Constent

1.The upper limit of η/s in the first-order phase transition

2.Spherical expansion with rotation in relativistic magnetohydrodynamics

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The upper limit of η/s in the first-order phase transition

1.Background

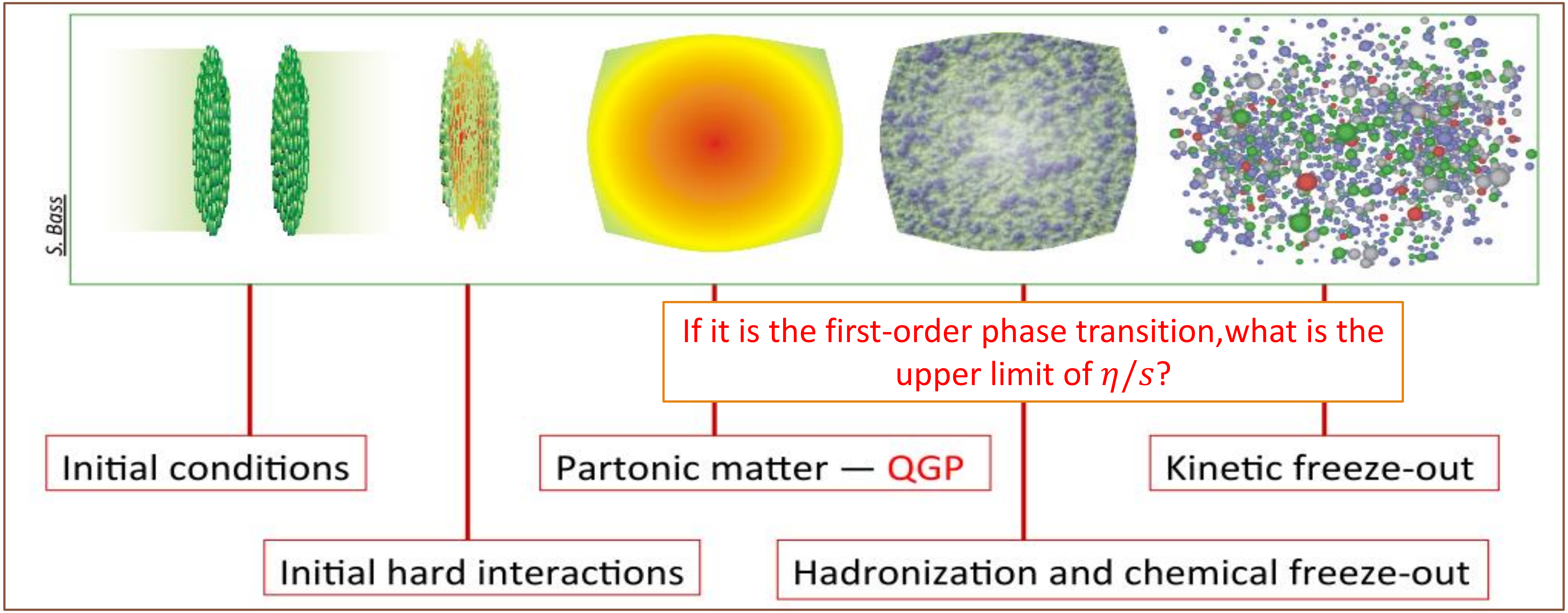
2.Derivation of the upper limit of η/s in the first-order phase transition

