Comparison of Different Realizations of Cooper-Frye Sampling with Conservation Laws

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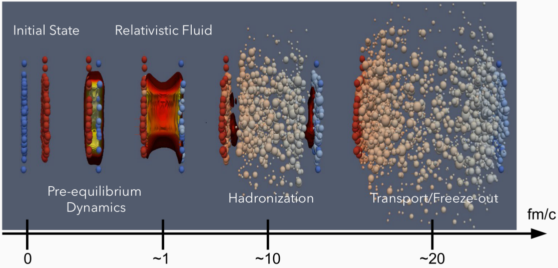
Introduction

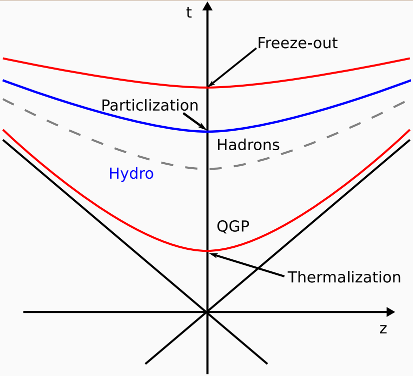
Heavy-ion collision:



- Hydrodynamics to describe hot and dense phase
- Transport approach to model hadronic and out of equilibrium phases
- Hybrid model to combine these two methods

Motivation

- Particlization: transition from relativistic fluid to single hadrons → Cooper-Frye procedure
- Most hybrid models do not apply event-by-event conservation laws but some (e.g. UrQMD) do



- In nature conservation laws are obeyed in every single event

⇒ Strict conservation during all stages for an event-by-event based model

Cooper-Frye procedure

Single-particle distribution for an expanding relativistic gas obeying the Boltzmann transport equation:

$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)p^\mu d\sigma_\mu \tag{1}$$

(Fred Cooper, Graham Frye, 1974)

Hydrodynamics $\rightarrow$ particlization:

- Going from global properties like p and ϵ of hydro to single particles with 4-momenta
- Build freeze-out hypersurface with a constant energy density ϵ or temperature T
- Use Cooper-Frye formula to generate particles and sample their momenta

Overview: 3 different sampling methods

1. Conventional Monte Carlo sampling
   - Cooper-Frye procedure with adaptive rejection sampling for momenta
   - Quantum numbers are conserved on average

(Implementation by Long-Gang Pang)

2. Mode sampling
   - Cooper-Frye sampling in 7 steps
   - Conserving global energy, baryon number, charge and strangeness event-by-event

(Petersen et al., arXiv:0806.1695v3 [nucl-th]; Huovinen, Petersen, arXiv:1206.3371v2 [nucl-th])

3. Own approach: Metropolis sampling
   - Cooper-Frye sampling using suppression factors to conserve baryon number, charge and strangeness
   - Energy and momentum conservation via rescaling

Conventional Monte Carlo sampling

Number of particle species i emitted from hypersurface element $d\sigma_\mu$:

$$dN_i = \frac{p^\mu d\sigma_\mu}{(2\pi\hbar)^3}\frac{d^3p}{p^0}f(p)\Theta(p^\mu d\sigma_\mu) \tag{2}$$

$$f(p) = \frac{1}{e^{\frac{p^\mu u_\mu - \mu_B}{T}} + \lambda} \tag{3}$$

Jüttner distribution ($\lambda = 0$) for all hadrons except pions

1. Integrate over momentum phase space to get dN_i and sum them up to get the total number of hadrons in the surface element: $dN = \sum_i dN_i$.
2. Use Poisson distribution with dN as the probability to determine the number of hadrons to be sampled

