The Baryon Diffusion Constant of a Hot Hadron gas - a Boltzmann Approach

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Stochastic process clearing out inhomogenities in particle-number densities
Maximizes entropy

Fick’s first law for dilute fluids

\[ \vec{j}(\vec{x}, t) = -D \vec{\nabla} n(\vec{x}, t) \]

Einstein, Annalen der Physik, vol. 322, no. 8 (1905), pp. 549-560
figure: https://de.wikipedia.org/wiki/Adolf_Fick
Transport Coefficients

Diffusion currents are **macroscopic observables**
Diffusion constant is a **transport coefficient**
Transport coefficients take their extremum at phase transitions:

Greif, Greiner, Denicol Phys. Rev. D 93 (2013) for electric conductivity
Rapidity Diffusion

In heavy-ion collisions: baryon stopping / diffusion of baryons to mid-rapidity

Wolschin, Phys. Rev., C 69 (2004), 024906
Calculating the **baryon** diffusion constant of a hot hadron gas nearby local equilibrium (**no** heavy-ion collisions)

Using **linear response** method proposed in (among others):

**main reference**: Greif, Greiner, Denicol Phys. Rev. D **93** (2016)

Investigating the response of a small volume of gas due to a small gradient in baryo-chemical potential
⇒ **neglecting non-linearities**
Assumptions

- Massive nucleon-pion-kaon gas with either constant or simplified GiBUU\(^1\) isotropic cross sections

- **Classical** particles $\Rightarrow$ Maxwell-Jüttner distribution

- Dilute gas with **elastic** $2 \leftrightarrow 2$ particle collisions $\Rightarrow$ Boltzmann approach

- **Small deviations** from (local) equilibrium and **small, constant gradients** in baryo-chemical potential $\Rightarrow$ **linear response**

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\(^1\)Greif, Greiner, Denicol Phys. Rev. D **93** (2016)
Relativistic Boltzmann equation for multi-component system

\[ p_i^\mu \frac{\partial f_i^p}{\partial x^\mu} = \hat{C}(p_i)[f^i_p] \]

Small deviations in equilibrium distribution:

\[ f^i_p = f(x_i, p_i) = f^i_{0p} + \delta f^i_p(x_i) \]

Maxwell-Jüttner distribution for classical particles:

\[ f^i_{0p} = \exp \left[ -\beta \left( u^\mu p_i^\mu - \mu_i \right) \right] \]
linearized Collision term (higher orders in deviation $\delta f$ neglected):

$$\hat{C}(p_i)[f^i_p] = \sum_{j=1}^{N_s} \int \frac{d^3p'_j}{(2\pi)^3 E_{jp'}} \frac{d^3k_i}{(2\pi)^3 E_{ik}} \frac{d^3k'_j}{(2\pi)^3 E_{jk'}} \gamma_{ij} s \sigma_{ij}(s, \theta) (2\pi)^6$$

$$\times \delta^{(4)} (k_i + k'_j - p_i - p'_j) f^i_0 f^j_0 \left[ \frac{\delta f^i_k}{f^i_0} + \frac{\delta f^j_{k'}}{f^j_{0k'}} - \frac{\delta f^i_p}{f^i_0} - \frac{\delta f^j_{p'}}{f^j_{0p'}} \right]$$

$\sigma_{ij}(s, \theta)$: Cross section of interaction between $i$-th and $j$-th particle species

$N_S$: number of particle species
Method

General particle-number current:

\[ N^\mu = n_0 u^\mu + j^\mu \]

**Assumptions**

- only **gradients in baryo-chemical potential** \( \mu_0 \)
- fluid velocity \( u^\mu = \) velocity energy current (**Landau frame**)
  \[ \Rightarrow j^\mu = \text{particle diffusion current} \]
- fluid neither accelerating nor expanding (gradients in \( u^\mu \) vanish)

Boltzmann equation with source term:

\[ p^\mu \partial_\mu \delta f_p^i + \lambda_i \beta_0 f_{0p}^i p_\nu \nabla_\nu \mu_0 = \hat{C}(p_i)[f_p^i] \]

source term: external "force field"
Fourier-transformed Boltzmann equation gives

$$\delta \tilde{f}^i_p = B^\nu \nabla_\nu \mu_0$$

$B^\nu$ fulfills Boltzmann equation divided by gradient in $\mu_0$. It is space-like and only dependent on energy and momentum.

**Ansatz**: power series in energy

$$B^\nu(p^i) = f_0^i \Delta^\nu_\mu p^\mu_i \sum_{n=0}^{\infty} a_n^{(i)} E_{ip}^n , \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Diffusion coefficient $D$:

$$D \propto \sum_{i=1}^{N_s} \lambda_i \int dP^i \Delta_{\mu\nu} p^\mu_i B^\nu$$

**Goal**

To calculate the coefficients $a_n^{(i)}$
Calculating the baryon diffusion constant for hot ($T = 80 - 250$ MeV) nucleon-pion-kaon gas

massive

assuming in general finite baryo-chemical potential and isotropic simplified GiBUU cross sections: $\Delta, \rho$ and $K^*$ resonances with Breit-Wigner shapes, all other: mean value


Calculations done with MATHEMATICA
Cross sections and temperature

- stronger interactions lead to smaller diffusion constants
- increase in temperature $\Rightarrow$ increase in total density of gas $\Rightarrow$ decreasing diffusion constant
Chemical potential and numerical issues

- Ratio of baryon and meson density also determines behavior
- Numerical issues for finite baryo-chemical potential
massive p-n-K-pi-Gas, $\sigma = 10$ mb, $T = 100$ MeV

massive p-n-K-pi-Gas, particle densities at $T = 100$ MeV

total
baryon
meson

The Baryon Diffusion Constant

Jan Fotakis (AG Greiner)
Comparison to diffusion in QGP


[3]: Aarts, Allton, Amato et al. , Gubser et al. JHEP 02, 186 (2015)
Issues

massive p-n-K-pi-Gas, $\sigma = 10$ mb

mu = 0 MeV
mu = 100 MeV
mu = 200 MeV
mu = 300 MeV

numerical issues at $T = 110 - 120$ MeV (low precision?)
Summary and conclusion

- Baryon diffusion: macroscopic observable of hadron gases related to rapidity diffusion and phase transition
- Linear response method in kinetic theory as proposed by Denicol, Rischke, Niemi, Greif et al. to calculate diffusion constant of nucleon-pion-kaon gas
- Diffusion constant is sensitive to total number-density, baryon- and meson-number-density ratios and baryon-number-density of hadron gas
- Approaches comparable values at critical temperature
- However: numerical issues at finite chemical potential
Conclusion and Outlook

Outlook

- Baryon Diffusion processes in heavy-ion collisions: e.g. how is rapidity diffusion connected to baryon diffusion?
- Baryon Diffusion for quantum gases
- Using more realistic baryo-chemical potential for calculations