3. Calculation in different cases

4.Results and discussion

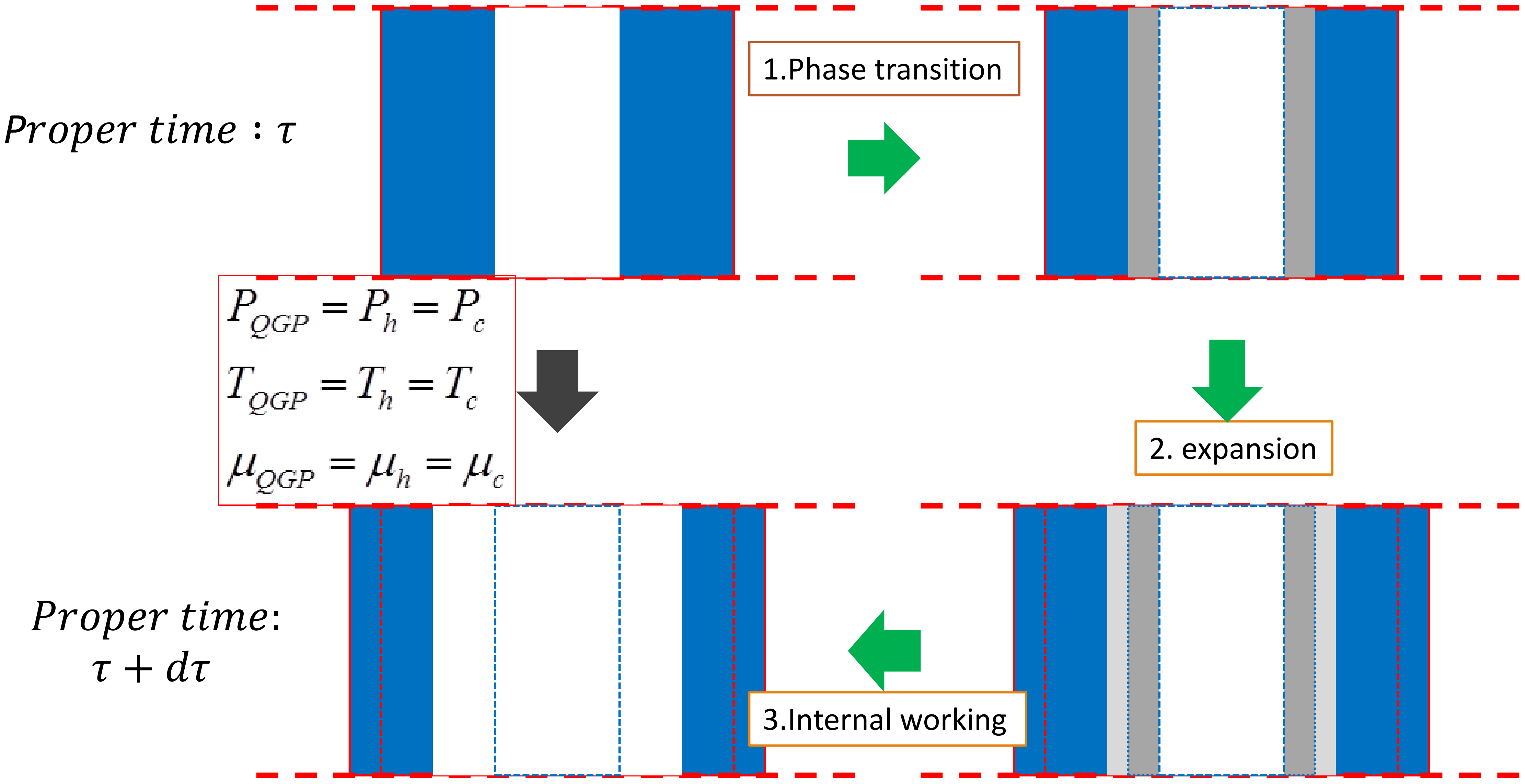
Background

Different stages of heavy ion collisions



If it is the first-order phase transition,what is the upper limit of η/s?

First-order phase transition



The energy balance of first-order phase transition

Phase transition

expansion

Increased energy of hadrons

$$e_{QGP}dV_{QGP} - (P_c + \bar{\pi}_{QGP}) f_{QGp} dV - (P_c + \bar{\pi}_h)(1 - f_{QGP}) dV = e_h dV_{\mathrm{h}}$$

$$\bar{\pi} = \bar{\pi}_{QGP} f_{QGP} + \bar{\pi}_{\mathrm{h}} (1 - f_{QGP}) \qquad dV_h = dV + dV_{QGP}$$

volume of phase transition

$$dV_{QGP} = \frac{e_h + P_c + \bar{\pi}}{e_{QGP} - e_h} dV > 0$$

$$e_h + P_c + \bar{\pi} > 0$$

$$\bar{\pi} = -\frac{\pi^{\mu\nu}\sigma_{\mu\nu}}{\nabla_\mu u^\mu}$$

$$\pi^{\mu\nu} = 2\eta \nabla^{<\mu} u^{\nu>}$$

$$\eta < \frac{(e_h + P_c)\nabla_\mu u^\mu}{2\nabla^{<\mu} u^{\nu>} \nabla_{<\mu} u_{\nu>}}$$

$$\frac{\eta}{s} < \frac{(e_h + P_c)\nabla_\mu u^\mu}{2s\nabla^{<\mu} u^{\nu>} \nabla_{<\mu} u_{\nu>}}\bigg|_{\tau=\tau_c}$$

$$\frac{e_h + P_c}{2s}$$

$$\frac{\nabla_\mu u^\mu}{\nabla^{<\mu} u^{\nu>} \nabla_{<\mu} u_{\nu>}}$$

The thermodynamical quantities of QGP and hadron

QGP

$$P_{QGP}=a(T,\mu_q,g_s)T^4-B$$

$$e_{QGP}=3a(T,\mu_q,g_s)T^4+B$$

$$\mathrm{n}_{\mathrm{QGP}}=(\frac{\partial P}{\partial\mu})_{\mathrm{T}},\mathrm{s}_{\mathrm{QGP}}=(\frac{\partial P}{\partial T})_{\mu}$$

$$\mathrm{a}(T,\mu_q,g_s)=\frac{\pi^2}{45}[8+\frac{21}{4}n_f+\frac{45}{2\pi^2}\sum_{q=1}^{n_f}(\frac{\mu_q^2}{T^2}+\frac{\mu_q^4}{2\pi^2T^4})]-$$
$$\frac{8}{144}g_s^2[3+\frac{5}{4}n_f+\frac{9}{2\pi^2}\sum_{q=1}^{n_f}(\frac{\mu_q^2}{T^2}+\frac{\mu_q^4}{2\pi^2T^4})]+$$
$$\frac{2}{3\pi^2}g_s^3[1+\frac{1}{6}n_f+\frac{1}{6}\sum_{q=1}^{n_f}(\frac{\mu_q^2}{\pi^2T^2})]$$

The running coupling

$$\alpha_s=\frac{g_s^2}{4\pi}=\frac{12\pi}{33-2n_f}[\ln(\frac{0.8\mu_q^2+15.6T^2}{\Lambda_s^2})]$$

hadron

$$P_h=\sum_i P_i^0(T,\tilde{\mu}_i),\tilde{\mu}_i=\mu_i-v_iP_h$$

$$e_h=\frac{\sum_i e_i^0(T,\tilde{\mu}_i)}{1+\sum_i n_i^0(T,\tilde{\mu}_i)v_i}$$

$$P_i^0(T,\tilde{\mu}_i)=\frac{d_i}{2\pi^2}\int\frac{k^4}{\sqrt{k^2+m_i^2}}dk[\exp(\frac{\sqrt{k^2+m_i^2}-\tilde{\mu}_i}{T})\pm1]^{-1}$$

$$n_i^0(T,\tilde{\mu}_i)=\frac{d_i}{2\pi^2}\int k^2dk[\exp(\frac{\sqrt{k^2+m_i^2}-\tilde{\mu}_i}{T})\pm1]^{-1}$$

$$e_i^0(T,\tilde{\mu}_i)=\frac{d_i}{2\pi^2}\int k^2\sqrt{k^2+m_i^2}dk[\exp(\frac{\sqrt{k^2+m_i^2}-\tilde{\mu}_i}{T})\pm1]^{-1}$$

Cleymans J, The hadronisation of a quark-gluon plasma Physik C Particles and Fields, 1993, 58(2):