Conventional Monte Carlo sampling

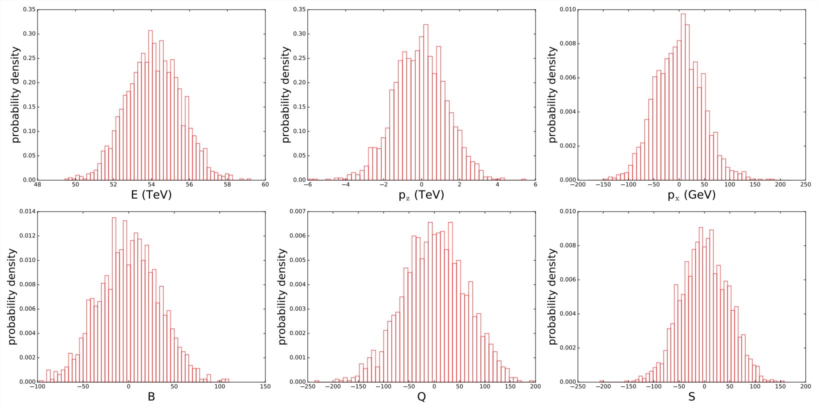
3. Draw the particle species with probabilities given by dN_i
4. Generate 4-momenta of the particles from the distribution function

$$f(p) = e^{-\left(\sqrt{p^2+m^2}-\mu_B\right)/T} \tag{4}$$

by using Monte Carlo method

\+ Possibility to force resonance decay into 2 or 3 daughter-particles

Conventional sampling: no event-by-event conservation



- In basic Cooper-Frye sampling the quantum numbers are only conserved on average

Calculation of hypersurface properties: energy and momentum

Energy-momentum tensor:

$$T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} - g^{\mu\nu}P$$

Energy density ϵ and pressure P are given by the equation of state (EOS) used in the hydrodynamic evolution

Total energy:

$$E = \int_{d\sigma} T^{\mu 0} d\sigma_{\mu}$$

Total longitudinal momentum:

$$p^3 = \int_{d\sigma} T^{\mu 3} d\sigma_{\mu}$$

Hypersurface: density, baryon number, charge, strangeness

X,y direction:

$$p^{1,2} = \int_{d\sigma} T^{\mu 1,2} d\sigma_\mu$$

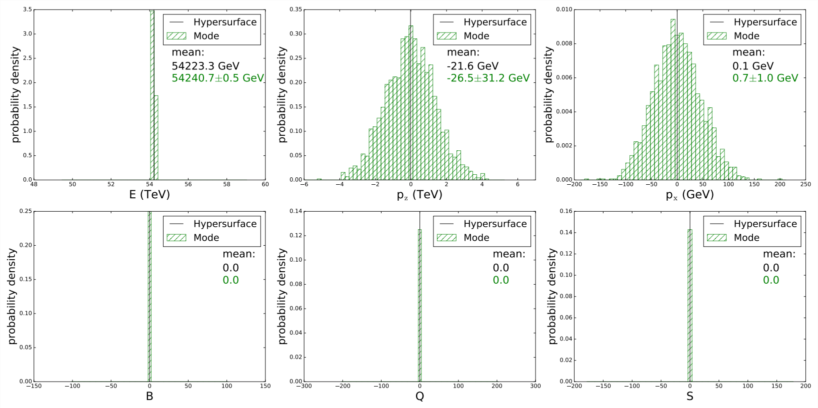
Particle density for species i:

$$n_i = \frac{g_i}{(2\pi\hbar)^3} \int p^2 f(p) dp$$

Net baryon number B, charge Q and strangeness S:

$$X = \int_{d\sigma} n_i x_i \; u^\mu d\sigma_\mu \;, \; x_i = B/Q/S \text{ of species } i$$

Mode sampling: event-by-event conservation



- Mode sampling conserves energy, baryon number, charge and strangeness

Metropolis sampling: B, Q, S conservation

- X = current total baryon number, charge, strangeness
- Treatment for baryons and charged/strange particles $x_i \neq 0$:

$\Delta X = |X_{surface} - X_{particles}|$
Uniform random number $r \in [0,1]$

$X_{particles} > X_{surface}$:

