Diffusion of conserved charges in relativistic heavy ion collisions

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Why is Diffusion Important?

High Baryon Density → BARYON DIFFUSION

Low Baryon Density

Baryon Density Gradient

HIC
The Evolution in (3+1)-Viscous Hydro


0-5% Au-Au collision at 19.7 GeV

Hydrodynamical evolution after 7.5 fm

Hadronization on next slide
Why is Diffusion Important

- At Low-Energy Heavy Ion Collisions (e.g. RHIC BES): diffusion could have great impact on dynamical evolution

Vanishing baryon diffusion coefficient

Large baryon diffusion coefficient

Description of Diffusion

- Early dynamical evolution of HIC modeled in Relativistic Dissipative Fluid-Dynamics
- For large evolution times: Navier-Stokes Theory applicable
- One conserved charge (q):

Particle 4-current: \( N_q^\mu = n_0 u^\mu + \kappa_q \nabla^\mu \left( \mu_q / T \right) \)

\( j_q^\mu \): Net-charge diffusion current

Net-charge diffusion coefficient

Gradient in thermal potential
\sim \text{Gradient in net-charge density}
Description of Diffusion

• In multi-component system with multiple conserved charges: particles can have any combination of charges (e.g. proton: electric and baryon charge)

• Net-charge diffusion currents effect each other

\[
\begin{pmatrix}
    j^\mu_B \\
    j^\mu_Q \\
    j^\mu_S
\end{pmatrix} =
\begin{pmatrix}
    \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\
    \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\
    \kappa_{SB} & \kappa_{SQ} & \kappa_{SS}
\end{pmatrix} \cdot
\begin{pmatrix}
    \nabla^\mu \alpha_B \\
    \nabla^\mu \alpha_Q \\
    \nabla^\mu \alpha_S
\end{pmatrix}
\]

Are the off-diagonal coefficients important?

Off-diagonal coefficients: Gradients of given charge can effect diffusion currents of other charges
The Chapman-Enskog Expansion

- Assume dilute Boltzmann gas with $N_s$ particle species and conserved baryon, strangeness and electric charge close to local equilibrium $\rightarrow$ describe with kinetic theory

\[ f^i_k = f^i_{0,k} + \epsilon f^i_{1,k} + O(\epsilon^2) \]

- Local equilibrium term
- Book-keeping parameter counts gradients

- Neglect non-linear contributions $\rightarrow$ Navier-Stokes limit
The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

\[ k_i^\mu \partial_\mu f^i_k = - \sum_{j=1}^{N_s} C_{ij} [f^i_k] \]

Chapman-Enskog expansion

\[ \varepsilon k_i^\mu \partial_\mu (f^i_{0k} + \varepsilon f^i_{0k}) \approx \varepsilon k_i^\mu \partial_\mu f^i_{0k} = -\varepsilon \sum_{j=1}^{N_s} C_{ij} [f^i_{1k}] \]

With linearized collision term:

\[ \sum_{j=1}^{N_s} C_{ij} [f^i_{1k}] = \sum_{j=1}^{N_s} \gamma_{ij} \int dK'_j dP_i dP_j W_{kk' \rightarrow pp'}^{ij} f^i_{0k} f^j_{0k'} \left( \frac{f^i_{1k}}{f^i_{0k}} + \frac{f^j_{1k'}}{f^j_{0k'}} - \frac{f^i_{1p}}{f^i_{0p}} - \frac{f^i_{1p'}}{f^i_{0p'}} \right) \]

Transition rate: contains (isotropic) cross sections = information of microscopic interactions
The Chapman-Enskog Expansion

Evaluating derivatives leads to source equation for deviation $f_{1k}^i$

$$k_i^\mu \partial_\mu f_{0k}^i = - \sum_{j=1}^{N_s} C_{ij} [f_{1k}^i]$$

Sum over all conserved charges $\rightarrow$ coupling of diffusion currents

Gradient in thermal potential

L.H.S. of eq. $\sim$ force term due to gradients in particle density $\rightarrow$ Navier Stokes currents
The Chapman-Enskog Expansion