D. H. Rischke, M. I. Gorenstein, H. Stoecker and W. Greiner,Z. Phys. C 51, 485 (1991).

Different forms of expansion

$$\frac{\eta}{s} < \frac{(e_h + P_c)}{2s} \frac{\nabla_{\mu} u^{\mu}}{\nabla^{<\mu} u^{\nu>} \nabla_{<\mu} u_{\nu>}} \Big|_{\tau=\tau_c}$$

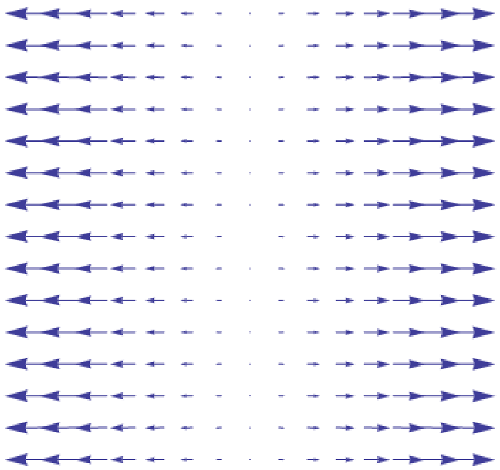
$$\frac{e_h + P_c}{2s}$$

$$\frac{\nabla_{\mu} u^{\mu}}{\nabla^{<\mu} u^{\nu>} \nabla_{<\mu} u_{\nu>}} \; ?$$

Bjorken

$$u^{\mu} = \gamma(1,0,0,v_z), \gamma = \frac{1}{\sqrt{1-v_z}}, v_z = \frac{z}{t}$$

J. D. Bjorken, Phys. Rev. D 27, 140 (1983).

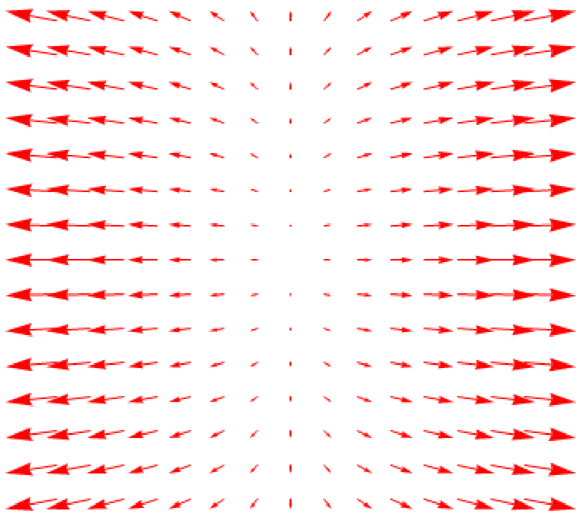


Different forms of expansion

Gubser

$$u^\mu = \gamma(1, v_\rho, 0,0) \qquad \gamma = \frac{1}{\sqrt{1 - v_\rho^2}}$$

$$v_\rho = \frac{2\tau\rho}{1 + \tau^2 + \rho^2}$$



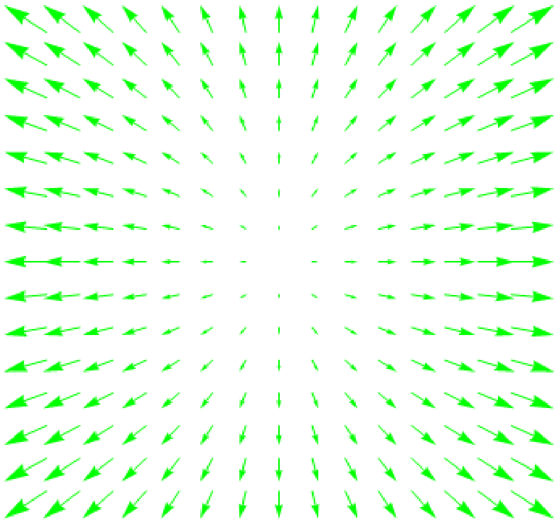
Gubser S S. Physical Review D, 2010, 82(8): 085027.

Hatta Y, Xiao B W, Yang D L

$$u_t = \frac{1}{t^2-\rho^2}\left(\frac{t^2(t^2-\rho^2+L^2+z^2)}{\sqrt{(t^2-\rho^2-L^2-z^2)^2+4L^2(t^2-\rho^2)}} + \rho^2\right),$$

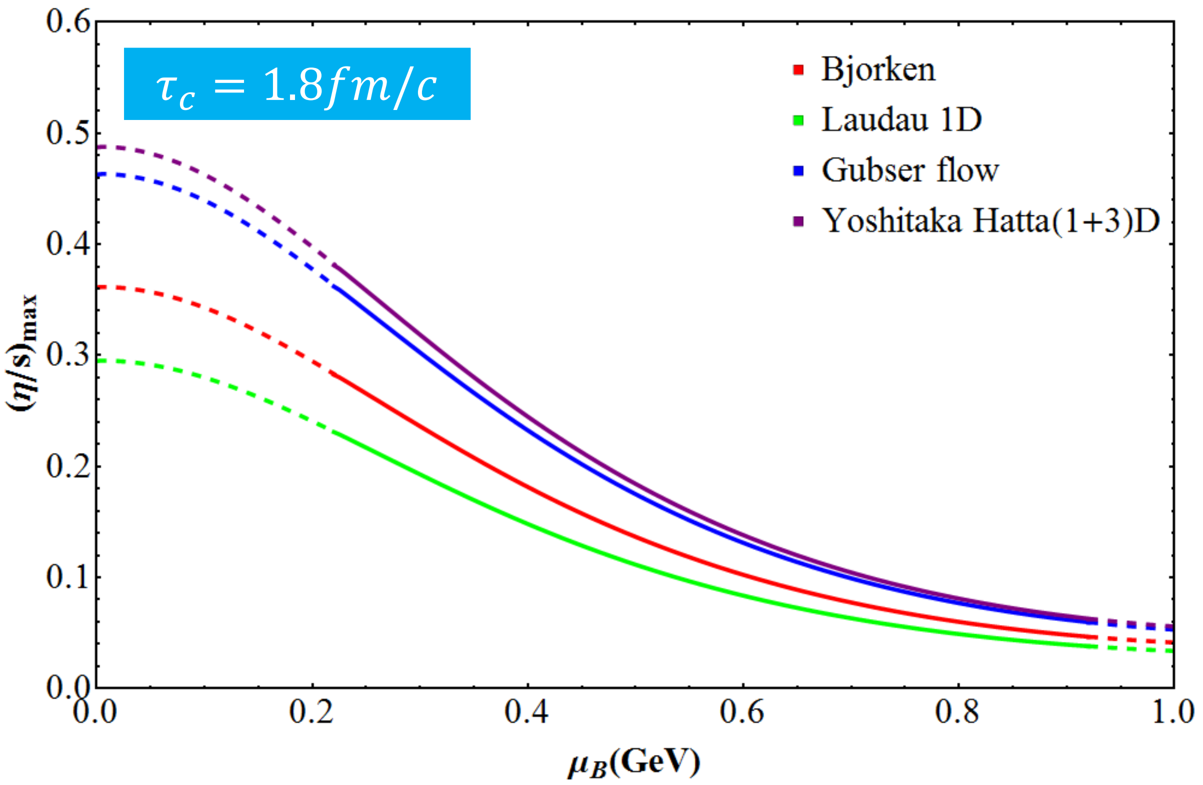
$$u_\rho = \frac{t\rho}{t^2-\rho^2}\left(\frac{t^2-\rho^2+L^2+z^2}{\sqrt{(t^2-\rho^2-L^2-z^2)^2+4L^2(t^2-\rho^2)}} + 1\right),$$