$x_i > 0 \rightarrow$ accept if $r \leq e^{-\Delta X}$
$x_i < 0 \rightarrow$ always accepted

$X_{particles} < X_{surface}$:

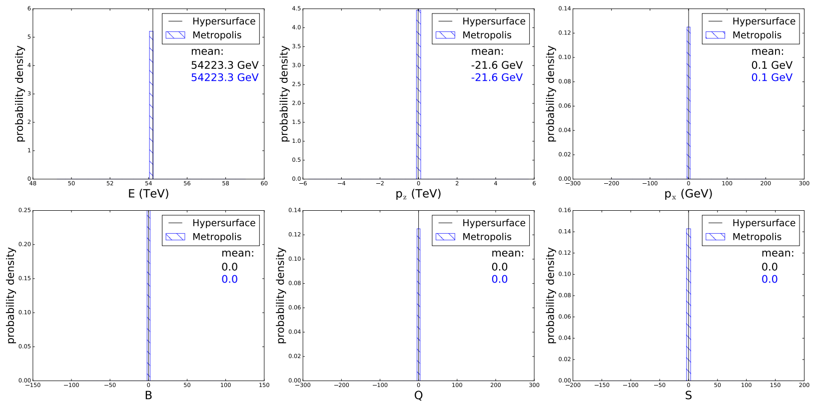
$x_i > 0 \rightarrow$ always accepted
$x_i < 0 \rightarrow$ accept if $r \leq e^{-\Delta X}$

Metropolis sampling: energy and momentum conservation

After sampling particles their momenta are rescaled to match the energy of the hypersurface:

1. Calculate total 4-momentum of the hypersurface $P_{hypersurface}$ and of the sampled particles $P_{particles}$
2. For each particle boost the particles 4-momentum to the global local rest frame ($v_{particles}$)
3. Find the factor $(1 + a)$ multiplied with their 3-momenta to match the hypersurface energy
4. Boost back with the velocity of the hypersurface $v_{hypersurface}$

Metropolis sampling: event-by-event conservation



- Here, additionally the 3-momentum is conserved

Au+Au 200 AGeV

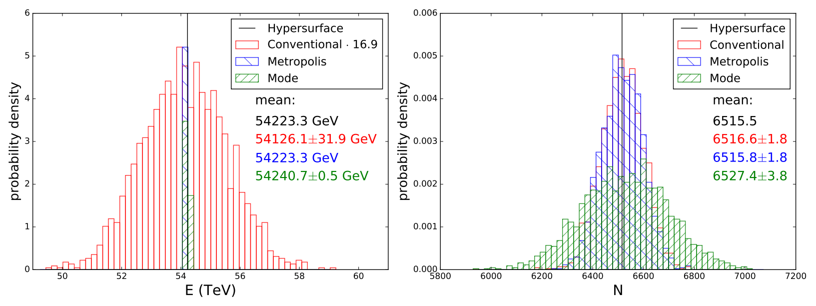
Simulate heavy-ion collision:

- Glauber initial conditions
- CLVisc 3D+1 smooth hydro evolution with s95p-PCE-v0 lattice QCD EOS
- Build hypersurface with $T_{\mathrm{frz}} = 137$ MeV

(Long-Gang Pang et al., arXiv:1411.7767v3 [hep-ph])

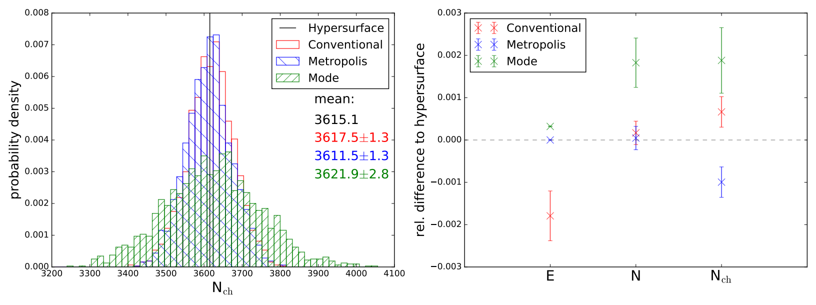
⇒ Sample 2000 events for each algorithm and analyse the generated particle ensembles

