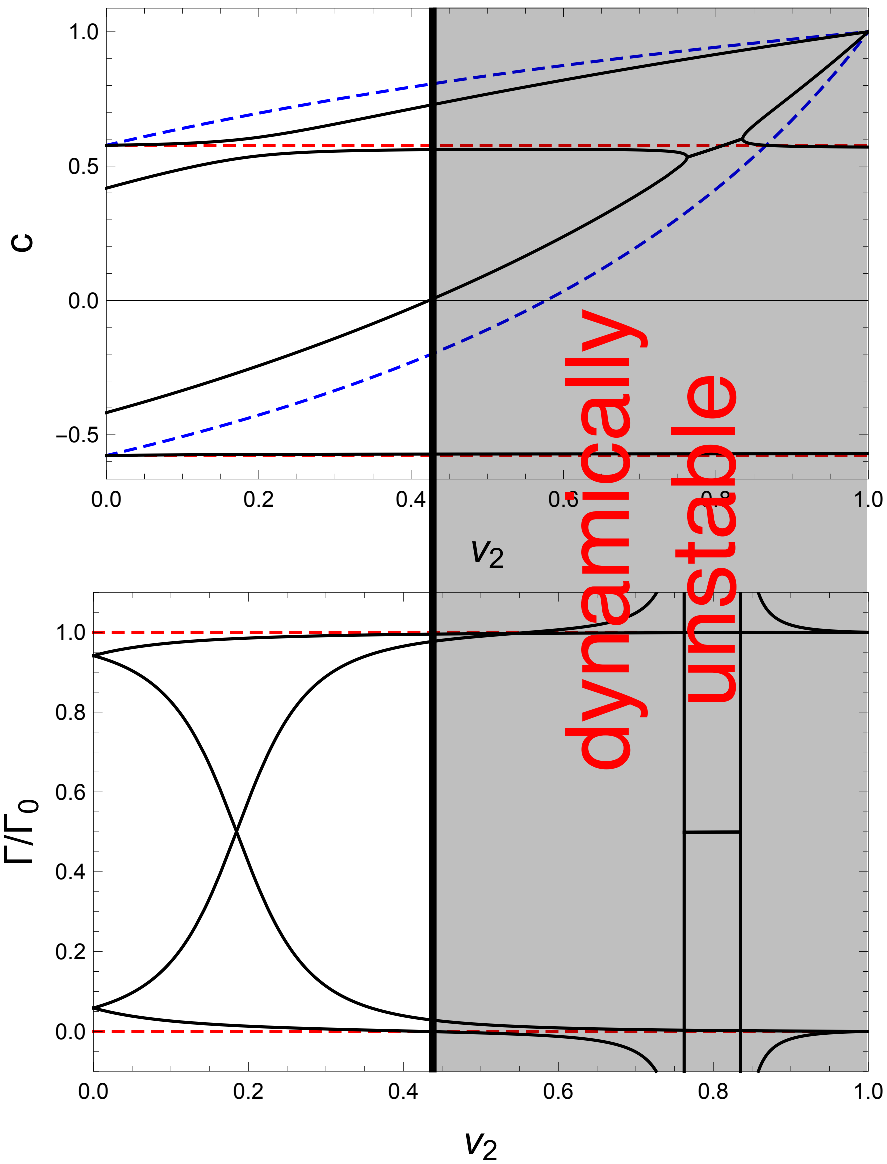




- "super-normal" system
- $T = 0$
- work in rest frame of normal fluid
- density coupling (no entrainment)