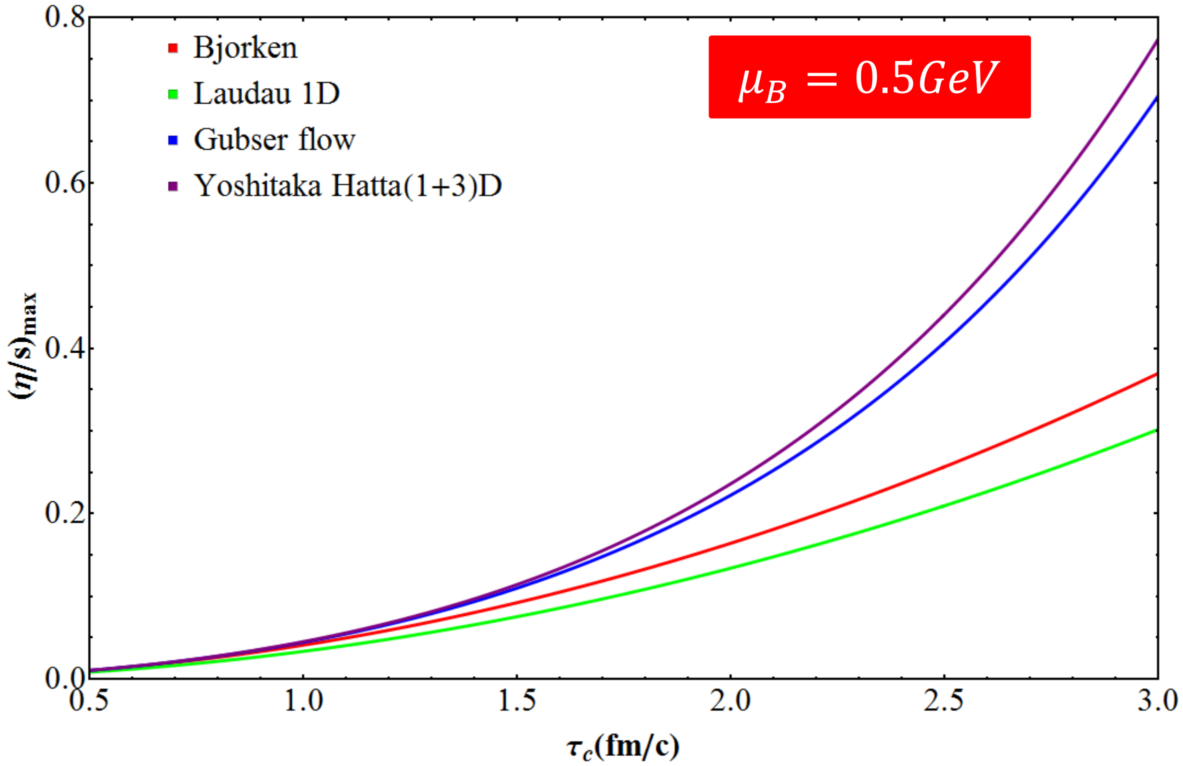
$$u_z = \frac{2zt}{\sqrt{(t^2-\rho^2-L^2-z^2)^2+4L^2(t^2-\rho^2)}}, \qquad u^\phi = 0$$



Hatta Y, Xiao B W, Yang D L. arXiv:1512.04221, 2015.

The upper limit of η/s in the first-order phase transition





Spherical expansion with rotation in the relativistic magnetohydrodynamics

1. Relativistic magnetohydrodynamics
2. Solution of the magnetohydrodynamics.
3. Effect of the magnetic field

Relativistic magnetohydrodynamics

Conservation laws in magentohydrodynamics.

$$\theta = d_\mu u^\mu = \nabla_\mu u^\mu$$
$$D = u^\mu d_\mu$$

$$\partial_\mu T^{\mu\nu} = 0, \partial_\mu n^\mu = 0,$$

Total energy-momentum

$$T^{\mu\nu} = T^{\mu\nu}{}_f + T^{\mu\nu}{}_{em} = \left(\epsilon + \frac{1}{2}b^2\right)u^\mu u^\nu + (\epsilon + p + b^2)g^{\mu\nu} - b^\mu b^\nu$$

The dual Faraday tensor

$$F^{*\mu\nu} = u^\mu b^\nu - u^\nu b^\mu$$

The conservation laws of the energy-momentum and faraday tensor

$$u^\mu d_\mu \left[\left(\epsilon + \frac{1}{2}b^2\right)u^\mu u^\nu + (\epsilon + p + b^2)g^{\mu\nu} - b^\mu b^\nu\right] = 0$$
$$u^\mu d_\mu [F^{*\mu\nu}] = 0$$