Au+Au 200 AGeV, energy and particle numbers



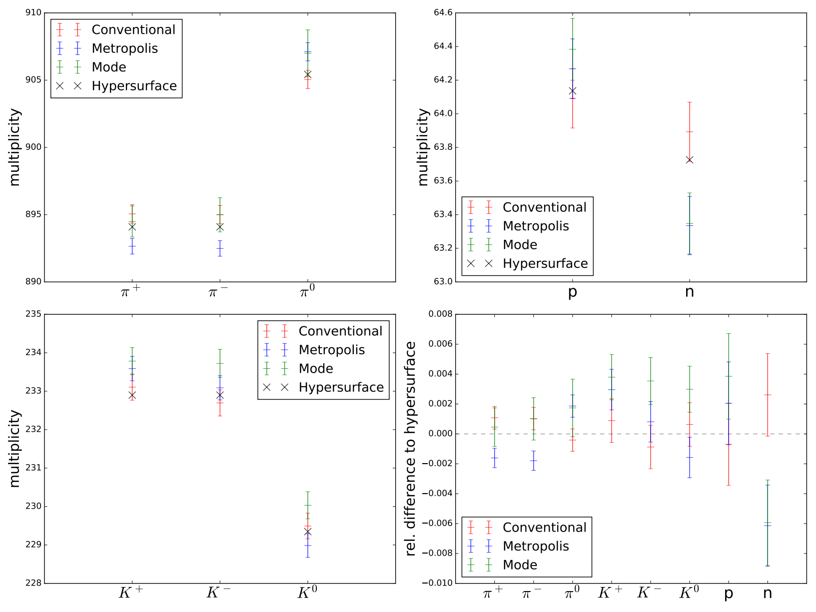
- Slightly more energy and particles for Mode sampling

Au+Au 200 AGeV, charged particles and relative difference

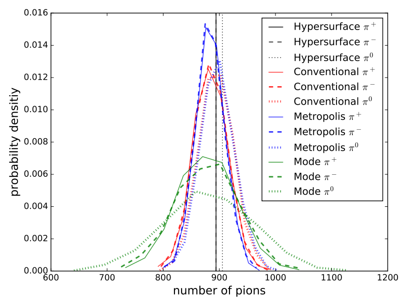


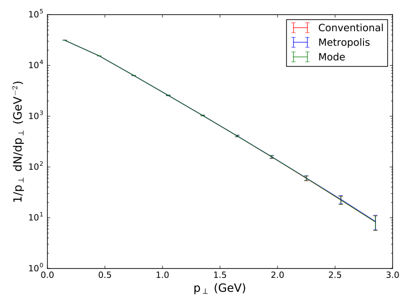
- More charged particles using Mode sampling
- Slightly less N_{ch} for Metropolis sampling

Au+Au 200 AGeV, multiplicities



Au+Au 200 AGeV, π-distribution, p_T-spectrum



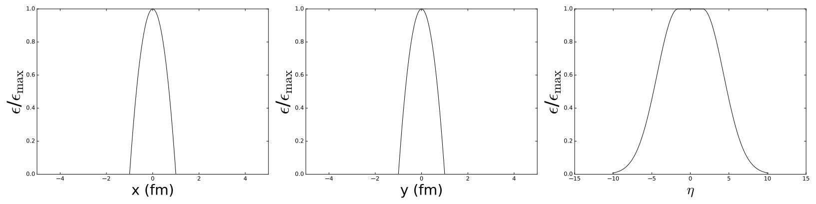


- Lower mass for non-charged pion:
  $m_{\pi^\pm} = 0.13957$ GeV, $m_{\pi^0} = 0.13498$ GeV
- Mode sampling shows a broader distribution for all three pion species