Dissipation renders energetic instability dynamical

• Two-fluid modes



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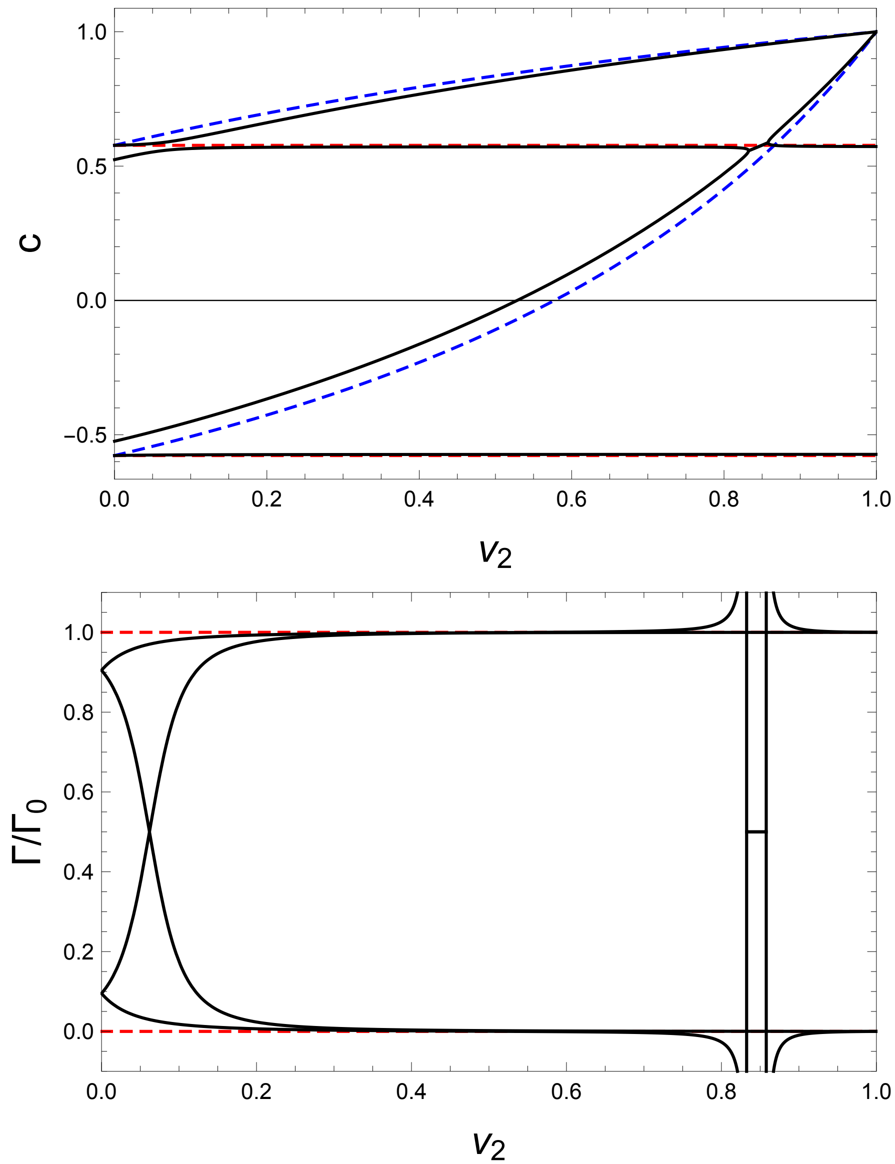
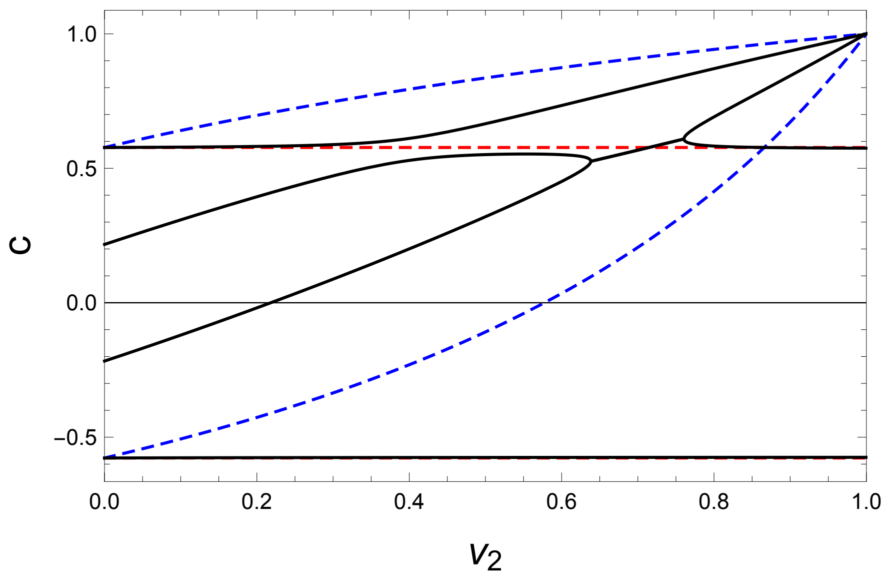