Diffusion currents in kinetic theory:

\[ j^\mu_q = \sum_{i=1}^{N_s} q_i \int dK \ k_i^{\langle \mu \rangle} f_{1k}^i = \sum_{q'} \kappa_{qq'} \nabla^\mu \left( \frac{\mu q'}{T} \right) \]

We want to calculate THIS

Navier-Stokes limit

In order to do so, we need to solve:

\[ \sum_{q \in \{B, S, Q\}} f_{0k}^i k_i^{\mu} \left( \frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla^\mu \left( \frac{\mu q}{T} \right) = -\sum_{j=1}^{N_s} C_{ij} [f_{1k}^i] \]
The Chapman-Enskog Expansion

\[ \sum_{q \in \{B,S,Q\}} f_{0k}^i k_i^\mu \left( \frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla \mu \left( \frac{\mu q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij} [f_{1k}^i] \]

Since collision term is linear in \( f_{1k}^i \) the solutions have the general form:

Scalar function in energy

\[ f_{1k}^i = \sum_q a_q^i k_i^\mu \nabla \mu \left( \frac{\mu q}{T} \right) \]

Expand coefficients in power series in energy:

\[ a_q^i = \sum_{m=0}^{\infty} a_{q,m}^i E_{ik}^m \]
The Chapman-Enskog Expansion

\[ \sum_{q \in \{B, S, Q\}} f_{0k}^{i} k_{i}^{\mu} \left( \frac{E_{ik} n_{q}}{\epsilon_{0} + P_{0}} - q_{i} \right) \nabla_{\mu} \left( \frac{\mu_{q}}{T} \right) = - \sum_{j=1}^{N_{s}} C_{ij} \left[ f_{1k}^{i} \right] \]

Truncate series at finite integer $M$ and calculate $n$-th moment of source equation $\rightarrow$ set of linear equations for expansion

Coefficients

Solutions of matrix equation $\rightarrow$ gives us $f_{1k}^{i}$

\[ \sum_{m=0}^{M} \sum_{j=1}^{N_{s}} \left( A_{nm}^{i} \delta^{ij} + C_{nm}^{ij} \right) a_{q,m}^{j} = b_{q,n}^{i} \]

moments of collision term $\rightarrow$ complicated integrals with information about microscopic interactions

Source term for diffusion
The Chapman-Enskog Expansion

\[ j_q^\mu = \sum_{i=1}^{N_s} q_i \int dK \ k_i^{(\mu)} f_{1k}^i = \sum_{q'} \kappa_{qq'} \nabla^\mu \left( \frac{\mu q'}{T} \right) \]

By comparing both sides we find:

\[ \kappa_{qq'} = \frac{1}{3} \sum_{i=1}^{N_s} q_i \sum_{m=0}^{M} a_{q',m}^i \int dK_i E_{ik}^m (m^2 - E_{ik}^2) f_{0k}^i \]

In our most detailed calculation: \( M = 1 \) and \( N_s = 19 \)
The Relaxation Time Approximation

Calculated for \( p, n, \bar{p}, \bar{n}, K, \pi \) gas (11 hadron species)

\[
\sum_{j=1}^{N_s} C_{ij} [f^i_{1k}] = -\frac{E_{ik}}{\tau} f^i_{1k}
\]

Relaxation time:

\[ \tau^{-1} = \frac{2}{3} n_{B,\text{tot}} \sigma_0 \]

Total baryon density

Constant cross section
Results

Hadronic resonance gas...

- Use 19 different, massive species: $\pi^{0,\pm}, K^{\pm,0,\bar{0}}, p, \bar{p}, n, \bar{n}, \Sigma^{0,\pm}, \bar{\Sigma}^{0,\pm}, \Lambda, \bar{\Lambda}$
- Isotropic cross sections

![Graph showing cross sections as a function of $\sqrt{s}$]

- Use PDG data
- Other cross sections: GiBUU, UrQMD or constant
Results

Simplified (conformal) QGP model...