⇒

$$D\left(\epsilon + \frac{1}{2}b^2\right) + (\epsilon + p + b^2)\theta + u_\mu b^\nu d_\nu b^\mu = 0$$
$$u^\mu d_\mu \left(u^\mu b^\nu - u^\nu b^\mu\right) = 0$$

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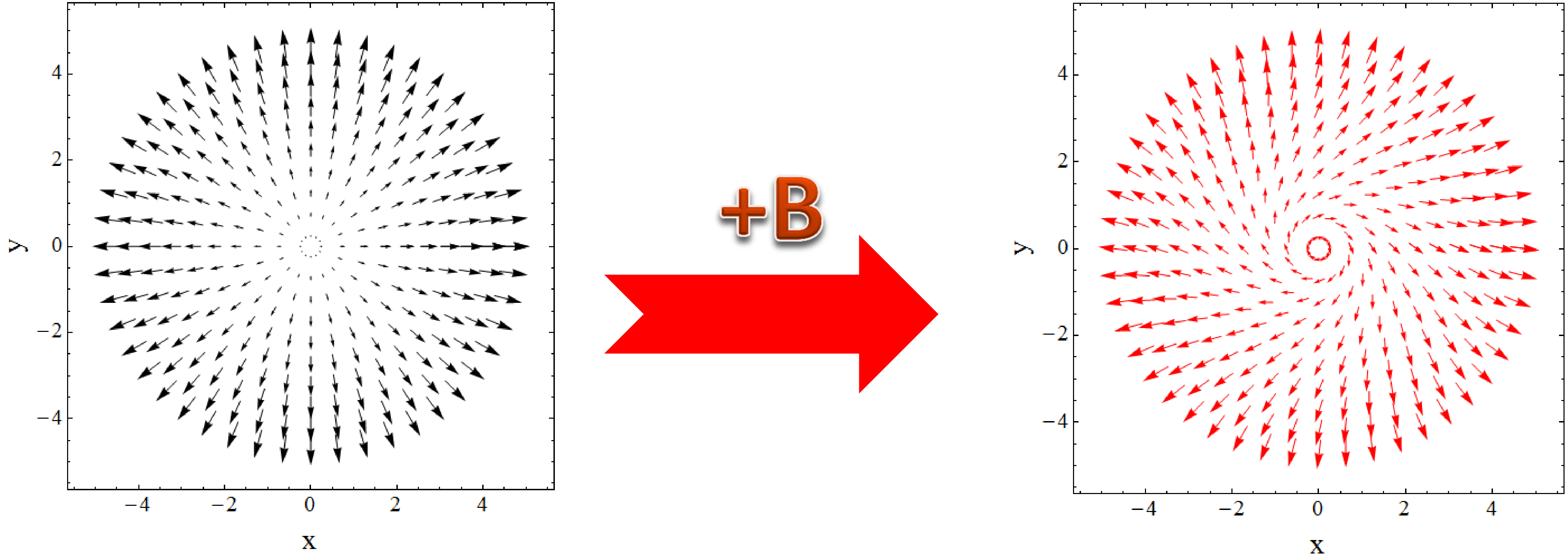


$$u_t = -\frac{L^2 + r^2 + t^2}{\sqrt{(L^2+r^2+t^2)^2 - 4r^2t^2}}$$

$$u_r = \frac{2tr}{\sqrt{(L^2+r^2+t^2)^2 - 4r^2t^2}}$$

$$u_t = -\frac{L^2 + r^2 + t^2}{\sqrt{(L^2+r^2+t^2)^2 - 4r^2t^2 - 4\omega^2L^2{x^2}_\perp}}$$

$$u_r = -\frac{2tr + 2\omega L(\vec{r} \times \vec{e_z})}{\sqrt{(L^2+r^2+t^2)^2 - 4r^2t^2 - 4\omega^2L^2{x^2}_\perp}}$$

Transformation of coordinate system

Minkowski coordinates

$x^{\mu} = (t, x, y, z)$

→

Global coordinates of AdS3

$\hat{x}^{\mu} = (\tau, \rho, \theta, \phi)$

The global coordinates of AdS_3,the metirc take the form

$$d\hat{s} = -cosh^2\rho d\tau^2 + d\rho^2 + sinh^2\rho d\theta^2 + d\phi^2$$

ρ is defined in terms of coordinates(t,r,$x_{\perp}$),

$$cosh\rho = \frac{1}{2Lx_{\perp}}\sqrt{(L^2 + (r+t)^2)(L^2 + (r-t)^2)}$$

$where\ x_{\perp}$=$\sqrt{x^2 + y^2}$, r=$\sqrt{x^2 + y^2 + z^2}$

Energy density and magnetic field intensity

$$u_t = -\frac{L^2 + r^2 + t^2}{\sqrt{(L^2+r^2+t^2)^2 - 4r^2t^2 - 4\omega^2L^2{x^2}_{\perp}}}$$

$$u_r = -\frac{2tr + 2\omega L(\vec{r} \times \vec{e_z})}{\sqrt{(L^2+r^2+t^2)^2 - 4r^2t^2 - 4\omega^2L^2{x^2}_{\perp}}}$$

$$u_\mu = -x_\perp \frac{d\hat{x}^\nu}{d\hat{x}^\mu}\hat{u}_\nu$$

- $\hat{u}_\tau = \frac{-cosh^2\rho}{(cosh^2\rho-\omega^2)^{\frac{1}{2}}}$
- $\hat{u}_\phi = \frac{\omega}{(cosh^2\rho-\omega^2)^{\frac{1}{2}}}$