p+p 13 TeV initialisation

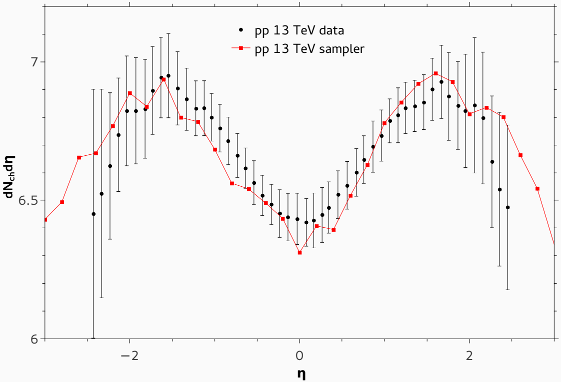
Initial energy density ϵ distribution for proton-proton collision to be run by CLVisc hydro:

- Hard spheres with radius R = 1 fm and distance to each other b



$\Rightarrow$ Generate hypersurfaces for different impact parameters b = 0.1, 0.2, ... , 2.0 fm

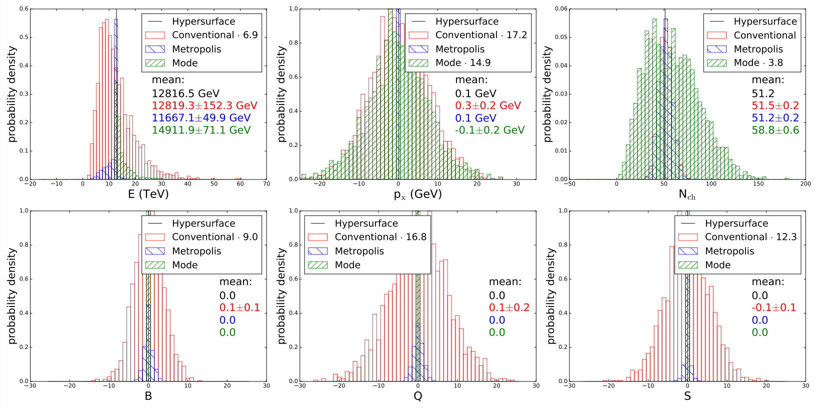
p+p 13 TeV initialisation



- Distribution of η from basic Cooper-Frye sampling with resonance decay compared to experimental data from ATLAS

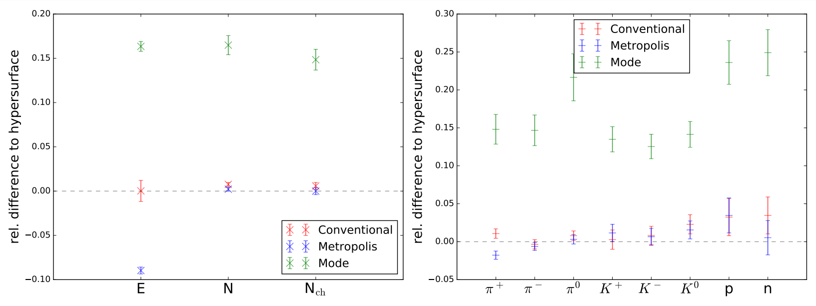
(ATLAS collaboration, arXiv:1606.01133v2 [hep-ex], 21 Sep 2016)

p+p 13 TeV, quantum numbers of central collision



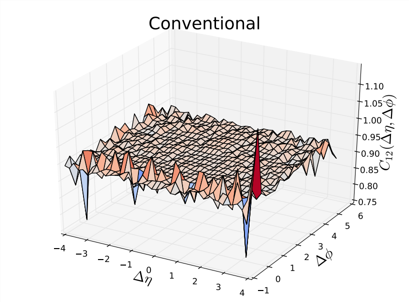
- In a small system both methods of energy conservation show some deviation from the hypersurface value
- B, Q, S conservation not 100 % accurate for Metropolis