- Use 7 massless species $u, \bar{u}, d, \bar{d}, s, \bar{s}, g$

- Simplified approach: Fix shear viscosity to express isotropic cross section in terms of temperature

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad \Rightarrow \quad \sigma_{tot} = \frac{0.716}{T^2}$$

Calculate diffusion coefficients for the hadron gas for $T < 160$ MeV and for higher temperatures in the simplified QGP model → phase transition area is **NOT** covered by our calculations
The diffusion matrix

\[
\begin{pmatrix}
\tilde{j}_B^\mu \\
\tilde{j}_Q^\mu \\
\tilde{j}_S^\mu
\end{pmatrix}
= 
\begin{pmatrix}
\kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\
\kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\
\kappa_{SB} & \kappa_{SQ} & \kappa_{SS}
\end{pmatrix}
\cdot
\begin{pmatrix}
\nabla^\mu \alpha_B \\
\nabla^\mu \alpha_Q \\
\nabla^\mu \alpha_S
\end{pmatrix}
\]

Diffusion matrix is symmetric! \(\Rightarrow\) Onsager Theorem holds
Baryon current

\[
\begin{pmatrix}
    \dot{j}_B^\mu \\
    \dot{j}_Q^\mu \\
    \dot{j}_S^\mu \\
\end{pmatrix}
= \begin{pmatrix}
    \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\
    \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\
    \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \\
\end{pmatrix} \cdot
\begin{pmatrix}
    \nabla^\mu \alpha_B \\
    \nabla^\mu \alpha_Q \\
    \nabla^\mu \alpha_S \\
\end{pmatrix}
\]

- Largest contribution
- Nearly constant at \( \mu_B = 600 \text{ MeV} \)
- So far only used coefficient

- Much smaller than others
- QGP-part vanishes at \( \mu_B = 0 \)
- Strong \( \mu_B \) dependence

- Negative contribution!
- Similar strength as \( \kappa_{BB} \)
- Could drastically reduce baryon current

\( \mu_B = 600 \text{ MeV} \)
\( \mu_B = 0 \)
Electric current

\[ j_\mu^Q = \kappa_{QB} \nabla_\mu \alpha_B + \kappa_{QQ} \nabla_\mu \alpha_Q + \kappa_{QS} \nabla_\mu \alpha_S \]

- Smaller than others
- QGP-part vanishes at \( \mu_B = 0 \)
- Strong \( \mu_B \) dependence
- \( \mu_B = 0 \) same as electric conductivity
- Only decreasing behavior in \( T \)
- QGP: strongest contribution
Strangeness current

\[ j_S^\mu = \kappa_{SB} \nabla^\mu \alpha_B + \kappa_{SQ} \nabla^\mu \alpha_Q + \kappa_{SS} \nabla^\mu \alpha_S \]

- **Negative contribution**
- Could also drastically reduce strange currents

- 1 Magnitude smaller than \( \kappa_{SS} \)
- Charged Kaons contribute to electric currents (see \( \kappa_{QQ} \))

- By far most important contribution
Conclusion

• First calculation of complete diffusion matrix of baryon, electric and strangeness charges in Navier-Stokes limit with first order Chapman-Enskog expansion
• Classical hadron gas with realistic isotropic cross sections and simple conformal QGP model were used

• HRG: dependence of coefficients on temperature and baryo-chemical potential
• Strong coupling of all gradients to (almost) all currents → large off-diagonal coefficients
• Suggestion: Off-diagonal terms should not be neglected!
• Can be used in (hydro) models
Outlook

• Calculation scheme can be used to calculate other Navier-Stokes coefficients

• Investigate effects in viscous hydro simulations → Observables?

• Compare to other models: SMASH? BAMPS? lQCD?