Minkowski coordinates

$x^\mu = (t, x, y, z)$

$$D\left(\epsilon + \frac{1}{2}b^2\right) + (\epsilon + p + b^2)\theta + u_\mu b^\nu d_\nu b^\mu = 0$$

$$u^\mu d_\mu \left(u^\mu b^\nu - u^\nu b^\mu\right) = 0$$

Global coordinates

$\hat{x}^\mu = (\tau, \rho, \theta, \phi)$

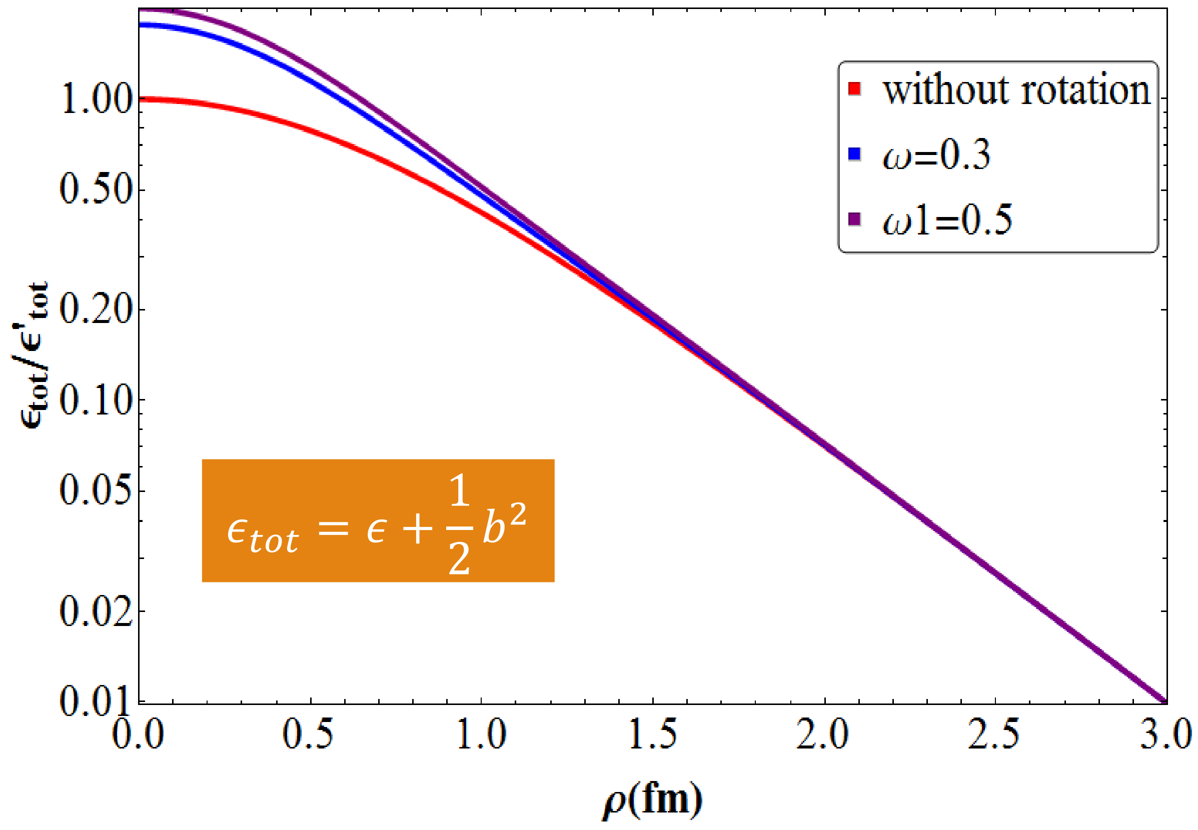
$$\hat{\epsilon} \propto \frac{1}{(cosh^2\rho-\omega^2)^2} + b0^2\frac{(csc\rho)^2+Log[1-2\omega^2+Cosh2\rho]}{2(1-2\omega^2+Cosh2\rho)^2}$$

$$\hat{b} \propto \frac{b0}{(1-2\omega^2+Cosh2\rho)}$$

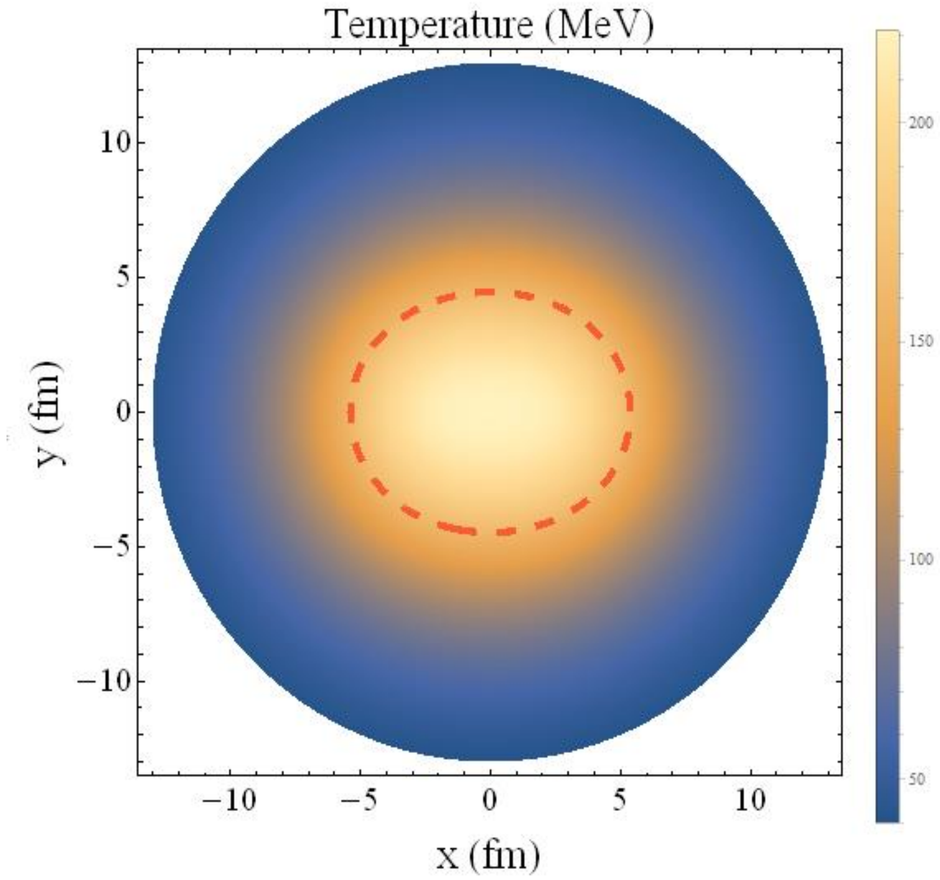
$$\epsilon \propto \frac{1}{((L^2+r^2+t^2)^2-4r^2t^2-4\omega^2L^2{x^2}_{\perp})^2} + B(r,t)$$