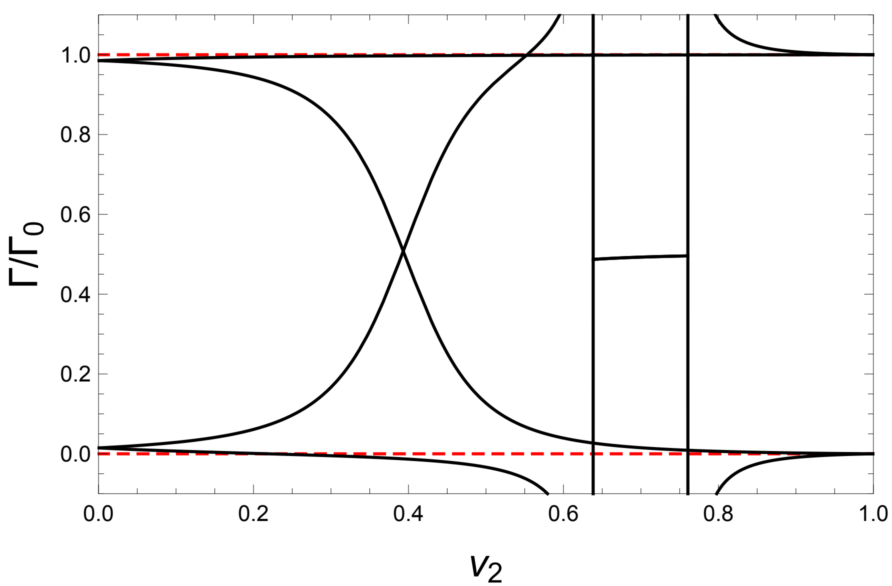
• small coupling

> Dissipation renders energetic instability dynamical

• Two-fluid modes

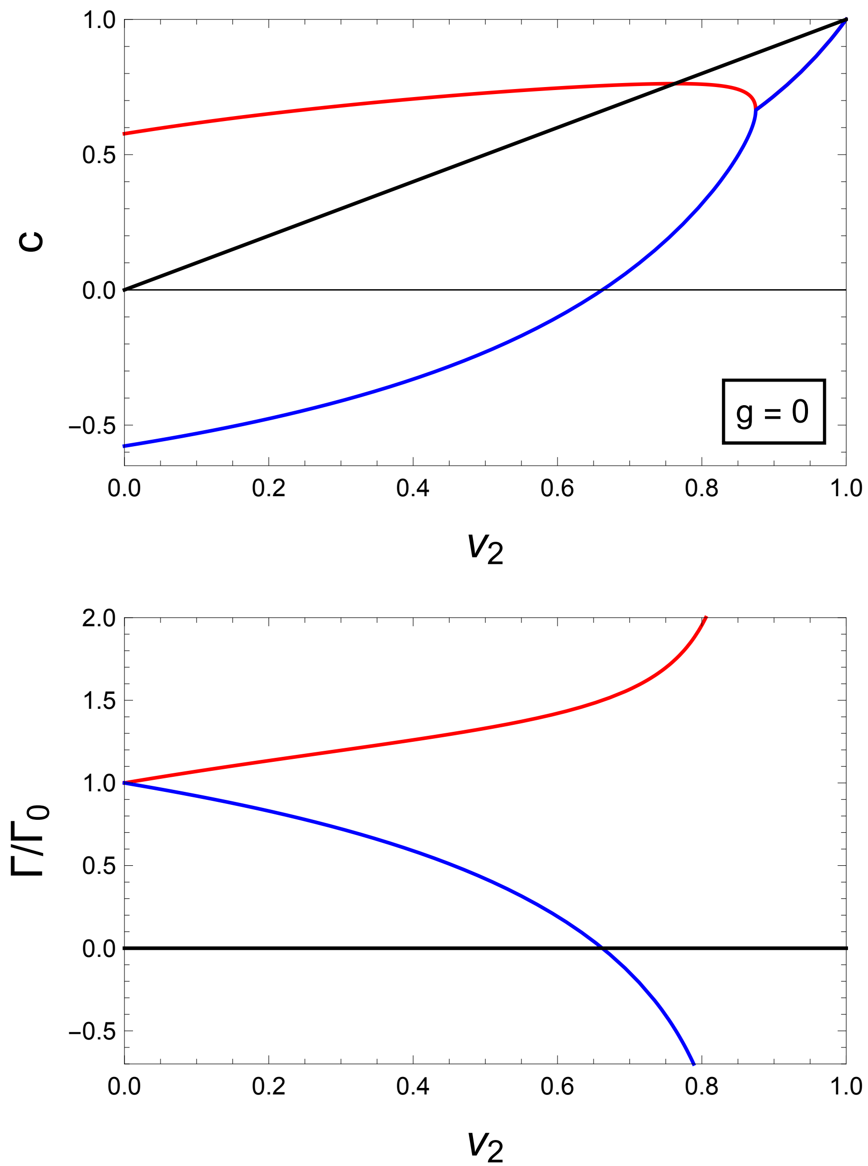


• large coupling



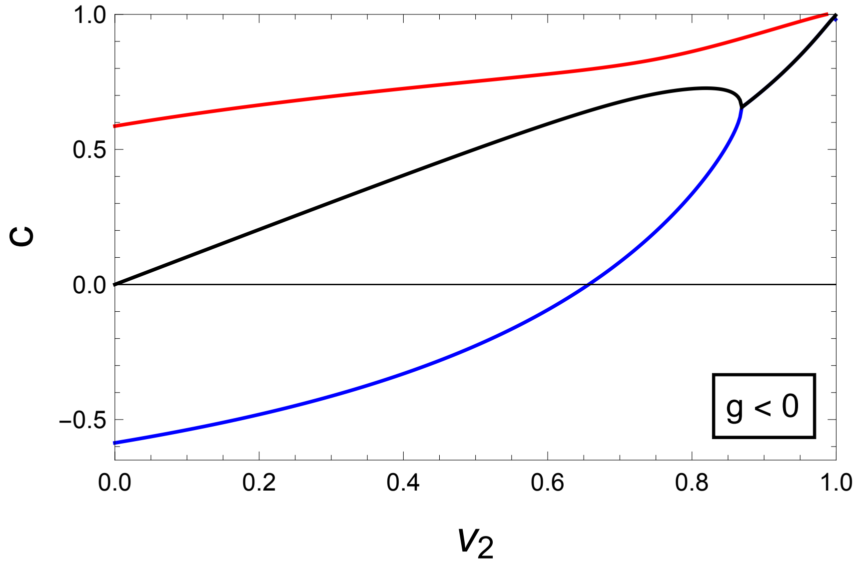
Dissipation renders energetic instability dynamical