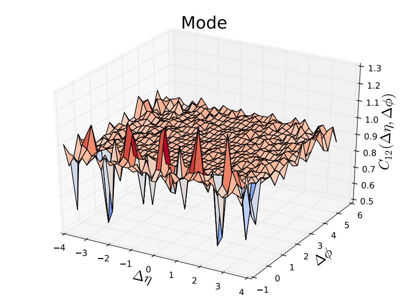
p+p 13 TeV, multiplicities

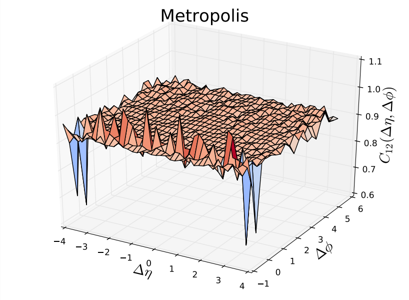


- Mode sampling produces more particles than the other two sampling methods

p+p 13 TeV, di-hadron correlation in central collision







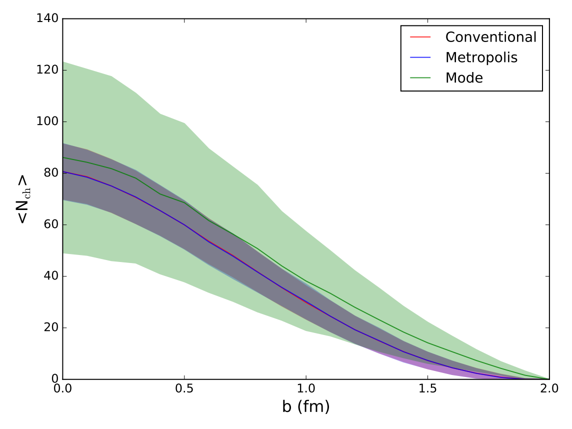
$$C_{12}(\Delta\eta,\Delta\Phi) = \frac{S(\Delta\eta,\Delta\Phi)}{B(\Delta\eta,\Delta\Phi)}$$

$$S(\Delta\eta,\Delta\Phi) = \left\langle \frac{1}{N_{\rm trig}} \frac{d^2N^{\rm same}}{d\Delta\eta d\Delta\phi} \right\rangle$$

$$B(\Delta\eta,\Delta\Phi) = \left\langle \frac{1}{N_{\rm trig}} \frac{d^2N^{\rm mixed}}{d\Delta\eta d\Delta\phi} \right\rangle$$

$N_{\rm trig}$: $N_{\rm ch}$, $|\eta| < 2$

p+p 13 TeV, N_{ch} for different impact parameters



- Centrality determination in proton-proton collisions is sensitive to the algorithm used

Summary

Three sampling methods:

- Conventional Cooper-Frye sampling
- Cooper-Frye with event-by-event conservation laws:
    - Mode sampling
    - Metropolis sampling

Differences:

- Mode sampling shows slightly higher energy and more particles
- Broader pion-distributions for Mode sampling
  $\rightarrow$ Compare to thermal distributions
- Small system:
    - Energy deviation for Mode and Metropolis sampling
    - Error in conserving quantum numbers for Metropolis sampling
    - Different dependence of $<\mathrm{N}_{\mathrm{ch}}>$ on b for Mode sampling

Thank you for your attention!

Mode sampling

- Calculate particle number dN to be sampled in a randomly chosen hypersurface element $d\sigma_\mu$
- Generate particles in 7 modes to conserve quantum numbers: Sample ...
  1. ... particles until energy is conserved, take only strange hadrons
  2. ... until strangeness is conserved
  3. ... non-strange hadrons until energy is conserved, take only baryons
  4. ... non-strange anti baryons until baryon number is conserved
  5. ... non-strange mesons until energy is conserved, take only positively charged particles
  6. ... non-strange, negatively charged mesons to conserve charge
  7. ... neutral non-strange mesons to conserve energy