$$b \propto \frac{1}{((L^2+r^2+t^2)^2-4r^2t^2-4\omega^2L^2{x^2}_{\perp})}$$

Energy denstiy and temperature



$$\epsilon_{tot} = \epsilon + \frac{1}{2}b^2$$



Summary and outlook

Summary

1.The value of $\eta/s < 0.8$ in first order phase transition .The upper limit of η/s in first order phase transition will decrease with the increase of chemical potential.

2. The rotation can be produced by magnetic field .The magnetic field has an effect on energy density and temperature, and in the center of the system it has a greater influence.

Outlook

1. Find more realistic solutions in the relativistic magnetohydrodynamics .
2. Study the characteristics of the vorticity in the magnetic field .

On-going work: Effect of electric field

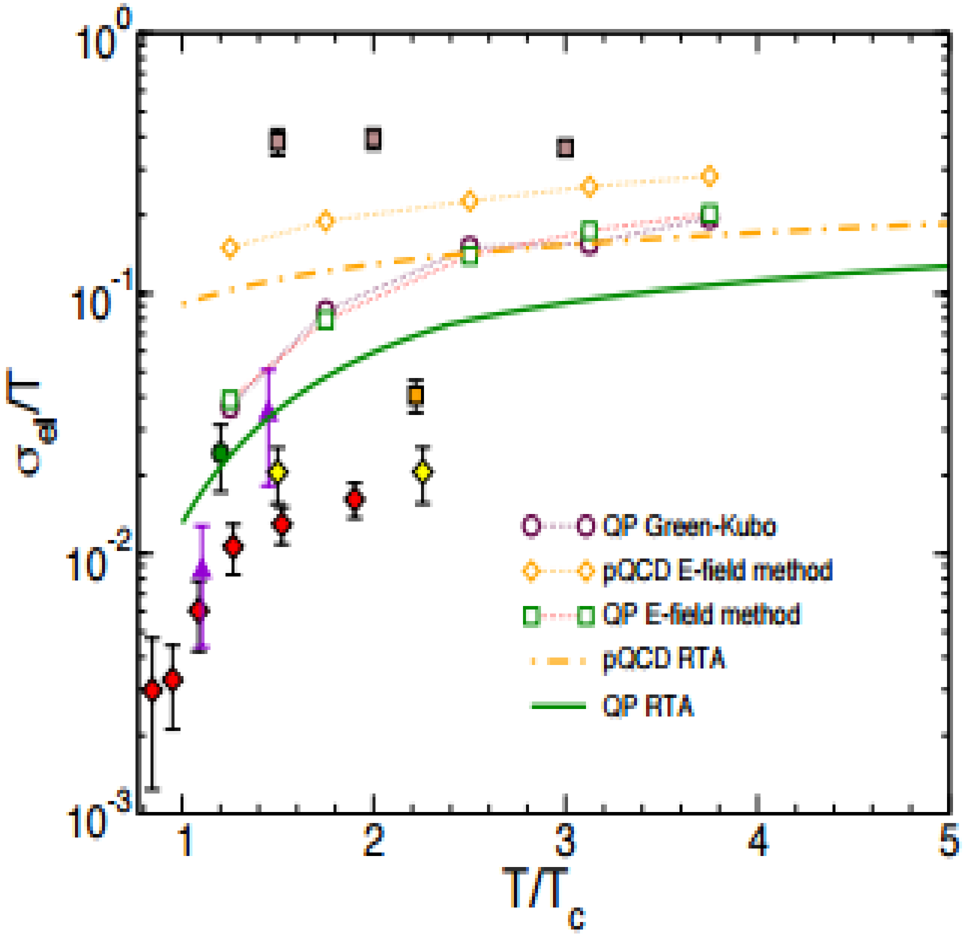
- Distribution function
- $f = f_0 + \delta f$

$$f_0 = e^{\frac{-u^\mu p_\mu}{T}}$$

- From the Boltzmann equation
- $k^\mu \frac{\partial}{\partial x^\mu} f + q k_\nu F^{\mu\nu} \frac{\partial}{\partial x^\mu} f = \sum C_{ij}(x^\mu, k^\mu)$
- $\delta f = \frac{1}{T} \frac{q\tau_{rel}}{P^\mu u_\mu} f_0 k_\nu E^\nu$ → • $\sigma = \frac{1}{3T} \sum_{k=1}^{M} q^2{}_k n_k \tau$ → $\sigma \propto \frac{1}{T}$
- From gauge theories
- $\delta f = \frac{q}{T^3} f_0 k_\nu E^\nu$ → • $\sigma = \frac{CT}{(e^2 \ln e^{-1})}$ → $\sigma \propto T$

Greif M, Bouras I, Xu Z, et al.IOP Publishing, 2015, 612(1): 012056.

Arnold P, Moore G D, Yaffe L G. Journal of High Energy Physics, 2000, 2000(11): 001.



Puglisi A, Plumari S, 2015, 612(1): 012057.