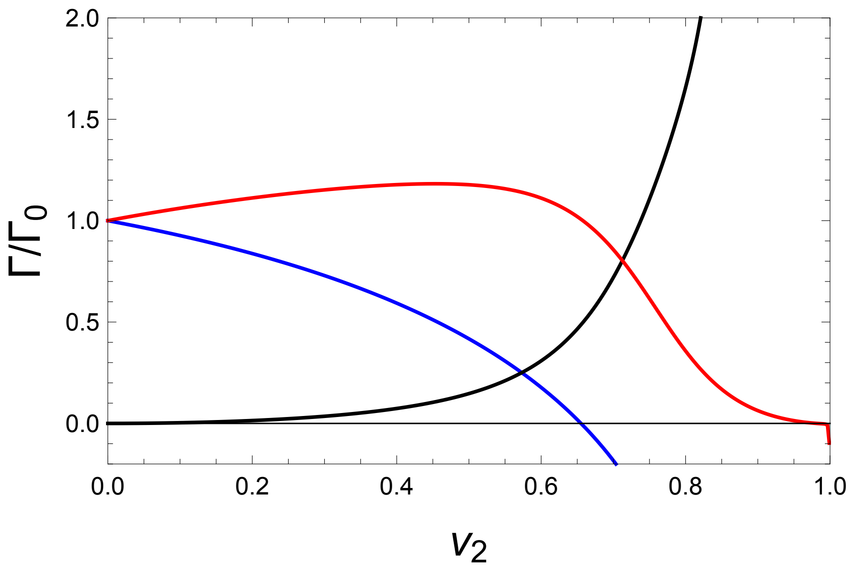
• Two-fluid modes: analogue of r-mode



- "normal-normal" system $\rightarrow$ no constraint on fluctuations
- consider $\delta\mu_1 = 0$ modes
- no entrainment $g = 0$: transverse mode $c = v_2 \cos\theta$ decouples

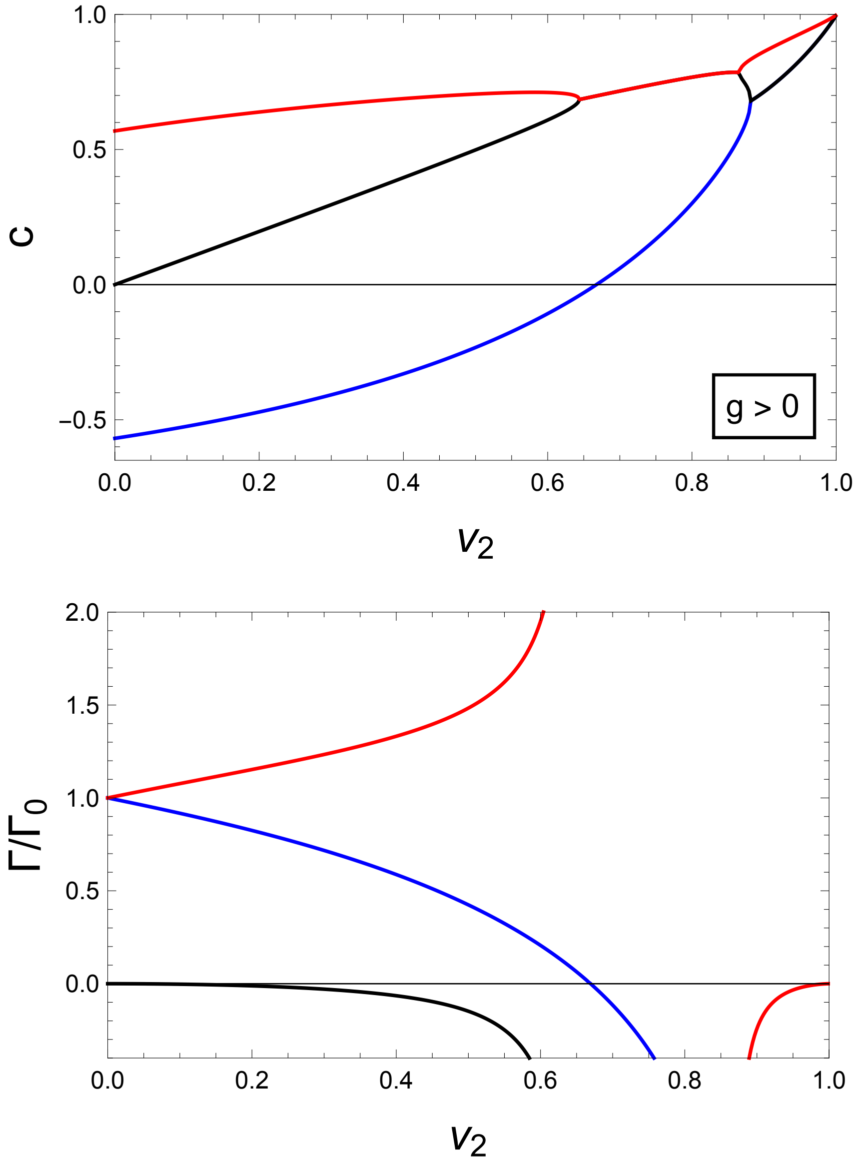
• Two-fluid modes: analogue of r-mode





- "normal-normal" system
  $\rightarrow$ no constraint on fluctuations
- consider $\delta\mu_1 = 0$ modes
- $g < 0$: "avoided crossing",
  no additional instability

• Two-fluid modes: analogue of r-mode



- "normal-normal" system
  $\to$ no constraint on fluctuations
- consider $\delta\mu_1 = 0$ modes
- $g > 0$: dynamical instability for arbitrarily small counterflow (just like r-mode instability!)

$$\Gamma \simeq -\frac{g(4\eta + 3\zeta)}{3\mu_1\mu_2}(v_2\cos\theta)^2$$

• Summary

- (relativistic) two-component superfluids exist in compact stars and can be created in the laboratory
- they show hydrodynamic instabilities (energetic and dynamical) in the presence of a sufficiently large relative flow
- energetic instability becomes dynamical through dissipation

• Outlook

- understand/exploit analogy to r-mode instability
- apply to superfluid at $T > 0$
  $\rightarrow$ use microscopic equation of state
- instabilities as trigger mechanism
  for vortex unpinning in pulsar glitches?
- go to second-order hydrodynamics
  $\rightarrow$ avoid identifying unphysical instabilities
- inhomogeneous ("striped") phases in response
  to energetic instability? effect on dynamical instability?
- time evolution of two-stream instability?