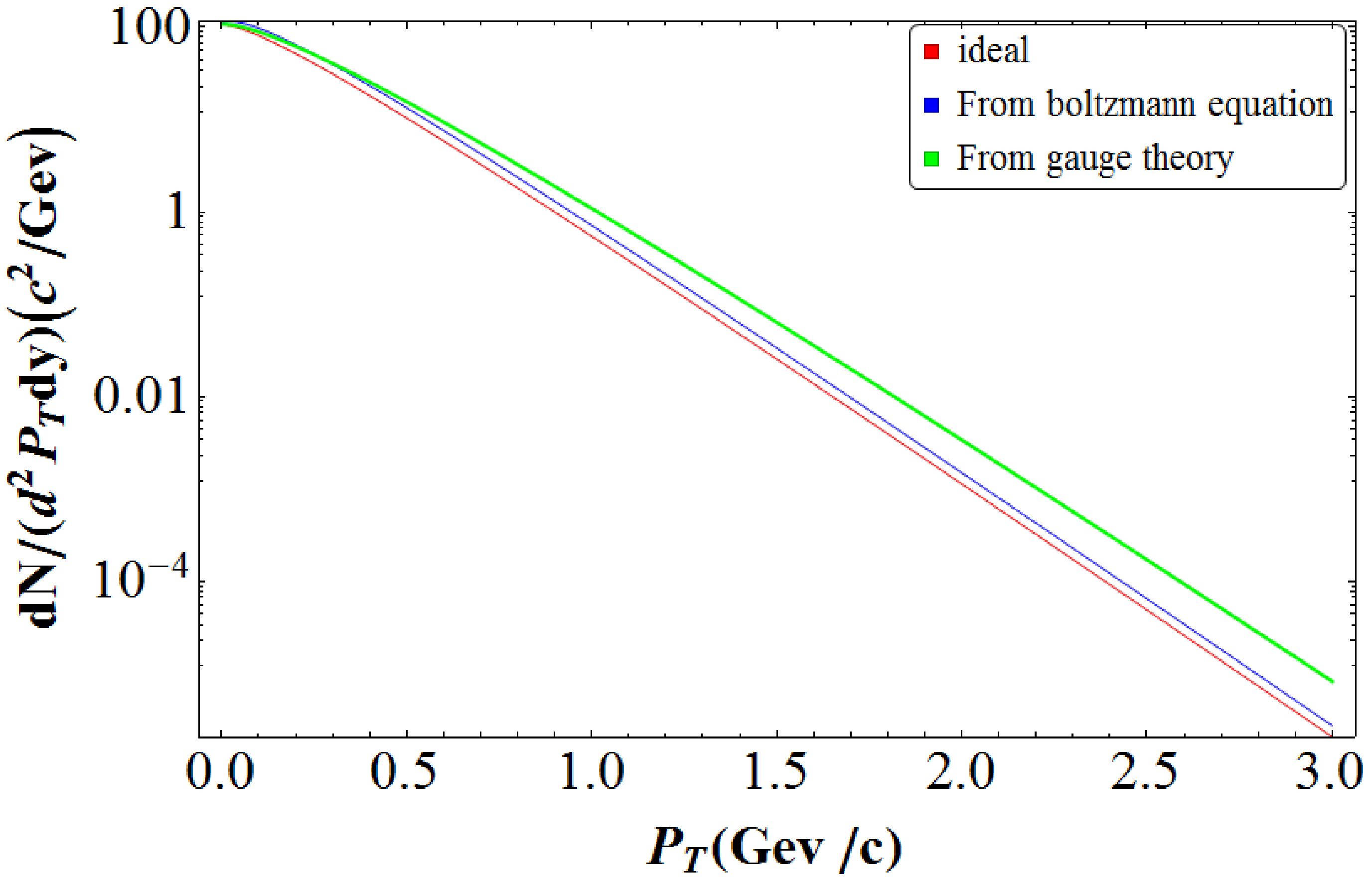


Fig.Pion spectra as a function of the transverse momentum

Vielen Dank für Ihre Aufmerksamkeit!

Relativistic viscous hydrondynamics

Relativistic energy and momentum equations

$$De+(e+P)\nabla_{\mu}u^{\mu}-\pi^{\mu\nu}\sigma_{\mu\nu}=0$$

$$\bar{\pi}=-\frac{\pi^{\mu\nu}\sigma_{\mu\nu}}{\nabla_{\mu}u^{\mu}}$$

$$P_{eff}=P+\bar{\pi}$$

$$De+(e+P_{\text{eff}})\nabla_{\mu}u^{\mu}=0$$

Viscous tensor

$$\pi^{\mu\nu}=2\eta\nabla^{<\mu}u^{\nu>}$$

$$\sigma_{\mu\nu}\equiv\nabla^{<\mu}u^{\nu>}=\frac{1}{2}(\partial^{\mu}u^{\nu}-u^{\mu}u^{\alpha}\partial_{\alpha}u^{\nu}+\partial^{\nu}u^{\mu}-u^{\nu}u^{\alpha}\partial_{\alpha}u^{\mu})$$
$$+\frac{1}{2}(\Delta^{\mu\alpha}u^{\beta}\Gamma^{\nu}_{\alpha\beta}+\Delta^{\nu\alpha}u^{\beta}\Gamma^{\mu}_{\alpha\beta})-\frac{\nabla_{\mu}u^{\mu}}{3}\Delta^{\mu\nu}$$

Convective derivative

$$D=u^{\mu}\partial_{\mu}$$

Viscous pressure

$$\bar{\pi}=-\frac{\pi^{\mu\nu}\sigma_{\mu\nu}}{\nabla_{\mu}u^{\mu}}$$

Gradient operator

$$\nabla_{\mu}u^{\mu}=\partial_{\mu}u^{\mu}+\Gamma^{\mu}_{\alpha\mu}u^{\alpha}$$

Christoffel symbols

$$\Gamma^{\mu}_{\alpha\beta}=\frac{1}{2}g^{\mu\nu}(\partial_{\beta}g_{\alpha\nu}-\partial_{\alpha}g_{\nu\beta}-\partial_{\nu}g_{\alpha\beta})$$

Projection operator

$$\Delta^{\mu\nu}=g^{\mu\nu}-u^{\mu}u^{\nu}$$

E. Molnàr, H. Niemi, D. H. Rischke,Euro. Phys. J. C 65, 615(2010).